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A Coordinated Direct AF/DF Relay-Aided NOMA Framework for Low Outage

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Abstract—This paper investigates the performance of a new framework for low-outage downlink non-orthogonal multiple access (NOMA) using a coordinated direct and relay transmission (CDRT) scheme with direct links to both the near-user (NU) and the far-user (FU). Both AF and DF relays are considered. In this framework, the NU combines the signals from BS and R at each stage of the successive interference cancellation (SIC) to attain good outage performance. For both NU and FU, the expressions for outage probability and throughput are derived in closed form. We also derive the high-SNR expressions for the outage probability to demonstrate that with the proposed framework, both users harness a diversity of two without feedback bits (this is the only framework to achieve this). We demonstrate that the choice of power allocation coefficient and target symbol rates is crucial to maximizing the NU throughput while ensuring a desired target FU throughput. We demonstrate that CDRT with the proposed framework outperforms known schemes in terms of outage probability, sum throughput, and energy efficiency. Moreover, we also show that optimal rate selection is important to maximize the EE. Monte Carlo simulations validate the accuracy of the derived analytical expressions.

Index Terms—Non-orthogonal multiple access (NOMA), coordinated direct and relay transmission (CDRT), energy efficiency (EE), amplify-and-forward (AF), decode-and-forward (DF).

I. INTRODUCTION

The proliferation of Internet of Things (IoT) and massive machine-type communication has augmented the demand for higher data rates, low latency, and high bandwidth efficiency in beyond 5G (B5G) and 6G networks [1]–[4]. Multiple access techniques used in older generations of wireless communication networks are insufficient to fulfill these stringent requirements on user density and network traffic. To meet the requirements of future wireless networks, several multiple access technologies were studied in the last decade. Among them, non-orthogonal multiple access (NOMA) has emerged as one of the most promising technologies for future wireless networks due to its high spectral efficiency (SE), low latency, and ability to facilitate massive connectivity [5]–[7].

In contrast to traditional orthogonal multiple access (OMA) schemes, NOMA facilitates concurrent transmission

to multiple users over the same spectral/time/spreading code resource using power domain multiplexing at the transmitter side and successive interference cancellation (SIC) at the receiver side. In downlink NOMA, the user with a poor channel condition is termed as the far user (FU), and the user with a better channel condition is termed as the near user (NU) [8]. The quality of service (QoS) and the coverage area can be further enhanced by using relayed NOMA (R-NOMA) where dedicated relay nodes are used [9]–[15]. In [9], [10], the performance of amplify-and-forward (AF) and decode-and-forward (DF) relaying schemes was compared for different relay selection criteria in an R-NOMA scheme. A novel relay selection strategy was proposed to ensure minimum outage probability in [11]. In [12], the authors analyzed the performance of energy harvesting based R-NOMA assuming that the harvested energy is augmented by regular battery energy. In [13], the effect of imperfect SIC and channel state information (CSI) errors were considered, and performance was analyzed in terms of bit error rate (BER), outage probability, and ergodic capacity. In particular, the authors also demonstrated that with DF relaying, R-NOMA provides better user fairness than cooperative NOMA (C-NOMA). The outage probability of a multi-antenna relay-based hybrid satellite-terrestrial network was investigated in [14]. In [15], the ergodic sum-rate performance of the R-NOMA system was analyzed for both AF and DF relaying schemes considering Nakagami-m fading channels.

C-NOMA is another approach to enhance the coverage and QoS in which the NU assists the information transmission to the FU. In [16], considering the direct link to both NU and FU, the performance of the C-NOMA system was investigated for both half-duplex and full-duplex relaying. Further, in [17] the FU performance was improved by applying OMA/C-NOMA switching in a multiuser scenario, whereas OMA/NOMA/C-NOMA switching with user selection was studied in [18] to improve the QoS in an underlay cognitive radio (CR) NOMA network. Further, in [19], the block error performance for short packet communication in a C-NOMA network was compared to conventional NOMA assuming Rayleigh flat fading channels. To improve the SE, successive user relaying in C-NOMA was discussed in [20].

The use of NOMA in coordinated direct and relay transmission (CDRT) [21]–[27] is another promising approach to improve the QoS at FU. In a typical CDRT-NOMA

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A portion of this work limited only to AF-NOMA in the absence of BS-FU direct link has been presented in IEEE PIMRC 2021.

signalling scheme, the base station (BS) shares a direct link to the NU, while a dedicated relay assists communication to the FU. IoT-assisted CDRT-NOMA was analyzed in terms of outage probability and ergodic sum capacity (ESC) in [21]. In [22], the SE and user fairness index of CDRT-NOMA were analyzed considering perfect and imperfect SIC at NU. Coordinated uplink transmission for the C-NOMA system was studied in [23], and the outage probability was derived considering Nakagami-m fading channels. NOMA-based CDRT and hybrid multiple access (HMA) protocols were studied in [24], and it was shown that NOMA-based CDRT achieves better ESC than HMA but with higher complexity. In [25] a joint downlink-uplink adaptive CDRT based NOMA network was investigated for both perfect and imperfect SIC, and closed-form expressions were derived for the outage probability, ESC, and energy efficiency (EE). Performance of CDRT-NOMA with a direct link to both users was investigated in [26], [28]. In [26], the performance of a DF relay aided CDRT-NOMA was investigated in the presence of a direct link to both the users. In [27], the performance of CDRT-NOMA was investigated with multiple FUs in terms of ergodic capacity and outage probability. However, the absence of R-NU and BS-FU links limits the performance of both NU and FU¹. [28] investigated the performance of fixed-gain AF relay-based CDRT-NOMA in the presence of the BS-FU link assuming sub-optimal selection combining at the FU. A detailed comparison of our work with existing literature on C-NOMA, R-NOMA, and CDRT-NOMA is provided in Table I.

A. Motivation and Contribution

In CDRT-NOMA, signalling occurs in two phases. In the first phase, BS broadcasts the superimposed symbols to the NU and the relay node, and NU decodes its own symbols after decoding and removing the FU's symbols using SIC. In the second phase, BS transmits a new set of symbols to NU, and the relay node forwards the decoded symbols to FU, which causes interference at NU. However, based on the decoding status of the FU's symbols in the first phase, NU can perform the interference cancellation. Existing CDRT-NOMA systems have the following limitations:

- The performance of NU depends a lot on the decoding status of the symbols in the first phase. If the BS-NU link fails in the first phase, then NU will not be able to decode the desired symbols together with the FU's symbols, and thus, the interference in the second phase cannot be canceled. Therefore, the decoding status in the first phase is a bottleneck for the NU's performance.
- The relay node has to decode the FU's symbols successfully to assist the information transmission to FU

¹Unlike [27], in this work, we do not consider multiple FUs as this may result in increased complexity at the NU receiver due to requirement of multiple SIC with combining at each stage. Note that multiple SIC will lead to SIC failure in some realizations, in which case NU might fail to decode some FU symbols and hence might lose in performance due to participation in NOMA signalling, which is contrary to the objective of this paper.

in the second phase. If the BS-R link goes into a deep fade, the FU will be in outage.

It is evident from the above that NU loses on performance due to participation in NOMA signalling. Also, the absence of the BS-FU direct link limits performance.

Due to participation in NOMA signalling, NU needs to decode the FU symbol first to perform SIC (complexity of the NU receiver clearly increases) before it can decode its own symbol. Depending on the channel condition, target rate, and power allocation factor, NU might occasionally fail to decode its own symbol due to insufficient signal-to-noise ratio (SNR) or unsuccessful decoding of the FU symbol. In such a scenario, the BS has to re-transmit the superimposed symbol, which causes a reduction in SE and EE. In C-NOMA, NU incurs a loss in SE (due to allocation of power to the FU symbols and the use of two signalling phases) and EE as well (it is required to expend energy for relaying the FU symbol). To date, virtually all existing literature on CDRT-NOMA focuses on AF and DF relaying in the absence of the BS-FU direct link. The availability of the BS-FU direct link enhances the QoS at FU and lowers the required energy consumption at the relay. In [26], DF relay-aided CDRT-NOMA in the presence of the BS-FU link has been investigated for three different cases a) fixed relaying where the DF relay always forwards the incoming signal from BS, b) selective DF where the relay only forwards after successful decoding of the incoming signal and requires two-bit feedback, and c) incremental selective DF relaying where the relayed link is used opportunistically and requires three-bit feedback. In particular, for the lower analytical complexity, the power allocation at the relay is considered such that after linearly combining the first and second phase signals, the FU symbols are canceled at the NU, and both the users recover their desired symbols. This is sub-optimal, and results in a diversity loss for both the users.

All the aforementioned works have laid a solid foundation for the use of C-NOMA and CDRT to improve the FU's performance, but there has been little effort to improve the NU's performance. It might not be possible to satisfy NU's constraints using these CDRT approaches (they only guarantee a diversity of one at the NU [29]). However, considering an AF or DF relay to assist FU and NU with optimal combining at both users improves the NU diversity. It results in improved QoS for both users. The NU will combine with SIC (in a framework discussed in this paper) but attains much better performance, which is necessary for many scenarios like multimedia and gaming applications that impose a low outage and higher link reliability requirements. We also discuss the optimization of NU performance while ensuring the desired performance at FU. *The novel contributions of this paper are as follows:*

- We investigate the performance of a new framework for CDRT based downlink NOMA network with the direct link to both NU and FU. The NU combines the incoming signals from BS and the relay at each stage of the SIC to enhance its performance. This

TABLE I: Comparison of literature on C-NOMA, R-NOMA, CDRT-NOMA with our work. N_r and N_t represent the number of transmit and receive antennas, respectively, while m stands for the m^{th} user and μ denotes the shape parameter for Nakagami- μ distribution.

	[16]	[24]	[25]	[22]	[13]	[14]	[15]	[21]	[20]	[19]	[23]	[28]	[26]	[27]	Our Work
Relayed NOMA	×	×	×	×	✓	✓	✓	×	×	×	×	×	×	×	×
Cooperative NOMA	✓	×	×	×	×	×	×	×	✓	✓	×	×	×	×	×
CDRT	×	✓	✓	✓	×	×	×	✓	×	×	✓	✓	✓	✓	✓
AF relaying	×	×	×	×	×	×	✓	×	×	×	×	✓	×	×	✓
DF relaying	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	×	✓	✓	✓
Rayleigh fading	✓	×	✓	×	✓	✓	×	✓	✓	✓	×	×	✓	✓	✓
Nakagami-m fading	×	✓	×	✓	×	✓	✓	×	×	×	✓	✓	×	×	×
Direct link to FU	✓	×	×	×	×	×	×	×	×	✓	✓	✓	✓	×	✓
Direct link to NU	×	×	×	✓	×	✓	×	✓	✓	✓	✓	✓	✓	✓	✓
Optimal combining at FU	✓	×	×	✓	×	×	×	×	×	✓	×	×	×	×	✓
Optimal combining at NU	×	×	×	×	×	×	×	×	×	×	×	×	×	×	✓
Energy Efficiency	✓	×	✓	✓	×	×	×	×	×	×	×	×	×	×	✓
Outage and/or throughput	✓	×	✓	×	✓	✓	✓	✓	✓	×	✓	✓	✓	✓	✓
BER/SER/BLER/Capacity	×	✓	✓	✓	✓	×	✓	✓	×	✓	×	×	×	✓	×
Power allocation at Source	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
Power allocation at Relay	-	✓	-	✓	✓	×	✓	×	×	×	✓	×	✓	✓	✓
Independent α_B and α_R	-	✓	-	✓	✓	×	×	×	×	×	×	×	×	×	✓
$\alpha_B < (1 - \alpha_B)$ or $\alpha_R < (1 - \alpha_R)$	✓	✓	✓	✓	×	✓	✓	✓	×	✓	×	✓	×	✓	×
Optimization of α_B	×	×	✓	×	×	×	×	×	✓	×	×	×	×	×	✓
τ_N^{max} while ensuring fixed τ_F	×	×	×	×	×	×	×	×	×	×	×	×	×	×	✓
Reward NU for cooperation	×	×	×	×	×	×	×	×	×	×	×	×	×	×	✓
Diversity order at NU	0 or 1	-	0 or 1	-	-	2	-	-	-	1	0	$m\mu$	1 or 2	×	2
Diversity order at FU	0/1/2	-	0 or 1	-	-	$\max(N_r, N_t)$	-	-	-	2	0	$m\mu$	1 or 2	×	2
Feedback bits required	-	-	-	-	-	-	-	-	-	-	-	-	1 or 3	×	-

incentivizes its participation in NOMA signalling. The FU also uses a combination of direct and relayed signals. Considering both AF and DF relaying, we present closed-form expressions for the outage probability and the throughput of both NU and FU with this new framework. Unlike other works [30], [31] on AF CDRT NOMA where (for analytical simplicity) the harmonic to min approximation is used, we derive the exact outage probability in closed form. The case when the BS-FU link is absent follows as a special case.

- We also derive highly accurate high-SNR expressions for the outage probability to demonstrate that combining direct and relayed signals at NU as well as FU (while performing SIC at NU) allows both NU and FU to harness a diversity of 2 with both AF and DF relaying. It is emphasized that this is the only framework to achieve this without feedback bits or incremental signalling. Clearly, the NU is rewarded for participation in NOMA signalling. However, in the absence of the BS-FU link, FU attains a diversity of only one.
- We show that how the choice of power allocation coefficients and target symbol rates are crucial to maximizing the NU throughput while guaranteeing the desired throughput at FU. Due to the intelligent combining

in different stages of the SIC at the NU, the power allocated to the NU symbols can be decreased, and that allocated power to the FU symbols can be increased, which ensures desired QoS at a more distant FU. We also present an approximate closed-form expression for the power allocation coefficient that maximizes NU throughput for a given FU throughput requirement. In addition to this, using the derived expressions for the optimum power allocation coefficients, an NU throughput maximization problem is formulated to determine the optimal NU and FU target rate pair.

- Further, we demonstrate that the proposed framework with CDRT (without using any feedback) always outperforms SDF CDRT-NOMA [26] (which uses two feedback bits) in terms of outage probability, sum throughput, and energy efficiency (EE) by a huge margin and also achieves better diversity at both the users. The additional complexity required for combining signals at the NU is very small.
- Moreover, we compare the performance of the proposed CDRT-NOMA framework to its OMA counterpart (with relaying) and observe that the proposed framework ensures higher EE, which is an important outcome of this paper. We further observe that the availability of the BS-FU link helps achieve a higher EE compared to

the case when the BS-FU link is absent. Also, optimal rate selection is important to maximize the EE.

The rest of this paper is structured as follows. Section II elaborates on the system model for AF as well as the DF relay-assisted CDRT-NOMA framework. Section III analyzes the performance with the proposed framework in terms of outage probability. Section IV discusses NU throughput and EE maximization. Numerical results based on mathematical analysis are compared to computer simulations in Section V. Finally, Section VI concludes this paper.

Notations: $\mathcal{CN}(0, \sigma^2)$ represents a zero-mean complex Gaussian distribution with variance σ^2 . $f_X(x)$ and $U(\cdot)$ respectively denote the probability density function (PDF) of a random variable (RV) X and the unit step function. $\mathcal{E}_1(\cdot)$, $K_1(\cdot, \cdot)$ and $\Gamma(a, x; b)$ denote the exponential integral of type 1, the modified Bessel function of the second kind and the generalized incomplete gamma function, respectively.

II. SYSTEM MODEL

As depicted in Fig. 1, we consider a CDRT downlink NOMA framework consisting of a base station B, a NU U_N , a FU U_F , and a relay station R. Both U_N and U_F have stringent outage QoS constraints. All nodes operate in the half-duplex mode and are equipped with a single antenna. Communication to U_N and U_F takes place in two signalling phases. In the first phase, B communicates to U_N , R, and U_F over direct links, while in the second phase, communication from R to U_N and U_F take place using either AF or DF mode of relaying.

The channel coefficients $h_{ij} \sim \mathcal{CN}(0, 1/\lambda_{ij})$ with $i \in \{B, R\}$, $j \in \{R, N, F\}$ are assumed to be independent and of quasi-static Rayleigh fading type, where $\lambda_{ij} = d_{ij}^m$, and m is the path-loss exponent. The additive zero-mean complex Gaussian noise at all the receiving nodes is assumed to be of variance σ^2 . Superscript "I" and "II" are used to represent first and second phase quantities.

A. AF Relaying

In the first phase, B transmits a superposition of unit-energy symbols s_N and s_F (of information rates R_N and R_F) intended respectively for U_N and U_F . The transmit power P_B is apportioned to U_N and U_F symbols in the ratio $\alpha_B : (1 - \alpha_B)$. Thus, the superimposed symbol can be expressed as $s_B = \sqrt{P_B \alpha_B} s_N + \sqrt{P_B (1 - \alpha_B)} s_F$ [32]. The sampled matched filter outputs at U_N , R and U_F are

$$y_N^I = s_B h_{BN} + w_N^I, \quad (1)$$

$$y_R^I = s_B h_{BR} + w_R^I, \quad (2)$$

$$y_F^I = s_B h_{BF} + w_F^I, \quad (3)$$

respectively, wherein w_N^I , w_R^I and w_F^I are the respective additive Gaussian noise samples. In accordance with the concept of NOMA, U_N first decodes the FU symbol s_F , and then performs SIC to decode the NU symbol s_N [33].

Using (1), the SINR Γ_{NF}^I to decode s_F , and the SNR Γ_N^I to decode s_N at U_N after SIC, are expressed as

$$\Gamma_{NF}^I = \frac{(1 - \alpha_B) \rho_B |h_{BN}|^2}{\alpha_B \rho_B |h_{BN}|^2 + 1} \quad (4)$$

$$\Gamma_N^I = \alpha_B \rho_B |h_{BN}|^2, \quad \text{provided } \Gamma_{NF}^I \geq \gamma_F, \quad (5)$$

where $\rho_B = P_B/\sigma^2$ represents the transmit SNR at B and $\gamma_F = 2^{R_F} - 1$ represents the threshold SNR at U_F . Using (3), the SINR Γ_{FF}^I to decode s_F at U_F can be expressed as

$$\Gamma_{FF}^I = \frac{(1 - \alpha_B) \rho_B |h_{BF}|^2}{\alpha_B \rho_B |h_{BF}|^2 + 1}. \quad (6)$$

Let P_R denote the available transmit power at R. In the second phase, R amplifies the incoming signal from the first phase and then forwards $s_R = \beta y_R^I$ to U_N and U_F as shown in Fig.1 (b), where $\beta = \sqrt{\frac{P_R}{\rho_B |h_{BR}|^2 + 1}}$ with $\rho_R = \frac{P_R}{\sigma^2}$. The signals received at U_N and U_F in the second phase are given by

$$y_N^{II} = \beta y_R^I h_{RN} + w_N^{II}, \quad (7)$$

$$y_F^{II} = \beta y_R^I h_{RF} + w_F^{II}, \quad (8)$$

respectively, where w_N^{II} and w_F^{II} are the additive noise samples at U_N and U_F . Similar to the first phase, using (7) the SINRs Γ_{NF}^{II} and Γ_{NN}^{II} to decode s_F and s_N (after SIC) are expressed as

$$\begin{aligned} \Gamma_{NF}^{II} &= \frac{\rho_B \beta^2 (1 - \alpha_B) |h_{BR}|^2 |h_{RN}|^2}{\rho_B \beta^2 \alpha_B |h_{BR}|^2 |h_{RN}|^2 + |h_{RN}|^2 \beta^2 + 1} \\ &= \frac{\rho_B \rho_R (1 - \alpha_B) |h_{BR}|^2 |h_{RN}|^2}{\rho_B \rho_R \alpha_B |h_{BR}|^2 |h_{RN}|^2 + \rho_R |h_{RN}|^2 + \rho_B |h_{BR}|^2 + 1}, \end{aligned} \quad (9)$$

$$\begin{aligned} \Gamma_{NN}^{II} &= \frac{\rho_B \beta^2 \alpha_B |h_{BR}|^2 |h_{RN}|^2}{\beta^2 |h_{RN}|^2 + 1} \\ &= \frac{\rho_B \rho_R \alpha_B |h_{BR}|^2 |h_{RN}|^2}{\rho_R |h_{RN}|^2 + \rho_B |h_{BR}|^2 + 1}, \quad \text{provided } \Gamma_{NF}^{II} > \gamma_F. \end{aligned} \quad (10)$$

Using (8), the SINR to decode s_F at U_F is given by

$$\begin{aligned} \Gamma_{FF}^{II} &= \frac{\rho_B \beta^2 (1 - \alpha_B) |h_{BR}|^2 |h_{RF}|^2}{\rho_B \beta^2 \alpha_B |h_{BR}|^2 |h_{RF}|^2 + |h_{RF}|^2 \beta^2 + 1} \\ &= \frac{\rho_B \rho_R (1 - \alpha_B) |h_{BR}|^2 |h_{RF}|^2}{\rho_B \rho_R \alpha_B |h_{BR}|^2 |h_{RF}|^2 + \rho_R |h_{RF}|^2 + \rho_B |h_{BR}|^2 + 1}. \end{aligned} \quad (11)$$

The signals from the first and second phases are combined at U_F . This paper uses an approach that deviates from all existing works to date and enables U_N to combine the signals in the two phases at each stage of the SIC. We show that this enables U_N to attain very good performance and harness a diversity of two without any feedback. This allows the network to be used for multimedia and other applications that impose strict outage QoS constraints. It is emphasized that none of the techniques suggested so far [26], [29] can ensure a diversity of two at the NU in the absence of any feedback². This improvement in diversity and

²Since the suggested combining based SIC at U_N improves performance when the direct links to U_N fails, the performance advantage is expected to be retained in the case when feedback is available.

Fig. 1: (a) System Model. (b) An illustration of the proposed CDRT transmission process, where $s_B = \sqrt{P_B \alpha_B} s_N + \sqrt{P_B (1 - \alpha_B)} s_F$. Further, in case of AF relaying $s_R = \beta (h_{BR} (\sqrt{P_B \alpha_B} s_N + \sqrt{P_B (1 - \alpha_B)} s_F) + w_R^I)$, and in case of DF relaying $s_R = \sqrt{P_R \alpha_R} s_N + \sqrt{P_R (1 - \alpha_R)} s_F$ if $\Gamma_{RF}^I \geq \gamma_F, \Gamma_{RN}^I \geq \gamma_N$ and $s_R = \sqrt{P_R} s_F$ if $\Gamma_{RF}^I > \gamma_F, \Gamma_{RN}^I < \gamma_N$.

throughput performance also serves as an incentive for U_N to participate in NOMA signalling. Clearly, U_N cannot attain this performance using traditional OMA. In this respect, this work is quite different from all existing works.

U_F combines y_F^I and y_F^{II} to decode s_F with SINR $\Gamma_{FF}^{COM} = \Gamma_{FF}^I + \Gamma_{FF}^{II}$. If s_F is decoded successfully in the first phase, U_N cancels the interference from y_N^{II} and then combines the first and second phase signals to decode s_N with SNR $\Gamma_{NN}^{COM} = \Gamma_{NN}^I + \Gamma_{NN}^{II}$. However, when U_N fails to decode s_F , it combines y_N^I and y_N^{II} to first decode s_F with SINR $\Gamma_{NF}^{COM} = \Gamma_{NF}^I + \Gamma_{NF}^{II}$, cancels interference, and then decodes s_N with SNR $\Gamma_{NN}^{COM} = \Gamma_{NN}^I + \Gamma_{NN}^{II}$.

B. DF relaying

As in AF relaying, B transmits the superposed signal s_B in the first phase. Therefore, the SINRs Γ_{NF}^I , Γ_{NN}^I and Γ_F^I remains same. Different from AF relaying, the DF relay decodes s_F and s_N by implementing SIC. The SINRs Γ_{RF}^I and Γ_{RN}^I (after SIC) to decode s_F and s_N at R can be expressed as

$$\Gamma_{RF}^I = \frac{(1 - \alpha_B) \rho_B |h_{BR}|^2}{\alpha_B \rho_B |h_{BR}|^2 + 1}, \quad (12)$$

$$\Gamma_{RN}^I = \alpha_B \rho_B |h_{BR}|^2 \quad \text{provided } \Gamma_{RF}^I \geq \gamma_F. \quad (13)$$

In the second phase, based on the decoding status of s_N and s_F , R transmits either superimposed signal s_R or only the FU symbol s_F as shown in Fig.1 (b). If R decodes both s_N and s_F successfully³, then it forwards the superimposed signal $s_R = \sqrt{P_R \alpha_R} s_N + \sqrt{P_R (1 - \alpha_R)} s_F$ to both U_N and U_F, where α_R and $(1 - \alpha_R)$ denotes the portion of power allocated to U_N and U_F, respectively. Note that DF relaying (unlike its AF counter part) allows different NOMA power allocations at B and R. The signals received

³In CDRT-NOMA with DF relaying, if R successfully decodes s_N and s_F , the power allocation takes place at both B and R, thus, DF provides more flexibility and a performance improvement can be expected in comparison to AF relaying.

at U_N and U_F can be expressed as $y_{N-S}^{II} = s_R h_{RN} + w_N^{II}$ and $y_{F-S}^{II} = s_R h_{RF} + w_F^{II}$, where $-S$ in the subscript is used to emphasize that a superposed signal is transmitted by R. Using y_{N-S}^{II} and y_{F-S}^{II} , the respective SINRs before and after SIC (Γ_{NF-S}^{II} and Γ_{NN-S}^{II}) to decode s_F and s_N at U_N are

$$\Gamma_{NF-S}^{II} = \frac{(1 - \alpha_R) \rho_R |h_{RN}|^2}{\alpha_R \rho_R |h_{RN}|^2 + 1}, \quad (14)$$

$$\Gamma_{NN-S}^{II} = \alpha_R \rho_R |h_{RN}|^2 \quad \text{provided } \Gamma_{NF-S}^{II} \geq \gamma_F. \quad (15)$$

Using y_{F-S}^{II} , the SINR Γ_{FF-S}^{II} at F to decode s_F is

$$\Gamma_{FF-S}^{II} = \frac{(1 - \alpha_R) \rho_R |h_{RF}|^2}{\alpha_R \rho_R |h_{RF}|^2 + 1}. \quad (16)$$

However, if R fails to decode s_N then it forwards only s_F . Now, the received signals at U_N and U_F are expressed as $y_{N-F}^{II} = \sqrt{P_R} s_F h_{RN} + w_N^{II}$ and $y_{F-F}^{II} = \sqrt{P_R} s_F h_{RF} + w_F^{II}$ respectively, where the subscript $-F$ is used to emphasize that R merely transmits s_F . Using y_{N-F}^{II} and y_{F-F}^{II} , the respective SINRs Γ_{NF-F}^{II} and Γ_{FF-F}^{II} to decode s_F at U_N and U_F can be expressed as

$$\Gamma_{NF-F}^{II} = \rho_R |h_{RN}|^2, \quad (17)$$

$$\Gamma_{FF-F}^{II} = \rho_R |h_{RF}|^2. \quad (18)$$

U_F combines y_F^I with y_{F-S}^{II} when s_N and s_F are decoded at R, or with y_{F-F}^{II} otherwise. The combined SINRs are $\Gamma_{FF-S}^{COM} = \Gamma_{FF}^I + \Gamma_{FF-S}^{II}$ or $\Gamma_{FF-F}^{COM} = \Gamma_{FF}^I + \Gamma_{FF-F}^{II}$ respectively. However, if s_F is decoded successfully in the first phase, U_N cancels interference from y_{N-S}^{II} or y_{N-F}^{II} and then combines the first and second phase signals to decode s_N with SNR $\Gamma_{NN-S}^{COM} = \Gamma_{NN}^I + \Gamma_{NN-S}^{II}$ or Γ_{NN}^I . When U_N fails to decode s_F , it combines y_N^I and y_{N-S}^{II} or y_{N-F}^{II} to first decode s_F with SINR $\Gamma_{NF-S}^{COM} = \Gamma_{NF}^I + \Gamma_{NF-S}^{II}$ or $\Gamma_{NF-F}^{COM} = \Gamma_{NF}^I + \Gamma_{NF-F}^{II}$. Subsequently, after cancelling the interference due to s_F , decoding of s_N takes place with SNR $\Gamma_{NN-S}^{COM} = \Gamma_{NN}^I + \Gamma_{NN-S}^{II}$ or Γ_{NN}^I . In this paper, we compare the performance of the

proposed framework with a relayed OMA scheme described below.

C. Relayed OMA Scheme

In the considered relayed OMA system, B transmits x_F to U_F , U_N , and R in the first signalling phase. In the second signalling phase, R forwards either the amplified (with AF relaying) or the decoded (with DF relaying) FU symbol, while B transmits x_N to NU. Using the decoded FU symbol in the first phase, NU cancels the interference caused by transmission by R before decoding. However, if NU fails to decode x_F in the first phase, then the signal received from R in the second phase acts as interference at U_N and x_N transmitted from B acts as interference at FU. Note that NU receives symbols in both time-slots in this OMA scheme with relaying. It can therefore be expected to attain high throughput. We observe in Section V that the proposed framework yields much better throughput performance than its relayed OMA counterpart.

III. OUTAGE PROBABILITY ANALYSIS

In this section, the outage performance of coordinated direct AF/DF relay assisted downlink NOMA is analyzed. We derive closed-form expressions of NU and FU outage probabilities considering AF or DF relaying. In order to determine the diversity order for U_N and U_F , the asymptotic expressions of user outage probabilities are also derived. For ease of exposition, we define $|h_{BR}|^2 = X$ ($\lambda_{BR} = \lambda_x$), $|h_{RF}|^2 = Y$ ($\lambda_{RF} = \lambda_y$), $|h_{BN}|^2 = Z$ ($\lambda_{BN} = \lambda_z$), $|h_{BF}|^2 = W$ ($\lambda_{BF} = \lambda_w$), and $|h_{RN}|^2 = V$ ($\lambda_{RN} = \lambda_v$). Note that the RVs X, Y, Z, W, V are independent and exponentially distributed with PDF $f_\Omega(\omega) = \lambda_\omega e^{-\lambda_\omega \omega}$ with $\Omega \in \{X, Y, Z, W, V\}$, $\omega \in \{x, y, z, w, v\}$.

A. AF Relaying

1) *FU outage probability*: The outage probability of U_F can be written as $p_F^{AF} = \Pr\{\Gamma_F^{COM} < \gamma_F\}$, where $\Gamma_F^{COM} = \Gamma_F^I + \Gamma_F^U$. Using (6) and (11), p_F^{AF} can be expressed as

$$\begin{aligned} p_F^{AF} &= \Pr\{\Gamma_{FF}^{COM} < \gamma_F\} \\ &= \Pr\left\{\frac{(1-\alpha_B)\rho_B W}{\alpha_B \rho_B W + 1} + \frac{\rho_B \rho_R (1-\alpha_B)XY}{\rho_B \rho_R \alpha_B XY + \rho_R Y + \rho_B X + 1} < \gamma_F\right\} \\ &= \Pr\left\{\rho_R Y [\rho_B X (\phi + \alpha_B \rho_B W (1 - \alpha_B + \phi)) \right. \\ &\quad \left. - (\gamma_F - \rho_B W \phi)] < (\gamma_F - \rho_B W \phi)(1 + \rho_B X)\right\}, \end{aligned} \quad (19)$$

wherein $\phi = 1 - \alpha_B(1 + \gamma_F)$. The sign of $(\gamma_F - \rho_B W \phi)$ depends on the range of W and different values of ϕ . Depending on $(\gamma_F - \rho_B W \phi) \geq 0$, the conditions for outage on ranges of X, Y and W with $\phi > 0$ ($\alpha_B < \frac{1}{1+\gamma_F}$), $\alpha_B - 1 < \phi < 0$ ($\frac{1}{1+\gamma_F} \leq \alpha_B < \frac{2}{2+\gamma_F}$) and $\phi < \alpha_B - 1$ ($\alpha_B \geq \frac{2}{2+\gamma_F}$) are listed in Table II, wherein

$$\begin{aligned} \zeta_{F_1}(x, w) &= \frac{(\gamma_F - \rho_B W \phi)(1 + \rho_B X)}{\rho_R(\rho_B X (\phi + \alpha_B \rho_B W (1 - \alpha_B + \phi)) - (\gamma_F - \rho_B W \phi))}, \\ \zeta_{F_1}(w) &= \frac{(\gamma_F - \rho_B W \phi)}{\rho_B(\phi + \alpha_B \rho_B W (1 - \alpha_B + \phi))}, \quad \chi_F = \frac{\gamma_F}{\phi \rho_B} \text{ and} \end{aligned}$$

TABLE II: U_F outage - ranges of Y, X and W for different values of α_B

$\alpha_B < \frac{1}{1+\gamma_F}$	$W < \chi_F$	$X < \zeta_{F_1}(W), Y > 0$
		$X \geq \zeta_{F_1}(W), Y < \zeta_{F_1}(X, W)$
$\frac{1}{1+\gamma_F} \leq \alpha_B < \frac{2}{2+\gamma_F}$	$W < \xi_1$	$X > 0, Y > 0$
	$W \geq \xi_1$	$X < \zeta_{F_1}(W), Y > 0$ $X \geq \zeta_{F_1}(W), Y < \zeta_{F_1}(X, W)$
$\alpha_B \geq \frac{2}{2+\gamma_F}$	Entire range of W, X and Y	

$\xi_1 = \frac{-\phi}{\alpha_B \rho_B (1 - \alpha_B + \phi)}$. Now, p_F^{AF} can be expressed as in (20). After some mathematical rearrangements (20) can be expressed as (21). We first solve for I_1 as follows:

$$I_1 = \int_0^{\chi_F} \int_{\zeta_{F_1}(w)}^{\infty} \exp\left(-\frac{\lambda_y \zeta_{F_1}(w)(1 + \rho_B x)}{\rho_R(x - \zeta_{F_1}(w))}\right) \lambda_x e^{-\lambda_x x} dx \lambda_w e^{-\lambda_w w} dw. \quad (22)$$

Substituting $x = x - \zeta_{F_1}(w)$ in the above and re-arranging terms, we get

$$\begin{aligned} I_1 &= \int_0^{\chi_F} \exp\left(-\zeta_{F_1}(w)\left(\lambda_x + \frac{\lambda_y \rho_B}{\rho_R}\right)\right) \int_0^{\infty} \lambda_x \\ &\quad \times \exp\left(-\lambda_x x - \frac{\lambda_y \zeta_{F_1}(w)[1 + \rho_B \zeta_{F_1}(w)]}{x \rho_R}\right) \lambda_w e^{-\lambda_w w} dx dw. \end{aligned} \quad (23)$$

Solving for the inner integral using [34, 3.324.1], we obtain

$$\begin{aligned} I_1 &= \int_0^{\chi_F} \exp\left(-\zeta_{F_1}(w)\left(\lambda_x + \frac{\lambda_y \rho_B}{\rho_R}\right)\right) 2 \sqrt{\mu_{F_1}(w)} \\ &\quad \times K_1\left(2 \sqrt{\mu_{F_1}(w)}\right) \lambda_w e^{-\lambda_w w} dw, \end{aligned} \quad (24)$$

where $\mu_{F_1}(w) = \frac{\lambda_x \lambda_y \zeta_{F_1}(w)[\rho_B \zeta_{F_1}(w) + 1]}{\rho_R}$. Following a similar approach, we derive an expression for I_2 . p_F^{AF} can then be expressed as

$$p_F^{AF} = \begin{cases} 1 - e^{-\lambda_w \chi_F} - \int_0^{\chi_F} \exp\left(-\zeta_{F_1}(w)\left(\lambda_x + \frac{\lambda_y \rho_B}{\rho_R}\right)\right) 2 \sqrt{\mu_{F_1}(w)} \\ \quad \times K_1\left(2 \sqrt{\mu_{F_1}(w)}\right) \lambda_w e^{-\lambda_w w} dw & \text{if } \alpha_B < \frac{1}{1+\gamma_F} \\ 1 - \int_{\xi_1}^{\infty} \exp\left(-\zeta_{F_1}(w)\left(\lambda_x + \frac{\lambda_y \rho_B}{\rho_R}\right)\right) 2 \sqrt{\mu_{F_1}(w)} \\ \quad \times K_1\left(2 \sqrt{\mu_{F_1}(w)}\right) \lambda_w e^{-\lambda_w w} dw & \text{if } \frac{1}{1+\gamma_F} \leq \alpha_B < \frac{2}{2+\gamma_F} \\ 1, & \text{otherwise.} \end{cases} \quad (25)$$

Unfortunately, the integrals in (25) do not admit a closed-form. We, therefore, present a highly accurate approximate expression for p_F^{AF} in the following Theorem.

Theorem 1. An accurate closed-form expression for FU

$$p_F^{AF} = \begin{cases} \int_0^{\chi_F} \int_{\zeta_{F_1}(w)}^{\infty} [1 - e^{-\lambda_y \zeta_{F_1}(x,w)}] \lambda_x e^{-\lambda_x x} \lambda_w e^{-\lambda_w w} dx dw + \int_0^{\chi_F} [1 - e^{-\lambda_x \zeta_{F_1}(w)}] \lambda_w e^{-\lambda_w w} dw & \text{if } \alpha_B < \frac{1}{1+\gamma_F} \\ 1 - e^{-\lambda_w \xi_1} + \int_{\xi_1}^{\infty} \int_{\zeta_{F_1}(w)}^{\infty} [1 - e^{-\lambda_y \zeta_{F_1}(x,w)}] \lambda_x \lambda_w e^{-\lambda_x x - \lambda_w w} dx dw + \int_{\xi_1}^{\infty} [1 - e^{-\lambda_x \zeta_{F_1}(w)}] \lambda_w e^{-\lambda_w w} dw & \text{if } \frac{1}{1+\gamma_F} \leq \alpha_B < \frac{2}{2+\gamma_F} \\ 1 & \text{otherwise,} \end{cases} \quad (20)$$

$$p_F^{AF} = \begin{cases} 1 - e^{-\lambda_w \chi_F} - \underbrace{\int_0^{\chi_F} \int_{\zeta_{F_1}(w)}^{\infty} \exp\left(-\frac{\lambda_y \zeta_{F_1}(w)(1+\rho_B x)}{\rho_R(x-\zeta_{F_1}(w))}\right) \lambda_x e^{-\lambda_x x} dx \lambda_w e^{-\lambda_w w} dw}_{I_1} & \text{if } \alpha_B < \frac{1}{1+\gamma_F} \\ 1 - \underbrace{\int_{\xi_1}^{\infty} \int_{\zeta_{F_1}(w)}^{\infty} \exp\left(-\frac{\lambda_y \zeta_{F_1}(w)(1+\rho_B x)}{\rho_R(x-\zeta_{F_1}(w))}\right) \lambda_x e^{-\lambda_x x} \lambda_w e^{-\lambda_w w} dx dw}_{I_2} & \text{if } \frac{1}{1+\gamma_F} \leq \alpha_B < \frac{2}{2+\gamma_F} \\ 1 & \text{otherwise.} \end{cases} \quad (21)$$

outage probability with AF relaying is given by

$$p_F^{AF} = \begin{cases} 1 - e^{-\lambda_w \chi_F} - e^{-\xi_1(\lambda_w + \frac{1}{\rho_R})} \left[\Gamma(1, -\lambda_w \xi_1; \vartheta_6) \right. \\ \left. - \Gamma(1, \lambda_w(\chi_F - \xi_1); \vartheta_6) \right] & \text{if } \alpha_B < \frac{1}{1+\gamma_F} \\ 1 - e^{-\xi_1(\lambda_w + \frac{1}{\rho_R})} \sqrt{4\vartheta_6} K_1(\sqrt{4\vartheta_6}) & \text{if } \frac{1}{1+\gamma_F} \leq \alpha_B < \frac{2}{2+\gamma_F} \\ 1 & \text{otherwise.} \end{cases} \quad (26)$$

Proof. Refer to Appendix A. ■

Using the expression obtained in (26), it is extremely difficult to establish the diversity order. We, therefore, derive a high-SNR approximation to p_F^{AF} in the following Lemma.

Lemma 1. A high-SNR approximation to p_F^{AF} is given by

$$p_F^{AF} \simeq \begin{cases} \left(\lambda_x + \frac{\lambda_y \rho_B}{\rho_R} \right) \xi_1 \left[1 - e^{-\lambda_w \chi_F} + (\xi_1 - \chi_F) \lambda_w e^{-\lambda_w \xi_1} \right. \\ \left. - [E_1(-\lambda_w \xi_1) - E_1(-\lambda_w \xi_1 + \lambda_w \chi_F)] \right] & \text{if } \alpha_B < \frac{1}{1+\gamma_F} \\ 1 - \frac{e^{-\lambda_w \xi_1}}{1 + \xi_1(\lambda_x + \lambda_y)} & \text{if } \frac{1}{1+\gamma_F} \leq \alpha_B < \frac{2}{2+\gamma_F} \\ 1 & \text{otherwise.} \end{cases} \quad (27)$$

Proof. Refer to Appendix B. ■

Remark 1. In the absence of direct link to U_F , the outage probability of U_F can be readily derived by substituting $W = 0$ into (19). After using a procedure similar to that used to derive (25), we obtain an exact closed-form expression for

p_F^{AF} as follows:

$$p_F^{AF} = 1 - \exp\left(-\frac{\chi_F}{\rho_R}(\lambda_x \rho_R + \lambda_y \rho_B)\right) \times 2\sqrt{\mu_F} K_1(2\sqrt{\mu_F}), \quad \text{if } \alpha_B < \frac{1}{1+\gamma_F} \quad (28)$$

where $\mu_F = \frac{\lambda_x \lambda_y \chi_F (1 + \rho_B \chi_F)}{\rho_R}$. The above equality holds only for $\alpha_B < \frac{1}{1+\gamma_F}$, otherwise $p_F^{AF} = 1$. Note that for higher values of ρ_B and ρ_R , μ_F and χ_F are very small. Therefore, applying $K_1(\theta) \simeq \frac{1}{2}\Gamma(1)(\frac{\theta}{2})^{-1}$ [35, 9.6.9] in (28), we obtain

$$p_F^{AF} \simeq 1 - \exp\left(-\chi_F \left[\lambda_x + \frac{\lambda_y \rho_B}{\rho_R}\right]\right). \quad (29)$$

Using $e^{-\theta} \simeq 1 - \theta$ in the above, a high-SNR approximation to p_F^{AF} is given by

$$p_F^{AF} \simeq \frac{\gamma_F [\rho_R \lambda_x + \rho_B \lambda_y]}{\rho_R \rho_B \phi}. \quad (30)$$

2) **NU Outage Probability:** Let $\Gamma_{NF}^{COM} = \Gamma_{NF}^I + \Gamma_{NF}^{II}$ and $\Gamma_{NN}^{COM} = \Gamma_{NN}^I + \Gamma_{NN}^{II}$. Due to the manner in which SIC is performed with combining, the outage probability of U_N is

$$p_N^{AF} = 1 - \Pr\left\{\Gamma_{NF}^{COM} \geq \gamma_F, \Gamma_{NN}^{COM} \geq \gamma_N\right\}. \quad (31)$$

Theorem 2. The NU outage probability can be written as in (32), where $\chi_N = \frac{\gamma_N}{\alpha_B \rho_B}$, $\chi_{N_1}(Z) = \frac{\gamma_N - \alpha_B Z \rho_B}{\rho_B \alpha_B}$, $\zeta_{N_1}(X, Z) = \frac{\chi_{N_1}(Z)(\rho_B X + 1)}{\rho_R(X - \chi_{N_1}(Z))}$, $\chi_{N_2}(Z) = \frac{\gamma_F - \phi Z \rho_B}{\rho_B [\phi + \alpha_B Z \rho_B (1 - \alpha_B + \phi)]}$ and $\zeta_{N_2}(X, Z) = \frac{\chi_{N_2}(Z)(\rho_B X + 1)}{\rho_R(X - \chi_{N_2}(Z))}$.

$$p_N^{AF} = \begin{cases} \underbrace{1 - \Pr\{Z > \max(\chi_F, \chi_N)\}}_{I_3} - \underbrace{\Pr\{V > \zeta_{N_1}(X, Z), X > \chi_{N_1}(Z), \chi_N > Z > \chi_F\}}_{I_4} \\ - \underbrace{\Pr\{V > \zeta_{N_1}(X, Z), X > \chi_{N_1}(Z), \min(\chi_F, \chi_N) > Z\}}_{I_5} & \text{if } \alpha_B < \frac{1}{2 + \gamma_F} \\ \\ \underbrace{1 - \Pr\{Z > \max(\chi_F, \chi_N)\}}_{I_3} - \underbrace{\Pr\{V > \zeta_{N_1}(X, Z), X > \chi_{N_1}(Z), \min(\chi_F, \chi_N) > Z\}}_{I_5} \\ - \underbrace{\Pr\{V > \zeta_{N_2}(X, Z), X > \chi_{N_2}(Z), \chi_F > Z > \chi_N\}}_{I_6} & \text{if } \frac{1}{2 + \gamma_F} \leq \alpha_B < \frac{1}{1 + \gamma_F} \\ \\ \underbrace{1 - \Pr\{V > \zeta_{N_2}(X, Z), X > \chi_{N_2}(Z), Z > \xi_1\}}_{I_7} & \text{if } \frac{1}{1 + \gamma_F} \leq \alpha_B < \frac{2}{2 + \gamma_F} \\ \\ 1, & \text{otherwise,} \end{cases} \quad (32)$$

Proof. Using (4), (5), (9) and (10) into (31), we obtain

$$\begin{aligned} p_N^{AF} &= 1 - \Pr \left\{ \frac{(1 - \alpha_B)\rho_B Z}{\alpha_B \rho_B Z + 1} + \frac{\rho_B \rho_R (1 - \alpha_B) X V}{\rho_B \rho_R \alpha_B X V + \rho_R V + \rho_B X + 1} \right. \\ &\quad \left. \geq \gamma_F, \alpha_B \rho_B Z + \frac{\rho_B \rho_R \alpha_B X V}{\rho_R V + \rho_B X + 1} \geq \gamma_N \right\} \\ &= 1 - \Pr \{ [V \rho_R (X \rho_B (\phi + \alpha_B \rho_B Z (1 - \alpha_B + \phi)) - (\gamma_F \\ &\quad - \phi \rho_B Z)) \geq (\gamma_F - \phi \rho_B Z)(1 + \rho_B X)], [V \rho_R (X \rho_B \alpha_B \\ &\quad - (\gamma_N - \alpha_B \rho_B Z)) \geq (\gamma_N - \alpha_B \rho_B Z)(1 + \rho_B X)] \}. \quad (33) \end{aligned}$$

In the above, depending on values of α_B and γ_F , two ranges of α_B exist i.e. $\alpha_B < \frac{1}{1 + \gamma_F}$ or $\alpha_B \geq \frac{1}{1 + \gamma_F}$. Note that the signs of $(\gamma_F - \phi \rho_B Z)$ and $(\gamma_N - \alpha_B \rho_B Z)$ depend on the range of Z . Based on the ranges of RVs V , X and Z , the conditions for non-outage probability of U_N for different values of α_B are listed in Table III. Now, p_N^{AF} can be expressed as

$$\begin{aligned} p_N^{AF} &= 1 - \Pr\{V > \zeta_{N_2}(X, Z), X > \chi_{N_2}(Z), Z > \xi_1\} U(-\phi) \\ &\quad - \left[\Pr\{Z > \max(\chi_N, \chi_F)\} + \Pr\{V > \zeta_{N_1}(X, Z), X > \chi_{N_1}(Z), \right. \\ &\quad \chi_N > Z > \chi_F\} U(\chi_N - \chi_F) + \Pr\{V > \zeta_{N_2}(X, Z), X > \chi_{N_2}(Z), \\ &\quad \chi_F > Z > \chi_N\} U(\chi_F - \chi_N) + \Pr\{V > \zeta_{N_1}(X, Z), X > \chi_{N_1}(Z), \\ &\quad \left. Z < \min(\chi_N, \chi_F)\} \right] U(\phi), \quad (34) \end{aligned}$$

where $\chi_{N_1}(Z)$, $\chi_{N_2}(Z)$, $\zeta_{N_1}(X, Z)$, $\zeta_{N_2}(X, Z)$, χ_F , χ_N and ϕ are defined in the line following (32). After performing mathematical rearrangements in (34) we obtain (32). ■

Theorem 3. An accurate closed-form expression for p_N^{AF} is given by (35), where $\vartheta_2 = \xi_1 \left(\xi_1 + \frac{\gamma_F}{\rho_B \phi} \right) C_\ell$, $C = C_\ell - \lambda_z$ and $C_\ell = \lambda_x + \frac{\lambda_z \rho_B}{\rho_R}$.

Proof. Refer to Appendix C. ■

Lemma 2. The high-SNR expression for p_N^{AF} is given by

Proof. Refer to Appendix C. ■

Lemma 3. The diversity orders D_F^{AF} and D_N^{AF} achieved at U_F and U_N are given by

$$\begin{aligned} D_F^{AF} &= \begin{cases} 2, & \text{if } \alpha_B < \frac{1}{1 + \gamma_F} \\ 1, & \text{if } \frac{1}{1 + \gamma_F} \leq \alpha_B < \frac{2}{2 + \gamma_F} \\ 0, & \text{otherwise.} \end{cases} \\ D_N^{AF} &= \begin{cases} 2, & \text{if } \alpha_B < \frac{1}{1 + \gamma_F} \\ 1, & \text{if } \frac{1}{1 + \gamma_F} \leq \alpha_B < \frac{2}{2 + \gamma_F} \\ 0, & \text{otherwise.} \end{cases} \quad (37) \end{aligned}$$

Proof. To derive the diversity order for U_F and U_N , we use the high-SNR expression of p_F^{AF} and p_N^{AF} given by (27) and (35), respectively. We consider $\rho_B = \rho_R = \rho$ without loss of generality. The diversity orders attained by U_F and U_N are given by $D_j^{AF} = -\lim_{\rho \rightarrow \infty} \frac{\log_2 p_j^{AF}(\rho)}{\log_2 \rho}$, $j \in \{N, F\}$. For $\rho \rightarrow \infty$, we use $e^{-\theta} = 1 - \theta$ and substitute for χ_F and ξ_1 . After neglecting higher order terms of $\frac{1}{\rho}$, we obtain diversity order as expressed in (37). The detailed proof is omitted due to paucity of space. ■

Remark 2. The NU also attains a diversity of two is significant and motivates it to participate in NOMA signalling.

Remark 3. In the absence of direct link to U_F , the diversity order D_F^{AF} is obtained by using $\rho_B = \rho_R = \rho$ into (30). The diversity order D_F^{AF} can be expressed as

$$D_F^{AF} = \begin{cases} 1, & \text{if } \alpha_B < \frac{1}{1 + \gamma_F} \\ 0, & \text{otherwise.} \end{cases} \quad (38)$$

However, the diversity order for U_N remains the same as expressed in (37).

TABLE III: U_N non-outage - ranges of Z , X and V for different values of α_B

$\alpha_B < \frac{1}{1+\gamma_F}$	$Z > \max(\chi_F, \chi_N)$	$V > 0, X > 0$
	$\chi_N > Z > \chi_F$	$V > \zeta_{N_1}(X, Z), X > \chi_{N_1}(Z)$
	$\chi_F > Z > \chi_N$	$V > \zeta_{N_2}(X, Z), X > \chi_{N_2}(Z)$
	$\min(\chi_F, \chi_N) > Z$	$V > \max(\zeta_{N_1}(X, Z), \zeta_{N_2}(X, Z)), X > \max(\chi_{N_1}(Z), \chi_{N_2}(Z))$ if $\chi_{N_1}(Z) > \chi_{N_2}(Z) \Rightarrow \zeta_{N_1}(X, Z) > \zeta_{N_2}(X, Z) \Rightarrow Z < \chi_N$ if $\chi_{N_1}(Z) > \chi_{N_2}(Z) \Rightarrow \zeta_{N_1}(X, Z) < \zeta_{N_2}(X, Z)$ not valid
$\frac{1}{1+\gamma_F} \leq \alpha_B < \frac{2}{2+\gamma_F}$	$Z > \xi_1$	$V > \zeta_{N_2}(X, Z), X > \chi_{N_2}(Z)$
$\alpha_B \geq \frac{2}{2+\gamma_F}$	Outage always occurs	

$$p_N^{AF} \simeq \begin{cases} 1 - e^{-\lambda_z \max(\chi_N, \chi_F)} - \lambda_z e^{-\chi_N C_\ell} \left[\frac{1 - e^{C(\chi_F - \chi_N)} + e^{C(\min(\chi_F, \chi_N) - \chi_N)} - e^{-C\chi_N}}{C} \right] & \text{if } \alpha_B < \frac{1}{2+\gamma_F} \\ 1 - e^{-\lambda_z \max(\chi_N, \chi_F)} - \lambda_z e^{-\chi_N C_\ell} \left[\frac{e^{C(\min(\chi_F, \chi_N) - \chi_N)} - e^{-C\chi_N}}{C} \right] - e^{\xi_1(C_\ell + \lambda_z)} \left[e^{-(\xi_1 + \chi_N)} - e^{-(\xi_1 + \chi_F)} \right] & \text{if } \frac{1}{2+\gamma_F} \leq \alpha_B < \frac{1}{1+\gamma_F} \\ -\lambda_z \vartheta_2 e^{\lambda_z \vartheta_2} \left[\mathcal{E}_1(\lambda_z(\chi_N + \xi_1 + \vartheta_2)) - \mathcal{E}_1(\lambda_z(\chi_F + \xi_1 + \vartheta_2)) \right] & \text{if } \frac{1}{1+\gamma_F} \leq \alpha_B < \frac{2}{2+\gamma_F} \\ 1 - e^{\xi_1(C_\ell + \lambda_z)} \left[e^{-2\xi_1} - \lambda_z \vartheta_2 e^{\lambda_z \vartheta_2} \left[\mathcal{E}_1(\lambda_z(2\xi_1 + \vartheta_2)) \right] \right] & \text{if } \frac{1}{1+\gamma_F} \leq \alpha_B < \frac{2}{2+\gamma_F} \\ 1, & \text{otherwise,} \end{cases} \quad (35)$$

$$p_N^{AF} \approx \begin{cases} 1 - e^{-\lambda_z \max(\chi_N, \chi_F)} - \frac{\lambda_z e^{-\chi_N C_\ell} \left[1 - e^{-C\chi_N} (1 + e^{C\chi_F} - e^{C\min(\chi_F, \chi_N)}) \right]}{C} & \text{if } \alpha_B < \frac{1}{2+\gamma_F} \\ 1 - e^{-\lambda_z \max(\chi_N, \chi_F)} - \frac{\lambda_z e^{-2\chi_N C_\ell} (e^{C\min(\chi_F, \chi_N)} - 1)}{C} + \frac{\lambda_z \vartheta_2 e^{-\lambda_z \xi_1} (e^{-\lambda_z \chi_N} - e^{-\lambda_z \chi_F})}{1 + \lambda_z(\chi_N + \xi_1 + \vartheta_2)} & \text{if } \frac{1}{2+\gamma_F} \leq \alpha_B < \frac{1}{1+\gamma_F} \\ -e^{\xi_1(C_\ell + \lambda_z - 1)} (e^{-\chi_N} - e^{-\chi_F}) & \text{if } \frac{1}{2+\gamma_F} \leq \alpha_B < \frac{1}{1+\gamma_F} \\ 1 - e^{\xi_1(\lambda_z - C_\ell)} + \frac{\lambda_z \vartheta_2 e^{\xi_1(C_\ell - \lambda_z)}}{1 + \lambda_z(\vartheta_2 + 2\xi_1)} & \text{if } \frac{1}{1+\gamma_F} \leq \alpha_B < \frac{2}{2+\gamma_F} \\ 1, & \text{otherwise,} \end{cases} \quad (36)$$

B. DF Relaying

1) *FU outage probability*: Let $\Gamma_{FF-S}^{COM} = \Gamma_{FF}^I + \Gamma_{FF-S}^{II}$ and $\Gamma_{FF-F}^{COM} = \Gamma_{FF}^I + \Gamma_{FF-F}^{II}$. FU is not in outage when a) both s_F and s_N are decoded at R ($\Gamma_{RF}^I \geq \gamma_F$, $\Gamma_{RN}^I \geq \gamma_N$) and FU SNR after combining is sufficient ($\Gamma_{FF-S}^{COM} \geq \gamma_F$), b) s_F can be decoded at R, but not s_N but SNR after combining is sufficient at FU ($\Gamma_{FF-F}^{COM} \geq \gamma_F$), and c) s_F cannot be decoded at R but $\Gamma_{FF}^I \geq \gamma_F$. The outage probability for U_F can be formulated as

$$p_F^{DF} = 1 - \Pr \left\{ \underbrace{\Gamma_{RF}^I \geq \gamma_F, \Gamma_{RN}^I \geq \gamma_N, \Gamma_{FF-S}^{COM} \geq \gamma_F}_{A_1} \right. \\ \left. - \Pr \left\{ \underbrace{\Gamma_{RF}^I \geq \gamma_F, \Gamma_{RN}^I < \gamma_N, \Gamma_{FF-F}^{COM} \geq \gamma_F}_{A_2} \right\} \right. \\ \left. - \Pr \left\{ \underbrace{\Gamma_{RF}^I < \gamma_F, \Gamma_{FF}^I \geq \gamma_F}_{A_3} \right\} \right\}, \quad (39)$$

In the following, we present closed-form expressions for FU outage.

Theorem 4. The exact closed-form expression of the outage probability for U_F is given by where $\phi_R = 1 - \alpha_R(1 + \gamma_F)$, $\vartheta_1 = \rho_B(\alpha_B(1 - \alpha_R) + \alpha_R\phi)$, $\varsigma_1 = \frac{\lambda_w \phi_R}{\vartheta_1}$ and $\varsigma_2 = \chi_F + \frac{\phi_R}{\vartheta_1}$

Proof. Refer to Appendix D. ■

The expressions obtained in (40) involve the generalized incomplete gamma function due to which the diversity cannot readily be established from them. We, therefore, present a highly accurate high-SNR approximation to p_N^{AF} in the following Lemma.

Lemma 4. A high-SNR approximation to p_F^{DF} is given by (41).

Proof. Refer to Appendix E. ■

Remark 4. In the absence of the direct link to U_F , we can use $W = 0$ in (39) to get

$$p_F^{DF} = \begin{cases} 1 - e^{-\lambda_w \chi_F} (1 - e^{-\lambda_x \chi_F}) - \left[e^{-\lambda_w \chi_F} + \left\{ \Gamma \left(1, \max(\varsigma_1, 0); \frac{-\lambda_y \phi \rho_B \varsigma_2}{\rho_R \vartheta_1 w} \right) - \Gamma \left(1, \lambda_w \varsigma_2; \frac{-\lambda_y \phi \rho_B \varsigma_2}{\rho_R \vartheta_1 w} \right) \right\} \exp \left(\varsigma_1 + \frac{\lambda_y \phi \rho_B}{\rho_R \vartheta_1} \right) \right] \\ \times e^{-\lambda_x \max(\chi_F, \chi_N)} - (e^{-\lambda_x \chi_F} - e^{-\lambda_x \chi_N}) \left[\exp \left(-\frac{\lambda_y \gamma_F}{\rho_R} \right) + \exp \left(\frac{\lambda_w}{\alpha_B \rho_B} + \frac{\lambda_y \phi}{\alpha_B \rho_R} \right) \left\{ \Gamma \left(1, \frac{\lambda_y \phi}{\alpha_B \rho_R}; \frac{\lambda_y \lambda_w (\phi + \alpha_B \gamma_F)}{\alpha_B^2 \rho_B \rho_R} \right) \right. \right. \\ \left. \left. - \Gamma \left(1, \frac{\lambda_y}{\rho_R} (\gamma_F + \frac{\phi}{\alpha_B}); \frac{\lambda_y \lambda_w (\phi + \alpha_B \gamma_F)}{\alpha_B^2 \rho_B \rho_R} \right) \right\} \right] & \text{if } \alpha_B < \frac{\gamma_N}{\gamma_N + \gamma_F + \gamma_F \gamma_N} \\ 1 - e^{-\lambda_w \chi_F} (1 - e^{-\lambda_x \chi_F}) - \left[e^{-\lambda_w \chi_F} + \left\{ \Gamma \left(1, \max(\varsigma_1, 0); \frac{-\lambda_y \phi \rho_B \varsigma_2}{\rho_R \vartheta_1 w} \right) - \Gamma \left(1, \lambda_w \varsigma_2; \frac{-\lambda_y \phi \rho_B \varsigma_2}{\rho_R \vartheta_1 w} \right) \right\} \right] \\ \times \exp \left(\varsigma_1 + \frac{\lambda_y \phi \rho_B}{\rho_R \vartheta_1} \right) e^{-\lambda_x \max(\chi_F, \chi_N)} & \text{if } \frac{\gamma_N}{\gamma_N + \gamma_F + \gamma_F \gamma_N} \leq \alpha_B < \frac{1}{1 + \gamma_F} \\ 1 & \text{otherwise,} \end{cases} \quad (40)$$

$$p_F^{DF} \simeq \begin{cases} 1 + e^{-\lambda_w \chi_F} (1 - e^{-\lambda_x \chi_F}) - e^{-\lambda_x \chi_N} \left[\exp \left(\varsigma_1 + \frac{\lambda_y \phi \rho_B}{\rho_R \vartheta_1} - \max(\varsigma_1, 0) \right) + e^{-\lambda_w \chi_F} \left(1 - \exp \left(\frac{\lambda_y \phi \rho_B}{\rho_R \vartheta_1} \right) \right) \right] \\ - (e^{-\lambda_x \chi_F} - e^{-\lambda_x \chi_N}) \left[\exp \left(\frac{\lambda_w}{\alpha_B \rho_B} \right) + \exp \left(-\frac{\lambda_y \gamma_F}{\rho_R} \right) \left(1 - \exp \left(\frac{\lambda_w}{\alpha_B \rho_B} \right) \right) \right] & \text{if } \alpha_B < \frac{\gamma_N}{\gamma_N + \gamma_F + \gamma_F \gamma_N} \\ 1 + e^{-\lambda_w \chi_F} (1 - e^{-\lambda_x \chi_F}) - e^{-\lambda_x \chi_F} \left[\exp \left(\varsigma_1 + \frac{\lambda_y \phi \rho_B}{\rho_R \vartheta_1} - \max(\varsigma_1, 0) \right) \right. \\ \left. + e^{-\lambda_w \chi_F} \left(1 - \exp \left(\frac{\lambda_y \phi \rho_B}{\rho_R \vartheta_1} \right) \right) \right] & \text{if } \frac{\gamma_N}{\gamma_N + \gamma_F + \gamma_F \gamma_N} \leq \alpha_B < \frac{1}{1 + \gamma_F} \\ 1 & \text{otherwise.} \end{cases} \quad (41)$$

$$p_F^{DF} = 1 - \Pr \{ X > \max(\chi_N, \chi_F), Y \phi_R \rho_R > \gamma_F \} \\ - \Pr \{ \chi_F < X < \chi_N, Y \rho_R > \gamma_F \}. \quad (42)$$

After solving (42) as in Theorem 4, an exact expression for p_F^{DF} is given by (43). Using $e^{-\theta} \simeq 1 - \theta$ in above and neglecting higher order terms of $\frac{1}{\rho_R}$, a high-SNR approximation to p_F^{DF} is given by

$$p_F^{DF} \simeq \begin{cases} \frac{\lambda_y \gamma_F}{\rho_R \phi_R} + \lambda_x \chi_F & \text{if } \alpha_B < \frac{1}{1 + \gamma_F} \\ 1 & \text{otherwise.} \end{cases} \quad (44)$$

2) *NU outage probability*: NU is not in outage when a) R decodes both s_F and s_N ($\Gamma_{RF}^I \geq \gamma_F$, $\Gamma_{RN}^I \geq \gamma_N$) and SNRs at NU (after combining) are sufficient to decode both s_F and s_N ($\Gamma_{NF-S}^{COM} \geq \gamma_F$, $\Gamma_{NN-S}^{COM} \geq \gamma_N$), b) R decodes s_F but fails to decode s_N ($\Gamma_{RF}^I \geq \gamma_F$, $\Gamma_{RN}^I < \gamma_N$), and SNRs to decode s_F (after combining) and s_N (from first phase) are sufficient at NU ($\Gamma_{NF-F}^{COM} \geq \gamma_F$, $\Gamma_{NN}^I \geq \gamma_N$), and c) R fails to decode s_F ($\Gamma_{RF}^I < \gamma_F$), but the first phase SNRs are sufficient to decode both s_F and s_N at NU ($\Gamma_{NF}^I \geq \gamma_F$, $\Gamma_{NN}^I \geq \gamma_N$). The outage probability for U_N can be formulated as

$$p_N^{DF} = 1 - \Pr \left\{ \underbrace{\Gamma_{RF}^I < \gamma_F, \Gamma_{NF}^I \geq \gamma_F, \Gamma_{NN}^I \geq \gamma_N}_{B_1} \right\} \\ - \Pr \left\{ \underbrace{\Gamma_{RF}^I \geq \gamma_F, \Gamma_{RN}^I < \gamma_N, \Gamma_{NF-F}^{COM} \geq \gamma_F, \Gamma_{NN}^I \geq \gamma_N}_{B_2} \right\} \\ - \Pr \left\{ \underbrace{\Gamma_{RF}^I \geq \gamma_F, \Gamma_{RN}^I \geq \gamma_N, \Gamma_{NF-S}^{COM} \geq \gamma_F, \Gamma_{NN-S}^{COM} \geq \gamma_N}_{B_3} \right\}, \quad (45)$$

where $\Gamma_{NF-F}^{COM} = \Gamma_{NF}^I + \Gamma_{NF-F}^{II}$, $\Gamma_{NF-S}^{COM} = \Gamma_{NF}^I + \Gamma_{NF-S}^{II}$ and $\Gamma_{NN-S}^{COM} = \Gamma_{NN}^I + \Gamma_{NN-S}^{II}$.

Theorem 5. The exact closed-form expression of the outage probability for U_N is given by (46), where $\vartheta_4 = \frac{\phi \alpha_R + \gamma_N \phi \alpha_R + \gamma_N \alpha_B (1 - \alpha_R) - \alpha_B \phi_R}{\rho_B \alpha_B (\phi \alpha_R + \alpha_B (1 - \alpha_R))}$, $\vartheta_5 = \frac{-\phi_R}{\vartheta_1}$, $\vartheta_1 = \rho_B (\phi \alpha_R + \alpha_B (1 - \alpha_R))$, $\tilde{\chi}_F = \frac{\lambda_x \lambda_y \phi \rho_B (\chi_F + \frac{\phi_R}{\vartheta_1})}{\vartheta_1}$ and $\tilde{\chi}_N = \min(\chi_F, \chi_N, \vartheta_4)$.

Proof. Refer to Appendix F. ■

In the following Lemma, we present a high-SNR approximate expression for p_N^{DF} to obtain the diversity order at NU.

Lemma 5. A high-SNR approximation to p_N^{DF} is given by (47), where $\epsilon_3 = \frac{\lambda_y}{\alpha_R \rho_R}$.

Proof. The proof is similar to that used to derive (41) and is therefore omitted. ■

Lemma 6. The diversity orders D_F^{DF} and D_N^{DF} attained by U_F and U_N can be expressed as

$$D_F^{DF} = \begin{cases} 2 & \text{if } \alpha_B < \frac{1}{1 + \gamma_F}, \alpha_R < \frac{1}{1 + \gamma_F} \\ 1 & \text{if } \alpha_B < \frac{1}{1 + \gamma_F}, \alpha_R \geq \frac{1}{1 + \gamma_F} \\ 0 & \text{otherwise.} \end{cases} \\ D_N^{DF} = \begin{cases} 2 & \text{if } \alpha_B < \frac{1}{1 + \gamma_F}, \alpha_R < \frac{1}{1 + \gamma_F} \\ 1 & \text{if } \alpha_B < \frac{1}{1 + \gamma_F}, \alpha_R \geq \frac{1}{1 + \gamma_F} \\ 0 & \text{otherwise.} \end{cases} \quad (48)$$

Proof. The diversity orders for U_F and U_N are obtained by using high-SNR expressions of p_F^{DF} and p_N^{DF} given by (41)

$$p_F^{DF} = \begin{cases} 1 - \exp\left(-\frac{\lambda_y \gamma_F}{\rho_R \phi_R} - \lambda_x \chi_N\right) - \exp\left(-\frac{\lambda_y \gamma_F}{\rho_R}\right) \left[e^{-\lambda_x \chi_F} - e^{-\lambda_x \chi_N}\right] & \text{if } \alpha_B < \frac{\gamma_N}{\gamma_N + \gamma_F + \gamma_F \gamma_N} \\ 1 - \exp\left(-\frac{\lambda_y \gamma_F}{\rho_R \phi_R} - \lambda_x \chi_F\right) & \text{if } \frac{\gamma_N}{\gamma_N + \gamma_F + \gamma_F \gamma_N} \leq \alpha_B < \frac{1}{1 + \gamma_F} \\ 1 & \text{otherwise.} \end{cases} \quad (43)$$

$$p_N^{DF} = \begin{cases} 1 - e^{-\lambda_z \chi_N} (1 - e^{-\lambda_x \chi_N}) - e^{-\lambda_x \chi_N} \left[e^{-\lambda_z \chi_N} - \frac{\lambda_z \alpha_R \rho_R e^{-\frac{\lambda_y \gamma_N}{\alpha_R \rho_R}}}{\lambda_y \alpha_B \rho_B - \lambda_z \alpha_R \rho_R} \left[\exp\left(\frac{\max(0, \vartheta_5) \lambda_y \alpha_B \rho_B}{\alpha_R \rho_R} - \lambda_z \max(0, \vartheta_5)\right) \right. \right. \\ \left. \left. - \exp\left(\frac{\lambda_y \gamma_N}{\alpha_R \rho_R} - \lambda_z \chi_N\right) + \exp\left(\frac{\lambda_y \alpha_B \gamma_F}{\alpha_R \rho_R \phi} - \lambda_z \chi_F\right) + \exp\left(\frac{\tilde{\chi}_N \lambda_y \alpha_B \rho_B}{\alpha_R \rho_R} - \tilde{\chi}_N \lambda_z\right) \right] + \exp\left(\frac{\phi_R \lambda_z \rho_R + \lambda_y \phi \rho_B}{\rho_R \vartheta_1}\right) \right. \\ \left. \times \left[\Gamma\left(1, \lambda_z \max(0, \chi_N - \vartheta_5); \tilde{\chi}_F\right) + \Gamma\left(1, \lambda_z \max(0, \vartheta_4 - \vartheta_5); \tilde{\chi}_F\right) - \Gamma\left(1, \lambda_z \chi_F - \vartheta_5 \lambda_z; \tilde{\chi}_F\right) \right. \right. \\ \left. \left. - \Gamma\left(1, \lambda_z (\chi_F - \vartheta_5); \tilde{\chi}_F\right) \right] \right] & \text{if } \alpha_B < \frac{\gamma_N}{\gamma_N + \gamma_F + \gamma_F \gamma_N} \\ 1 - e^{-\lambda_z \chi_F} (1 - e^{-\lambda_x \chi_F}) - e^{-\lambda_x \chi_F} \left[e^{-\lambda_z \chi_F} - \frac{\lambda_z \alpha_R \rho_R e^{-\frac{\lambda_y \gamma_N}{\alpha_R \rho_R}}}{\lambda_y \alpha_B \rho_B - \lambda_z \alpha_R \rho_R} \left[\exp\left(\frac{\max(0, \vartheta_5) \lambda_y \alpha_B \rho_B}{\alpha_R \rho_R} - \lambda_z \max(0, \vartheta_5)\right) \right. \right. \\ \left. \left. - \exp\left(\frac{\tilde{\chi}_N \lambda_y \alpha_B \rho_B}{\alpha_R \rho_R} - \tilde{\chi}_N \lambda_z\right) \right] + \exp\left(\frac{\phi_R \lambda_z \rho_R + \lambda_y \phi \rho_B}{\rho_R \vartheta_1}\right) \left[\Gamma\left(1, \lambda_z \max(0, \vartheta_4 - \vartheta_5); \tilde{\chi}_F\right) - \Gamma\left(1, \lambda_z (\chi_F - \vartheta_5); \tilde{\chi}_F\right) \right. \right. \\ \left. \left. + \Gamma\left(1, \lambda_z \max(0, \chi_N - \vartheta_5); \tilde{\chi}_F\right) - \Gamma\left(1, \lambda_z \chi_N - \vartheta_5 \lambda_z; \tilde{\chi}_F\right) \right] \right] & \text{if } \frac{\gamma_N}{\gamma_N + \gamma_F + \gamma_F \gamma_N} \leq \alpha_B < \frac{1}{1 + \gamma_F} \\ 1 & \text{otherwise,} \end{cases} \quad (46)$$

$$p_N^{DF} \simeq \begin{cases} 1 - e^{-\lambda_z \chi_N} (1 - e^{-\lambda_x \chi_N}) - e^{-\lambda_x \chi_N} \left[e^{-\lambda_z \chi_N} + \frac{\lambda_z e^{-\varepsilon_3 \gamma_N}}{\varepsilon_3 \alpha_B \rho_B - \lambda_z} \left[\exp(\varepsilon_3 \gamma_N - \lambda_z \chi_N) - \exp\left(\frac{\varepsilon_3 \alpha_B \gamma_F}{\phi} - \lambda_z \chi_F\right) \right. \right. \\ \left. \left. + \exp(\min(\chi_F, \vartheta_4) \varepsilon_3 \alpha_B \rho_B - \lambda_z \min(\chi_F, \vartheta_4)) + \max(0, \vartheta_5) \varepsilon_3 \alpha_B \rho_B - \lambda_z \max(0, \vartheta_5) \right] \right] & \text{if } \alpha_B < \frac{\gamma_N}{\gamma_N + \gamma_F + \gamma_F \gamma_N} \\ 1 - e^{-\lambda_z \chi_F} (1 - e^{-\lambda_x \chi_F}) - e^{-\lambda_x \chi_F} \left[e^{-\lambda_z \chi_F} - \frac{\lambda_z e^{-\varepsilon_3 \gamma_N}}{\varepsilon_3 \alpha_B \rho_B - \lambda_z} \left[\exp(\max(0, \vartheta_5) \varepsilon_3 \alpha_B \rho_B - \max(0, \vartheta_5) \lambda_z) \right. \right. \\ \left. \left. - \exp(\min(\chi_N, \vartheta_4) \varepsilon_3 \alpha_B \rho_B - \lambda_z \min(\chi_N, \vartheta_4)) \right] \right] & \text{if } \frac{\gamma_N}{\gamma_N + \gamma_F + \gamma_F \gamma_N} \leq \alpha_B < \frac{1}{1 + \gamma_F} \\ 1 & \text{otherwise.} \end{cases} \quad (47)$$

and (47), respectively. We substitute $\rho_B = \rho_R = \rho$ and solve for $D_j^{DF} = -\lim_{\rho \rightarrow \infty} \frac{\log_2 p_j^{DF}(\rho)}{\log_2 \rho}$, $j \in \{N, F\}$. Details are omitted due to paucity of space. ■

Remark 5. In the absence of direct link to U_F , the diversity order D_F^{DF} is obtained by using $\rho_B = \rho_R = \rho \rightarrow \infty$ into (43) and using linear approximations to the exponential terms. The diversity order D_F^{DF} can be expressed as

$$D_F^{DF} = \begin{cases} 1 & \text{if } \alpha_B < \frac{1}{1 + \gamma_F}, \alpha_R < \frac{1}{1 + \gamma_F} \\ 0 & \text{otherwise.} \end{cases} \quad (49)$$

However, the diversity order for U_N remains the same as in (48).

IV. NU THROUGHPUT AND EE MAXIMIZATION

NU needs to decode the FU symbol, perform SIC to decode its own symbol, and might therefore incur some perfor-

mance loss. For this reason, maximizing the NU throughput while guaranteeing the desired FU target throughput $\tilde{\tau}$ is well motivated. We do so in this section. The FU and NU throughput in bits per channel use (bpcu) is defined as

$$\tau_j^i = \frac{1}{2} R_j (1 - p_j^i), \quad j \in \{N, F\}, i \in \{AF, DF\} \quad (50)$$

where p_F^{AF} , p_N^{AF} , p_F^{DF} and p_N^{DF} are as in (26), (35), (40) and (46) respectively.

Careful choice of power allocation coefficient α_B and symbol rates R_F and R_N is crucial to attaining good throughput at U_N and U_F . With DF relays, α_R can also be chosen. However, it will be demonstrated in Section V that a large range of α_R values result in the same throughput performance. The reason for this is that α_R is only used when both s_F and s_N are decoded correctly at R (which in turn depends on α_B). We, therefore, focus only on the choice of α_B , R_N , and R_F .

We first consider the case when the B- U_F link is absent.

Noting that the NU benefits when the maximum value of α_B is chosen, we derive α_B^{opt} by using (30) and (44) into (50) to get

$$\alpha_B^{AF^{opt}} \simeq \frac{1}{1 + \gamma_F} - \frac{\gamma_F R_F (\lambda_x \rho_R + \lambda_y \rho_B)}{\rho_B \rho_R (1 + \gamma_F) (R_F - 2\tau_F^i)}, \quad (51)$$

$$\alpha_B^{DF^{opt}} \simeq \frac{\gamma_F \lambda_y \rho_B - (\alpha_R + \alpha_R \gamma_F - 1) (\gamma_F \lambda_x + (\frac{2\tilde{\tau}}{R_F} - 1) \rho_B) \rho_R}{\rho_B (1 + \gamma_F) (\gamma_F \lambda_y - \rho_R (\frac{2\tilde{\tau}}{R_F} - 1) (\alpha_R + \alpha_R \gamma_F - 1))}. \quad (52)$$

The NU throughput for AF and DF relaying schemes can be obtained by substituting $\alpha_B = \alpha_B^{opt}$ into expressions for τ_N^i . Clearly, τ_N^i is now a function only of symbol rates R_N and R_F (and therefore γ_N and γ_F). Thus, careful choice of symbol rates is essential for maximizing the NU throughput. Let R_N^{opt} and R_F^{opt} denote the optimal symbol rates that maximize τ_N^i . Now, the optimization problem is formulated as

$$R_N^{opt}, R_F^{opt} = \arg \max_{R_N, R_F} \tau_N^i(R_N, R_F) \quad \text{such that } \alpha = \alpha_B^{opt}. \quad (53)$$

Solving the above joint optimization problem analytically is difficult. However, numerical techniques can be used to pick the optimal R_N and R_F .

In the presence of the B- U_F link, a three-dimensional search is required to determine α_B , R_N and R_F while ensuring that $\tau_F^i \geq \tilde{\tau}$.

EE is an important performance metric for any communication system. It is defined as [16]:

$$\eta_E^i = \frac{\text{Sum throughput}}{\text{Total energy consumed}}, \quad i \in \{AF, DF\}, \quad (54)$$

where 'sum throughput' represents the sum of τ_N^i and τ_F^i . Let T denote the signalling duration. Let $E_B = \frac{P_B T}{2}$ and $E_R = \frac{P_R T}{2}$. Using τ_N^i and τ_F^i from (50), the EE of the system is expressed as

$$\eta_E^i = \frac{\tau_{sum}^i}{E_B + E_R}, \quad \text{where} \quad \tau_{sum}^i = \tau_F^i + \tau_N^i. \quad (55)$$

Since the sum throughput τ_{sum}^i is function of symbol rates R_N and R_F , careful choice of symbol rates is essential for maximizing the EE. Let R_N^{opt} and R_F^{opt} denote the optimal symbol rates that maximize η_E^i . The EE optimization problem can be formulated as

$$R_N^{opt}, R_F^{opt} = \arg \max_{R_N, R_F} \eta_E^i(R_N, R_F) \quad (56)$$

Obtaining a closed-form solution to the above joint optimization problem is extremely difficult. However, a two-dimensional search can be used to find the optimal R_N and R_F .

V. SIMULATIONS AND NUMERICAL RESULTS

In this section we present computer simulations to confirm accuracy of the derived analytical expressions and derive useful insights. Unless mentioned otherwise, the considered system parameters are as follows: $d_{BR} = d_{RF} = d_{RN} = 2$,

$d_{BN} = 1.2$, $d_{BF} = 4$, and $m = 4$. Also σ^2 is normalized to unity [17]. Further, we assume that $P_B = P_R = P$ ($\rho_B = \rho_R = \rho$) [16] and symbol rates $R_N = R_F = R$ ($\gamma_N = \gamma_F = \gamma_T$). We also compare the performance of the proposed framework with its relayed OMA counterpart described in Section II.

In Fig.2 and Fig.3, we plot p_N^{AF} and p_F^{AF} versus transmit SNR (ρ) for different values of α_B and R . The accuracy of derived analytical expressions in (26), (28) and (35) can be clearly confirmed by simulations. Also the derived high-SNR approximations in (30) and (27) are quite accurate. With decreasing α_B , more power is allocated to U_F , and p_F^{AF} therefore continuously improves as shown in Fig.2. It is evident from Fig.3 that an initial increase in α_B significantly improves p_N^{AF} due to successful decoding of x_F and x_N at U_N , while further increase in α_B deteriorates p_N^{AF} as U_N fails to decode x_F . From Fig.2 and Fig.3 it is clear that (unlike in all CDRT schemes proposed so far) combining of incoming signals from B and R at U_N as well as U_F in the proposed CDRT framework ensures a full diversity of 2 to be harnessed (as derived in Lemma 3 for the case of AF relaying) by both users. However, in the absence of the $B-U_F$ link, a diversity order of 1 is attained at U_F (as noted in Remark 3). The switching of diversity order from 2 to 1 and 0 can also be observed when changing α_B from 0.1, 0.3, and 0.4 for $R = 2$. Therefore, a careful choice of power allocation coefficient is important to attain full diversity for both users.

Fig.4 and Fig.5 depict the variation of p_N^{DF} and p_F^{DF} versus ρ for different values of α_B , α_R and R . Simulations confirm the accuracy of the analytical expressions derived in (40), (43) and (46) respectively. Also, the approximate high-SNR expressions for outage probability derived in (44) and (41) are quite accurate. A decrease in α_B and α_R signifies that more power is allocated to U_F , and p_F^{DF} therefore continuously improves with decreasing α_B and α_R as shown in Fig.4. Clearly, from Fig.5, p_N^{DF} significantly improves with increasing α_B initially (due to successful decoding of x_F and x_N at U_N), while further increase in α_B degrades p_N^{DF} (U_N fails to decode x_F). Similar to the case of AF relaying, full diversity order of two can be attained (derived in Lemma 6) for both U_N and U_F in the presence of the direct link. While in the absence of a direct link to U_F , a diversity of order 1 can be achieved (as noted in Remark 5), while a diversity of 2 is achieved by U_N .

Fig.6 shows the variation of τ_F^i versus ρ at $R = 1, 2$. Here, we compare both DF and AF relaying schemes considering optimal power allocation coefficients α_B and α_R . In the presence of the $B-U_F$ link, performance with DF is superior to that of AF for lower SNRs. At high-SNRs, the performance of both AF and DF is comparable. However, in the absence of the $B-U_F$ link, performance with DF is always superior. AF relays are often preferred due to their simplicity and can be seen to result in good performance except at low SNRs. Fig.7 depicts τ_N^i versus ρ for $R = 1, 2$. For a fair comparison of AF and DF relaying, optimal power allocation coefficients are used at B and R. We observe

Fig. 2: p_F^{AF} vs. ρ . Fig. 3: p_N^{AF} vs. ρ . Fig. 4: p_F^{DF} vs. ρ . Fig. 5: p_N^{DF} vs. ρ . Fig. 6: τ_F^{AF} or τ_F^{DF} vs. ρ . Fig. 7: τ_N^{AF} or τ_N^{DF} vs. ρ .

that both AF and DF achieve almost similar performance in terms of NU throughput⁴.

Fig.8 depicts the variation of τ_N^{AF} or τ_F^{AF} versus α_B for $R = 2, 3$. When α_B is very small, more power is allocated to U_F and τ_F^{AF} therefore is maximum at $\alpha_B = 0$. With increase in α_B , τ_F^{AF} gradually decreases and becomes 0 for $\alpha_B \geq \frac{2}{2+\gamma_F}$ due to unsuccessful decoding of s_F at U_F . However, in the absence of the $B-U_F$ link, τ_F^{AF} becomes zero for $\alpha_B \geq \frac{1}{1+\gamma_F}$. Therefore, the presence of $B-U_F$ link significantly improves the FU performance and allows a wider range of valid power allocations. On the other hand τ_N^{AF} increases initially and attains a maximum value (at $\alpha_B = \frac{1}{1+\gamma_F}$). Further increase in α_B decreases the probability of successful decoding of s_F at U_N , and τ_N^{AF} therefore also decreases.

In Fig.9, τ_N^{DF} or τ_F^{DF} is plotted versus α_B (considering optimal α_R) and α_R (considering optimal α_B) for $R = 1.5, 2.5$. As with AF relaying, τ_F^{DF} is maximum at $\alpha_B = 0$ and then decreases to 0 with increasing α_B . τ_N^{DF} increases initially, attains a maximum value at $\alpha_B = \frac{\gamma_N}{\gamma_N + \gamma_F + \gamma_{F_N}}$, and then

decreases to 0 (the SINR to decode s_F at U_N decreases). It can be observed that the presence of the $B-U_F$ significantly improves the FU throughput. Also, both τ_F^{DF} and τ_N^{DF} change significantly with α_B while the change in α_R does not significantly change the throughput⁵. Hence, an optimal choice of α_B is important. For this reason, we study the variation of EE w.r.t α_B only. From Fig.8 and Fig.9, it can be clearly seen that CDRT-NOMA with AF relaying provides a wider range of valid power allocations in comparison to DF relaying for both U_N and U_F . The presence of the $B-U_F$ widens the valid range of power allocations for U_F with AF relaying (but not with DF relaying).

Fig.10 shows the variation of NU and FU throughput with respect to the d_{BN} to d_{BF} ratio (we vary d_{BN} while fixing $d_{BF} = 4$ to vary $\frac{d_{BN}}{d_{BF}}$) for AF and DF relaying, respectively. For AF as well as DF relaying, FU throughput is constant due to fixed d_{BF} , whereas NU throughput increases with increasing d_{BN} to d_{BF} ratio and saturates after a certain value due to the fixed target rate. Also, for lower distance ratios, the considered CDRT-NOMA with DF relaying performs

⁴Although DF relay R offers an additional degree of freedom in terms of α_R (power allocation at R), AF achieves performance almost similar to it. This is because in CDRT-NOMA with DF relaying, power allocation at R takes place if and only if R decodes both FU and NU symbols successfully.

⁵Since transmission from R depends on successful decoding of s_F , power allocation at S is crucial. Also, at R, power allocation is required only if both s_F and s_N get successfully decoded. Therefore, an optimal choice of α_B is more important in comparison to α_R .

Fig. 8: τ_N^{AF} or τ_F^{AF} vs. α_B with $\rho = 30$ dB.

Fig. 9: τ_N^{DF} or τ_F^{DF} vs. α_B with $\rho = 30$ dB.

Fig. 10: τ_N^{AF} or τ_F^{AF} vs. the ratio of d_{BN} to d_{BF} .

Fig. 11: τ_N^{AF} or τ_F^{AF} vs. the ratio of d_{BR} to d_{RN} .

Fig. 12: F_{τ}^{AF} vs. R for different α_B and ρ .

Fig. 13: F_{τ}^{DF} vs. R for different α_B and ρ .

better than with AF relaying, whereas both DF and AF show almost similar performance for high distance ratios.

Fig.11 depicts the variation of NU and FU throughput with respect to the d_{BR} to d_{RN} ratio (to vary $\frac{d_{BR}}{d_{RN}}$, we vary d_{BR} while fixing $d_{RN} = 2$) for AF and DF relaying, respectively. In both the relaying schemes, the FU throughput initially increases with increasing $\frac{d_{BR}}{d_{RN}}$, attains a maximum value, and then decreases to zero. Therefore, the relay location significantly impacts the FU's performance.

Fig.12 and Fig.13 depict user fairness⁶ versus target rate of AF and DF relay-based CDRT-NOMA, respectively. User fairness depends on the choice of target rate and NOMA power allocation factor. In both cases, the fairness decreases with increasing α_B . On the other hand, increasing transmit SNR results in increased fairness. Also, high transmit SNR ensures a wider range of valid target rates to maintain user

fairness.

In Fig.14, we plot the sum throughput $\tau_{sum}^i = \tau_F^i + \tau_N^i$ versus $R = R_N = R_F$ using optimal α_B and α_R . With increase in SNR τ_{sum}^i also increases and τ_{sum}^i is a quasi-concave function of R , and a unique value R^{opt} exists at which τ_{sum}^i is maximum. Clearly, CDRT-NOMA with DF outperforms that with AF in terms of sum throughput.

Fig.15 illustrates the effect of transmit power ($P_B = P_R = P$) on the EE of AF or DF relay-aided CDRT-NOMA. It can be observed that at $\rho = 30$ dB and $R = 2$, with $P = 2W$ $\eta_E^{AF} = 0.955$ and $\eta_E^{DF} = 0.945$, while at $P = 3W$, $\eta_E^{AF} = 0.637$ and $\eta_E^{DF} = 0.630$. Thus, EE improves with decreasing P . In addition to this, we observe that for the mid-SNR range, DF outperforms AF, while at low as well as at high-SNRs, both achieve similar performance in terms of EE. However, in the absence of the $B-U_F$ link, DF outperforms AF for a wider range of SNRs, and the gap is more than when the $B-U_F$ link is present.

In Fig.16, we plot η_E^{AF} and η_E^{DF} versus α_B with $R = 1$ and $\rho = 30$ dB for $P = 1W$ and $P = 2W$. We also compare the EE of CDRT-NOMA with its relayed OMA counterpart. It is evident that EE of both AF and DF relay-aided NOMA

⁶In NOMA signalling, due to apportioning of power between users, the user performance differs and, a user may have better performance in comparison to others. To maintain user fairness, the performance gap among users should not be large. Similar to [13], we also define the proportional fairness in terms of the ratio of FU to NU throughput as $F_{\tau}^i = \frac{\tau_F^i}{\tau_N^i}$ where $i \in \{AF, DF\}$.

Fig. 14: τ_{sum}^{AF} or τ_{sum}^{DF} vs. R . Fig. 15: η_E^{AF} or η_E^{DF} vs. ρ . Fig. 16: η_E^{AF} or η_E^{DF} vs. α_B . Fig. 17: η_E^{AF} or η_E^{DF} vs. α_B . Fig. 18: τ_N^{AF} vs. R_N . Fig. 19: τ_N^{DF} vs. R_N .

in CDRT is higher than that of relayed OMA. This is large because the signals from both B and R are combined at NU and FU. It can be seen that in the OMA case, EE with DF is better than that with AF, whereas in CDRT-NOMA, AF is more energy efficient. We also show that the $B-U_F$ link significantly improves the EE (the gap is larger with AF than DF).

In Fig.17, we plot the variation of η_E^{AF} and η_E^{DF} versus α_B for fixed transmit power ($P = 3W$), $\rho = 30$ dB and $R = 1, 2, R^{opt}$. Clearly, the energy efficiencies of AF as well as DF increase as we increase the symbol rate from 1 to 2, and the improvement is huge when the optimal symbol rate (R^{opt}) is used. Further, this is true for the entire range of the power allocation parameter. We also observe that EE of relay-aided CDRT-NOMA is significantly higher than that of OMA.

Fig.18 and Fig.19 depict τ_N^{AF} and τ_N^{DF} versus R_N for NOMA in CDRT with AF and DF relaying, respectively for different $\tau_F = \tilde{\tau}$ and $\rho = 25$ dB. Fig.18 and Fig.19 are plotted for optimum values of α_B derived in (51) and (52), respectively. It can be seen that τ_N increases with decreasing $\tilde{\tau}$. In case of both DF and AF aided NOMA in CDRT, we observe that by optimally choosing R_N , the NU throughput can be maximized while guaranteeing the

desired FU throughput $\tilde{\tau}$. Also, selecting an optimal rate ($R_F = R_F^{opt}$) helps to provide huge gain in NU throughput in comparison to any fixed rate R_F . Therefore, the NU throughput attains a maximum performance at (R_N^{opt}, R_F^{opt}) . Clearly, jointly optimizing R_N and R_F (as in Section IV) is of utmost importance. It can also be observed that presence of the $B-U_F$ significantly improves the NU performance.

In Fig.20, Fig.21 and Fig.22, we compare Since the SDF CDRT scheme (with 2-bit feedback) presented in [26] linearly combines the first and second phase signals with orthogonal power allocation at BS and R. FU symbols are canceled at NU, and both the users recover their desired symbols. Due to this, no SIC is required at NU, and analytical complexity is reduced. However, the scheme is sub-optimal and results in a diversity loss for both users. On the other hand, our framework requires no feedback and uses combining at both users with SIC at NU. This results in a diversity order of 2 with improved performance at both the users, but with a minor increase in complexity for combining. the performance of proposed AF as well as DF relay-aided CDRT-NOMA with SDF CDRT-NOMA of [26] in terms of outage probability, sum throughput, and EE, respectively. It is clear from Fig.20 that for optimal power allocation, the proposed AF, as well as DF relay-aided

Fig. 20: P_N^i or P_F^i vs. ρ . Fig. 21: τ_{sum}^i vs. R . Fig. 22: η_E^i vs. α_B .

CDRT-NOMA, outperform the SDF CDRT-NOMA scheme of [26] by a huge margin. The SDF CDRT-NOMA achieves a diversity order of one, whereas the proposed framework can achieve a diversity of two at both the users. It is clear from Fig.21 that the proposed almost doubles the sum throughput in comparison to the SDF CDRT-NOMA scheme of [26]. Fig.22 illustrates that the proposed framework is also more energy-efficient.

VI. CONCLUSION

In this paper, we analyzed the performance of a new framework for downlink non-orthogonal multiple access (NOMA) in a coordinated direct and relay transmission (CDRT) scheme with direct links to the near-user (NU) and the far-user (FU). In this framework, NU combines the direct and relayed signals while performing successive interference cancellation, which considerably improves its outage performance. Considering either amplify-and-forward (AF) or decode-and-forward (DF) relaying, closed-form expressions for outage probability and throughput were derived for both NU and FU. Further, we have also derived the high-SNR approximations to the outage probability and shown that the proposed framework enables both NU and FU to attain a diversity of two without feedback, which is a significant advancement over all existing schemes. This scheme is very useful in multimedia and other applications where the users have stringent outage constraints. It was also demonstrated that the performance of NU with AF and DF relaying is similar in the entire SNR range. For FU, in the presence of the BS-FU link, DF outperformed AF at low SNRs while at high SNRs, both resulted in similar performance. However, DF always outperformed AF in the absence of the BS-FU link. Further, we demonstrated how the choice of power allocation coefficient and target symbol rates are crucial to attain maximum NU throughput while guaranteeing the desired target throughput at FU. We compared the performance of the proposed framework with selective decode-and-forward (SDF) CDRT-NOMA in terms of three metrics: outage probability, sum throughput, and energy efficiency (EE) and demonstrated that the proposed framework, without using any feedback, provides huge

performance gain in terms of all three metrics and helps to secure an additional diversity order at both the users. Furthermore, by comparing the performance of the proposed CDRT based NOMA system to its OMA counterpart, we demonstrated that the proposed system provides higher EE, and the presence of BS-FU link significantly improves the system performance. We also demonstrated that optimal rate selection is essential to maximize EE.

APPENDIX A

Proof of Theorem 1: Substituting $\zeta_{F_1}(w)$ from line following (20) into (24), I_1 can be expressed as

$$I_1 = \int_0^{\lambda_F} \lambda_w \exp \left(\frac{-\xi_1(w - \chi_F)(\lambda_x \rho_R + \lambda_y \rho_B)}{\rho_R(w - \xi_1)} - \lambda_w w \right) \times 2 \sqrt{\frac{-\xi_1(w - \chi_F)\lambda_x \lambda_y}{\rho_R(w - \xi_1)}} K_1 \left(2 \sqrt{\frac{-\xi_1(w - \chi_F)\lambda_x \lambda_y}{\rho_R(w - \xi_1)}} \right) dw. \quad (57)$$

For large values of ρ_B and ρ_R , the argument inside the Bessel function becomes very small throughout the range of SNRs (as verified in Section V). We therefore use $K_1(\theta) \simeq \frac{1}{2}\Gamma(1)(\frac{\theta}{2})^{-1}$ [35, 9.6.9]. Using $\lambda_w(w - \xi_1) = w$ we obtain

$$I_1 \simeq e^{-\xi_1(\lambda_w + \frac{1}{\rho_R})} \int_0^{\lambda_F} \exp \left(-w - \frac{\lambda_w \xi_1 (\xi_1 - \chi_F)(\lambda_x \rho_R + \lambda_y \rho_B)}{\rho_R w} \right) dw. \quad (58)$$

Using $\int_0^\infty t^{a-1} e^{-t-\frac{b}{t}} dt = \Gamma(a, x; b)$ [36, eq. (4)] in the above, we obtain

$$I_1 \simeq e^{-\xi_1(\lambda_w + \frac{1}{\rho_R})} \left[\Gamma(1, \xi_1 \lambda_w; \vartheta_6) - \Gamma(1, \lambda_w(\chi_F - \xi_1); \vartheta_6) \right], \quad (59)$$

where $\vartheta_6 = \frac{\lambda_w \xi_1 (\xi_1 - \chi_F)(\lambda_x \rho_R + \lambda_y \rho_B)}{\rho_R}$. Similarly, we solve for I_2 as

$$I_2 \simeq e^{-\xi_1(\lambda_w + \frac{1}{\rho_R})} \int_0^\infty \exp \left(-w - \frac{\lambda_w \xi_1 (\xi_1 - \chi_F)(\lambda_x \rho_R + \lambda_y \rho_B)}{\rho_R w} \right) dw. \quad (60)$$

Using $\int_0^\infty \exp(-\frac{b}{4t} - at)dt = \sqrt{\frac{b}{a}} K_1(\sqrt{ab})$ [34, 3.324.1] in the above, we obtain

$$I_2 \simeq e^{-\xi_1(\lambda_w + \frac{1}{\rho_R})} \sqrt{4\vartheta_6} K_1(\sqrt{4\vartheta_6}). \quad (61)$$

After substituting I_1 and I_2 from (59) and (61) into (21), we obtain (26).

APPENDIX B

Proof of Lemma 1: After some mathematical arrangements, (23) can be expressed as

$$I_1 = \int_0^{\chi_F} \exp\left(-\zeta_{F_1}(w)\left(\lambda_x + \frac{\lambda_y \rho_B}{\rho_R}\right)\right) \times \int_0^\infty \lambda_x \frac{\exp(-\lambda_x x)}{\exp\left(\frac{\lambda_y \zeta_{F_1}(w)[1 + \rho_B \zeta_{F_1}(w)]}{x \rho_R}\right)} \lambda_w e^{-\lambda_w w} dx dw. \quad (62)$$

For large ρ_B and ρ_R , we use the linear approximation to the exponential term in the denominator to obtain

$$I_1 \simeq \int_0^{\chi_F} \exp\left(-\zeta_{F_1}(w)\left(\lambda_x + \frac{\lambda_y \rho_B}{\rho_R}\right)\right) \left[\int_0^\infty \lambda_x e^{-\lambda_x x} dx - \xi_2(w) \int_0^\infty \frac{\lambda_x e^{-\lambda_x x}}{x + \xi_2(w)} dx \right] \lambda_w e^{-\lambda_w w} dw, \quad (63)$$

where $\xi_2(w) \simeq \frac{\lambda_y \zeta_{F_1}(w)[1 + \rho_B \zeta_{F_1}(w)]}{\rho_B}$. Using [34, 3.352.6] in the above, we obtain

$$I_1 \simeq \int_0^{\chi_F} \exp\left(-\zeta_{F_1}(w)\left(\lambda_x + \frac{\lambda_y \rho_B}{\rho_R}\right)\right) (1 - \xi_2(w) \lambda_x) \times e^{\xi_2(w) \lambda_x} \mathcal{E}_1(\xi_2(w) \lambda_x) \lambda_w e^{-\lambda_w w} dw. \quad (64)$$

Further using $e^{\xi_2(w) \lambda_x} \mathcal{E}_1(\xi_2(w) \lambda_x) = \frac{1}{1 + \xi_2(w) \lambda_x}$ [35, 5.1.19] and simplifying, we obtain

$$I_1 \simeq \int_0^{\chi_F} \frac{\lambda_w e^{-\lambda_w w}}{1 + \xi_2(w) \lambda_x} \exp\left(-\zeta_{F_1}(w)\left(\lambda_x + \frac{\lambda_y \rho_B}{\rho_R}\right)\right) dw. \quad (65)$$

Substituting for $\zeta_{F_1}(w)$ from the line following (25) and neglecting higher-order terms of $1/\rho_B$, we get

$$I_1 \simeq 1 - e^{-\lambda_w \chi_F} - \left(\lambda_x + \frac{\lambda_y \rho_B}{\rho_R}\right) \xi_1 \left[\int_0^{\chi_F} \lambda_w e^{-\lambda_w w} dw + (\xi_1 - \chi_F) \lambda_w \int_0^{\chi_F} \frac{e^{-\lambda_w w}}{w - \xi_1} dw \right]. \quad (66)$$

We use [34, 3.352.1] to obtain

$$I_1 \simeq 1 - e^{-\lambda_w \chi_F} - \left(\lambda_x + \frac{\lambda_y \rho_B}{\rho_R}\right) \xi_1 \left[1 - e^{-\lambda_w \chi_F} + (\xi_1 - \chi_F) \times \lambda_w e^{-\lambda_w \xi_1} [\mathcal{E}_1(-\lambda_w \xi_1) - \mathcal{E}_1(-\lambda_w \xi_1 + \lambda_w \chi_F)] \right]. \quad (67)$$

Similarly, the expression for I_2 is given by

$$I_2 \simeq \frac{e^{-\lambda_w \xi_1}}{1 + \xi_1(\lambda_x + \lambda_y)}. \quad (68)$$

After substituting for I_1 and I_2 from (67) and (68) into (21), the high-SNR approximation to p_F^{AF} is given by (27).

APPENDIX C

Proof of Theorem 3: Using (32), $p_N^{AF} = 1 - (I_5 + I_3)U(\phi) - I_4 U(\phi - \alpha_B) - I_6[U(\phi) - U(\phi - \alpha_B)] - I_7 U(-\phi)$. We first solve for $I_4 = \Pr\{V > \zeta_{N_1}(X, Z), X > \chi_{N_1}(Z), \chi_F < Z < \chi_N\}$ as follows:

$$I_4 = \int_{\chi_F}^{\chi_N} \int_{\chi_{N_1}(z)}^\infty \int_{\zeta_{N_1}(x,z)}^\infty f_V(v) f_X(x) f_Z(z) dv dx dz = \int_{\chi_F}^{\chi_N} \int_{\chi_{N_1}(z)}^\infty \lambda_x \lambda_z \exp\left(-\lambda_v \zeta_{N_1}(x, z) - \lambda_x x - \lambda_z z\right) dx dz. \quad (69)$$

Substituting $x = x - \chi_{N_1}(z)$ in the above and rearranging, we obtain

$$I_4 = \int_{\chi_F}^{\chi_N} \int_0^\infty \exp\left(-\chi_{N_1}(z)\left[\lambda_x + \frac{\lambda_y \rho_B}{\rho_R}\right]\right) \frac{\lambda_x e^{-\lambda_x x} \lambda_z e^{-\lambda_z z}}{\exp\left(\frac{\mu_{N_1}(z)}{\lambda_x x}\right)} dx dz, \quad (70)$$

where $\mu_{N_1}(z) = \frac{\lambda_x \lambda_y \chi_{N_1}(z)[\rho_B \chi_{N_1}(z) + 1]}{\rho_R}$. Solving the above using [34, 3.324.1], we obtain

$$I_4 = \int_{\chi_F}^{\chi_N} e^{-\chi_{N_1}(z)\left(\lambda_x + \frac{\lambda_y \rho_B}{\rho_R}\right)} 2\sqrt{\mu_{N_1}(z)} K_1\left(2\sqrt{\mu_{N_1}(z)}\right) \lambda_z e^{-\lambda_z z} dz. \quad (71)$$

The above integral does not admit a closed-form. To derive a highly accurate closed-form expression (as with I_1), we use $K_1(\theta) \simeq \frac{1}{2}\Gamma(1)(\frac{\theta}{2})^{-1}$ [35, 9.6.9] to obtain

$$I_4 \simeq \lambda_z e^{-C_\ell \chi_N} \int_{\chi_F - \chi_N}^0 e^{C_\ell z} dz, \quad (72)$$

where $C = C_\ell - \lambda_z$ with $C_\ell = \lambda_x + \frac{\lambda_y \rho_B}{\rho_R}$. After solving the above integral, I_4 is obtained as

$$I_4 \simeq \lambda_z e^{-\chi_N C_\ell} \left[\frac{1 - e^{C(\chi_F - \chi_N)}}{C} \right]. \quad (73)$$

Using a similar approach, I_5 , I_6 and I_7 are obtained as

$$I_5 \simeq \frac{\lambda_z e^{-\chi_N C_\ell}}{C} \left[e^{C(\min(\chi_F, \chi_N) - \chi_N)} - e^{-C \chi_N} \right], \quad (74)$$

$$I_6 \simeq e^{\xi_1(C_\ell + \lambda_z)} \left[e^{-(\xi_1 + \chi_N)} e^{-(\xi_1 + \chi_F)} \lambda_z \vartheta_2 e^{\lambda_z \vartheta_2} \times [\mathcal{E}_1(\lambda_z(\chi_N + \xi_1 + \vartheta_2)) - \mathcal{E}_1(\lambda_z(\chi_F + \xi_1 + \vartheta_2))] \right], \quad (75)$$

$$I_7 \simeq e^{\xi_1(C_\ell + \lambda_z)} \left[e^{-2\xi_1} - \lambda_z \vartheta_2 e^{\lambda_z \vartheta_2} [\mathcal{E}_1(\lambda_z(2\xi_1 + \vartheta_2))] \right], \quad (76)$$

respectively, where $\vartheta_2 = \xi_1 \left(\xi_1 + \frac{\gamma_F}{\rho_B \phi} \right) C_\ell$. Finally, I_3 can be derived as

$$I_3 = \Pr\{Z > \max(\chi_F, \chi_N)\} = e^{-\lambda_z \max(\chi_F, \chi_N)}. \quad (77)$$

After substituting the derived expressions of I_3 , I_4 , I_5 , I_6 and I_7 from (77), (73), (74), (75) and (76), respectively, into (32), p_N^{AF} is expressed as in (35).

Proof of Lemma2: Using standard upper bound $e^x \mathcal{E}_1(x) \approx \frac{1}{1+x}$ [35, 5.1.19] into (75) and (76), and using some algebraic simplifications we obtain

$$I_6 \approx \frac{\lambda_z \vartheta_2 e^{-\lambda_z \xi_1} (e^{-\lambda_z \chi_N} - e^{-\lambda_z \chi_F})}{1 + \lambda_z (\chi_N + \xi_1 + \vartheta_2)} - e^{\xi_1 (C_\ell + \lambda_z - 1)} (e^{-\lambda_z \chi_N} - e^{-\lambda_z \chi_F}), \quad (78)$$

$$\text{and } I_7 \approx e^{\xi_1 (\lambda_z - C_\ell)} + \frac{\lambda_z \vartheta_2 e^{\xi_1 (C_\ell - \lambda_z)}}{1 + \lambda_z (\vartheta_2 + 2\xi_1)}. \quad (79)$$

Note that the derived expressions for I_3 , I_4 and I_5 in (77), (73) and (74) are already simplified. Therefore, substituting I_3 to I_7 into (32), we obtain the high-SNR approximated expression for p_N^{AF} as in (36).

APPENDIX D

Proof of Theorem 4: Using (6), (12), (13) and (16) into (39), $\Pr\{A_1\}$ can be expressed as

$$\begin{aligned} \Pr\{A_1\} &= \Pr\left\{ \frac{(1 - \alpha_B)\rho_B X}{\alpha_B \rho_B X + 1} \geq \gamma_F, \alpha_B \rho_B X \geq \gamma_N, \right. \\ &\quad \left. \frac{(1 - \alpha_R)\rho_R Y}{\alpha_R \rho_R Y + 1} + \frac{(1 - \alpha_B)\rho_B W}{\alpha_B \rho_B W + 1} \geq \gamma_F \right\} \\ &= \Pr\{X \geq \max(\chi_F, \chi_N)\} \left[\Pr\{W \geq \chi_F\} \right. \\ &\quad \left. + \Pr\left\{ \chi_F > W > \max\left(0, \frac{-\phi_R}{\rho_B(\alpha_B(1 - \alpha_R) + \alpha_R \phi)}\right) \right. \right. \\ &\quad \left. \left. \times Y \geq \frac{(\gamma_F - \phi_B W)}{\rho_R(\phi_R + (1 - \alpha_R)\alpha_B \rho_B W + \alpha_R \phi_B W)} \right\} \right]. \quad (80) \end{aligned}$$

Since the RVs W , X and Y are independent and have an exponential distribution, we obtain

$$\begin{aligned} \Pr\{A_1\} &= e^{-\lambda_x \max(\chi_F, \chi_N)} \left[e^{-\lambda_w \chi_F} \right. \\ &\quad \left. + \int_{\max(0, \frac{-\phi_R}{\vartheta_1})}^{\chi_F} \lambda_w \exp\left(\frac{\lambda_y \phi_B \rho_B (w - \chi_F)}{\rho_R \vartheta_1 (w + \frac{\phi_R}{\vartheta_1})} - \lambda_w w\right) dw \right]. \quad (81) \end{aligned}$$

After substituting $\lambda_w (w + \frac{\phi_R}{\vartheta_1}) = w$ and solving, we obtain

$$\begin{aligned} \Pr\{A_1\} &= e^{-\lambda_x \max(\chi_F, \chi_N)} \left[e^{-\lambda_w \chi_F} + \exp\left(\frac{\lambda_w \phi_R}{\vartheta_1} + \frac{\lambda_y \phi_B \rho_B}{\rho_R \vartheta_1}\right) \right. \\ &\quad \times \left. \int_{\max(\frac{\lambda_w \phi_R}{\vartheta_1}, 0)}^{\lambda_w (\chi_F + \frac{\phi_R}{\vartheta_1})} \exp\left(\frac{-\lambda_y \lambda_w \phi_B (\frac{\phi_R}{\vartheta_1} + \chi_F)}{\rho_R \vartheta_1 w} - w\right) dw \right]. \quad (82) \end{aligned}$$

Using an approach similar to that used to derive (58), we obtain

$$\begin{aligned} \Pr\{A_1\} &= e^{-\lambda_x \max(\chi_F, \chi_N)} \left[e^{-\lambda_w \chi_F} + \exp\left(\frac{\lambda_w \phi_R}{\vartheta_1} + \frac{\lambda_y \phi_B \rho_B}{\rho_R \vartheta_1}\right) \right. \\ &\quad \times \left\{ \Gamma\left(1, \max\left(\frac{\lambda_w \phi_R}{\vartheta_1}, 0\right); \frac{-\lambda_y \lambda_w \phi_B (\frac{\phi_R}{\vartheta_1} + \chi_F)}{\rho_R \vartheta_1}\right) \right. \\ &\quad \left. \left. - \Gamma\left(1, \lambda_w \left(\chi_F + \frac{\phi_R}{\vartheta_1}\right); \frac{-\lambda_y \lambda_w \phi_B (\frac{\phi_R}{\vartheta_1} + \chi_F)}{\rho_R \vartheta_1}\right) \right\} \right]. \quad (83) \end{aligned}$$

Similarly, the expression for $\Pr\{A_2\}$ is obtained as

$$\begin{aligned} \Pr\{A_2\} &= (e^{-\lambda_x \chi_F} - e^{-\lambda_x \chi_N}) \left[e^{-\frac{\lambda_y \gamma_F}{\rho_R}} + \exp\left(\frac{\lambda_w}{\alpha_B \rho_B} + \frac{\lambda_y \phi}{\alpha_B \rho_R}\right) \right. \\ &\quad \times \left\{ \Gamma\left(1, \frac{\lambda_y \phi}{\alpha_B \rho_R}; \frac{\lambda_y \lambda_w (\phi + \alpha_B \gamma_F)}{\alpha_B^2 \rho_B \rho_R}\right) \right. \\ &\quad \left. \left. - \Gamma\left(1, \frac{\lambda_y}{\rho_R} \left(\gamma_F + \frac{\phi}{\alpha_B}\right); \frac{\lambda_y \lambda_w (\phi + \alpha_B \gamma_F)}{\alpha_B^2 \rho_B \rho_R}\right) \right\} \right]. \quad (84) \end{aligned}$$

Further, using (6), (12) and (13), $\Pr\{A_3\}$ of (39) can be expressed as

$$\Pr\{A_3\} = e^{-\lambda_w \chi_F} (1 - e^{-\lambda_x \chi_F}). \quad (85)$$

Using (83), (84) and (85) in (39), p_F^{DF} is obtained as in (40).

APPENDIX E

Proof of Lemma 4: For higher values of ρ_B and ρ_R , we use $e^{-\frac{a}{x}} = \frac{1}{1 + \frac{a}{x}}$ in (82) to obtain

$$\begin{aligned} \Pr\{A_1\} &\simeq e^{-\lambda_x \max(\chi_F, \chi_N)} \left[e^{-\lambda_w \chi_F} + \exp\left(\frac{\lambda_w \phi_R}{\vartheta_1} + \frac{\lambda_y \phi_B \rho_B}{\rho_R \vartheta_1}\right) \right. \\ &\quad \times \left. \int_{\max(\frac{\lambda_w \phi_R}{\vartheta_1}, 0)}^{\lambda_w (\chi_F + \frac{\phi_R}{\vartheta_1})} \left(1 - \frac{\kappa}{w + \kappa}\right) e^{-w} dw \right], \quad (86) \end{aligned}$$

where $\kappa = \frac{\lambda_y \lambda_w \phi_B \rho_B (\frac{\phi_R}{\vartheta_1} + \chi_F)}{\rho_R \vartheta_1}$. Solving the above integral using [34, 3.352.3], we obtain

$$\begin{aligned} \Pr\{A_1\} &\simeq e^{-\lambda_x \max(\chi_F, \chi_N)} \left[\exp\left(\frac{\lambda_w \phi_R}{\vartheta_1} + \frac{\lambda_y \phi_B \rho_B}{\rho_R \vartheta_1}\right) \left\{ \exp\left(-\max\left(\frac{\lambda_w \phi_R}{\vartheta_1}, 0\right)\right) c - \exp\left(-\lambda_w \left(\chi_F + \frac{\phi_R}{\vartheta_1}\right)\right) - \kappa e^\kappa \left[\mathcal{E}_1\left(\max\left(\frac{\lambda_w \phi_R}{\vartheta_1}, 0\right) + \kappa\right) - \mathcal{E}_1\left(\lambda_w \left(\chi_F + \frac{\phi_R}{\vartheta_1}\right) + \kappa\right) \right] \right\} \right. \\ &\quad \left. + e^{-\lambda_w \chi_F} \right]. \quad (87) \end{aligned}$$

At high-SNRs, $\kappa \propto \frac{1}{\rho_B^2}$. We therefore neglect the product term of κe^κ . Now $\Pr\{A_1\}$ is expressed as

$$\begin{aligned} \Pr\{A_1\} &\simeq e^{-\lambda_x \max(\chi_F, \chi_N)} \left[\exp\left(\frac{\lambda_w \phi_R}{\vartheta_1} + \frac{\lambda_y \phi_B \rho_B}{\rho_R \vartheta_1} - \max\left(\frac{\lambda_w \phi_R}{\vartheta_1}, 0\right)\right) \right. \\ &\quad \left. + e^{-\lambda_w \chi_F} \left(1 - \exp\left(\frac{\lambda_y \phi_B \rho_B}{\rho_R \vartheta_1}\right)\right) \right]. \quad (88) \end{aligned}$$

Similarly, solving for A_2 , we obtain

$$\begin{aligned} \Pr\{A_2\} &\simeq (e^{-\lambda_x \chi_F} - e^{-\lambda_x \chi_N}) \left[\exp\left(\frac{\lambda_w}{\alpha_B \rho_B}\right) + \exp\left(-\frac{\lambda_y \gamma_F}{\rho_R}\right) \right. \\ &\quad \left. \left(1 - \exp\left(\frac{\lambda_w}{\alpha_B \rho_B}\right)\right) \right]. \quad (89) \end{aligned}$$

Substituting for $\Pr\{A_1\}$ and $\Pr\{A_2\}$ from (87) and (89) into (39), we obtain p_F^{DF} given by (41).

APPENDIX F

Proof of Theorem 5: Using (4), (5), (12) and (13) in (45), $\Pr\{B_1\}$ can be expressed as

$$\begin{aligned}\Pr\{B_1\} &= \Pr\left\{\Gamma_{RF}^I < \gamma_F, \Gamma_{NF}^I \geq \gamma_F, \Gamma_{NN}^I \geq \gamma_N\right\} \\ &= \Pr\left\{\frac{(1-\alpha_B)\rho_B X}{\alpha_B \rho_B X + 1} < \gamma_F, \frac{(1-\alpha_B)\rho_B Z}{\alpha_B \rho_B Z + 1} \geq \gamma_F, \alpha_B \rho_B Z \geq \gamma_N\right\} \\ &= \Pr\{X < \chi_F, Z \geq \max(\chi_F, \chi_N)\} U(\phi) \\ &= e^{-\lambda_z \max(\chi_F, \chi_N)} [1 - e^{-\lambda_x \chi_F}] U(\phi). \quad (90)\end{aligned}$$

Using (45), $\Pr\{B_2\}$ is given by

$$\begin{aligned}\Pr\{B_2\} &= \Pr\left\{\Gamma_{RF}^I \geq \gamma_F, \Gamma_{RN}^I < \gamma_N, \Gamma_{NF-F}^{COM} \geq \gamma_F, \Gamma_{NN}^I \geq \gamma_N\right\} \\ &= \Pr\left\{\chi_F < X < \chi_N, \rho_B Z \left(\underbrace{(1-\alpha_B) - \alpha_B(\gamma_F - \rho_R V)}_{(\phi + \alpha_B \rho_R V)}\right) \right. \\ &\quad \left. > (\gamma_F - \rho_R V), Z \geq \chi_N\right\} U(\phi) \\ &= \Pr\left\{\chi_F < X < \chi_N, V > \frac{\gamma_F}{\rho_R}, Z \geq \chi_N\right\} + \Pr\left\{\chi_F < X \right. \\ &\quad \left. < \chi_N, V < \frac{\gamma_F}{\rho_R}, Z \geq \max\left(\chi_N, \frac{\gamma_F - \rho_R V}{\phi + \alpha_B \phi_R V}\right)\right\}. \quad (91)\end{aligned}$$

To satisfy $\chi_N > \chi_F$, $\max\left(\chi_N, \frac{\gamma_F - \rho_R V}{\phi + \alpha_B \phi_R V}\right) = \chi_N$. Using this fact, the above can be simplified as

$$\begin{aligned}\Pr\{B_2\} &= \Pr\{\chi_F < X < \chi_N, Z \geq \chi_N\} \\ &= e^{-\lambda_z \chi_N} (e^{-\lambda_x \chi_F} - e^{-\lambda_x \chi_N}) U\left(\phi - \frac{\alpha_B \gamma_F}{\gamma_N}\right). \quad (92)\end{aligned}$$

Further, the expression for $\Pr\{B_3\}$ is calculated as

$$\begin{aligned}\Pr\{B_3\} &= \Pr\left\{\Gamma_{RF}^I \geq \gamma_F, \Gamma_{RN}^I \geq \gamma_N, \Gamma_{NF-S}^{COM} \geq \gamma_F, \Gamma_{NN-S}^{COM} \geq \gamma_N\right\} \\ &= \Pr\left\{X > \max(\chi_F, \chi_N), \rho_R V \left((1-\alpha_R)(1+\alpha_B \rho_B Z) - \alpha_R(\gamma_F - \phi \rho_B Z)\right) \right. \\ &\quad \left. > (\gamma_F - \phi \rho_B Z), V \geq \frac{\gamma_N - \alpha_B \rho_B Z}{\alpha_R \rho_B}, Z < \chi_N\right\}. \quad (93)\end{aligned}$$

Using the above, the conditions for outage on ranges of X , V and Z with $\phi > 0$ and $\phi < 0$ are listed in Table IV. Now, $\Pr\{B_3\}$ can be expressed as

$$\begin{aligned}\Pr\{B_3\} &= \left[\Pr\{Z > \max(\chi_F, \chi_N)\} + \Pr\left\{\chi_F < z < \chi_N, V \right. \right. \\ &\quad \left. \left. \geq \tilde{\chi}_{N_1}(z)\right\} + \Pr\left\{\chi_F > Z > \max(\chi_N, \vartheta_5), V \geq \tilde{\chi}_{N_1}\right\} \right. \\ &\quad \left. + \Pr\left\{V \geq \tilde{\chi}_{N_1}(Z), \min(\chi_F, \chi_N) > Z > \max(\vartheta_5, \vartheta_4)\right\} \right. \\ &\quad \left. + \Pr\left\{\min(\chi_F, \chi_N, \vartheta_4) > Z > \max(0, \vartheta_5), \right. \right. \\ &\quad \left. \left. V \geq \tilde{\chi}_{N_2}(Z)\right\}\right] \Pr\{X > \max(\chi_F, \chi_N)\}, \quad (94)\end{aligned}$$

where $\tilde{\chi}_{N_1}(Z) = \frac{\gamma_F - \phi \rho_B Z}{\rho_R(\phi_R + \rho_B Z(\phi \alpha_R + \alpha_B(1-\alpha_R)))}$ and $\tilde{\chi}_{N_2}(Z) = \frac{\gamma_N - \alpha_B \rho_B Z}{\alpha_R \rho_B}$. After averaging over the PDFs of RVs V ,

X and Z , we obtain $\Pr\{B_3\}$ as in (95), where $\omega = \frac{-\lambda_v(\gamma_F - \phi \rho_B z)}{\rho_R(\phi_R + \rho_B z(\phi \alpha_R + \alpha_B(1-\alpha_R)))}$, $\tilde{\chi}_N = \min(\chi_F, \chi_N, \vartheta_4)$. After some mathematical rearrangement, the above can be expressed as

$$\begin{aligned}\Pr\{B_3\} &= e^{-\lambda_x \max(\chi_F, \chi_N)} \left[e^{-\lambda_z \max(\chi_F, \chi_N)} + \frac{\lambda_z e^{-\varepsilon_3 \gamma_N}}{\varepsilon_3 \alpha_B \rho_B - \lambda_z} \times \right. \\ &\quad \left[\exp(\varepsilon_3 \alpha_B \rho_B \chi_N - \lambda_z \chi_N) - \exp(\varepsilon_3 \alpha_B \rho_B \chi_F - \lambda_z \chi_F) \right] \\ &\quad + \frac{\lambda_z e^{-\varepsilon_3 \gamma_N}}{\varepsilon_3 \alpha_B \rho_B - \lambda_z} \left[\exp(\varepsilon_3 \alpha_B \rho_B \tilde{\chi}_N - \lambda_z \tilde{\chi}_N) - \exp(\varepsilon_3 \alpha_B \rho_B \right. \\ &\quad \left. \max(0, \frac{-\phi_R}{\vartheta_1}) - \lambda_z \max(0, \frac{-\phi_R}{\vartheta_1})) \right] \\ &\quad + \underbrace{\int_{\max(\chi_N, \frac{-\phi_R}{\vartheta_1})}^{\chi_F} \exp\left(\frac{\lambda_v \phi \rho_B}{\rho_R \vartheta_1} \times \frac{z - \chi_F}{z + \frac{\phi_R}{\vartheta_1}}\right) \lambda_z e^{-\lambda_z z} dz}_{I_8} \\ &\quad \left. + \underbrace{\int_{\max(\frac{-\phi_R}{\vartheta_1}, \vartheta_4)}^{\min(\chi_F, \chi_N)} \exp\left(\frac{\lambda_v \phi \rho_B}{\rho_R \vartheta_1} \times \frac{z - \chi_F}{z + \frac{\phi_R}{\vartheta_1}}\right) \lambda_z e^{-\lambda_z z} dz}_{I_9} \right]. \quad (96)\end{aligned}$$

where $\varepsilon_3 = \frac{\lambda_v}{\alpha_R \rho_R}$. Substituting $\lambda_z(z + \frac{\phi_R}{\vartheta_1}) = z$, and using [36, eq. (4)], I_8 and I_9 are obtained as

$$\begin{aligned}I_8 &= \exp\left(\frac{\phi_R \lambda_z \rho_R + \lambda_v \phi \rho_B}{\rho_R \vartheta_1}\right) \left[\Gamma\left(1, \lambda_z \max(0, \chi_N + \frac{\phi_R}{\vartheta_1}); \tilde{\chi}_F\right) \right. \\ &\quad \left. - \Gamma\left(1, \lambda_z(\chi_F + \frac{\phi_R}{\vartheta_1}); \tilde{\chi}_F\right) \right], \quad (97)\end{aligned}$$

$$\begin{aligned}I_9 &= \exp\left(\frac{\phi_R \lambda_z \rho_R + \lambda_v \phi \rho_B}{\rho_R \vartheta_1}\right) \left[\Gamma\left(1, \lambda_z \max(0, \vartheta_4 + \frac{\phi_R}{\vartheta_1}); \tilde{\chi}_F\right) \right. \\ &\quad \left. - \Gamma\left(1, \lambda_z \min(\chi_N, \chi_F) + \frac{\phi_R \lambda_z}{\vartheta_1}; \tilde{\chi}_F\right) \right], \quad (98)\end{aligned}$$

where $\tilde{\chi}_F = \frac{\lambda_z \lambda_v \phi \rho_B (\chi_F + \frac{\phi_R}{\vartheta_1})}{\vartheta_1}$. Using I_8 and I_9 from (97) and (98) into (96), $\Pr\{B_3\}$ is expressed as

$$\begin{aligned}\Pr\{B_3\} &= e^{-\lambda_x \max(\chi_F, \chi_N)} \left[e^{-\lambda_z \max(\chi_F, \chi_N)} + \frac{\lambda_z e^{-\varepsilon_3 \gamma_N}}{\varepsilon_3 \alpha_B \rho_B - \lambda_z} \right. \\ &\quad \times \left[\exp(\varepsilon_3 \alpha_B \rho_B \chi_N - \lambda_z \chi_N) - \exp(\varepsilon_3 \alpha_B \rho_B \chi_F - \lambda_z \chi_F) \right. \\ &\quad + \frac{\exp(\varepsilon_3 \alpha_B \rho_B \tilde{\chi}_N)}{\exp(\tilde{\chi}_N \lambda_z)} - \frac{\exp\left(\varepsilon_3 \alpha_B \rho_B \max(0, \frac{-\phi_R}{\vartheta_1})\right)}{\exp(\max(0, \frac{-\phi_R}{\vartheta_1}) \lambda_z)} \left. \right] \\ &\quad + \exp\left(\frac{\phi_R \lambda_z \rho_R + \lambda_v \phi \rho_B}{\rho_R \vartheta_1}\right) \left[\Gamma\left(1, \lambda_z \max(0, \chi_N + \frac{\phi_R}{\vartheta_1}); \tilde{\chi}_F\right) \right. \\ &\quad - \Gamma\left(1, \lambda_z(\chi_F + \frac{\phi_R}{\vartheta_1}); \tilde{\chi}_F\right) + \Gamma\left(1, \lambda_z \max(0, \vartheta_4 + \frac{\phi_R}{\vartheta_1}); \tilde{\chi}_F\right) \\ &\quad \left. - \Gamma\left(1, \lambda_z \min(\chi_N, \chi_F) + \frac{\phi_R \lambda_z}{\vartheta_1}; \tilde{\chi}_F\right) \right] \right]. \quad (99)\end{aligned}$$

Substituting (90), (92) and (99) into (45), p_N^{DF} is obtained as in (46).

TABLE IV: B_3 - ranges of V , X and Z for different values of α_B

$\alpha_B < \frac{1}{1+\gamma_F}$	$Z > \max(\chi_F, \chi_N)$	$V > 0, X > \max(\chi_F, \chi_N)$
	$\chi_N > Z > \chi_F$	$V > \tilde{\chi}_{N_2}(Z), X > \max(\chi_F, \chi_N)$
	$\chi_F > Z > \max(\chi_N, \vartheta_5)$	$V > \tilde{\chi}_{N_1}(Z), X > \max(\chi_F, \chi_N)$
	$\min(\chi_F, \chi_N) > Z > \vartheta_5$	$V > \max(\tilde{\chi}_{N_1}(Z), \tilde{\chi}_{N_2}(Z)), X > \max(\chi_F, \chi_N)$ if $\tilde{\chi}_{N_1}(Z) \geq \tilde{\chi}_{N_2}(Z) \Rightarrow Z \geq \vartheta_4$
$\alpha_B \geq \frac{1}{1+\gamma_F}$	No range of X , V and Z is valid	

$$\Pr\{B_3\} = e^{-\lambda_x \max(\chi_F, \chi_N)} \left[e^{-\lambda_z \max(\chi_F, \chi_N)} + \int_{\chi_F}^{\chi_N} \exp\left(\frac{-\lambda_v(\gamma_N - \alpha_B \rho_B z)}{\alpha_R \rho_R}\right) \lambda_z e^{-\lambda_z z} dz + \int_{\max(0, -\frac{\phi_R}{\vartheta_1})}^{\chi_N} \exp\left(\frac{-\lambda_v(\gamma_N - \alpha_B \rho_B z)}{\alpha_R \rho_R}\right) \lambda_z e^{-\lambda_z z} dz \right. \\ \left. \times \lambda_z e^{-\lambda_z z} dz + \int_{\max(\chi_N, -\frac{\phi_R}{\vartheta_1})}^{\chi_F} \exp\left(\frac{-\lambda_v(\gamma_F - \phi \rho_B z)}{\rho_R(\phi_R + \rho_B z \omega)}\right) \lambda_z e^{-\lambda_z z} dz + \int_{\max(-\frac{\phi_R}{\vartheta_1}, \vartheta_4)}^{\min(\chi_F, \chi_N)} \exp\left(\frac{-\lambda_v(\gamma_F - \phi \rho_B z)}{\rho_R(\phi_R + \rho_B z \omega)}\right) \lambda_z e^{-\lambda_z z} dz \right], \quad (95)$$

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