A Theory for Electromagnetic Radiation and Electromagnetic Coupling

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ABSTRACT In the proposed theory, the total electromagnetic energy of a radiator is separated into three parts: a Coulomb-velocity energy, a radiative energy, and a macroscopic Schott energy. The Coulomb-velocity energy is considered to be attached to the sources as the same in the charged particle theory. It becomes zero as soon as its sources have disappeared. The radiative energy leaves the radiator and propagates to the surrounding space. The macroscopic Schott energy continues to exist for a short time after the sources have disappeared. It is a kind of oscillating energy and is considered to be responsible for energy exchange between the reactive energy and the radiative energy, performing like the Schott energy in the charged particle theory. As the Poynting vector describes the total power flux density related to the total electromagnetic energy, it should include the contributions of the real radiative power and a pseudo power flow caused by the fluctuation of the reactive energy. The energies involved in the electromagnetic mutual coupling are interpreted in a similar way. In the theory, all energies are defined with explicit expressions in which the vector potential plays an important role. The time domain formulation and the frequency domain formulation of the theory are in consistent with each other. The theory is also verified with Hertzian dipole. Numerical examples demonstrate that the theory may provide insightful interpretation for electromagnetic radiation and mutual coupling problems.

INDEX TERMS Reactive energy, Schott energy, radiative energy, electromagnetic coupling, Poynting vector

I. INTRODUCTION

The electromagnetic radiation and coupling problems have been intensively investigated for more than a hundred years. It is a little bit strange that there is still no widely accepted formulation for evaluating the stored reactive energy and Q factor of radiators [1]-[14]. The main difficulty comes from the fact that there is no clear definition in macroscopic electromagnetic theory for the reactive electromagnetic energy and the radiative electromagnetic energy. It is commonly known in classical charged particle theory that the fields associated with charged particles can be divided into Coulomb fields, velocity fields and radiative fields [16][17]. The energy carried by Coulomb fields and velocity fields is referred to as Coulomb-velocity energy in this paper. The radiative fields are generated by acceleration of charged particles, emitting radiative energy to the surrounding space. The Coulomb-velocity fields and energy are considered to be attached to the charged particles, or simply speaking, they appear and disappear with the charged particles. On the contrary, after being radiated by the charged particles, the radiative fields and energy depart from the sources and propagate to the remote infinity. They exist after their generation sources have disappeared and can couple with other sources they encounter in their journey. Schott energy was first introduced in 1912 by Schott [18]-[20]. It is reversible and is responsible for energy exchange between the Coulomb-velocity energy and the radiative energy. Although it is natural to consider that the reactive energy in macroscopic electromagnetics is similar to the Coulomb-velocity energy and the Schott energy, no successful attempt has been found or well accepted to handle the reactive energy in this manner.

On the other hand, Poynting vector is widely considered as the electromagnetic power flux density [21]. Poynting Theorem describes the relationship between the Poynting vector, the varying rate of the total electromagnetic energy densities, and the work rate done by the exciting source. It provides an intuitive description of the propagation of the electromagnetic energy. However, interpreting the Poynting vector as the electromagnetic power flux density has always been controversial [22]-[32], and some researchers have pointed out that Poynting Theorem may have not been used in the correct way in some situations [33][34]. This difficulty is largely due to the fact that it is not easy to separate the real radiative power flux from the Poynting vector.

It is known that the Poynting Theorem is not convenient to use for evaluating the reactive energy stored by radiators in an open space [5][13], which has been investigated for decades. For harmonic fields, the total electromagnetic energy obtained by integrating the conventional energy densities of $(0.5\mathbf{D}\bullet\mathbf{E})$ and $(0.5\mathbf{B}\bullet\mathbf{H})$ over the infinite three-dimensional volume is infinite because it includes the radiative energy and the reactive energy. For harmonic fields over the time interval $(-\infty < t < \infty)$, the radiative energy occupies the whole space and is infinitely large [14]. Some researchers suggested that those fields associated with the propagating waves should not contribute to the stored reactive energy. The reactive energy can be made finite by subtracting from the total energy density an additional term associated with the radiative power. However, it is not easy to give a general expression for that term because the propagation patterns are quite different for different radiators [1] [5].

Based on these observations, the macroscopic electromagnetic radiation issue is revisited and a new energy-power balance equation at a certain instant time is proposed. It is based on the Poynting relation, only with some substitution and reorganization that can be derived from Maxwell equations. The new equation gives an intuitive and reasonable demonstration that the Poynting vector does not only contain the radiative power flux density but also the pseudo power flux caused by the fluctuation of the reactive energy.

In Section II, a definition for the reactive electromagnetic energy is proposed based on the hypothesis that the reactive energy in the macroscopic electromagnetics bears the same characteristics as the Coulomb-velocity energy and the Schott energy: (i) it is attached to the sources. It disappears simultaneously with its sources or soon after its sources have disappeared; (ii) the definition is in consistent with the stored energy associated with static charges and static currents; (iii) the reactive energy has properties different to the radiative energy, but its fluctuation performs like the radiative fields and propagates in free space at the light velocity. Based on these considerations, it seems natural to define the Coulomb-velocity energy alone as the reactive energy. However, further analysis shows that this choice will lead to disagreement in the radiative electric energy and the radiative magnetic energy. In order to overcome this inconsistency, a special term has to be introduced, which can be demonstrated to be exactly the Schott energy in the charged particle theory [18][20] by applying the Lienard-Wiechert potentials [32] to a moving charge [35]. As a consequence, the Poynting vector is divided into two vectors. One vector accounts for the power flux density associated with the radiative energy, the other vector accounts for the effect of the fluctuation of the reactive energy.

A closely related issue is the electromagnetic mutual coupling, which plays a very important role in many systems. Efficient and accurate analysis of electromagnetic mutual coupling is still a challenging issue [36]. In Section III, the theory is extended for handling multiple radiators. We can aggregate all radiators together and treat them as a single larger radiator, similar to an antenna array. The mutual electromagnetic coupling energies are defined in the same way. One radiator may exert electromagnetic coupling to other sources through its potentials instead of fields. The key issue involved in the electromagnetic mutual coupling is the same as that in the electromagnetic radiation problem.

This paper focuses on discussing the time domain formulation associated with pulse radiators, which is hopefully to provide a more insightful understanding to the electromagnetic radiation and mutual coupling process. It will be shown in Section IV that, just as expected, by applying the formulation to harmonic fields, the corresponding formulation in frequency domain can be obtained straightforwardly and is exactly the same as that proposed in [14], which verifies that the time domain formulation and the frequency domain formulation of the theory are in consistent with each other.

Hertzian dipole with harmonic excitation is a standard validation example for these situations because the exact solutions for its fields and potentials are available both in time domain and in frequency domain, together with a well-established equivalent circuit model. In Section VI, all the expressions of the electromagnetic energies and powers corresponding to the Hertzian dipole are derived. They are exactly in agreement with those obtained with circuit model.

Two kinds of time domain formulations for this issue can be found in published literatures. One was proposed by Shlivinski and Heyman [2][3], the other was proposed by Vandenbosch [6][7]. The first one is an approximate method, the second one is sometimes not in consistent with its counterpart formulation in frequency domain. In Section VII, a loop pulse radiator and a Yagi antenna are analyzed with the proposed theory. They are not for comparison with the other two time domain formulations but for the purpose to show what we can do with the proposed expressions. Numerical examples for comparison among various formulations in frequency domain can be found in [14][15].

The theory is briefly discussed in Section VIII, where it is concluded that the theory is neither a static limit formulation nor a kind of updated version of the Carpernter formulation [24].

II. FORMULATIONS FOR REACTIVE AND RADIATIVE ENERGIES

Consider in free space a radiator with charge density $\rho(\mathbf{r}_1,t)$ and $\mathbf{J}(\mathbf{r}_1,t)$, $\mathbf{r}_1 \in V_s$. Denote

$$W_\rho(t) = \int_{V_s} \frac{1}{2}\rho(\mathbf{r}_1,t)\phi(\mathbf{r}_1,t)d\mathbf{r}_1 \quad (1)$$

$$W_J(t)=\int_{V_s}\frac{1}{2}\mathbf{J}(\mathbf{r}_1,t)\bullet\mathbf{A}(\mathbf{r}_1,t)d\mathbf{r}_1 \tag{2}$$

$W_\rho(t)$ and $W_J(t)$ have the dimension of energy. The scalar potential $\phi(\mathbf{r},t)$ and the vector potential $\mathbf{A}(\mathbf{r},t)$ evaluated at the observation point $\mathbf{r}$ and the time t are defined in their usual way,

$$\phi(\mathbf{r},t)=\int_{V_s}\frac{\rho(\mathbf{r}_1,t')}{4\pi\varepsilon_0 R_1}d\mathbf{r}_1 \tag{3}$$

$$\mathbf{A}(\mathbf{r},t)=\mu_0\int_{V_s}\frac{\mathbf{J}(\mathbf{r}_1,t')}{4\pi R_1}d\mathbf{r}_1 \tag{4}$$

In the above equations, $t'=t-R/c$ is the retarded time, and c is the light velocity in vacuum and $R_1=|\mathbf{r}-\mathbf{r}_1|$ is the distance between the two positions. μ_0 and ε_0 are respectively the permeability and permittivity in free space. The potentials have to satisfy the Lorentz Gauge, and their reference zero points are put at the infinity.

From Maxwell equations, the electric energy density and the magnetic energy density can be transformed to

$$\frac{1}{2}\mathbf{D}\bullet\mathbf{E}=\frac{1}{2}\rho\phi-\frac{1}{2}\nabla\bullet(\mathbf{D}\phi)-\frac{1}{2}\mathbf{D}\bullet\frac{\partial\mathbf{A}}{\partial t} \tag{5}$$

$$\frac{1}{2}\mathbf{B}\bullet\mathbf{H}=\frac{1}{2}\mathbf{J}\bullet\mathbf{A}+\frac{1}{2}\frac{\partial\mathbf{D}}{\partial t}\bullet\mathbf{A}-\frac{1}{2}\nabla\bullet(\mathbf{H}\times\mathbf{A}) \tag{6}$$

where $\mathbf{E}$, $\mathbf{H}$ are the electromagnetic fields, and $\mathbf{D}$, $\mathbf{B}$ are the flux densities. Integrating (5) over a domain $V_a\supset V_s$ and making use of (1) gives

$$W_\rho(t)=\int_{V_a}\left(\frac{1}{2}\mathbf{D}\bullet\mathbf{E}+\frac{1}{2}\mathbf{D}\bullet\frac{\partial\mathbf{A}}{\partial t}\right)d\mathbf{r}_1+\oint_{S_a}\frac{1}{2}\phi\mathbf{D}\bullet\hat{\mathbf{n}}dS \tag{7}$$

where S_a is the surface enclosing V_a with outward normal unit $\hat{\mathbf{n}}$. (7) shows that $W_\rho(t)$ is basically an electric energy and can be separated into two parts: one part is stored in the domain V_a, the other part passes through S_a and resides in the region outside V_a. Recalling that $\lim_{r\to\infty}(\mathbf{D}\cdot\hat{\mathbf{r}})\sim O(1/r^2)$ and $\lim_{r\to\infty}\phi\sim O(1/r)$, where $\hat{\mathbf{r}}$ is the unit radial vector, the surface integral at the RHS of (7) approaches zero at S_∞ with $r\to\infty$. Therefore, the energy defined by $W_\rho(t)$ really has the meaning of being stored in the space with no energy leaking to the infinity. Furthermore, it can be checked that $W_\rho(t)$ satisfies the three terms listed in the previously specified hypothesis. Therefore, it seems not improper to define $W_\rho(t)$ as the reactive electric energy of the pulse radiator,

$$W^e_{react}(t)=\int_{V_\infty}\left(\frac{1}{2}\mathbf{D}\bullet\mathbf{E}+\frac{1}{2}\mathbf{D}\bullet\frac{\partial\mathbf{A}}{\partial t}\right)d\mathbf{r}_1=W_\rho(t) \tag{8}$$

Let $W^e_{tot}(t)=\int_{V_\infty}(0.5\mathbf{D}\bullet\mathbf{E})d\mathbf{r}_1$ denote the total electric energy. Since $W^e_{tot}(t)=W^e_{rad}(t)+W^e_{react}(t)$, it is natural to define the total radiative electric energy as

$$W^e_{rad}(t)=\int_{V_\infty}\left(-\frac{1}{2}\mathbf{D}\bullet\frac{\partial\mathbf{A}}{\partial t}\right)d\mathbf{r}_1 \tag{9}$$

It's important to emphasize that when the charge source has disappeared, the total reactive electric energy $W^e_{react}(t)$ becomes zero, but it only means that the volume integral of (8) is zero. There may still exist nonzero electromagnetic fields in the space.

Integrating (6) over the domain $V_a\supset V_s$ and making use of (2) gives

$$W_J(t)=\int_{V_a}\left(\frac{1}{2}\mathbf{B}\bullet\mathbf{H}-\frac{1}{2}\frac{\partial\mathbf{D}}{\partial t}\bullet\mathbf{A}\right)d\mathbf{r}_1+\oint_{S_a}\left(\frac{1}{2}\mathbf{H}\times\mathbf{A}\right)\bullet\hat{\mathbf{n}}dS \tag{10}$$

which shows that $W_J(t)$ is basically a magnetic energy. However, it is not proper to directly define $W_J(t)$ as the reactive magnetic energy. Firstly, the surface integral in (10) is usually a bounded but nonzero value at S_∞ since $\lim_{r\to\infty}(\mathbf{H}\times\mathbf{A})\bullet\hat{\mathbf{r}}\sim O(1/r^2)$. Therefore, $W_J(t)$ is not an energy purely stored in the whole space V_∞ because it contains a part of energy leaking to the infinity, which is related to the electromagnetic radiation. Secondly, in vacuum, the total radiative electric energy of a radiator should equal its total radiative magnetic energy. Denote the total magnetic energy as $W^m_{tot}(t)=\int_{V_a}(0.5\mathbf{B}\bullet\mathbf{H})d\mathbf{r}_1$. If $W_J(t)$ is defined as the reactive magnetic energy, it can be checked from (10) that the resultant radiative magnetic energy, which is $W^m_{tot}(t)-W_J(t)$, will not equal the corresponding radiative electric energy $W^e_{rad}(t)$. Thirdly, as has been verified in our previous works [37][38], in the case of the Hertzian dipole, the reactive electric energy defined by $W^e_{react}(t)$ is exactly in agreement with the electric energy stored in the capacitor in its equivalent circuit model proposed by Chu [39]. However, the reactive magnetic energy calculated with $W_J(t)$ does not exactly equal the magnetic energy stored in the equivalent inductor. Only their time averaged values are equal. Taking into account of these facts, the definition of the reactive magnetic energy of a radiator is modified by making the total radiative magnetic energy equal the total radiative electric energy. Explicitly, we define

$$W^m_{react}(t)=W^m_{tot}(t)-W^m_{rad}(t)=\int_{V_\infty}\left(\frac{1}{2}\mathbf{B}\bullet\mathbf{H}+\frac{1}{2}\mathbf{D}\bullet\frac{\partial\mathbf{A}}{\partial t}\right)d\mathbf{r}_1 \tag{11}$$

wherein $W^m_{rad}(t)=W^e_{rad}(t)$ is applied. Making use of (6), the reactive magnetic energy can be evaluated with

$$W^m_{react}(t)=\int_{V_s}\left(\frac{1}{2}\mathbf{J}\bullet\mathbf{A}\right)d\mathbf{r}_1+\int_{V_\infty}\frac{1}{2}\frac{\partial}{\partial t}(\mathbf{D}\bullet\mathbf{A})\,d\mathbf{r}_1 -\oint_{S_\infty}\left(\frac{1}{2}\mathbf{H}\times\mathbf{A}\right)\bullet\hat{\mathbf{n}}dS \tag{12}$$

There is an additional volume integral in the reactive magnetic energy, which is defined as the macroscopic Schott energy,

$$W_S(t)=\int_{V_\infty}\frac{1}{2}\frac{\partial}{\partial t}(\mathbf{D}\bullet\mathbf{A})d\mathbf{r}_1 \tag{13}$$

In order to reveal the property of the term $W_S(t)$, recall the expression for the electric flux density as follows,

$$\begin{aligned}\mathbf{D}(\mathbf{r},t)&=-\varepsilon_0\nabla\phi(\mathbf{r},t)-\varepsilon_0\frac{\partial}{\partial t}\mathbf{A}(\mathbf{r},t)\\&=-\int_{V_s}\int_{-\infty}^{\infty}\rho(\mathbf{r}_1,t_1)\nabla G(t-t_1-R_1/c)dt_1d\mathbf{r}_1\\&\quad-\frac{1}{c^2}\int_{V_s}\int_{-\infty}^{\infty}\mathbf{J}(\mathbf{r}_1,t_1)\dot{G}(t-t_1-R_1/c)dt_1d\mathbf{r}_1\end{aligned} \tag{14}$$

where the superscript "˙" means derivative with respect to time. The time domain Green's function can be expressed with the Dirac delta function,

$$G_1(\mathbf{r},\mathbf{r}_1;t-R_1/c)=\frac{\delta(t-R_1/c)}{4\pi R_1} \tag{15}$$

Substituting (4) and (14) into $W_S(t)$ yields the integration over source region

$$\begin{aligned}W_S(t)&=\int_{V_\infty}\frac{1}{2}\frac{\partial}{\partial t}(\mathbf{D}\bullet\mathbf{A})d\mathbf{r}\\&=-\mu_0\int_{V_\infty}\int_{V_s}\int_{V_s}\left\{\begin{bmatrix}\int_{-\infty}^{\infty}\rho(\mathbf{r}_1,t_1)\nabla G_1dt_1\\+c^{-2}\int_{-\infty}^{\infty}\mathbf{J}(\mathbf{r}_1,t_1)\dot{G}_1dt_1\end{bmatrix}\right.\\&\qquad\left.\bullet\int_{-\infty}^{\infty}\mathbf{J}(\mathbf{r}_2,t_2)G_2dt_2\right\}d\mathbf{r}_1d\mathbf{r}_2d\mathbf{r}\end{aligned} \tag{16}$$

where $G_{1,2}=G(t-t_{1,2}-R_{1,2}/c)$ and $R_{1,2}=|\mathbf{r}-\mathbf{r}_{1,2}|$. With the derivation detailed in the Appendix, the integral can be explicitly expressed by an integration over the source region,

$$\begin{aligned}W_S(t)=&-\frac{1}{8\pi\varepsilon_0}\int_{V_s}\int_{V_s}\frac{1}{r_{21}}\int_{t-r_{21}/c}^{t}\times\\&\begin{bmatrix}\rho(\mathbf{r}_1,\tau)\dot{\rho}\left(\mathbf{r}_2,2t-\tau-\frac{r_{21}}{c}\right)+\\c^{-2}\dot{\mathbf{J}}\left(\mathbf{r}_1,2t-\tau-\frac{r_{21}}{c}\right)\bullet\mathbf{J}(\mathbf{r}_2,\tau)\end{bmatrix}d\tau d\mathbf{r}_2d\mathbf{r}_1\end{aligned} \tag{17}$$

where $r_{21}=|\mathbf{r}_2-\mathbf{r}_1|$. Note that $\rho(\mathbf{r}_1,t_1)$, $\mathbf{J}(\mathbf{r}_1,t_1)$ and $\rho(\mathbf{r}_2,t_2)$, $\mathbf{J}(\mathbf{r}_2,t_2)$ stand for the sources at $(\mathbf{r}_1,t_1)$ and $(\mathbf{r}_2,t_2)$, respectively. They are the same function related to the same radiator. For a pulse source in $[0,T]$, as checked in the Appendix, the integral becomes zero when $t\geq T+r_{21,\max}/2c$, where $r_{21,\max}$ is the largest distance between two source points. This means that after the sources have disappeared, although $\partial(\mathbf{D}\bullet\mathbf{A})/\partial t$ is not zero everywhere in the space, its volume integral over the whole space, i.e., $W_S(t)$, soon becomes zero.

The reactive electromagnetic energy is the sum of the reactive electric energy defined in (8) and the reactive magnetic energy defined in (11),

$$W_{react}(t)=\int_{V_\infty}\left(\frac{1}{2}\mathbf{D}\bullet\mathbf{E}+\frac{1}{2}\mathbf{B}\bullet\mathbf{H}+\mathbf{D}\bullet\frac{\partial\mathbf{A}}{\partial t}\right)d\mathbf{r}_1 \tag{18}$$

For pulse radiator, the surface integral in (12) is zero as the fields never reach S_∞. Hence, the reactive energy is numerically equal to

$$\begin{aligned}W_{react}(t)&=\int_{V_s}\left(\frac{1}{2}\mathbf{J}\bullet\mathbf{A}+\frac{1}{2}\rho\phi\right)d\mathbf{r}_1+\int_{V_\infty}\frac{1}{2}\frac{\partial}{\partial t}(\mathbf{D}\bullet\mathbf{A})d\mathbf{r}_1\\&=W_{\rho J}(t)+W_S(t)\end{aligned} \tag{19}$$

where $W_{\rho J}(t)=W_\rho(t)+W_J(t)$. It is the Coulomb-velocity energy.

The total radiative energy is the sum of the radiative electric energy and the radiative magnetic energy, which is

$$W_{rad}(t)=W_{rad}^e(t)+W_{rad}^m(t)=-\int_{V_\infty}\left(\mathbf{D}\bullet\frac{\partial\mathbf{A}}{\partial t}\right)d\mathbf{r}_1 \tag{20}$$

For static electromagnetic fields, the radiative energy is zero, and the reactive electric (magnetic) energy is exactly the stored electric (magnetic) energy associated with the static sources.

Introduce a principal radiative energy as

$$W_{rad0}(t)=\int_{V_\infty}\frac{1}{2}\left(\frac{\partial\mathbf{D}}{\partial t}\bullet\mathbf{A}-\mathbf{D}\bullet\frac{\partial\mathbf{A}}{\partial t}\right)d\mathbf{r}_1 \tag{21}$$

The radiative energy can then be divided into two parts,

$$W_{rad}(t)=W_{rad0}(t)-W_S(t) \tag{22}$$

With these definitions, the total electromagnetic energy can be expressed with

$$\begin{aligned}W_{tot}(t)&=W_{react}(t)+W_{rad}(t)\\&=W_{rad0}(t)+W_{\rho J}(t)\\&=W_{\rho J}(t)+W_{rad}(t)+W_S(t)\end{aligned} \tag{23}$$

As shown in the Appendix, the principal radiative energy can be evaluated with the integration over the source region,

$$\begin{aligned}W_{rad0}(t)=&\frac{1}{8\pi\varepsilon_0}\int_{V_s}\int_{V_s}\frac{1}{r_{21}}\int_{r_{21}/c}^{t}\\&\left\{\begin{matrix}\begin{bmatrix}\dot{\rho}(\mathbf{r}_1,\tau)\rho(\mathbf{r}_2,\tau-r_{21}/c)\\-\dot{\rho}(\mathbf{r}_1,\tau-r_{21}/c)\rho(\mathbf{r}_2,\tau)\end{bmatrix}\\+c^{-2}\begin{bmatrix}\mathbf{J}(\mathbf{r}_1,\tau)\dot{\mathbf{J}}(\mathbf{r}_2,\tau-r_{21}/c)\\-\mathbf{J}(\mathbf{r}_1,\tau-r_{21}/c)\dot{\mathbf{J}}(\mathbf{r}_2,\tau)\end{bmatrix}\end{matrix}\right\}d\tau d\mathbf{r}_2d\mathbf{r}_1\end{aligned} \tag{24}$$

It can be checked that for a pulse source over $[0,T]$, $W_{rad0}(t)=W_{rad0}(T)$ for $t\geq T$. However, as seen from (22), the total radiative energy continues to vary in a small time period $[T,T+r_{21,\max}/2c]$ due to the effect of $W_S(t)$.

For a pulse radiator with sources existing in $[0,T]$, its total energy can be divided into $W_{tot}(t)=W_{\rho J}(t)+W_{rad0}(t)$, as shown in (23). This is what we had proposed in our previous works [37][38]. However, careful examination shows that, for oscillating pulses, $W_{rad0}(t)$ does not exactly equal the radiative energy but only approximately equals its time averaged value. The oscillating component of the radiative energy is lost, as will be demonstrated later in the examples. At the same time, $W_J(t)$ does not exactly reflect the stored magnetic energy for a Hertzian dipole. We have revisited the issue of the electromagnetic radiation of a moving charged particle [16][17] and finally realized that radiative fields can interact with other sources before they completely leave the source area. The interaction causes oscillatory energy exchange. Therefore, we introduce the macroscopic Schott energy $W_S(t)$ in our formulation to account for the energy exchange between $W_{\rho J}(t)$ and $W_{rad0}(t)$, and divide the total energy of a radiator into a reactive energy $W_{react}(t) = W_{\rho J}(t) + W_S(t)$ and a radiative energy $W_{rad}(t) = W_{rad0}(t) - W_S(t)$, where the Schott energy $W_S(t)$ plays the role of energy exchanging. It can be verified that the Coulomb-velocity energy $W_{\rho J}(t)$ is strictly attached to the sources and appears/disappears simultaneously with the sources. However, the Schott energy $W_S(t)$ does not disappear simultaneously with its sources. It continues to remain nonzero within the period of $\left[T, T + r_{21,\max}/2c\right]$ and then disappears. Since the sources have disappeared for $t \geq T$, the nonzero Schott energy $W_S(t)$ cannot be absorbed. Because of energy conservation, the only possible way is that $W_S(t)$ converts to radiative energy, corresponding to the change of the radiative energy in this time interval after the sources have disappeared. As is discussed in [20], the radiative energy is always nonnegative and it describes an irreversible loss of energy, while the Schott energy can change reversibly. On the other hand, for $t \geq T + r_{21,\max}/2c$, although $W_S(t) = 0$, its integrand is not necessary to be zero everywhere. It induces an energy oscillation and contributes to the Poynting vector.

The Poynting Theorem correctly describes the relationship between the work rate done by the source, the total electromagnetic energy in region $V_a \supseteq V_s$ containing the source, and the total electromagnetic power flux crossing the boundary S_a of the region,

$$-\int_{V_s} \mathbf{J} \bullet \mathbf{E} d\mathbf{r}_1 = \frac{\partial}{\partial t} \int_{V_a} \left(\frac{1}{2} \mathbf{D} \bullet \mathbf{E} + \frac{1}{2} \mathbf{B} \bullet \mathbf{H} \right) d\mathbf{r}_1 + \oint_{S_a} \mathbf{S} \bullet \hat{\mathbf{n}} dS \quad (25)$$

where the Poynting vector $\mathbf{S} = \mathbf{E} \times \mathbf{H}$ is conventionally regarded as the power flux density. With the definition of (18) and (20), it can be rewritten as

$$\begin{aligned} -\int_{V_s} \mathbf{J} \bullet \mathbf{E} d\mathbf{r}_1 &= \frac{\partial}{\partial t} \int_{V_a} \left(\frac{1}{2} \mathbf{D} \bullet \mathbf{E} + \frac{1}{2} \mathbf{B} \bullet \mathbf{H} + \mathbf{D} \bullet \frac{\partial \mathbf{A}}{\partial t} \right) d\mathbf{r}_1 \\ &+ \int_{V_a} \left(-\mathbf{D} \bullet \frac{\partial \mathbf{A}}{\partial t} \right) d\mathbf{r}_1 + \oint_{S_a} \mathbf{S} \bullet \hat{\mathbf{n}} dS \end{aligned} \quad (26)$$

which clearly implies that the Poynting vector contains the contribution from the propagation of the radiative energy and the fluctuation of the reactive energy.

Now we will show that $W_{rad0}(t)$ associated with a bounded volume is a convenient quantity for engineering application. Substituting (5) and (6) into (25) and reorganizing it gives

$$\begin{aligned} &-\int_{V_s} \mathbf{J} \bullet \mathbf{E} d\mathbf{r}_1 - \frac{\partial}{\partial t} W_{\rho J}(t) = \frac{\partial}{\partial t} \int_{V_a} \frac{1}{2} \left(\frac{\partial \mathbf{D}}{\partial t} \bullet \mathbf{A} - \mathbf{D} \bullet \frac{\partial \mathbf{A}}{\partial t} \right) d\mathbf{r}_1 \\ &+ \oint_{S_a} \left[\mathbf{E} \times \mathbf{H} - \frac{1}{2} \frac{\partial}{\partial t} \left(\mathbf{H} \times \mathbf{A} + \mathbf{D} \phi \right) \right] \bullet \hat{\mathbf{n}} dS \end{aligned} \quad (27)$$

For the sake of convenience, a new vector is introduced for the integrand of the surface integral in (27),

$$\mathbf{S}_{rad0}(\mathbf{r}, t) = \mathbf{E} \times \mathbf{H} - \frac{\partial}{\partial t} \left(\frac{1}{2} \mathbf{H} \times \mathbf{A} + \frac{1}{2} \mathbf{D} \phi \right) \quad (28)$$

It has to be noted that $\mathbf{S}_{rad0}$ is not the radiative power density. Denote its surface integral as

$$P_{Srad}(t) = \oint_{S_a} \mathbf{S}_{rad0} \bullet \hat{\mathbf{n}} dS \quad (29)$$

The total work done by the source is

$$W_{exc}(t) = -\int_{-\infty}^{t} \int_{V_s} \mathbf{J}(\mathbf{r}_1, \tau) \bullet \mathbf{E}(\mathbf{r}_1, \tau) d\mathbf{r}_1 d\tau \quad (30)$$

Integrating both side of (27) gives

$$W_{exc}(t) - W_{\rho J}(t) = W_{rad0}(t) + \int_0^t P_{Srad}(\tau) d\tau \quad (31)$$

where $W_{rad0}(t)$ is defined using (21) but with the integration domain replaced by V_a. Accordingly, $P_{Srad}(t)$ can be interpreted as the principal radiative power associated with the principal radiative energy $W_{rad0}(t)$ passing through the observation surface S_a. Since it is not easy to find an explicit expression for the total radiative power passing through the observation surface, the principal radiative power $P_{Srad}(t)$ can provide a good measurement for it. As shown in the Hertzian dipole and the other examples, the principal radiative power $P_{Srad}(t)$ gives a kind of time averaged value of the total radiative power passing through the observation surface.

For $t \geq T + r_{21,\max}/2c$, we have $W_{\rho J}(t) = W_S(t) = 0$. The total radiative energy can be expressed with the temporal integration of $P_{Srad}(t)$ on an arbitrary observation surface enclosing the radiator,

$$W_{rad}(t) = W_{rad0}(t) = \int_{t_{\min}}^{t_{\max}} P_{Srad}(t) dt = W_{exc}(T) \quad (32)$$

For pulse sources, $P_{Srad}(t)$ has nonzero values only over period $(t_{\min} < t < t_{\max})$, in which $t_{\min}$ and $t_{\max}$ are respectively the earliest and the latest time that the fields pass through the observation surface. As a special case, we may choose $V_a = V_s$, and put the observation surface S_a close to the surface of the sources. Assuming that all radiative fields coming out of S_a no longer interact with the sources in V_s and ignoring the radiative energy stored in V_s, we may obtain the power coming out of the surface of the sources as

$$P_{Srad0}(t) = -\int_{V_s}\left[\mathbf{J}\bullet\mathbf{E} + \frac{\partial}{\partial t}\left(\frac{1}{2}\rho\phi + \frac{1}{2}\mathbf{J}\bullet\mathbf{A}\right)\right]d\mathbf{r}_1 \quad (33)$$

Apparently, $P_{Srad}(t)$ at different observation surfaces are not expected to be equal, but their integrations over the time interval $(t_{\min} < t < t_{\max})$ are equal, and are approximately equal to that of $P_{Srad0}(t)$ since all the radiative energy of the pulse source in vacuum will eventually pass through the observation surface and propagate to infinity.

III. MUTUAL COUPLINGS

Consider a group of N radiators in vacuum. The i-th radiator has source $(\mathbf{J}_i, \rho_i)$ in region V_{si}. The reactive electromagnetic energy coupled from source-j to source-i is expressed as

$$\begin{aligned} W_{ij}^{react}(t) &= \int_{V_{si}}\left(\frac{1}{2}\rho_i\phi_j + \frac{1}{2}\mathbf{J}_i\bullet\mathbf{A}_j + \frac{1}{2}\frac{\partial}{\partial t}\left(\mathbf{D}_i\bullet\mathbf{A}_j\right)\right)d\mathbf{r}_i \\ &= \frac{1}{8\pi\varepsilon_0}\int_{V_{si}}\int_{V_{sj}}\frac{1}{r_{ij}}\times \\ &\left\{\begin{array}{l} \rho_i(\mathbf{r}_i,t)\rho_j(\mathbf{r}_j,t-r_{ij}/c) \\ +c^{-2}\mathbf{J}_i(\mathbf{r}_i,t)\bullet\mathbf{J}_j(\mathbf{r}_j,t-r_{ij}/c) - \\ \int_{t-r_{ij}/c}^{t}\left[\begin{array}{l}\rho_i(\mathbf{r}_i,\tau)\dot{\rho}_j\left(\mathbf{r}_j,2t-\tau-\dfrac{r_{ij}}{c}\right) \\ +c^{-2}\dot{\mathbf{J}}_i\left(\mathbf{r}_i,2t-\tau-\dfrac{r_{ij}}{c}\right)\bullet\mathbf{J}_j(\mathbf{r}_j,\tau)\end{array}\right]d\tau \end{array}\right\}d\mathbf{r}_j d\mathbf{r}_i \end{aligned} \quad (34)$$

where $\mathbf{A}_{i,j}$ and $\phi_{i,j}$ are respectively the vector potential and the scalar potential generated by the source $(\rho_{i,j}, \mathbf{J}_{i,j})$.

The mutual coupled reactive energies also include the Schott energies, which may be denoted by $W_{Sij}(t)$. It is straightforward to check that the total reactive energy of the system is

$$W_{tot}^{react}(t) = \sum_{i=1}^{N}\sum_{j=1}^{N}W_{ij}^{react}(t) \quad (35)$$

It contains the self reactive energies ($i = j$) and the mutual coupled reactive energies ($i \neq j$). Note that conventional formulations may be difficult to be extended to contain multiple radiators because it is difficult to determine the coordinate origin and the subtraction term.

The mutual radiative energy from source-j to source-i is

$$W_{ij}^{rad}(t) = \int_{V_{si}}\left(-\mathbf{D}_i\bullet\frac{\partial\mathbf{A}_j}{\partial t}\right)d\mathbf{r}_i \quad (36)$$

which is the energy radiated by radiator-i when it is excited by radiator-j.

In circuit theory, mutual coupling is usually referred to the coupling between the energies stored in inductors or capacitors, not including the losses dissipated by resistors. As the mutual radiative energy is a kind of radiation loss to the radiator, it is reasonable to consider that electromagnetic mutual coupling energies only include the mutual reactive energies.

The electromagnetic radiation and coupling problem of two pulse radiators is illustrated in Fig.1. We only analyze the radiation of radiator-1 in the region V_{s1}. It induces a Coulomb-velocity field carrying Coulomb-velocity energy and emits a radiation field carrying radiative energy to the surrounding space. Specifically, we consider a small part of source in the internal region of V_{s1} denoted by the red star in Fig.1. The radiative fields by the red star source interact with other sources in the source region V_{s1} when they propagate through the source region to the outside space. Part of the radiative energy is transferred to the sources they have encountered. A nonzero Schott energy appears in this period corresponding to energy exchange. Only after they have completely left the source region, the radiative fields by the red star source can propagate to far region with a constant radiative energy, until they reach radiator-2 and interact with the sources there. The radiative fields from other sources in V_{s1} experience the similar journey, and carry the radiative energy of radiator-1, inducing a real electromagnetic radiative power flow.

On the other hand, the reactive energy of radiator-1 affects radiator-2 through mutual coupling. When the reactive energy of radiator-1 varies with time, the effect does not reach radiator-2 simultaneously. The fluctuation of the reactive energy causes a pseudo power flow and travels with the real power flow to radiator-2.

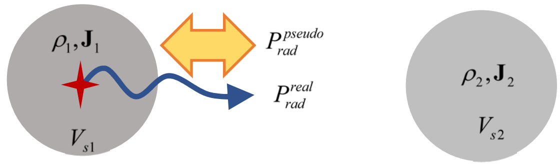
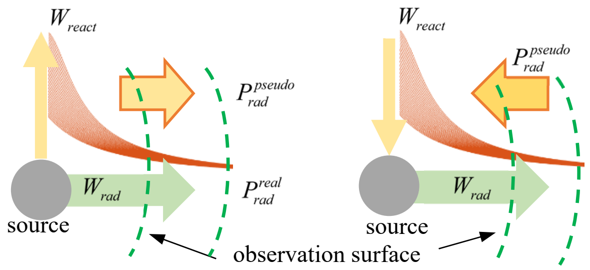


Fig.1 Electromagnetic radiation and mutual coupling of two radiators.

Poynting vector represents the total power flow, including the real radiative power flow $P_{rad}^{real}(t)$ and the pseudo power flow $P_{rad}^{pseudo}(t)$. As shown in Fig.2, the real power flow always propagates always from its source, crossing the observation surface from left to right. However, the direction of the pseudo power flow is reversible. It crosses the observation surface from left to right when the reactive energy of the source increases, and from right to left when the reactive energy decreases.



(a) (b)

Fig.2 Electromagnetic power flow. (a) Direction when the reactive energy increases. (b) Direction when the reactive energy decreases.

In the radiation process of a radiator, mutual coupling occurs between different part of sources in the same radiator. The source distribution is not only dependent on the excitations at the feeding port, but also dependent on the mutual coupling. The total reactive energy and the total radiative energy of the radiator have taken into account all these mutual coupling effect. If the radiator contains several separated parts, each part may have different types of sources, the radiation process can be analyzed in the same way. A good example is an antenna array. It can be handled as a radiation problem if we take the whole array as a single radiator. It is a radiation and mutual coupling problem if we want to reveal the mutual couplings among different units of the array.

IV. RADIATION OF HARMONIC SOURCES

For harmonic fields with time convention of $e^{j\omega t}$, the radiation is assumed to last temporally from $-\infty$ to $+\infty$, so the radiative energy is infinitely large. The Poynting theorem can be applied to describe the balance between the time averaged powers and the varying rate of the energies,

$$-\frac{1}{2}\int_{V_s}\mathbf{J}^*\cdot\mathbf{E}d\mathbf{r}_1 = 2j\omega\int_{V_a}\left[\frac{1}{4}\mathbf{B}\cdot\mathbf{H}^* - \frac{1}{4}\mathbf{E}\cdot\mathbf{D}^*\right]d\mathbf{r}_1 + \frac{1}{2}\oint_{S_a}\mathbf{E}\times\mathbf{H}^*\cdot\hat{\mathbf{n}}dS \tag{37}$$

from which the time averaged radiative power at infinity can be evaluated with source distributions,

$$\left(P_{rad}\right)_{av} = \mathrm{Re}\left\{\frac{1}{2}\oint_{S_\infty}\mathbf{E}\times\mathbf{H}^*\cdot\hat{\mathbf{n}}dS\right\} = -\mathrm{Re}\left\{\frac{1}{2}\int_{V_s}\mathbf{E}\cdot\mathbf{J}^*d\mathbf{r}_1\right\} \tag{38}$$

The same symbols are used for the corresponding phasors for the sake of simplicity.

However, the evaluation of the reactive energies in conventional formulation requires to subtract the radiative energy from the total energy. Since both the energies are unbounded, all those formulations based on energy subtraction are not always satisfactory.

With the theory proposed here, the power balance can be evaluated within any domain enclosed by an observation surface S_a containing the source region V_s,

$$-\int_{V_s}\mathbf{J}^*\cdot\mathbf{E}d\mathbf{r}_1 = 2j\omega\int_{V_s}\left(\frac{1}{4}\rho^*\phi + \frac{1}{4}\mathbf{J}^*\cdot\mathbf{A}\right)d\mathbf{r}_1 + \oint_{S_a}\left[\frac{1}{2}\mathbf{E}\times\mathbf{H}^* - j\omega\left(\frac{1}{4}\mathbf{H}^*\times\mathbf{A} + \frac{1}{4}\mathbf{D}^*\phi\right)\right]\cdot\hat{\mathbf{n}}dS \tag{39}$$

The time averaged radiative power crossing the observation surface can be obtained using the radiative power flux vector $\mathbf{S}_{Srad}$ or the source distributions,

$$\left(P_{rad}\right)_{av} = \mathrm{Re}\left\{\oint_{S_a}\left[\frac{1}{2}\mathbf{E}\times\mathbf{H}^* - \frac{j\omega}{4}\left(\mathbf{H}^*\times\mathbf{A} + \mathbf{D}^*\phi\right)\right]\cdot\hat{\mathbf{n}}dS\right\} = -\mathrm{Re}\left\{\int_{V_s}\frac{1}{2}\mathbf{J}^*\cdot\mathbf{E}d\mathbf{r}_1\right\} \tag{40}$$

Note that, with (40), the observation surface is not required to approach infinity for evaluating the radiative power. It can be checked that the result is in consistent with that obtained using the Poynting vector since it has been proved in [14] that

$$\mathrm{Re}\left\{\oint_{S_\infty} j\omega\left(\frac{1}{4}\mathbf{H}^*\times\mathbf{A} + \frac{1}{4}\mathbf{D}^*\phi\right)\cdot\hat{\mathbf{n}}dS\right\} = 0 \tag{41}$$

The time averaged reactive energy can be calculated with the fields and the vector potential,

$$\left(W_{react}\right)_{av} = \mathrm{Re}\left\{\int_{V_\infty}\left(\frac{1}{4}\mathbf{E}\cdot\mathbf{D}^* + \frac{1}{4}\mathbf{B}\cdot\mathbf{H}^* + \frac{1}{2}j\omega\mathbf{D}^*\cdot\mathbf{A}\right)\right\} \tag{42}$$

It is easy to verify that $\left(W_S\right)_{av} = 0$, so the time averaged reactive energy can be alternatively calculated using the source-potential products as follows

$$\left(W_{react}\right)_{av} = \mathrm{Re}\left\{\int_{V_s}\left(\frac{1}{4}\phi^*\rho + \frac{1}{4}\mathbf{J}^*\cdot\mathbf{A}\right)d\mathbf{r}_1\right\} \tag{43}$$

It can be checked that $\left(W_{Sij}\right)_{av} = 0$. Therefore, the time averaged mutual coupled reactive electromagnetic energies are found to be

$$\left(W_{ij}^{react}\right)_{av} = \mathrm{Re}\int_{V_{si}}\left(\frac{1}{4}\rho_i\phi_j^* + \frac{1}{4}\mathbf{J}_i^*\cdot\mathbf{A}_j\right)d\mathbf{r}_i \tag{44}$$

As expected, $\left(W_{ij}^{react}\right)_{av} = \left(W_{ji}^{react}\right)_{av}$ holds for mutual coupling in free space.

VI. HERTZIAN DIPOLE

A Hertzian dipole locating at the origin is analyzed to show the energy/power balance relationship. The moment of the dipole is assumed to be $ql\cos\omega t$, the scalar potential and the vector potential of which can be readily derived from the Hertzian potential $\Pi = \left(ql/4\pi r\right)\cos\left(\omega t - kr\right)$ [38][41],

$$\mathbf{A} = -\frac{\omega\mu_0 ql}{4\pi r}\sin\left(\omega t - kr\right)\left(\hat{\mathbf{r}}\cos\theta - \hat{\boldsymbol{\theta}}\sin\theta\right) \tag{45}$$

$$\varphi = \frac{\omega^2\mu_0 ql}{4\pi}\cos\theta\left[\frac{1}{k^2r^2}\cos\left(\omega t - kr\right) - \frac{1}{kr}\sin\left(\omega t - kr\right)\right] \tag{46}$$

from which the fields are found to be

$$\mathbf{E}=\frac{k^2ql}{4\pi\varepsilon_0 r}\left\{\begin{array}{l}\hat{\mathbf{r}}2\cos\theta\dfrac{1}{kr}\left[\begin{array}{l}\dfrac{1}{kr}\cos(\omega t-kr)\\-\sin(\omega t-kr)\end{array}\right]\\+\hat{\boldsymbol{\theta}}\sin\theta\left[\begin{array}{l}\left(\dfrac{1}{k^2r^2}-1\right)\cos(\omega t-kr)\\-\dfrac{1}{kr}\sin(\omega t-kr)\end{array}\right]\end{array}\right\} \tag{47}$$

$$\mathbf{H}=-\frac{\omega kql}{4\pi r}\sin\theta\left[\frac{1}{kr}\sin(\omega t-kr)+\cos(\omega t-kr)\right]\hat{\boldsymbol{\varphi}} \tag{48}$$

As is known, the Hertzian dipole is a point source and its total reactive energy is infinite. A common strategy is to evaluate the integrals (8) and (11) in the whole space excluding a small sphere with radius *a*. The results are listed below,

$$\begin{aligned}W_{react}^{e}(t)&=\int_{V_\infty-V_a}\left(\frac{1}{2}\mathbf{D}\bullet\mathbf{E}+\frac{1}{2}\mathbf{D}\bullet\frac{\partial\mathbf{A}}{\partial t}\right)d\mathbf{r}_1\\&=\alpha_0\left[\begin{array}{l}\dfrac{1}{k^3a^3}+\dfrac{1}{ka}+\left(\dfrac{1}{k^3a^3}-\dfrac{1}{ka}\right)\cos2(\omega t-ka)\\-\dfrac{2}{k^2a^2}\sin2(\omega t-ka)\end{array}\right]\end{aligned} \tag{49}$$

$$\begin{aligned}W_{react}^{m}(t)&=\int_{V_\infty-V_a}\left(\frac{1}{2}\mathbf{B}\bullet\mathbf{H}+\frac{1}{2}\mathbf{D}\bullet\frac{\partial\mathbf{A}}{\partial t}\right)d\mathbf{r}_1\\&=\frac{2\alpha_0}{ka}\sin^2(\omega t-ka)\end{aligned} \tag{50}$$

The principal radiative power evaluated at a spherical observation surface is

$$P_{Srad}(t)=\oint_{S_a}\mathbf{S}_{Srad}\bullet\hat{\mathbf{n}}dS=2\omega\alpha_0 \tag{51}$$

It is a constant value independent of the radius of the sphere, clearly indicating that the total radiative power associated with $W_{rad0}(t)$ crossing any concentric spherical surface is the same.

The surface integral of the Poynting vector on the spherical surface S_a is calculated to be

$$\begin{aligned}P_{pv}(t)&=\oint_{S_a}\mathbf{S}\bullet\hat{\mathbf{n}}dS=2\omega\alpha_0\left[1+\cos2(\omega t-ka)\right]\\&+2\omega\alpha_0\left[\begin{array}{l}\left(\dfrac{2}{ka}-\dfrac{1}{k^3a^3}\right)\sin2(\omega t-ka)\\-\dfrac{2}{k^2a^2}\cos2(\omega t-ka)\end{array}\right]\end{aligned} \tag{52}$$

which varies with the radius of the surface due to the effect of the reactive energy. As expected, the time average of $P_{pv}(t)$ equals that of $P_{Srad}(t)$.

Since the Hertzian dipole is a point source, its fields propagate radially and cross all concentric spherical observation surfaces with light velocity. Therefore, the radiative energy per unit time near the spherical surface S_a can be considered as the real radiative power crossing S_a,

$$\begin{aligned}P_{rad}^{real}(t)dt&=\int_S\int_a^{a+cdt}\left(-\mathbf{D}\bullet\frac{\partial\mathbf{A}}{\partial t}\right)drdS\\&=cdt\int_{S_a}\left(-\mathbf{D}\bullet\frac{\partial\mathbf{A}}{\partial t}\right)dS\end{aligned} \tag{53}$$

from which the real radiative power is found to be

$$P_{rad}^{real}(t)=2\omega\alpha_0\left[1+\cos2(\omega t-ka)\right] \tag{54}$$

the amplitude of which is not dependent on the radius of the observation surface. It is readily to recognize from (52) that it is the first term in the Poynting power $P_{pv}(t)$. The other terms of $P_{pv}(t)$ in (52) compose the pseudo power flow, which decreases with the distance to the dipole.

The time averaged energies are listed below for readers' reference,

$$\begin{cases}(W_m)_{av}=\alpha_0\left(\dfrac{1}{ka}\right)\\(W_e)_{av}=\alpha_0\left(\dfrac{1}{k^3a^3}+\dfrac{1}{ka}\right)\end{cases} \tag{55}$$

The Q factor of the dipole is then calculated to be

$$Q=\frac{2\omega(W_e)_{av}}{(P_{rad})_{av}}=\frac{1}{k^3a^3}+\frac{1}{ka} \tag{56}$$

which is exactly in agreement with the result shown in [42].

The well-established equivalent circuit model proposed by Chu [39] for Hertzian dipole is shown in Fig.3. Assume that the current in the radiation resistor at the interface of $r=a$ is $i_R=I_0\cos(\omega t-ka)$. The energies stored in the capacitor and the inductor can be derived to be

$$\begin{cases}W_C(t)=\dfrac{I_0^2}{4\omega}\left[\begin{array}{l}\dfrac{1}{ka}+\dfrac{1}{k^3a^3}+\left(\dfrac{1}{k^3a^3}-\dfrac{1}{ka}\right)\cos2(\omega t-ka)\\-\dfrac{2}{(ka)^2}\sin2(\omega t-ka)\end{array}\right]\\W_L(t)=\dfrac{I_0^2}{2\omega}\left(\dfrac{1}{ka}\sin^2(\omega t-ka)\right)\end{cases} \tag{57}$$

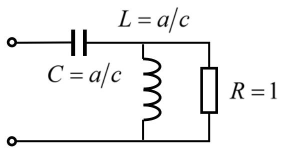


Fig.3 Equivalent circuit model for Hertzian dipole radiation.

If we choose $I_0^2=4\omega\alpha_0$, it can be readily verified that $W_C(t)=W_e(t)$ and $W_L(t)=W_m(t)$. This exact agreement gives a good support to the proposed theory.

The integration regions for $W_{rad}(t)$, $W_{rad0}(t)$ and $W_S(t)$ are all modified in a similar way. They are found to be

$$W_{rad}(t) = -\int_{V_\infty - V_a} \mathbf{D}\bullet\frac{\partial \mathbf{A}}{\partial t}dV = 2k\alpha_0 \lim_{r\to\infty}(r-a) \\ +\alpha_0\left[\sin 2(\omega t - ka) - \lim_{r\to\infty}\sin 2(\omega t - kr)\right] \tag{58}$$

$$W_{rad0}(t) = \int_{V_\infty - V_a} \frac{1}{2}\left(\frac{\partial \mathbf{D}}{\partial t}\bullet\mathbf{A} - \mathbf{D}\bullet\frac{\partial \mathbf{A}}{\partial t}\right)dV \\ = 2\alpha_0 k \lim_{r\to\infty}(r-a) \tag{59}$$

$$W_S(t) = \int_{V_\infty - V_a} \frac{\partial}{\partial t}\left(\frac{1}{2}\mathbf{D}\bullet\mathbf{A}\right)dV \\ = -\alpha_0\left[\sin 2(\omega t - ka) - \lim_{r\to\infty}\sin 2(\omega t - kr)\right] \tag{60}$$

With the wave travels to infinity, the principal radiative energy $W_{rad0}(t)$ monotonically increases with the radius, revealing that the radiative rate is always positive. The Schott energy $W_S(t)$ oscillates in the propagation with a zero average value. Its amplitude remains constant in this case.

VII. Numerical Results

A. Solenoidal Loop

The radiation of a solenoidal loop current is analyzed. The solenoidal surface current on a ring is described by $\mathbf{J}_s(\mathbf{r},t) = \mathbf{f}(\mathbf{r})I(t)$ [A/m], as shown in Fig.4. Here we choose

$$\mathbf{f}(\mathbf{r}) = 1.0\hat{\varphi} \tag{61}$$

The inner and outer radius of the ring is 0.08m and 0.1m, respectively. The temporal function is a modulated Gaussian pulse,

$$I(t) = \begin{cases} e^{-\gamma^2}\sin\omega t, & 0 \le t \le T \\ 0, & \text{else} \end{cases} \tag{62}$$

with $\omega = 2\pi \times 10^{10}$, $\gamma = 2\sqrt{5}(t - 0.5T)/T$, and $T = 1\text{ns}$. Therefore, both its initial and final reactive energy are zero. Two spherical surfaces with radius of 0.2m and 10m are chosen as the observation surfaces, with their centers coinciding with that of the source. They are labeled by sphere-1 and sphere-2, respectively. The principal radiative energy passing through sphere-1 and sphere-2 are calculated with integration of $P_{Srad}(t)$, as expressed in (32). $W_{pv}(t)$ is the integration of the Poynting vector power passing through the observation surface,

$$W_{pv}(t) = \int_0^t P_{Spv}(\tau)d\tau = \int_0^t \oint_{sphere-1,2} \mathbf{S}(\mathbf{r}_1,\tau)\cdot\hat{\mathbf{n}}d\mathbf{r}_1 d\tau \tag{63}$$

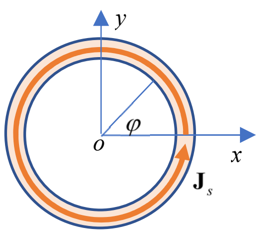
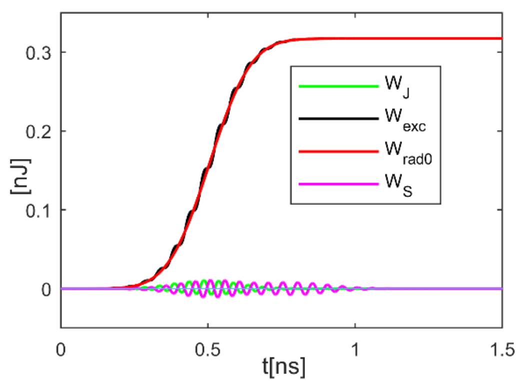


Fig. 4 Solenoidal loop current.

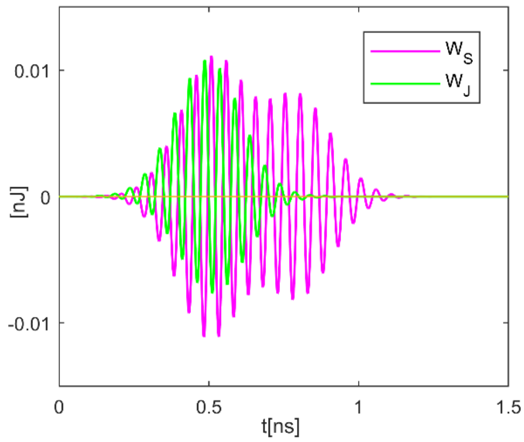
The excitation energy, the principal radiative energy and the energy evaluated with Poynting vector are shown in Fig. 5(a). In this case, the reactive energy includes the contribution from the current alone since the current is solenoidal and its corresponding charge is zero, so it is denoted as W_J in the figures. W_J oscillates with the source and admits negative values periodically. It is acceptable because the reactive energy is dependent on the potentials, which are values relative to their reference zero points. When the current varies and changes its direction periodically, the retarded vector potential in the source region lags behind and may point in direction opposite to that of the current, causing negative values. Note that the Schott energy in the charged particle theory may also be negative [20][43]. The Schott energy is plotted in Fig.5(a) as well, and is zoomed-in in Fig.5(b) together with W_J. It can be seen that the Schott energy oscillates like W_J but continues to exist for about 0.33ns after the source has disappeared at 1ns.

The energies passing through sphere-1 are shown in Fig. 5(c). The smallest and the largest distance between the source and sphere-1 are respectively 0.1m and 0.3m. The total radiative energy passing through sphere-1 at t=2ns is equal to that evaluated at the source region.

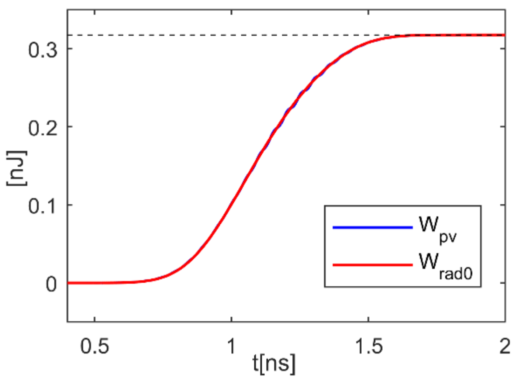
The excitation power, the principal radiative power and the time varying rate of the reactive energy are shown in Fig. 6(a). The powers crossing sphere-1 and sphere-2 are shown in Fig.6(b) and (c), respectively. $P_{Srad}(t)$ varies smoothly and remains positive. The Poynting power contains ripples coming from $P_{Sreact}(t)$, which gradually decreases with the propagation distance.



(a)

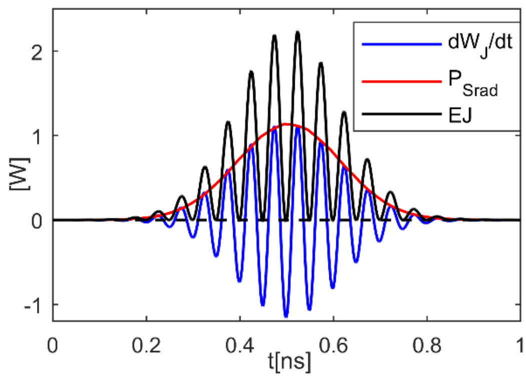


(b)

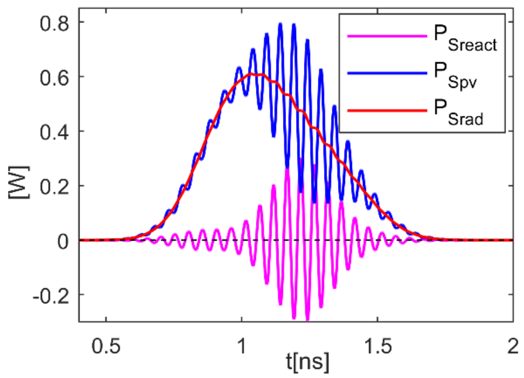


(c)

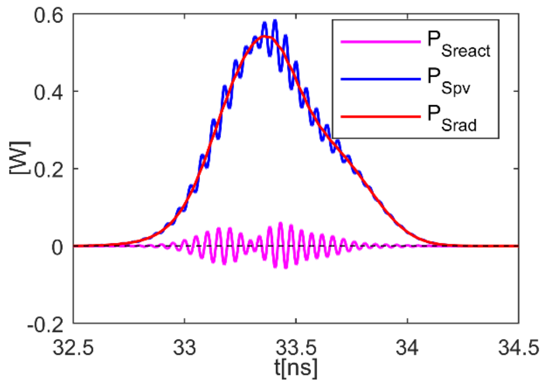
Fig.5 The energies of the loop current. (a) The energies evaluated in the source region. (b) The zoom-in figure for W_J and W_S . (c) The energies crossing sphere-1.



(a)



(b)



(c)

Fig.6 The powers of the loop current. (a) The powers evaluated in the source region. (b) The powers crossing sphere-1. (c) The powers crossing sphere-2.

B. *Thin Plate Yagi Antenna*

The geometrical structure and parameters of the Yagi antenna is shown in Fig.7. It consists of 3 PEC plates with zero thickness: a dipole in the middle, a reflector in the left and a director in the right. The width of the plate is 2mm. The dipole is fed at its center with a Delta-gap voltage source of

$$V_{feed}(t) = 1.0\sin(\omega t)[\mathrm{V}], t \geq 0 \tag{64}$$

where $\omega = 2\pi \times 1.5 \times 10^8$, corresponding to 150MHz.

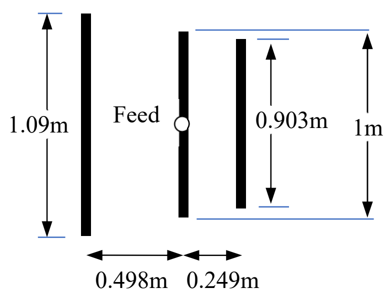


Fig.7 Thin plate Yagi antenna with 3 PEC plates.

The first step is to calculate the surface current by solving the surface electric field integral equation (EFIE) with marching-on in time scheme (MOT) [40][44]. The plates are triangularly meshed, and the surface current is expanded with RWG basis functions [45]. There are 53, 59, and 50 RWGs on the left, middle and right plate, respectively. The Delt-gap voltage feeding is put on the common edge of the RWG in the middle of the dipole. The time step of the MOT is 2.67ps. The second step is to evaluate the energies and powers using the obtained surface currents and the expressions we have proposed. When calculating the Schott energy and the principle radiative energy, the integration interval of the innermost integral is dependent on the distance between two points. It may become much smaller than the time step and should be handled carefully to get satisfactory numerical accuracy. For $r_{ij} = 0$, we can use the L'Hospital rule to find the limit of the innermost integral.

The principal radiative powers of the 3 plates are calculated separately with (33), with the integration domain respectively replaced by those of the three plates. The results are shown in Fig.8. The Coulomb-velocity energies are shown in Fig.9. As shown in the figures, the principal radiative power of the dipole is always positive. It radiates electromagnetic energy to the space from the beginning. However, the principal radiative powers of the reflector and the director are negative at the beginning, which means that they absorb energy from the dipole to generate reactive energy at the transition stage. When the radiation enters the steady state, the reactive energies tend to become steady stable. Since the PEC reflector and the PEC director are passive elements, they do not radiate by themself, but only scatter all the electromagnetic energies they received. Consequently, their principal radiative powers are zero at the steady state.

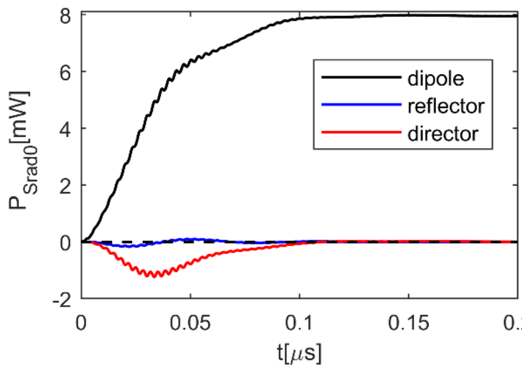


Fig.8 The principal radiative power passing through the surface of each plate.

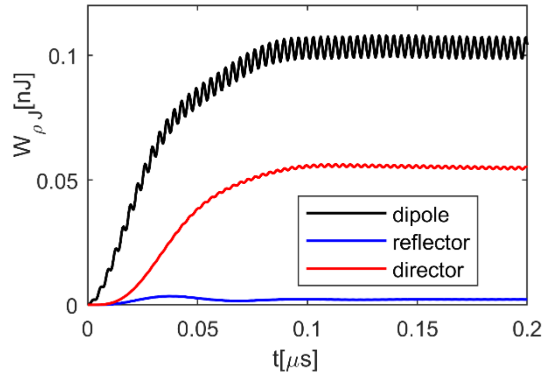


Fig.9 The Coulomb-velocity energy of each plate.

The total energies are plotted in Fig.10. After a short transition stage, the radiation approaches steady state. It can be seen that the Schott energy gradually becomes an oscillation with approximately uniform amplitude and the same period of the excitation. It brings equiripples to the total reactive energy and the total radiative energy. The total reactive energy tends to become steady and bounded, while the radiative energy increases approximately in a linear way.

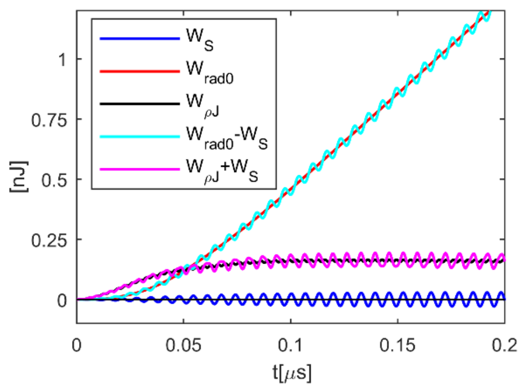


Fig.10 The total energies of the Yagi antenna.

The principal radiative power and the electromagnetic power calculated with Poynting vector are evaluated on an observation spherical surface with radius of 2m, the center of which locates at the feeding point of the Yagi antenna. The results are illustrated in Fig.11.

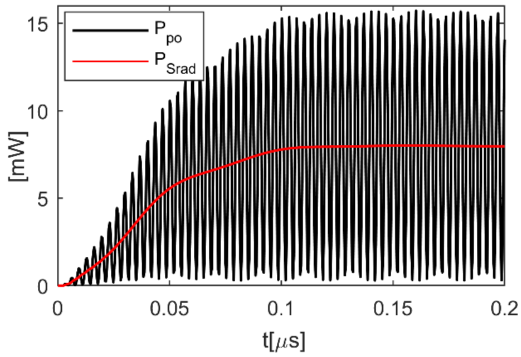


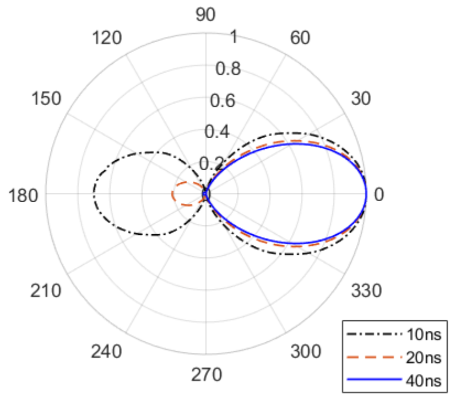
Fig.11 Electromagnetic powers passing through the observation surface.

The directivity of the antenna inevitably varies in the transition stage. The directivity patterns in the E-plane and the H-plane at 10ns, 20ns, and 40ns are depicted in Fig.12. As can be seen, in the beginning, the passive reflector and the director absorb more electromagnetic energies than the energies they received. The radiation is mainly determined by the center dipole, and the pattern is much like that of a single dipole. The antenna performs like a Yagi antenna only after the two passive plates have achieved a balance between their absorbed and scattered powers.

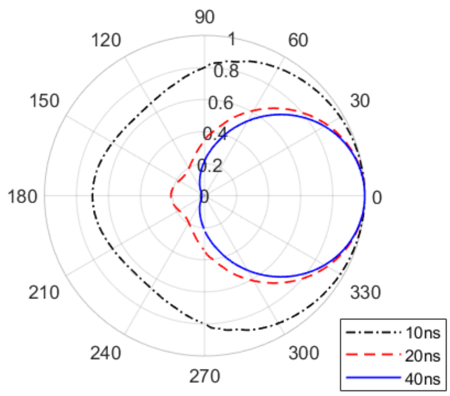
In the steady state, we can evaluate the Q factor of the radiator with formula as follows

$$Q(t) \approx \frac{2\pi}{T} \frac{W_{\rho J}(t)}{P_{Srad0}(t)} \tag{65}$$

For comparison, the Q factor for $t > 0.15\mu s$ is calculated to be 18.5, while the Q factor obtained with the frequency domain formulation described in [15] is 17.5 at 150MHz.



(a)



(b)

Fig.12 Evolution of the radiation pattern. (a) E-plane. (b) H-plane.

VIII. CONCLUSION AND DISCUSSIONS

Some issues concerning with the electromagnetic radiation and mutual couplings remain ambiguous or even controversial for decades long, especially the definitions for the reactive energy. This theory proposed clear definitions and explicit expressions for the reactive energy and the radiative energy of a radiator. The introduction of the macroscopic Schott energy makes it possible to separate the radiative energy and the reactive energy in a reasonable manner. Consequently, a new form of power balance equation is given based on the Poynting relation so that the Poynting vector is divided into two parts, respectively accounting for the contribution from the radiative energy propagation and the fluctuation of the reactive energy. The newly defined principal reactive energy $W_{rad0}(t)$ and its flux, the principal radiative power $P_{Srad}(t)$, can characterize the main property of the radiative energy. Furthermore, they can be numerically evaluated more efficiently, so are the mutual electromagnetic coupling energies defined with potentials.

As is pointed out in the introduction, the main problem of the issue is how to separate the radiative energy and the reactive energy. We began with the hypothesis that the reactive energy performs like the Coulomb-velocity energy. This provides us a clue and starting base to separate the electromagnetic energy into three parts, namely, the Coulomb-velocity energy $W_{\rho J}(t)$, the macroscopic Schott energy $W_S(t)$, and the radiative energy $W_{rad}(t)$. Then we derived the explicit and accurate expressions for them and verified that with a pulse radiator. All the expressions are strictly derived from Maxwell equations with no approximations. Retarded time is included in them and they are no static limit although they may look alike at a first glance.

The basic theory is for time varying pulse radiators. We also provided formulae for harmonic waves. Unlike Vandenbosch formulation in which the results obtained with time domain formulation sometimes may not agree with the results obtained with the frequency domain formulation, in the theory proposed here, the results in time domain and frequency domain are completely in consistent because they are respectively directly derived from the time domain Maxwell equations and the frequency domain Maxwell equations. The electromagnetic fields in frequency domain and time domain can be converted with Fourier Transform, which is a common sense in computational electromagnetics.

It has been discussed in previous sections that the electromagnetic radiation and mutual coupling issue are closely related and can be handled in the same manner. If let $i = j$ in (34) and (36), we get (18) and (20), which are the expressions for a single radiator. The terms in these expressions are all dot-products of two vectors: those at the right side are potentials, and those at the left side are source densities or their fields. In systems with electromagnetic mutual couplings, the potentials are associated with the radiators that impose the couplings, while the left quantities are associated with the radiators affected by the couplings.

The theory is completely different from the Carpenter formulation [24]. In Carpenter formulation, it was proposed to use the Coulomb-velocity energy alone as the total electromagnetic energy and to replace the Poynting Theorem with a new equation. The formulation, as well as the power flow vector $\phi\mathbf{J}$ by Slepian [46], was pointed out to be mathematically flawed by Dr. Endean [47]. In our theory, we combine the Coulomb-velocity energy and the Schott energy together to form the reactive energy. The theory does not suffer from the mathematical flaw since there is no modification to the total electromagnetic energy and the Poynting Theorem.

APPENDIX

Equation (17) and (24) can be obtained using the method given in [6]. It is required to evaluate the following key integral associated with two source point $\mathbf{r}_1$ and $\mathbf{r}_2$,

$$\begin{aligned} I &= \int_{V_\infty} G\left(\tau_1 - R_1/c\right) G\left(\tau_2 - R_2/c\right) d\mathbf{r} \\ &= \frac{c^2}{16\pi^2} \int_{V_\infty} \frac{1}{R_1 R_2} \delta\left(c\tau_1 - R_1\right) \delta\left(c\tau_2 - R_2\right) d\mathbf{r} \end{aligned} \tag{66}$$

where $\tau_{1,2} = t - t_{1,2}$, $R_{1,2} = \left|\mathbf{r} - \mathbf{r}_{1,2}\right|$. The value of the integral has been given in the equation (32) in [6]. Here we provide an alternative rigorous proof. In the spherical coordinates, choose $\mathbf{r}_1$ as the origin, and put $\mathbf{r}_2$ on $+z$ axis. Therefore, we can write $\mathbf{r}_2 = r_{21}\hat{\mathbf{z}}$, $R_1 = |\mathbf{r}| = r$, and

$$R_2 = \left|\mathbf{r} - \mathbf{r}_2\right| = \sqrt{r^2 - 2rr_{21}\cos\theta + r_{21}^2} \ .$$

Since the integrand is symmetric, we have

$$\begin{aligned} I &= \frac{c^2}{8\pi} \int_0^\pi \int_0^\infty \frac{1}{rR_2} \delta\left(c\tau_1 - r\right) \delta\left(c\tau_2 - R_2\right) r^2 \sin\theta dr d\theta \\ &= \frac{c^2}{8\pi} \int_{-1}^{1} \frac{c\tau_1}{R_2} \delta\left(c\tau_2 - R_2\right) d\cos\theta \\ &= -\frac{c^2}{8\pi} \int_{\left|c\tau_1 - r_{21}\right|}^{c\tau_1 + r_{21}} \frac{c\tau_1}{R_2} \delta\left(c\tau_2 - R_2\right) \left(-\frac{R_2}{c\tau_1 r_{21}}\right) dR_2 = \frac{c^2}{8\pi r_{21}} \end{aligned} \tag{67}$$

where $dR_2 = -c\tau_1 r_{21}/R_2 \, d\cos\theta$ is used. The integration range of $\left(\tau_1, \tau_2\right)$ for nonzero I is determined by

$$\left|c\tau_1 - r_{21}\right| \le c\tau_2 \le c\tau_1 + r_{21} \tag{68}$$

which is exactly the same as eq. (33) in [6].

Next, we take the first term of $W_S(t)$ as an example to show the derivation of (17). Rearranging the integration order gives

$$\begin{aligned} W_S^\rho(t) &= -\mu_0 \frac{1}{2} \frac{\partial}{\partial t} \int_{V_s} \int_{V_s} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \times \\ &\mathbf{J}\left(\mathbf{r}_1, t_1\right) \rho\left(\mathbf{r}_2, t_2\right) \bullet \int_{V_\infty} G_1 \nabla G_2 d\mathbf{r} dt_2 dt_1 d\mathbf{r}_2 d\mathbf{r}_1 \end{aligned} \tag{69}$$

Making use of the identity $G_1 \nabla G_2 = \nabla\left(G_1 G_2\right) - \nabla G_1 G_2$ and $\mathbf{J}_1 \bullet \nabla G_1 = -\mathbf{J}_1 \bullet \nabla_1 G_1 = G_1 \nabla_1 \bullet \mathbf{J}_1 - \nabla_1 \bullet \left(\mathbf{J}_1 G_1\right)$, and ignoring the surface integrals at S_∞, we get

$$\begin{aligned} W_S^\rho(t) &= -\mu_0 \frac{\partial}{\partial t} \int_{V_s} \int_{V_s} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \times \\ &\nabla_1 \bullet \mathbf{J}\left(\mathbf{r}_1, t_1\right) \rho\left(\mathbf{r}_2, t_2\right) \int_{V_\infty} G_1 G_2 d\mathbf{r} dt_2 dt_1 d\mathbf{r}_2 d\mathbf{r}_1 \\ &= \frac{\mu_0 c^2}{8\pi} \frac{\partial}{\partial t} \int_{V_s} \int_{V_s} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{1}{r_{21}} \nabla_1 \bullet \mathbf{J}\left(\mathbf{r}_1, t_1\right) \rho\left(\mathbf{r}_2, t_2\right) dt_2 dt_1 d\mathbf{r}_2 d\mathbf{r}_1 \\ &= -\frac{\mu_0 c^2}{16\pi} \frac{\partial}{\partial t} \int_{V_s} \int_{V_s} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{1}{r_{21}} \dot{\rho}\left(\mathbf{r}_1, t_1\right) \rho\left(\mathbf{r}_2, t_2\right) dt_1 dt_2 d\mathbf{r}_2 d\mathbf{r}_1 \end{aligned} \tag{70}$$

Performing the double integration $\left(\iint dt_1 dt_2\right)$ on the region limited by (68), and dividing the inner integration into three sub-regions gives

$$\begin{aligned} W_S^\rho(t) &= -\frac{\mu_0 c^2}{16\pi} \frac{\partial}{\partial t} \int_{V_s} \int_{V_s} \frac{1}{r_{21}} \times \\ &\left\{ \begin{aligned} &\int_0^{r_{21}/c} \rho\left(\mathbf{r}_2, t_1 + \frac{r_{21}}{c}\right) \rho\left(\mathbf{r}_1, t_1\right) dt_1 \\ &+ \int_{r_{21}/c}^{t - r_{21}/c} \begin{bmatrix} \rho\left(\mathbf{r}_2, t_1 + \frac{r_{21}}{c}\right) \\ -\rho\left(\mathbf{r}_2, t_1 - \frac{r_{21}}{c}\right) \end{bmatrix} \rho\left(\mathbf{r}_1, t_1\right) dt_1 \\ &\int_{t - r_{21}/c}^{t} \begin{bmatrix} \rho\left(\mathbf{r}_2, 2t - t_1 - \frac{r_{21}}{c}\right) \\ -\rho\left(\mathbf{r}_2, t_1 - \frac{r_{21}}{c}\right) \end{bmatrix} \rho\left(\mathbf{r}_1, t_1\right) dt_1 \end{aligned} \right\} d\mathbf{r}_2 d\mathbf{r}_1 \\ &= -\frac{\mu_0 c^2}{8\pi} \int_{V_s} \int_{V_s} \int_{t - r_{21}/c}^{t} \begin{bmatrix} \frac{1}{r_{21}} \rho\left(\mathbf{r}_1, t_1\right) \times \\ \dot{\rho}\left(\mathbf{r}_2, 2t - t_1 - \frac{r_{21}}{c}\right) \end{bmatrix} dt_1 d\mathbf{r}_2 d\mathbf{r}_1 \end{aligned} \tag{71}$$

Replacing t_1 by τ and evaluating the second term in a similar way we can get (17). The nonzero range can be determined by noting that the sources exist within $[0, T]$ and at least one of the source terms is zero for $t > T + r_{21,\max}/2c$.

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