A Coordinated Direct AF/DF Relay-Aided NOMA Framework for Low Outage

Anand Jee, *Student Member, IEEE*, Kamal Agrawal, *Student Member, IEEE* and
Shankar Prakriya, *Senior Member, IEEE*

Abstract

This paper investigates the performance of a framework for low-outage downlink non-orthogonal multiple access (NOMA) signalling using a coordinated direct and relay transmission (CDRT) scheme with direct links to both the near-user (NU) and the far-user (FU). Both amplify-and-forward (AF) and decode-and-forward (DF) relaying are considered. In this framework, NU and FU combine the signals from BS and R to attain good outage performance and harness a diversity of two without any need for feedback. For the NU, this serves as an incentive to participate in NOMA signalling. For both NU and FU, expressions for outage probability and throughput are derived in closed form. High-SNR approximations to the outage probability are also presented. We demonstrate that the choice of power allocation coefficient and target rate is crucial to maximize the NU performance while ensuring a desired FU performance. We demonstrate performance gain of the proposed scheme over selective decode-and-forward (SDF) CDRT-NOMA in terms of three metrics: outage probability, sum throughput and energy efficiency. Further, we demonstrate that by choosing the target rate intelligently, the proposed CDRT NOMA scheme ensures higher energy efficiency (EE) in comparison to its orthogonal multiple access counterpart. Monte Carlo simulations validate the derived expressions.

Index Terms

Non-orthogonal multiple access (NOMA), coordinated direct and relay transmission (CDRT), energy efficiency (EE), amplify-and-forward (AF), decode-and-forward (DF).

I. INTRODUCTION

The proliferation of internet of things (IoT) and massive machine type communication has augmented the demand for higher data rates, low latency and high bandwidth efficiency in beyond 5G (B5G) and

A. Jee, K. Agrawal and S. Prakriya are with the Department of Electrical Engineering, Indian Institute of Technology Delhi (IIT Delhi), New Delhi-110016, India (e-mail: {anandjee7, kamal.agrawal, shankar}@ee.iitd.ac.in).

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6G networks [1]–[4]. Multiple access techniques used in older generations of wireless communication networks are not sufficient to fulfil these stringent requirements on user density and network traffic. In order to meet the requirements of future wireless networks, several multiple access technologies were studied in the last decade. Among them non-orthogonal multiple access (NOMA) has emerged as one of the most promising technology for future wireless networks due to its high spectral efficiency (SE), low latency, and its ability to facilitate massive connectivity [5]–[7].

In contrast to traditional orthogonal multiple access (OMA) schemes, NOMA facilitates concurrent transmission to multiple users over the same spectral/time/spreading code resource using power domain multiplexing at the transmitter side and successive interference cancellation (SIC) at the receiver side. In downlink NOMA, the user with a poor channel condition is termed as the far user (FU), and is allocated a larger fraction of the transmit power in comparison to the user with a better channel condition i.e. the near user (NU) [8]. The quality of service at the FU (as well as the coverage area) can be further enhanced by using relayed NOMA (R-NOMA) [9]–[12] and cooperative NOMA (C-NOMA) [13]–[16]. In C-NOMA, NU assists the information transmission to FU whereas in R-NOMA, dedicated nodes are used for relaying to FU. In [9], [17], the performance of amplify-and-forward (AF) and decode-and-forward (DF) relaying schemes was compared for different relay selection criteria in an R-NOMA framework. Further, in [18], the block error performance for short packet communication in a C-NOMA network was compared to that in conventional NOMA assuming Rayleigh flat fading channels. To improve the SE, successive user relaying in C-NOMA was discussed in [19].

The use of NOMA in coordinated direct and relay transmission (CDRT) [20]–[25] is another promising approach to improve the QoS at FU. In a typical NOMA CDRT signalling scheme, the base station (BS) shares a direct link to the NU, while communication to the FU is assisted by a dedicated relay. In [20], the capacity of NOMA CDRT was analyzed. In [21], [22], capacity gain at FU was achieved along with improved SE. IoT-assisted NOMA CDRT was analyzed in terms of outage probability and ergodic sum capacity in [23]. In [24], the SE and user fairness index of CDRT NOMA were derived considering perfect and imperfect SIC at NU. Coordinated uplink transmission for cooperative NOMA system was studied in [25]. To date, existing literature on NOMA CDRT focuses on AF as well as DF relaying in the absence of the BS-FU direct link. The availability of BS-FU direct link not only enhances the quality of service (QoS) at FU but also lowers the required energy consumption at the relay. In [26], DF relay-aided CDRT NOMA in presence of BS-FU link has been investigated for three different cases a) fixed relaying where the DF relay always forwards the incoming signal from BS, b) selective DF where the relay only forwards after successful decoding

of the incoming signal, and c) incremental selective DF relaying where the relayed link is used opportunistically, and requires a three-bit feedback. Further, in [27], performance of fixed-gain AF relay based CDRT NOMA was investigated in the presence of the BS-FU link assuming sub-optimal selection combining at the FU.

A. Motivation and Contribution

Due to participation in NOMA signalling, NU needs to decode the FU symbol first to perform SIC (complexity of the NU receiver clearly increases) before it can decode its own symbol. Depending on the channel condition as well as the target rate and power allocation factor, NU might occasionally fail to decode its own symbol due to insufficient signal-to-noise ratio (SNR) or unsuccessful decoding of the FU symbol. In such a scenario the BS has to re-transmit the superimposed symbol, which causes reduction in SE as well as energy efficiency (EE). In C-NOMA NU incurs loss in SE (due to allocation of power to the FU symbols, and use of two signalling phases), and EE as well (it is required to expend energy for relaying the FU symbol).

All the aforementioned works [10], [13], [15]–[17], [21]–[24], [26], [27] have laid a solid foundation for the use of C-NOMA and CDRT to improve the FU's performance, but there has been little effort to improve the NU's performance. Several multimedia-type applications impose a low outage QoS, and it might not be possible to satisfy NU's constraints using these CDRT approaches (they only guarantee a diversity of one at the NU [21]). However, considering an AF or DF relay to assist FU as well as NU with optimal combining at both users not only improves the NU diversity, but also results in improved QoS at both the users. The NU will be required to perform combining with SIC (in a framework discussed in this paper), but attains much better performance, which is required in many scenarios. We also discuss optimization of NU performance while ensuring a desired performance at FU. The key contributions of this paper are as follows:

- Considering both AF and DF relaying, we investigate the performance of a new framework for CDRT based downlink NOMA network with direct link to both NU and FU. In the considered framework, BS transmits NOMA signal to relay, NU and FU in the first signalling phase, while in the second signalling phase, R communicates to both NU as well as FU. The NU combines the incoming signals from BS and relay to enhance its performance as an incentive for its participation in NOMA signalling.
- For both NU and FU, considering both AF and DF relaying, we present expressions for outage probability and throughput in closed-form. Unlike other works [28], [29] on AF CDRT NOMA

where (for analytical simplicity) the harmonic to min approximation is used, we derive the exact outage probability in closed form.

- Our simulations show that the performance of NU with AF and DF relaying is almost the same throughout the range of SNRs. For FU, in the presence of BS-FU link, DF performs better than that of AF at low SNRs and at high SNRs, both achieve similar performance. However, DF always outperforms AF in the absence of the BS-FU link. We also demonstrate that CDRT NOMA with AF relaying allows a wider range of valid power allocations in comparison to DF relaying for both NU and FU.
- We also derive highly accurate high-SNR expressions for the outage probability to demonstrate that combining direct and relayed signals at NU as well as FU (while performing SIC at NU) allows both NU and FU to harness a diversity of 2 with both AF and DF relaying. Clearly, the NU is rewarded for participation in NOMA signalling. However, in the absence of the BS-FU link, FU attains a diversity of only one.
- Next, We show that how the choice of power allocation coefficient and target symbol rates are crucial to maximize the NU throughput while guaranteeing a desired target throughput at FU. We also present an approximate closed-form expression for power allocation coefficient that maximizes NU throughput for a given FU throughput requirement. In addition to this, we also formulate a NU throughput maximization problem and determine the optimal NU and FU target rate pair.
- We also observe that in DF-assisted CDRT NOMA (unlike in the AF case), both BS and R transmit the superimposed symbols (combination of NU and FU symbols) in first and second phase of signalling, respectively. However, the power allocation at BS significantly affects the system performance in comparison to the power allocation at R.
- Further, we observe that the proposed scheme (without using any feedback) always outperforms SDF CDRT-NOMA [26] in terms of outage probability, sum throughput and energy efficiency by a huge margin and also achieves better diversity at both the users.
- Moreover, we compare the performance of the proposed CDRT-NOMA framework to its OMA counterpart (with relaying) and observe that the proposed system ensures higher energy efficiency (EE). We further observe that the availability of BS-FU link helps in achieving a higher EE in comparison to the case when BS-FU link is absent. Also, optimal rate selection is important to maximize the EE.

The rest of this paper is structured as follows. Section II elaborates on the system model for AF

as well as DF relay assisted CDRT NOMA framework. Section III analyzes the performance with the proposed framework in terms of outage probability. Section IV discusses NU throughput and energy efficiency maximization. Numerical results based on mathematical analysis are compared to computer simulations in Section V. Finally, Section VI concludes this paper.

Notations: $\mathcal{CN}(0, \sigma^2)$ represents a zero-mean complex Gaussian distribution with variance σ^2 . $f_X(x)$ and $U(\cdot)$ respectively denote the probability density function (PDF) of a random variable (RV) X and the unit step function. $\mathcal{E}_1(\cdot)$, $K_1(\cdot, \cdot)$ and $\Gamma(a, x; b)$ denote the exponential integral of type 1, the modified Bessel function of the second kind and the generalized incomplete gamma function, respectively.

II. SYSTEM MODEL

As depicted in Fig. 1, we consider a CDRT downlink NOMA framework consisting of a base station B, a near user U_N , a far user U_F and a relay station R. Both U_N and U_F have stringent outage QoS constraints. All nodes operate in the half-duplex (HD) mode and are equipped with a single antenna. Communication to U_N and U_F takes place in two signalling phases. In the first phase, B communicates to U_N , R and U_F over direct links, while in the second phase communication from R to U_N as well as U_F takes place using either AF or DF mode of relaying.

The channel coefficients $h_{ij} \sim \mathcal{CN}(0, 1/\lambda_{ij})$ with $i \in \{B, R\}$, $j \in \{R, N, F\}$ are assumed to be independent and of quasi-static Rayleigh fading type, where $\lambda_{ij} = d_{ij}^m$, and m is the path-loss exponent. The additive zero-mean complex Gaussian noise at all the receiving nodes is assumed to be of variance σ^2 . Superscript "I" and "II" are used to represent first and second phase quantities.

Fig. 1: (a) System Model. (b) An illustration of the proposed CDRT transmission process.

A. Amplify-and-Forward Relaying

In the first phase, B transmits a superposition of unit-energy symbols s_N and s_F (of information rates R_N and R_F) intended respectively for U_N and U_F . The transmit power P_B is apportioned to

U_N and U_F symbols in the ratio $\alpha_B : (1 - \alpha_B)$. Thus, the superimposed symbol can be expressed as $s_B = \sqrt{P_B \alpha_B} s_N + \sqrt{P_B (1 - \alpha_B)} s_F$. The sampled matched filter outputs at U_N , R and U_F are

$$y_N^I = s_B h_{BN} + w_N^I, \quad y_R^I = s_B h_{BR} + w_R^I \quad \text{and} \quad y_F^I = s_B h_{BF} + w_F^I \quad (1)$$

respectively, wherein w_N^I , w_R^I and w_F^I are the respective additive Gaussian noise samples. In accordance with the concept of NOMA, U_N first decodes the FU symbol s_F , and then performs SIC to decode the NU symbol s_N [30]. Using (1), the signal-to-interference-plus-noise ratio (SINR) Γ_{NF}^I to decode s_F , and the signal-to-noise ratio (SNR) Γ_N^I to decode s_N at U_N after SIC, are expressed as

$$\Gamma_{NF}^I = \frac{(1 - \alpha_B) \rho_B |h_{BN}|^2}{\alpha_B \rho_B |h_{BN}|^2 + 1} \quad \text{and} \quad \Gamma_{NN}^I = \alpha_B \rho_B |h_{BN}|^2, \quad \text{provided } \Gamma_{NF}^I \geq \gamma_F, \quad (2)$$

where $\rho_B = P_B / \sigma^2$ represents the transmit SNR at B and $\gamma_F = 2^{R_F} - 1$ represents the threshold SNR at U_F . Using (1), the SINR Γ_{FF}^I to decode s_F at U_F can be expressed as

$$\Gamma_{FF}^I = \frac{(1 - \alpha_B) \rho_B |h_{BF}|^2}{\alpha_B \rho_B |h_{BF}|^2 + 1}. \quad (3)$$

Let P_R denote the available transmit power at R . In the second phase, R amplifies the incoming signal from the first phase and then forwards $s_R = \beta y_R^I$ to U_N and U_F as shown in Fig.1 (b), where $\beta = \sqrt{\frac{\rho_R}{\rho_B |h_{BR}|^2 + 1}}$ with $\rho_R = \frac{P_R}{\sigma^2}$. The signals received at U_N and U_F in the second phase are given by

$$y_N^{II} = \beta y_R^I h_{RN} + w_N^{II} \quad \text{and} \quad y_F^{II} = \beta y_R^I h_{RF} + w_F^{II}, \quad (4)$$

respectively, where w_N^{II} and w_F^{II} are the additive noise samples at U_N and U_F . Similar to the first phase, using (4) the SINRs Γ_{NF}^{II} and Γ_{NN}^{II} to decode s_F and s_N (after SIC) are expressed as

$$\Gamma_{NF}^{II} = \frac{\rho_B \beta^2 (1 - \alpha_B) |h_{BR}|^2 |h_{RN}|^2}{\rho_B \beta^2 \alpha_B |h_{BR}|^2 |h_{RN}|^2 + |h_{RN}|^2 \beta^2 + 1} = \frac{\rho_B \rho_R (1 - \alpha_B) |h_{BR}|^2 |h_{RN}|^2}{\rho_B \rho_R \alpha_B |h_{BR}|^2 |h_{RN}|^2 + \rho_R |h_{RN}|^2 + \rho_B |h_{BR}|^2 + 1}, \quad (5)$$

$$\Gamma_{NN}^{II} = \frac{\rho_B \beta^2 \alpha_B |h_{BR}|^2 |h_{RN}|^2}{\beta^2 |h_{RN}|^2 + 1} = \frac{\rho_B \rho_R \alpha_B |h_{BR}|^2 |h_{RN}|^2}{\rho_R |h_{RN}|^2 + \rho_B |h_{BR}|^2 + 1}, \quad \text{provided } \Gamma_{NF}^{II} > \gamma_F. \quad (6)$$

Using (4), the SINR to decode s_F at U_F is given by

$$\Gamma_{FF}^{II} = \frac{\rho_B \beta^2 (1 - \alpha_B) |h_{BR}|^2 |h_{RF}|^2}{\rho_B \beta^2 \alpha_B |h_{BR}|^2 |h_{RF}|^2 + |h_{RF}|^2 \beta^2 + 1} = \frac{\rho_B \rho_R (1 - \alpha_B) |h_{BR}|^2 |h_{RF}|^2}{\rho_B \rho_R \alpha_B |h_{BR}|^2 |h_{RF}|^2 + \rho_R |h_{RF}|^2 + \rho_B |h_{BR}|^2 + 1}. \quad (7)$$

The signals from first and second phases are combined at U_F . In this paper, we use an approach that deviates from all existing works to date and enable U_N to combine the signals in the two phases at each stage of the SIC. We show that this enables U_N to attain very good performance and harness a diversity of two without any need for feedback. This allows the network to be used for multimedia and other applications that impose strict outage QoS constraints. It is emphasized that none of the techniques suggested so far [21], [26] can ensure a diversity of two at the near-user in the absence of

any feedback¹. This improvement in diversity as well as in throughput performance also serves as an incentive for U_N to participate in NOMA signalling. Clearly, U_N cannot attain this performance using traditional OMA. In this respect, this work is quite different from all existing works.

U_F combines y_F^I and y_F^{II} to decode s_F with SINR $\Gamma_{FF}^{COM} = \Gamma_{FF}^I + \Gamma_{FF}^{II}$. If s_F is decoded successfully in the first phase, U_N cancels the interference from y_N^I and then combines the first and second phase signals to decode s_N with SINR $\Gamma_{NN}^{COM} = \Gamma_{NN}^I + \Gamma_{NN}^{II}$. However, when U_N fails to decode s_F , it combines y_N^I and y_N^{II} to first decode s_F with SINR $\Gamma_{NF}^{COM} = \Gamma_{NF}^I + \Gamma_{NF}^{II}$, cancels interference, and then decodes s_N with SINR $\Gamma_{NN}^{COM} = \Gamma_{NN}^I + \Gamma_{NN}^{II}$.

B. Decode-and-Forward relaying

As in AF relaying, B transmits the superposed signal s_B in the first phase. Therefore, the SINRs Γ_{NF}^I , Γ_{NN}^I and Γ_F^I remains same. Different from AF relaying, the DF relay decodes s_F and s_N by implementing SIC. The SINRs Γ_{RF}^I and Γ_{RN}^I (after SIC) to decode s_F and s_N at R can be expressed as

$$\Gamma_{RF}^I = \frac{(1 - \alpha_B)\rho_B|h_{BR}|^2}{\alpha_B\rho_B|h_{BR}|^2 + 1} \quad \text{and} \quad \Gamma_{RN}^I = \alpha_B\rho_B|h_{BR}|^2 \quad \text{provided} \quad \Gamma_{RF}^I \geq \gamma_F. \quad (8)$$

In the second phase, based on the decoding status of s_N and s_F , R transmits either superimposed signal s_R or only the far user symbol s_F as shown in Fig.1 (b). If R decodes both s_N and s_F successfully², then it forwards the superimposed signal $s_R = \sqrt{P_R\alpha_R}s_N + \sqrt{P_R(1 - \alpha_R)}s_F$ to both U_N and U_F , where α_R and $(1 - \alpha_R)$ denotes the portion of power allocated to U_N and U_F , respectively. Note that DF relaying (unlike its AF counter part) allows different NOMA power allocations at B and R. The signals received at U_N and U_F can be expressed as $y_{N-S}^{II} = s_R h_{RN} + w_N^{II}$ and $y_{F-S}^{II} = s_R h_{RF} + w_F^{II}$, where $-S$ in the subscript is used to emphasize that a superposed signal is transmitted by R. Using y_{N-S}^{II} , the respective SINRs before and after SIC (Γ_{NF-S}^{II} and Γ_{NN-S}^{II}) to decode s_F and s_N at U_N are

$$\Gamma_{NF-S}^{II} = \frac{(1 - \alpha_R)\rho_R|h_{RN}|^2}{\alpha_R\rho_R|h_{RN}|^2 + 1} \quad \text{and} \quad \Gamma_{NN-S}^{II} = \alpha_R\rho_R|h_{RN}|^2 \quad \text{provided} \quad \Gamma_{NF-S}^{II} \geq \gamma_F. \quad (9)$$

Using y_{F-S}^{II} , the SINR Γ_{FF-S}^{II} at F to decode s_F is

$$\Gamma_{FF-S}^{II} = \frac{(1 - \alpha_R)\rho_R|h_{RF}|^2}{\alpha_R\rho_R|h_{RF}|^2 + 1}. \quad (10)$$

However, if R fails to decode s_N then it forwards only s_F . Now, the received signals at U_N and U_F are expressed as $y_{N-F}^{II} = \sqrt{P_R}s_F h_{RN} + w_N^{II}$ and $y_{F-F}^{II} = \sqrt{P_R}s_F h_{RF} + w_F^{II}$ respectively, where the subscript $-F$ is used to emphasize that R merely transmits s_F . Using y_{N-F}^{II} and y_{F-F}^{II} , the respective SINRs Γ_{NF-F}^{II} and Γ_{FF-F}^{II} to decode s_F at U_N and U_F can be expressed as

¹Since the suggested combining based SIC at U_N improves performance when the direct links to U_N fails, the performance advantage is expected to be retained in the case when feedback is available.

²In CDRT NOMA with DF relaying, if R successfully decodes s_N and s_F , the power allocation takes place at both B and R, thus, DF provides more flexibility and a performance improvement can be expected in comparison to AF relaying.

$$\Gamma_{NF-F}^{II} = \rho_R |h_{RN}|^2 \quad \text{and} \quad \Gamma_{FF-F}^{II} = \rho_R |h_{RF}|^2. \quad (11)$$

U_F combines y_F^I with y_{F-S}^{II} when s_N and s_F are decoded at R, or with y_{F-F}^{II} otherwise. The combined SINRs are $\Gamma_{FF-S}^{COM} = \Gamma_{FF}^I + \Gamma_{FF-S}^{II}$ or $\Gamma_{FF-F}^{COM} = \Gamma_{FF}^I + \Gamma_{FF-F}^{II}$ respectively. However, if s_F is decoded successfully in the first phase, U_N cancels interference from y_{N-S}^{II} or y_{N-F}^{II} and then combines the first and second phase signals to decode s_N with SNR $\Gamma_{NN-S}^{COM} = \Gamma_{NN}^I + \Gamma_{NN-S}^{II}$ or Γ_{NN}^I . When U_N fails to decode s_F , it combines y_N^I and y_{N-S}^{II} or y_{N-F}^{II} to first decode s_F with SINR $\Gamma_{NF-S}^{COM} = \Gamma_{NF}^I + \Gamma_{NF-S}^{II}$ or $\Gamma_{NF-F}^{COM} = \Gamma_{NF}^I + \Gamma_{NF-F}^{II}$. Subsequently, after cancelling the interference due to s_F , decoding of s_N takes place with SNR $\Gamma_{NN-S}^{COM} = \Gamma_{NN}^I + \Gamma_{NN-S}^{II}$ or Γ_{NN}^I . In this paper we compare performance of the proposed scheme with a relayed OMA scheme described below.

C. Relayed OMA Scheme

In the considered relayed OMA system, B transmits x_F to U_F , U_N and R in the first signalling phase. In the second signalling phase R forwards either the amplified (with AF relaying) or the decoded (with DF relaying) FU symbol, while B transmits x_N to NU. Using the decoded FU symbol in the first phase, NU cancels the interference caused by transmission by R before decoding. However, if NU fails to decode x_F in the first phase, then the signal received from R in the second phase acts as interference at U_N and x_N transmitted from B acts as interference at FU. Note that NU receives symbols in both time-slots in this OMA scheme with relaying. It can therefore be expected to attain high throughput. We observe in Section V that the proposed scheme yields much better throughput performance as compared to its relayed OMA counterpart.

III. OUTAGE PROBABILITY ANALYSIS

In this section, the outage performance of coordinated direct AF/DF relay assisted downlink NOMA is analyzed. We derive closed-form expressions of near and far user outage probabilities considering AF or DF relaying. In order to determine the diversity order for U_N and U_F , the asymptotic expressions of user outage probabilities are also derived. For ease of exposition, we define $|h_{BR}|^2 = X$ ($\lambda_{BR} = \lambda_x$), $|h_{RF}|^2 = Y$ ($\lambda_{RF} = \lambda_y$), $|h_{BN}|^2 = Z$ ($\lambda_{BN} = \lambda_z$), $|h_{BF}|^2 = W$ ($\lambda_{BF} = \lambda_w$), and $|h_{RN}|^2 = V$ ($\lambda_{RN} = \lambda_v$). Note that the RVs X, Y, Z, W, V are independent and exponentially distributed with PDF $f_\Omega(\omega) = \lambda_\omega e^{-\lambda_\omega \omega}$ with $\Omega \in \{X, Y, Z, W, V\}$, $\omega \in \{x, y, z, w, v\}$.

A. Amplify-and-Forward Relaying

1) *Far user outage probability*: The outage probability of U_F can be written as $p_F^{AF} = \Pr\{\Gamma_F^{COM} < \gamma_F\}$, where $\Gamma_F^{COM} = \Gamma_F^I + \Gamma_F^{II}$. Using (3) and (7), p_F^{AF} can be expressed as

$$p_F^{AF} = \Pr\left\{\Gamma_{FF}^{COM} < \gamma_F\right\} = \Pr\left\{\frac{(1 - \alpha_B)\rho_B W}{\alpha_B \rho_B W + 1} + \frac{\rho_B \rho_R (1 - \alpha_B) XY}{\rho_B \rho_R \alpha_B XY + \rho_R Y + \rho_B X + 1} < \gamma_F\right\}$$

$$= \Pr \left\{ \rho_R Y [\rho_B X (\phi + \alpha_B \rho_B W (1 - \alpha_B + \phi)) - (\gamma_F - \rho_B W \phi)] < (\gamma_F - \rho_B W \phi) (1 + \rho_B X) \right\}, \quad (12)$$

wherein $\phi = 1 - \alpha_B (1 + \gamma_F)$. The sign of $(\gamma_F - \rho_B W \phi)$ depends on the range of W and different values of ϕ . Depending on $(\gamma_F - \rho_B W \phi) \geq 0$, the conditions for outage on ranges of X , Y and W with $\phi > 0$ ($\alpha_B < \frac{1}{1+\gamma_F}$), $\alpha_B - 1 < \phi < 0$ ($\frac{1}{1+\gamma_F} \leq \alpha_B < \frac{2}{2+\gamma_F}$) and $\phi < \alpha_B - 1$ ($\alpha_B \geq \frac{2}{2+\gamma_F}$) are listed in Table I, wherein $\zeta_{F_1}(x, w) = \frac{(\gamma_F - \rho_B w \phi)(1 + \rho_B x)}{\rho_R(\rho_B x(\phi + \alpha_B \rho_B w(1 - \alpha_B + \phi)) - (\gamma_F - \rho_B w \phi))}$, $\zeta_{F_1}(w) = \frac{(\gamma_F - \rho_B w \phi)}{\rho_B(\phi + \alpha_B \rho_B w(1 - \alpha_B + \phi))}$, $\chi_F = \frac{\gamma_F}{\phi \rho_B}$ and $\xi_1 = \frac{-\phi}{\alpha_B \rho_B (1 - \alpha_B + \phi)}$. Now, p_F^{AF} can be expressed as

$$p_F^{AF} = \begin{cases} \int_0^{\chi_F} \int_{\zeta_{F_1}(w)}^{\infty} [1 - e^{-\lambda_y \zeta_{F_1}(x, w)}] \lambda_x e^{-\lambda_x x} \lambda_w e^{-\lambda_w w} dx dw + \int_0^{\chi_F} [1 - e^{-\lambda_x \zeta_{F_1}(w)}] \lambda_w e^{-\lambda_w w} dw & \text{if } \alpha_B < \frac{1}{1+\gamma_F} \\ 1 - e^{-\lambda_w \xi_1} + \int_{\xi_1}^{\infty} \int_{\zeta_{F_1}(w)}^{\infty} [1 - e^{-\lambda_y \zeta_{F_1}(x, w)}] \lambda_x \lambda_w e^{-\lambda_x x - \lambda_w w} dx dw + \int_{\xi_1}^{\infty} [1 - e^{-\lambda_x \zeta_{F_1}(w)}] \lambda_w e^{-\lambda_w w} dw & \text{if } \frac{1}{1+\gamma_F} \leq \alpha_B < \frac{2}{2+\gamma_F} \\ 1 & \text{otherwise,} \end{cases} \quad (13)$$

After some mathematical rearrangement (13) can be expressed as

$$p_F^{AF} = \begin{cases} 1 - e^{-\lambda_w \chi_F} - \underbrace{\int_0^{\chi_F} \int_{\zeta_{F_1}(w)}^{\infty} \exp\left(-\frac{\lambda_y \zeta_{F_1}(w)(1 + \rho_B x)}{\rho_R(x - \zeta_{F_1}(w))}\right) \lambda_x e^{-\lambda_x x} dx \lambda_w e^{-\lambda_w w} dw}_{I_1} & \text{if } \alpha_B < \frac{1}{1+\gamma_F} \\ 1 - \underbrace{\int_{\xi_1}^{\infty} \int_{\zeta_{F_1}(w)}^{\infty} \exp\left(-\frac{\lambda_y \zeta_{F_1}(w)(1 + \rho_B x)}{\rho_R(x - \zeta_{F_1}(w))}\right) \lambda_x e^{-\lambda_x x} \lambda_w e^{-\lambda_w w} dx dw}_{I_2} & \text{if } \frac{1}{1+\gamma_F} \leq \alpha_B < \frac{2}{2+\gamma_F} \\ 1 & \text{otherwise.} \end{cases} \quad (14)$$

We first solve for I_1 as follows:

$$I_1 = \int_0^{\chi_F} \int_{\zeta_{F_1}(w)}^{\infty} \exp\left(-\frac{\lambda_y \zeta_{F_1}(w)(1 + \rho_B x)}{\rho_R(x - \zeta_{F_1}(w))}\right) \lambda_x e^{-\lambda_x x} dx \lambda_w e^{-\lambda_w w} dw. \quad (15)$$

TABLE I: U_F outage - ranges of Y , X and W for different values of α_B

$\alpha_B < \frac{1}{1+\gamma_F}$	$W < \chi_F$	$X < \zeta_{F_1}(W), Y > 0$
		$X \geq \zeta_{F_1}(W), Y < \zeta_{F_1}(X, W)$
$\frac{1}{1+\gamma_F} \leq \alpha_B < \frac{2}{2+\gamma_F}$	$W < \xi_1$	$X > 0, Y > 0$
	$W \geq \xi_1$	$X < \zeta_{F_1}(W), Y > 0$ $X \geq \zeta_{F_1}(W), Y < \zeta_{F_1}(X, W)$
$\alpha_B \geq \frac{2}{2+\gamma_F}$	Entire range of W , X and Y	

Substituting $x = x - \zeta_{F_1}(w)$ in the above and re-arranging terms, we get

$$I_1 = \int_0^{\chi_F} \exp\left(-\zeta_{F_1}(w)\left(\lambda_x + \frac{\lambda_y \rho_B}{\rho_R}\right)\right) \int_0^\infty \lambda_x \exp\left(-\lambda_x x - \frac{\lambda_y \zeta_{F_1}(w)[1 + \rho_B \zeta_{F_1}(w)]}{x \rho_R}\right) \lambda_w e^{-\lambda_w w} dx dw. \quad (16)$$

Solving for the inner integral using [31, 3.324.1], we obtain

$$I_1 = \int_0^{\chi_F} \exp\left(-\zeta_{F_1}(w)\left(\lambda_x + \frac{\lambda_y \rho_B}{\rho_R}\right)\right) 2 \mu_{F_1}(w) K_1(2 \mu_{F_1}(w)) \lambda_w e^{-\lambda_w w} dw, \quad (17)$$

where $\mu_{F_1}(w) = \frac{\lambda_x \lambda_y \zeta_{F_1}(w)[\rho_B \zeta_{F_1}(w) + 1]}{\rho_R}$. Following a similar approach, we derive an expression for I_2 . p_F^{AF} can then be expressed as

$$p_F^{AF} = \begin{cases} 1 - e^{-\lambda_w \chi_F} - \int_0^{\chi_F} \exp\left(-\zeta_{F_1}(w)\left(\lambda_x + \frac{\lambda_y \rho_B}{\rho_R}\right)\right) 2 \mu_{F_1}(w) K_1(2 \mu_{F_1}(w)) \lambda_w e^{-\lambda_w w} dw & \text{if } \alpha_B < \frac{1}{1 + \gamma_F} \\ 1 - \int_{\xi_1}^\infty \exp\left(-\zeta_{F_1}(w)\left(\lambda_x + \frac{\lambda_y \rho_B}{\rho_R}\right)\right) 2 \mu_{F_1}(w) K_1(2 \mu_{F_1}(w)) \lambda_w e^{-\lambda_w w} dw & \text{if } \frac{1}{1 + \gamma_F} \leq \alpha_B < \frac{2}{2 + \gamma_F} \\ 1, & \text{otherwise,} \end{cases} \quad (18)$$

where $\mu_{F_1}(w) = \frac{\lambda_x \lambda_y \zeta_{F_1}(w)[\rho_B \zeta_{F_1}(w) + 1]}{\rho_R}$. Unfortunately, the integrals in (18) do not admit a closed form. We therefore present a highly accurate approximate expression for p_F^{AF} in the following theorem.

Theorem 1. *An accurate closed-form expression for FU outage probability with AF relaying is given by*

$$p_F^{AF} = \begin{cases} 1 - e^{-\lambda_w \chi_F} - e^{-\xi_1(\lambda_w + \frac{1}{\rho_R})} \left[\Gamma(1, -\lambda_w \xi_1; \vartheta_6) - \Gamma(1, \lambda_w(\chi_F - \xi_1); \vartheta_6) \right] & \text{if } \alpha_B < \frac{1}{1 + \gamma_F} \\ 1 - e^{-\xi_1(\lambda_w + \frac{1}{\rho_R})} \sqrt{4 \vartheta_6} K_1(\sqrt{4 \vartheta_6}) & \text{if } \frac{1}{1 + \gamma_F} \leq \alpha_B < \frac{2}{2 + \gamma_F} \\ 1 & \text{otherwise.} \end{cases} \quad (19)$$

Proof. Refer to Appendix A. ■

Using the expression obtained in (19), it is extremely difficult to establish the diversity order. We therefore derive a high-SNR approximation to p_F^{AF} in the following Lemma.

Lemma 1. *A high SNR approximation to p_F^{AF} is given by*

$$p_F^{AF} \simeq \begin{cases} \left(\lambda_x + \frac{\lambda_y \rho_B}{\rho_R}\right) \xi_1 \left[1 - e^{-\lambda_w \chi_F} + (\xi_1 - \chi_F) \lambda_w e^{-\lambda_w \xi_1} [\mathcal{E}_1(-\lambda_w \xi_1) - \mathcal{E}_1(-\lambda_w \xi_1 + \lambda_w \chi_F)] \right] & \text{if } \alpha_B < \frac{1}{1 + \gamma_F} \\ 1 - \frac{e^{-\lambda_w \xi_1}}{1 + \xi_1(\lambda_x + \lambda_y)} & \text{if } \frac{1}{1 + \gamma_F} \leq \alpha_B < \frac{2}{2 + \gamma_F} \\ 1 & \text{otherwise.} \end{cases} \quad (20)$$

Proof. Refer to Appendix B. ■

Remark 1. In the absence of direct link to U_F , the outage probability of U_F can be readily derived by substituting $W = 0$ into (12). After using a procedure similar to that used to derive (18), we obtain an exact closed-form expression for p_F^{AF} as follows:

$$p_F^{AF} = 1 - \exp\left(-\frac{\chi_F}{\rho_R}(\lambda_x \rho_R + \lambda_y \rho_B)\right) 2\sqrt{\mu_F} K_1(2\sqrt{\mu_F}), \quad \text{if } \alpha_B < \frac{1}{1+\gamma_F} \quad (21)$$

where $\mu_F = \frac{\lambda_x \lambda_y \chi_F (1 + \rho_B \chi_F)}{\rho_R}$. The above equality holds only for $\alpha_B < \frac{1}{1+\gamma_F}$, otherwise $p_F^{AF} = 1$. Note that for higher values of ρ_B and ρ_R , μ_F and χ_F are very small. Therefore, applying $K_1(\theta) \simeq \frac{1}{2}\Gamma(1)(\frac{\theta}{2})^{-1}$ [32, 9.6.9] in (21), we obtain

$$p_F^{AF} \simeq 1 - \exp\left(-\chi_F \left[\lambda_x + \frac{\lambda_y \rho_B}{\rho_R}\right]\right). \quad (22)$$

Using $e^{-\theta} \simeq 1 - \theta$ in the above, a high SNR approximation to p_F^{AF} is given by

$$p_F^{AF} \simeq \frac{\gamma_F [\rho_R \lambda_x + \rho_B \lambda_y]}{\rho_R \rho_B \phi}. \quad (23)$$

2) *Near user Outage Probability:* Let $\Gamma_{NF}^{COM} = \Gamma_{NF}^I + \Gamma_{NF}^{II}$ and $\Gamma_{NN}^{COM} = \Gamma_{NN}^I + \Gamma_{NN}^{II}$. Due to the manner in which SIC is performed with combining, the outage probability of U_N is

$$p_N^{AF} = 1 - \Pr\left\{\Gamma_{NF}^{COM} \geq \gamma_F, \Gamma_{NN}^{COM} \geq \gamma_N\right\}. \quad (24)$$

Theorem 2. The near user outage probability can be written as follows:

$$p_N^{AF} = \begin{cases} 1 - \underbrace{\Pr\{Z > \max(\chi_F, \chi_N)\}}_{I_3} - \underbrace{\Pr\{V > \zeta_{N_1}(X, Z), X > \chi_{N_1}(Z), \chi_N > Z > \chi_F\}}_{I_4} \\ - \underbrace{\Pr\{V > \zeta_{N_1}(X, Z), X > \chi_{N_1}(Z), \min(\chi_F, \chi_N) > Z\}}_{I_5} & \text{if } \alpha_B < \frac{1}{2+\gamma_F} \\ 1 - \underbrace{\Pr\{Z > \max(\chi_F, \chi_N)\}}_{I_3} - \underbrace{\Pr\{V > \zeta_{N_1}(X, Z), X > \chi_{N_1}(Z), \min(\chi_F, \chi_N) > Z\}}_{I_5} \\ - \underbrace{\Pr\{V > \zeta_{N_2}(X, Z), X > \chi_{N_2}(Z), \chi_F > Z > \chi_N\}}_{I_6} & \text{if } \frac{1}{2+\gamma_F} \leq \alpha_B < \frac{1}{1+\gamma_F} \\ 1 - \underbrace{\Pr\{V > \zeta_{N_2}(X, Z), X > \chi_{N_2}(Z), Z > \xi_1\}}_{I_7} & \text{if } \frac{1}{1+\gamma_F} \leq \alpha_B < \frac{2}{2+\gamma_F} \\ 1, & \text{otherwise,} \end{cases} \quad (25)$$

where $\chi_N = \frac{\gamma_N}{\alpha_B \rho_B}$, $\chi_{N_1}(Z) = \frac{\gamma_N - \alpha_B Z \rho_B}{\rho_B \alpha_B}$, $\zeta_{N_1}(X, Z) = \frac{\chi_{N_1}(Z)(\rho_B X + 1)}{\rho_R(X - \chi_{N_1}(Z))}$, $\chi_{N_2}(Z) = \frac{\gamma_F - \phi Z \rho_B}{\rho_B[\phi + \alpha_B Z \rho_B(1 - \alpha_B + \phi)]}$ and $\zeta_{N_2}(X, Z) = \frac{\chi_{N_2}(Z)(\rho_B X + 1)}{\rho_R(X - \chi_{N_2}(Z))}$.

Proof. Using (2), (5) and (6) into (24), we obtain

TABLE II: U_N non-outage - ranges of Z , X and V for different values of α_B

$\alpha_B < \frac{1}{1+\gamma_F}$	$Z > \max(\chi_F, \chi_N)$	$V > 0, X > 0$
	$\chi_N > Z > \chi_F$	$V > \zeta_{N_1}(X, Z), X > \chi_{N_1}(Z)$
	$\chi_F > Z > \chi_N$	$V > \zeta_{N_2}(X, Z), X > \chi_{N_2}(Z)$
	$\min(\chi_F, \chi_N) > Z$	$V > \max(\zeta_{N_1}(X, Z), \zeta_{N_2}(X, Z)), X > \max(\chi_{N_1}(Z), \chi_{N_2}(Z))$ if $\chi_{N_1}(Z) > \chi_{N_2}(Z) \Rightarrow \zeta_{N_1}(X, Z) > \zeta_{N_2}(X, Z) \Rightarrow Z < \chi_N$ if $\chi_{N_1}(Z) > \chi_{N_2}(Z) \Rightarrow \zeta_{N_1}(X, Z) < \zeta_{N_2}(X, Z)$ not valid
$\frac{1}{1+\gamma_F} \leq \alpha_B < \frac{2}{2+\gamma_F}$	$Z > \xi_1$	$V > \zeta_{N_2}(X, Z), X > \chi_{N_2}(Z)$
$\alpha_B \geq \frac{2}{2+\gamma_F}$	Outage always occurs	

$$\begin{aligned}
p_N^{AF} &= 1 - \Pr \left\{ \frac{(1-\alpha_B)\rho_B Z}{\alpha_B \rho_B Z + 1} + \frac{\rho_B \rho_R (1-\alpha_B) X V}{\rho_B \rho_R \alpha_B X V + \rho_R V + \rho_B X + 1} \geq \gamma_F, \alpha_B \rho_B Z + \frac{\rho_B \rho_R \alpha_B X V}{\rho_R V + \rho_B X + 1} \geq \gamma_N \right\}, \\
&= 1 - \Pr \{ [V \rho_R (X \rho_B (\phi + \alpha_B \rho_B Z (1 - \alpha_B + \phi))) - (\gamma_F - \phi \rho_B Z)] \geq (\gamma_F - \phi \rho_B Z) (1 + \rho_B X) \}, \\
&\quad [V \rho_R (X \rho_B \alpha_B - (\gamma_N - \alpha_B \rho_B Z) \geq (\gamma_N - \alpha_B \rho_B Z) (1 + \rho_B X) \}]. \tag{26}
\end{aligned}$$

In the above, depending on values of α_B and γ_F , two ranges of α_B exist i.e. $\alpha_B < \frac{1}{1+\gamma_F}$ or $\alpha_B \geq \frac{1}{1+\gamma_F}$. Note that the signs of $(\gamma_F - \phi \rho_B Z)$ and $(\gamma_N - \alpha_B \rho_B Z)$ depend on the range of Z . Based on the ranges of RVs V , X and Z , the conditions for non-outage probability of U_N for different values of α_B are listed in Table II. Now, p_N^{AF} can be expressed as

$$\begin{aligned}
p_N^{AF} &= 1 - \Pr \{ V > \zeta_{N_2}(X, Z), X > \chi_{N_2}(Z), Z > \xi_1 \} U(-\phi) - \left[\Pr \{ Z > \max(\chi_N, \chi_F) \} \right. \\
&\quad + \Pr \{ V > \zeta_{N_1}(X, Z), X > \chi_{N_1}(Z), \chi_N > Z > \chi_F \} U(\chi_N - \chi_F) + \Pr \{ V > \zeta_{N_2}(X, Z), X > \chi_{N_2}(Z), \\
&\quad \left. \chi_F > Z > \chi_N \} U(\chi_F - \chi_N) + \Pr \{ V > \zeta_{N_1}(X, Z), X > \chi_{N_1}(Z), Z < \min(\chi_N, \chi_F) \} \right] U(\phi), \tag{27}
\end{aligned}$$

where $\chi_{N_1}(Z)$, $\chi_{N_2}(Z)$, $\zeta_{N_1}(X, Z)$, $\zeta_{N_2}(X, Z)$, χ_F , χ_N and ϕ are defined in the line following (25). After performing mathematical rearrangements in (27) we obtain (25). $\blacksquare$

Theorem 3. An accurate closed-form expression for p_N^{AF} is given by

$$p_N^{AF} \simeq \begin{cases} 1 - e^{-\lambda_z \max(\chi_N, \chi_F)} - \lambda_z e^{-\lambda_N C_\ell} \left[\frac{1 - e^{C(\chi_F - \chi_N)} + e^{C(\min(\chi_F, \chi_N) - \chi_N)} - e^{-C\chi_N}}{C} \right] & \text{if } \alpha_B < \frac{1}{2 + \gamma_F} \\ 1 - e^{-\lambda_z \max(\chi_N, \chi_F)} - \lambda_z e^{-\lambda_N C_\ell} \left[\frac{e^{C(\min(\chi_F, \chi_N) - \chi_N)} - e^{-C\chi_N}}{C} \right] - e^{\xi_1(C_\ell + \lambda_z)} \left[e^{-(\xi_1 + \chi_N)} - e^{-(\xi_1 + \chi_F)} \right] & \text{if } \frac{1}{2 + \gamma_F} \leq \alpha_B < \frac{1}{1 + \gamma_F} \\ -\lambda_z \vartheta_2 e^{\lambda_z \vartheta_2} \left[\mathcal{E}_1(\lambda_z(\chi_N + \xi_1 + \vartheta_2)) - \mathcal{E}_1(\lambda_z(\chi_F + \xi_1 + \vartheta_2)) \right] & \text{if } \frac{1}{1 + \gamma_F} \leq \alpha_B < \frac{2}{2 + \gamma_F} \\ 1 - e^{\xi_1(C_\ell + \lambda_z)} \left[e^{-2\xi_1} - \lambda_z \vartheta_2 e^{\lambda_z \vartheta_2} \left[\mathcal{E}_1(\lambda_z(2\xi_1 + \vartheta_2)) \right] \right] & \text{otherwise,} \\ 1, & \end{cases} \tag{28}$$

where $\vartheta_2 = \xi_1 \left(\xi_1 + \frac{\gamma_F}{\rho_B \phi} \right) C_\ell$, $C = C_\ell - \lambda_z$ and $C_\ell = \lambda_x + \frac{\lambda_v \rho_B}{\rho_R}$.

Proof. Refer to Appendix C. ■

Lemma 2. The diversity orders D_F^{AF} and D_N^{AF} achieved at U_F and U_N are given by

$$D_F^{AF} = \begin{cases} 2, & \text{if } \alpha_B < \frac{1}{1+\gamma_F} \\ 1, & \text{if } \frac{1}{1+\gamma_F} \leq \alpha_B < \frac{2}{2+\gamma_F} \\ 0, & \text{otherwise.} \end{cases}, \quad D_N^{AF} = \begin{cases} 2, & \text{if } \alpha_B < \frac{1}{1+\gamma_F} \\ 1, & \text{if } \frac{1}{1+\gamma_F} \leq \alpha_B < \frac{2}{2+\gamma_F} \\ 0, & \text{otherwise.} \end{cases} \quad (29)$$

Proof. To derive the diversity order for U_F and U_N , we use the high-SNR expression of p_F^{AF} and p_N^{AF} given by (20) and (28), respectively. We consider $\rho_B = \rho_R = \rho$ without loss of generality. The diversity orders attained by U_F and U_N are given by $D_j^{AF} = -\lim_{\rho \rightarrow \infty} \frac{\log_2 p_j^{AF}(\rho)}{\log_2 \rho}$, $j \in \{N, F\}$. For $\rho \rightarrow \infty$, we use $e^{-\theta} = 1 - \theta$ and substitute for χ_F and ξ_1 . After neglecting higher order terms of $\frac{1}{\rho}$, we obtain diversity order as expressed in (29). The detailed proof is omitted due to paucity of space. ■

Remark 2. The fact that the near-user also attains a diversity of two is significant, and motivates it to participate in NOMA signalling.

Remark 3. In the absence of direct link to U_F , the diversity order D_F^{AF} is obtained by using $\rho_B = \rho_R = \rho$ into (23). The diversity order D_F^{AF} can be expressed as

$$D_F^{AF} = \begin{cases} 1, & \text{if } \alpha_B < \frac{1}{1+\gamma_F} \\ 0, & \text{otherwise.} \end{cases} \quad (30)$$

However, the diversity order for U_N remains the same as expressed in (29).

B. Decode-and-Forward Relaying

1) *Far user outage probability:* Let $\Gamma_{FF-S}^{COM} = \Gamma_{FF}^I + \Gamma_{FF-S}^{II}$ and $\Gamma_{FF-F}^{COM} = \Gamma_{FF}^I + \Gamma_{FF-F}^{II}$. FU is not in outage when a) both s_F and s_N are decoded at R ($\Gamma_{RF}^I \geq \gamma_F$, $\Gamma_{RN}^I \geq \gamma_N$) and FU SNR after combining is sufficient ($\Gamma_{FF-S}^{COM} \geq \gamma_F$), b) s_F can be decoded at R, but not s_N but SNR after combining is sufficient at FU ($\Gamma_{FF-F}^{COM} \geq \gamma_F$), and c) s_F cannot be decoded at R but $\Gamma_{FF}^I \geq \gamma_F$. The outage probability for U_F can be formulated as

$$p_F^{DF} = 1 - \underbrace{\Pr\{\Gamma_{RF}^I \geq \gamma_F, \Gamma_{RN}^I \geq \gamma_N, \Gamma_{FF-S}^{COM} \geq \gamma_F\}}_{A_1} - \underbrace{\Pr\{\Gamma_{RF}^I \geq \gamma_F, \Gamma_{RN}^I < \gamma_N, \Gamma_{FF-F}^{COM} \geq \gamma_F\}}_{A_2} - \underbrace{\Pr\{\Gamma_{RF}^I < \gamma_F, \Gamma_{FF}^I \geq \gamma_F\}}_{A_3}, \quad (31)$$

In the following, we present closed-form expressions for FU outage.

Theorem 4. The exact closed-form expression of the outage probability for U_F is given by

$$p_F^{DF} = \begin{cases} 1 - e^{-\lambda_w \chi_F} (1 - e^{-\lambda_x \chi_F}) - \left[e^{-\lambda_w \chi_F} + \left\{ \Gamma \left(1, \max(\varsigma_1, 0); \frac{-\lambda_y \phi \rho_B \varsigma_2}{\rho_R \vartheta_1 w} \right) - \Gamma \left(1, \lambda_w \varsigma_2; \frac{-\lambda_y \phi \rho_B \varsigma_2}{\rho_R \vartheta_1 w} \right) \right\} \right. \\ \times \exp \left(\varsigma_1 + \frac{\lambda_y \phi \rho_B}{\rho_R \vartheta_1} \right) \left. e^{-\lambda_x \max(\chi_F, \chi_N)} - (e^{-\lambda_x \chi_F} - e^{-\lambda_x \chi_N}) \left[\exp \left(-\frac{\lambda_y \gamma_F}{\rho_R} \right) + \exp \left(\frac{\lambda_w}{\alpha_B \rho_B} + \frac{\lambda_y \phi}{\alpha_B \rho_R} \right) \right] \right. \\ \times \left. \left\{ \Gamma \left(1, \frac{\lambda_y \phi}{\alpha_B \rho_R}; \frac{\lambda_y \lambda_w (\phi + \alpha_B \gamma_F)}{\alpha_B^2 \rho_B \rho_R} \right) - \Gamma \left(1, \frac{\lambda_y}{\rho_R} (\gamma_F + \frac{\phi}{\alpha_B}); \frac{\lambda_y \lambda_w (\phi + \alpha_B \gamma_F)}{\alpha_B^2 \rho_B \rho_R} \right) \right\} \right] & \text{if } \alpha_B < \frac{\gamma_N}{\gamma_N + \gamma_F + \gamma_F \gamma_N} \\ 1 - e^{-\lambda_w \chi_F} (1 - e^{-\lambda_x \chi_F}) - \left[e^{-\lambda_w \chi_F} + \left\{ \Gamma \left(1, \max(\varsigma_1, 0); \frac{-\lambda_y \phi \rho_B \varsigma_2}{\rho_R \vartheta_1 w} \right) - \Gamma \left(1, \lambda_w \varsigma_2; \frac{-\lambda_y \phi \rho_B \varsigma_2}{\rho_R \vartheta_1 w} \right) \right\} \right. \\ \times \exp \left(\varsigma_1 + \frac{\lambda_y \phi \rho_B}{\rho_R \vartheta_1} \right) \left. e^{-\lambda_x \max(\chi_F, \chi_N)} \right] & \text{if } \frac{\gamma_N}{\gamma_N + \gamma_F + \gamma_F \gamma_N} \leq \alpha_B < \frac{1}{1 + \gamma_F} \\ 1 & \text{otherwise,} \end{cases} \quad (32)$$

where $\phi_R = 1 - \alpha_R(1 + \gamma_F)$, $\vartheta_1 = \rho_B(\alpha_B(1 - \alpha_R) + \alpha_R \phi)$, $\varsigma_1 = \frac{\lambda_w \phi_R}{\vartheta_1}$ and $\varsigma_2 = \chi_F + \frac{\phi_R}{\vartheta_1}$

Proof. Refer to Appendix D. ■

The expressions obtained in (32) involve the generalized incomplete gamma function due to which the diversity cannot readily be established from them. We therefore present a highly accurate high-SNR approximation to p_F^{AF} in the following Lemma.

Lemma 3. A high SNR approximation to p_F^{DF} is given by

$$p_F^{DF} \simeq \begin{cases} 1 + e^{-\lambda_w \chi_F} (1 - e^{-\lambda_x \chi_F}) - e^{-\lambda_x \chi_N} \left[\exp \left(\varsigma_1 + \frac{\lambda_y \phi \rho_B}{\rho_R \vartheta_1} - \max(\varsigma_1, 0) \right) + e^{-\lambda_w \chi_F} \left(1 - \exp \left(\frac{\lambda_y \phi \rho_B}{\rho_R \vartheta_1} \right) \right) \right] \\ - (e^{-\lambda_x \chi_F} - e^{-\lambda_x \chi_N}) \left[\exp \left(\frac{\lambda_w}{\alpha_B \rho_B} \right) + \exp \left(-\frac{\lambda_y \gamma_F}{\rho_R} \right) \left(1 - \exp \left(\frac{\lambda_w}{\alpha_B \rho_B} \right) \right) \right] & \text{if } \alpha_B < \frac{\gamma_N}{\gamma_N + \gamma_F + \gamma_F \gamma_N} \\ 1 + e^{-\lambda_w \chi_F} (1 - e^{-\lambda_x \chi_F}) - e^{-\lambda_x \chi_F} \left[\exp \left(\varsigma_1 + \frac{\lambda_y \phi \rho_B}{\rho_R \vartheta_1} - \max(\varsigma_1, 0) \right) \right. \\ \left. + e^{-\lambda_w \chi_F} \left(1 - \exp \left(\frac{\lambda_y \phi \rho_B}{\rho_R \vartheta_1} \right) \right) \right] & \text{if } \frac{\gamma_N}{\gamma_N + \gamma_F + \gamma_F \gamma_N} \leq \alpha_B < \frac{1}{1 + \gamma_F} \\ 1 & \text{otherwise.} \end{cases} \quad (33)$$

Proof. Refer to Appendix E. ■

Remark 4. In the absence of the direct link to U_F , we can use $W = 0$ in (31) to get

$$p_F^{DF} = 1 - \Pr \{ X > \max(\chi_N, \chi_F), Y \phi \rho_R > \gamma_F \} - \Pr \{ \chi_F < X < \chi_N, Y \rho_R > \gamma_F \}. \quad (34)$$

After solving (34) as in Theorem 4, an exact expression for p_F^{DF} is given by

$$p_F^{DF} = \begin{cases} 1 - \exp \left(-\frac{\lambda_y \gamma_F}{\rho_R \phi_R} - \lambda_x \chi_N \right) - \exp \left(-\frac{\lambda_y \gamma_F}{\rho_R} \right) \left[e^{-\lambda_x \chi_F} - e^{-\lambda_x \chi_N} \right] & \text{if } \alpha_B < \frac{\gamma_N}{\gamma_N + \gamma_F + \gamma_F \gamma_N} \\ 1 - \exp \left(-\frac{\lambda_y \gamma_F}{\rho_R \phi_R} - \lambda_x \chi_F \right) & \text{if } \frac{\gamma_N}{\gamma_N + \gamma_F + \gamma_F \gamma_N} \leq \alpha_B < \frac{1}{1 + \gamma_F} \\ 1 & \text{otherwise.} \end{cases} \quad (35)$$

Using $e^{-\theta} \simeq 1 - \theta$ in above and neglecting higher order terms of $\frac{1}{\rho_R}$, a high-SNR approximation to p_F^{DF} is given by

$$p_F^{DF} \simeq \begin{cases} \frac{\lambda_y \gamma_F}{\rho_R \phi_R} + \lambda_x \chi_F & \text{if } \alpha_B < \frac{1}{1 + \gamma_F} \\ 1 & \text{otherwise.} \end{cases} \quad (36)$$

2) *Near user outage probability*: NU is not in outage when a) R decodes both s_F and s_N ($\Gamma_{RF}^I \geq \gamma_F$, $\Gamma_{RN}^I \geq \gamma_N$) and SNRs at NU (after combining) are sufficient to decode both s_F and s_N ($\Gamma_{NF-S}^{COM} \geq \gamma_F$, $\Gamma_{NN-S}^{COM} \geq \gamma_N$), b) R decodes s_F but fails to decode s_N ($\Gamma_{RF}^I \geq \gamma_F$, $\Gamma_{RN}^I < \gamma_N$), and SNRs to decode s_F (after combining) and s_N (from first phase) are sufficient at NU ($\Gamma_{NF-F}^{COM} \geq \gamma_F$, $\Gamma_{NN}^I \geq \gamma_N$), and c) R fails to decode s_F ($\Gamma_{RF}^I < \gamma_F$), but the first phase SNRs are sufficient to decode both s_F and s_N at NU ($\Gamma_{NF}^I \geq \gamma_F$, $\Gamma_{NN}^I \geq \gamma_N$). The outage probability for U_N can be formulated as

$$p_N^{DF} = 1 - \Pr \left\{ \underbrace{\Gamma_{RF}^I < \gamma_F, \Gamma_{NF}^I \geq \gamma_F, \Gamma_{NN}^I \geq \gamma_N}_{B_1} \right\} - \Pr \left\{ \underbrace{\Gamma_{RF}^I \geq \gamma_F, \Gamma_{RN}^I < \gamma_N, \Gamma_{NF-F}^{COM} \geq \gamma_F, \Gamma_{NN}^I \geq \gamma_N}_{B_2} \right\} - \Pr \left\{ \underbrace{\Gamma_{RF}^I \geq \gamma_F, \Gamma_{RN}^I \geq \gamma_N, \Gamma_{NF-S}^{COM} \geq \gamma_F, \Gamma_{NN-S}^{COM} \geq \gamma_N}_{B_3} \right\}, \quad (37)$$

where $\Gamma_{NF-F}^{COM} = \Gamma_{NF}^I + \Gamma_{NF-F}^{II}$, $\Gamma_{NF-S}^{COM} = \Gamma_{NF}^I + \Gamma_{NF-S}^{II}$ and $\Gamma_{NN-S}^{COM} = \Gamma_{NN}^I + \Gamma_{NN-S}^{II}$.

Theorem 5. The exact closed-form expression of the outage probability for U_N is given by

$$p_N^{DF} = \begin{cases} 1 - e^{-\lambda_z \chi_N} (1 - e^{-\lambda_x \chi_N}) - e^{-\lambda_x \chi_N} \left[e^{-\lambda_z \chi_N} - \frac{\lambda_z \alpha_R \rho_R e^{-\frac{\lambda_v \gamma_N}{\alpha_R \rho_R}}}{\lambda_v \alpha_B \rho_B - \lambda_z \alpha_R \rho_R} \left[\exp \left(\frac{\max(0, \vartheta_5) \lambda_v \alpha_B \rho_B}{\alpha_R \rho_R} - \lambda_z \max(0, \vartheta_5) \right) \right. \right. \\ \left. \left. - \exp \left(\frac{\lambda_v \gamma_N}{\alpha_R \rho_R} - \lambda_z \chi_N \right) + \exp \left(\frac{\lambda_v \alpha_B \gamma_F}{\alpha_R \rho_R \phi} - \lambda_z \chi_F \right) \right] + \exp \left(\frac{\tilde{\chi}_N \lambda_v \alpha_B \rho_B}{\alpha_R \rho_R} - \tilde{\chi}_N \lambda_z \right) \right] + \exp \left(\frac{\phi_R \lambda_z \rho_R + \lambda_v \phi \rho_B}{\rho_R \vartheta_1} \right) \\ \times \left[\Gamma \left(1, \lambda_z \max(0, \chi_N - \vartheta_5); \tilde{\chi}_F \right) + \Gamma \left(1, \lambda_z \max(0, \vartheta_4 - \vartheta_5); \tilde{\chi}_F \right) - \Gamma \left(1, \lambda_z \chi_F - \vartheta_5 \lambda_z; \tilde{\chi}_F \right) \right. \\ \left. - \Gamma \left(1, \lambda_z (\chi_F - \vartheta_5); \tilde{\chi}_F \right) \right] & \text{if } \alpha_B < \frac{\gamma_N}{\gamma_N + \gamma_F + \gamma_F \gamma_N} \\ 1 - e^{-\lambda_z \chi_F} (1 - e^{-\lambda_x \chi_F}) - e^{-\lambda_x \chi_F} \left[e^{-\lambda_z \chi_F} - \frac{\lambda_z \alpha_R \rho_R e^{-\frac{\lambda_v \gamma_N}{\alpha_R \rho_R}}}{\lambda_v \alpha_B \rho_B - \lambda_z \alpha_R \rho_R} \left[\exp \left(\frac{\max(0, \vartheta_5) \lambda_v \alpha_B \rho_B}{\alpha_R \rho_R} - \lambda_z \max(0, \vartheta_5) \right) \right. \right. \\ \left. \left. - \exp \left(\frac{\tilde{\chi}_N \lambda_v \alpha_B \rho_B}{\alpha_R \rho_R} - \tilde{\chi}_N \lambda_z \right) \right] + \exp \left(\frac{\phi_R \lambda_z \rho_R + \lambda_v \phi \rho_B}{\rho_R \vartheta_1} \right) \left[\Gamma \left(1, \lambda_z \max(0, \vartheta_4 - \vartheta_5); \tilde{\chi}_F \right) - \Gamma \left(1, \lambda_z (\chi_F - \vartheta_5); \tilde{\chi}_F \right) \right. \right. \\ \left. \left. + \Gamma \left(1, \lambda_z \max(0, \chi_N - \vartheta_5); \tilde{\chi}_F \right) - \Gamma \left(1, \lambda_z \chi_N - \vartheta_5 \lambda_z; \tilde{\chi}_F \right) \right] \right] & \text{if } \frac{\gamma_N}{\gamma_N + \gamma_F + \gamma_F \gamma_N} \leq \alpha_B < \frac{1}{1 + \gamma_F} \\ 1 & \text{otherwise,} \end{cases} \quad (38)$$

where $\vartheta_4 = \frac{\phi \alpha_R + \gamma_N \phi \alpha_R + \gamma_N \alpha_B (1 - \alpha_R) - \alpha_B \phi_R}{\rho_B \alpha_B (\phi \alpha_R + \alpha_B (1 - \alpha_R))}$, $\vartheta_5 = \frac{-\phi_R}{\vartheta_1}$, $\vartheta_1 = \rho_B (\phi \alpha_R + \alpha_B (1 - \alpha_R))$, $\tilde{\chi}_F = \frac{\lambda_z \lambda_v \phi \rho_B (\chi_F + \frac{\phi_R}{\vartheta_1})}{\vartheta_1}$

and $\tilde{\chi}_N = \min(\chi_F, \chi_N, \vartheta_4)$.

Proof. Refer to Appendix F. ■

In the following lemma, we present a high-SNR approximate expression for p_N^{DF} to obtain the diversity order at NU.

Lemma 4. A high-SNR approximation to p_N^{DF} is given by

$$p_N^{DF} \simeq \begin{cases} 1 - e^{-\lambda_z \chi_N} (1 - e^{-\lambda_x \chi_N}) - e^{-\lambda_x \chi_N} \left[e^{-\lambda_z \chi_N} + \frac{\lambda_z e^{-\varepsilon_3 \gamma_N}}{\varepsilon_3 \alpha_B \rho_B - \lambda_z} \left[\exp(\varepsilon_3 \gamma_N - \lambda_z \chi_N) - \exp\left(\frac{\varepsilon_3 \alpha_B \gamma_F}{\phi} - \lambda_z \chi_F\right) \right. \right. \\ \left. \left. + \exp(\min(\chi_F, \vartheta_4) \varepsilon_3 \alpha_B \rho_B - \lambda_z \min(\chi_F, \vartheta_4) + \max(0, \vartheta_5) \varepsilon_3 \alpha_B \rho_B - \lambda_z \max(0, \vartheta_5)) \right] \right] & \text{if } \alpha_B < \frac{\gamma_N}{\gamma_N + \gamma_F + \gamma_F \gamma_N} \\ 1 - e^{-\lambda_z \chi_F} (1 - e^{-\lambda_x \chi_F}) - e^{-\lambda_x \chi_F} \left[e^{-\lambda_z \chi_F} - \frac{\lambda_z e^{-\varepsilon_3 \gamma_N}}{\varepsilon_3 \alpha_B \rho_B - \lambda_z} \left[\exp(\max(0, \vartheta_5) \varepsilon_3 \alpha_B \rho_B - \max(0, \vartheta_5) \lambda_z) \right. \right. \\ \left. \left. - \exp(\min(\chi_N, \vartheta_4) \varepsilon_3 \alpha_B \rho_B - \lambda_z \min(\chi_N, \vartheta_4)) \right] \right] & \text{if } \frac{\gamma_N}{\gamma_N + \gamma_F + \gamma_F \gamma_N} \leq \alpha_B < \frac{1}{1 + \gamma_F} \\ 1 & \text{otherwise.} \end{cases} \quad (39)$$

where $\varepsilon_3 = \frac{\lambda_v}{\alpha_R \rho_R}$.

Proof. Proof is similar to that used to derive (33), and is therefore omitted. ■

Lemma 5. The diversity orders D_F^{DF} and D_N^{DF} attained by U_F and U_N can be expressed as

$$D_F^{DF} = \begin{cases} 2 & \text{if } \alpha_B < \frac{1}{1 + \gamma_F}, \alpha_R < \frac{1}{1 + \gamma_F} \\ 1 & \text{if } \alpha_B < \frac{1}{1 + \gamma_F}, \alpha_R \geq \frac{1}{1 + \gamma_F} \\ 0 & \text{otherwise.} \end{cases}, \quad D_N^{DF} = \begin{cases} 2 & \text{if } \alpha_B < \frac{1}{1 + \gamma_F}, \alpha_R < \frac{1}{1 + \gamma_F} \\ 1 & \text{if } \alpha_B < \frac{1}{1 + \gamma_F}, \alpha_R \geq \frac{1}{1 + \gamma_F} \\ 0 & \text{otherwise.} \end{cases} \quad (40)$$

Proof. The diversity orders for U_F and U_N are obtained by using high-SNR expressions of p_F^{DF} and p_N^{DF} given by (33) and (39), respectively. We substitute $\rho_B = \rho_R = \rho$ and solve for $D_j^{DF} = -\lim_{\rho \rightarrow \infty} \frac{\log_2 p_j^{DF}(\rho)}{\log_2 \rho}$, $j \in \{N, F\}$. Details are omitted due to paucity of space. ■

Remark 5. In the absence of direct link to U_F , the diversity order D_F^{DF} is obtained by using $\rho_B = \rho_R = \rho \rightarrow \infty$ into (35) and using linear approximations to the exponential terms. The diversity order D_F^{DF} can be expressed as

$$D_F^{DF} = \begin{cases} 1 & \text{if } \alpha_B < \frac{1}{1 + \gamma_F}, \alpha_R < \frac{1}{1 + \gamma_F} \\ 0 & \text{otherwise.} \end{cases} \quad (41)$$

However, the diversity order for U_N remains the same as in (40).

IV. NU THROUGHPUT AND ENERGY EFFICIENCY MAXIMIZATION

NU needs to decode the FU symbol and perform SIC to decode its own symbol, and might therefore incur some performance loss. For this reason, maximizing the NU throughput while guaranteeing a desired FU target throughput $\tilde{\tau}$ is well motivated. We do so in this section. The FU and NU throughput in bits per channel use (bpcu) are defined as

$$\tau_j^i = \frac{1}{2} R_j (1 - p_j^i), \quad j \in \{N, F\}, i \in \{AF, DF\} \quad (42)$$

where p_F^{AF} , p_N^{AF} , p_F^{DF} and p_N^{DF} are as in (19), (28), (32) and (38) respectively.

Careful choice of power allocation coefficient α_B and symbol rates R_F and R_N is crucial to attaining good throughput at U_N and U_F . With DF relays, α_R can also be chosen. However it will be demonstrated in Section V that a large range of α_R values result in the same throughput performance. The reason for this is that α_R is only used when both s_F and s_N are decoded correctly at R (which in turn depends on α_B). We therefore focus only on choice of α_B , R_N and R_F .

We first consider the case when the B- U_F link is absent. Noting that the NU benefits when the maximum value of α_B is chosen, we derive α_B^{opt} by using (23) and (36) into (42) to get

$$\alpha_B^{AF^{opt}} \simeq \frac{1}{1 + \gamma_F} - \frac{\gamma_F R_F (\lambda_x \rho_R + \lambda_y \rho_B)}{\rho_B \rho_R (1 + \gamma_F) (R_F - 2\tau_F^T)}, \quad (43)$$

$$\alpha_B^{DF^{opt}} \simeq \frac{\gamma_F \lambda_y \rho_B - (\alpha_R + \alpha_R \gamma_F - 1) (\gamma_F \lambda_x + (\frac{2\tilde{\tau}}{R_F} - 1) \rho_B) \rho_R}{\rho_B (1 + \gamma_F) (\gamma_F \lambda_y - \rho_R (\frac{2\tilde{\tau}}{R_F} - 1) (\alpha_R + \alpha_R \gamma_F - 1))}. \quad (44)$$

The NU throughput for AF and DF relaying schemes can be obtained by substituting $\alpha_B = \alpha_B^{opt}$ into expressions for τ_N^i . Clearly, τ_N^i is now a function only of symbol rates R_N and R_F (and therefore γ_N and γ_F). Thus, careful choice of symbol rates is essential for maximizing the NU throughput. Let R_N^{opt} and R_F^{opt} denote the optimal symbol rates that maximize τ_N^i . Now, the optimization problem is formulated as

$$R_N^{opt}, R_F^{opt} = \arg \max_{R_N, R_F} \tau_N^i(R_N, R_F) \quad (45)$$

Solving the above joint optimization problem analytically is difficult. However, numerical techniques can be used to pick the optimal R_N and R_F .

In the presence of the B- U_F link, a three-dimensional search is required to determine α_B , R_N and R_F while ensuring that $\tau_F^i \geq \tilde{\tau}$.

Energy efficiency is an important performance metric for any communication system. It is defined as [14]:

$$\eta_E^i = \frac{\text{Sum throughput}}{\text{Total energy consumed}}, \quad i \in \{AF, DF\}, \quad (46)$$

where 'sum throughput' represents the sum of τ_N^i and τ_F^i . Let T denote the signalling duration. Let $E_B = \frac{P_B T}{2}$ and $E_R = \frac{P_R T}{2}$. Using τ_N^i , and τ_F^i from (42), the EE of the system is expressed as

$$\eta_E^i = \frac{\tau_{sum}^i}{E_B + E_R}, \quad \text{where} \quad \tau_{sum}^i = \tau_F^i + \tau_N^i. \quad (47)$$

Since the sum throughput τ_{sum}^i is function of symbol rates R_N and R_F , careful choice of symbol rates is essential for maximizing the EE. Let R_N^{opt} and R_F^{opt} denote the optimal symbol rates that maximize η_E^i . The EE optimization problem can be formulated as

$$R_N^{opt}, R_F^{opt} = \arg \max_{R_N, R_F} \eta_E^i(R_N, R_F) \quad (48)$$

Obtaining a closed-form solution to the above joint optimization problem is extremely difficult. However, a two dimensional search can be used to find the optimal R_N and R_F .

V. SIMULATIONS AND NUMERICAL RESULTS

In this section we present computer simulations to confirm accuracy of the derived analytical expressions and derive useful insights. Unless mentioned otherwise, the considered system parameters are as follows: $d_{BR} = d_{RF} = d_{RN} = 2$, $d_{BN} = 1.2$, $d_{BF} = 4$, and $m = 4$. Also σ^2 is normalized to unity [15]. Further, we assume that $P_B = P_R = P$ ($\rho_B = \rho_R = \rho$) [14] and symbol rates $R_N = R_F = R$ ($\gamma_N = \gamma_F = \gamma_T$). We also compare the performance of the proposed schemes with its relayed OMA counterpart described in Section II.

In Fig.2 and Fig.3, we plot p_N^{AF} and p_F^{AF} versus transmit SNR (ρ) for different values of α_B and R . The accuracy of derived analytical expressions in (19), (21) and (28) can be clearly confirmed by simulations. Also the derived high-SNR approximations in (23) and (20) are quite accurate. With decreasing α_B , more power is allocated to U_F , and p_F^{AF} therefore continuously improves as shown

Fig. 2: p_F^{AF} vs. ρ .

Fig. 3: p_N^{AF} vs. ρ .

Fig. 4: p_F^{DF} vs. ρ .

Fig. 5: p_N^{DF} vs. ρ . Fig. 6: τ_F^{AF} or τ_F^{DF} vs. ρ . Fig. 7: τ_N^{AF} or τ_N^{DF} vs. ρ .

in Fig.2. It is evident from Fig.3 that an initial increase in α_B significantly improves p_N^{AF} due to successful decoding of x_F and x_N at U_N , while further increase in α_B deteriorates p_N^{AF} as U_N fails to decode x_F . From Fig.2 and Fig.3 it is clear that (unlike in all CDRT schemes proposed so far) combining of incoming signals from B and R at U_N as well as U_F in the proposed CDRT framework ensures a full diversity of 2 to be harnessed (as derived in Lemma 2 for the case of AF relaying) by both users. However, in the absence of the $B-U_F$ link, diversity order of 1 is attained at U_F (as noted in Remark 3). The switching of diversity order from 2 to 1 and 0 can also be observed when changing α_B from 0.1, 0.3 and 0.4 for $R=2$. Therefore, a careful choice of power allocation coefficient is important to attain full diversity at both users.

Fig.4 and Fig.5 depict the variation of p_N^{DF} and p_F^{DF} versus ρ for different values of α_B , α_R and R . Simulations confirm the accuracy of the analytical expressions derived in (32), (35) and (38) respectively. Also, the approximate high-SNR expressions for outage probability derived in (36) and (33) are quite accurate. A decrease in α_B and α_R signifies that more power is allocated to U_F , and p_F^{DF} therefore continuously improves with decreasing α_B and α_R as shown in Fig.4. Clearly, from Fig.5, p_N^{DF} significantly improves with increasing α_B initially (due to successful decoding of x_F and x_N at U_N), while further increase in α_B degrades p_N^{AF} (U_N fails to decode x_F). Similar to the case of AF relaying, full diversity order of two can be attained (derived in Lemma 5) for both U_N and U_F in the presence of the direct link. While in the absence of direct link to U_F , a diversity of order 1 can be achieved (as noted in Remark 5), while diversity of 2 is achieved by U_N .

Fig.6 shows the variation of τ_F^i versus ρ at $R=1,2$. Here, we compare both DF and AF relaying schemes considering optimal power allocation coefficients α_B and α_R . In the presence of the $B-U_F$ link, performance with DF is superior to that of AF for lower SNRs. At high SNRs, the performance of both AF and DF is comparable. However, in the absence of $B-U_F$ link, performance with DF is

always superior. AF relays are often preferred due to their simplicity, and can be seen to result in good performance except at low SNRs. Fig.7 depicts τ_N^i versus ρ for $R = 1, 2$. For fair comparison of AF and DF relaying, optimal power allocation coefficients are used at B and R. We observe that both AF and DF achieve almost similar performance in terms of near-user throughput³.

Fig. 8: τ_N^{AF} or τ_F^{AF} vs. α_B with $\rho = 30$ dB .

Fig. 9: τ_N^{DF} or τ_F^{DF} vs. α_B with $\rho = 30$ dB .

Fig.8 depicts the variation of τ_N^{AF} or τ_F^{AF} versus α_B for $R = 2, 3$. When α_B is very small, more power is allocated to U_F and τ_F^{AF} therefore is maximum at $\alpha_B = 0$. With increase in α_B , τ_F^{AF} gradually decreases and becomes 0 for $\alpha_B \geq \frac{2}{2+\gamma_F}$ due to unsuccessful decoding of s_F at U_F . However, in the absence of the $B - U_F$ link, τ_F^{AF} becomes zero for $\alpha_B \geq \frac{1}{1+\gamma_F}$. Therefore, the presence of $B - U_F$ link significantly improves the FU performance and allows a wider range of valid power allocations. On the other hand τ_N^{AF} increases initially and attains a maximum value (at $\alpha_B = \frac{1}{1+\gamma_F}$). Further increase in α_B decreases the probability of successful decoding of s_F at U_N , and τ_N^{AF} therefore also decreases.

In Fig.9, τ_N^{DF} or τ_F^{DF} is plotted versus α_B (considering optimal α_R) and α_R (considering optimal α_B) for $R = 1.5, 2, 2.5$. As with AF relaying, τ_F^{DF} is maximum at $\alpha_B = 0$ and then decreases to 0 with increasing α_B . τ_N^{DF} increases initially, attains a maximum value at $\alpha_B = \frac{\gamma_N}{\gamma_N + \gamma_F + \gamma_F \gamma_N}$, and then decreases to 0 (the SINR to decode s_F at NU decreases). It can be observed that presence of the $B - U_F$ significantly improves the FU throughput. Also, both τ_F^{DF} and τ_N^{DF} change significantly with α_B while change in α_R does not significantly change the throughput⁴. Hence, an optimal choice of α_B is important. For this reason, we study variation of energy efficiency w.r.t α_B only. From Fig.8 and Fig.9, it can be clearly seen that CDRT NOMA with AF relaying provides wider range of valid power

³Although DF relay R offers additional degree of freedom in terms of α_R (power allocation at R), AF achieves performance almost similar to it. This is because in CDRT NOMA with DF relaying, power allocation at R takes place if and only if R decodes both FU and NU symbols successfully.

⁴Since transmission from R depends on successful decoding of s_F , power allocation at S is crucial. Also at R power allocation is required only if both s_F and s_N get successfully decoded, an optimal choice of α_B is therefore more important in comparison to α_R .

allocations in comparison to DF relaying for both U_N and U_F . The presence of the $B - U_F$ widens the valid range of power allocations for U_F with AF relaying (but not with DF relaying).

In Fig.10, we plot sum throughput $\tau_{sum}^i = \tau_F^i + \tau_N^i$ versus $R = R_N = R_F$ using optimal α_B and α_R . With increase in SNR τ_{sum}^i also increases and τ_{sum}^i is a quasi-concave function of R , and a unique value R^{opt} exists at which τ_{sum}^i is maximum. Clearly, CDRT NOMA with DF outperforms that with AF in terms of sum throughput.

Fig.11 illustrates the effect of transmit power ($P_B = P_R = P$) on the energy efficiency of AF or DF relay-aided CDRT NOMA. It can be observed that at $\rho = 30$ dB and $R = 2$, with $P = 2W$ $\eta_E^{AF} = 0.955$ and $\eta_E^{DF} = 0.945$, while at $P = 3W$, $\eta_E^{AF} = 0.637$ and $\eta_E^{DF} = 0.630$. Thus, EE improves with decreasing P . In addition to this we observe that for mid-SNR range DF outperforms AF, while at low as well as at high SNRs both achieve similar performance in terms of EE. However, in the absence of the $B - U_F$ link, DF outperforms AF for a wider range of SNRs and the gap is more as compared to the case when the $B - U_F$ link is present.

In Fig.12, we plot η_E^{AF} and η_E^{DF} versus α_B with $R = 1$ and $\rho = 30$ dB for $P = 1W$ and $P = 2W$. We also compare the EE of NOMA CDRT with its relayed OMA counterpart. It is evident that EE of both AF and DF relay-aided NOMA in CDRT are higher than that of relayed OMA. This is largely because of the fact that the signals from both B and R are combined at NU as well as FU. It can be seen that in the OMA case, EE with DF is better than that with AF, whereas in CDRT NOMA, AF is more energy efficient. We also show that the presence of the $B - U_F$ link significantly improves the EE (the gap is larger with AF as compared to DF).

In Fig.13, we plot the variation of η_E^{AF} and η_E^{DF} versus α_B for fixed transmit power ($P = 3W$), $\rho = 30$ dB and $R = 1, 2, R^{opt}$. Clearly, the energy efficiencies of AF as well as DF increase as we increase the

Fig. 10: τ_{sum}^{AF} or τ_{sum}^{DF} vs. R .

Fig. 11: η_E^{AF} or η_E^{DF} vs. ρ .

Fig. 12: η_E^{AF} or η_E^{DF} vs. α_B .

Fig. 13: η_E^{AF} or η_E^{DF} vs. α_B . Fig. 14: τ_N^{AF} vs. R_N . Fig. 15: τ_N^{DF} vs. R_N . Fig. 16: P_N^i vs. ρ . Fig. 17: τ_{sum}^i vs. R . Fig. 18: η_E^i vs. α_B .

symbol rate from 1 to 2, and the improvement is huge when the optimal symbol rate (R^{opt}) is used. Further, this is true for the entire range of the power allocation parameter. We also observe that EE of relay-aided NOMA CDRT is significantly higher than that of OMA.

Fig.14 and Fig.15 depict τ_N^{AF} and τ_N^{DF} versus R_N for NOMA in CDRT with AF and DF relaying, respectively for different $\tau_F = \tilde{\tau}$ and $\rho = 25$ dB. It can be seen that τ_N increases with decreasing $\tilde{\tau}$. In case of both DF and AF aided NOMA in CDRT, we observe that by optimally choosing R_N , the NU throughput can be maximized while guaranteeing the desired FU throughput $\tilde{\tau}$. Also, selecting an optimal rate ($R_F = R_F^{opt}$) helps to provide huge gain in NU throughput in comparison to any fixed rate R_F . Therefore, the NU throughput attains a maximum performance at (R_N^{opt}, R_F^{opt}) . Clearly, jointly optimizing R_N and R_F (as in Section IV) is of utmost importance. It can also be observed that presence of the $B - U_F$ significantly improves the NU performance.

In Fig.16, Fig.17 and Fig.18, we compare the performance of proposed AF as well as DF relay-aided

CDRT NOMA with selective decode and forward (SDF) CDRT NOMA of [26] in terms of outage probability, sum throughput and energy efficiency, respectively. It is clear from Fig.16 that for optimal power allocation, the proposed AF as well as DF relay-aided CDRT NOMA outperform the SDF CDRT NOMA framework of [26] by a huge margin. The SDF CDRT NOMA achieves diversity order of one, whereas, proposed schemes can achieve a diversity of two at both the users. It is clear from Fig.17 that the proposed scheme almost doubles the sum throughput in comparison to SDF-CDRT NOMA framework of [26]. Fig.18 illustrates that the proposed scheme is also more energy efficient.

VI. CONCLUSION

In this paper, we analyzed the performance of new framework for downlink non-orthogonal multiple access (NOMA) in a coordinated direct and relay transmission (CDRT) scheme with direct link to both near-user (NU) and far-user (FU). In this framework, NU performs combining of direct and relayed signals while performing successive interference cancellation, which considerably improves NU outage performance, and enables it to attain a diversity of two without feedback. This scheme is very useful in multimedia and other applications where the NU has stringent outage constraints. The FU also can attain a diversity of two. Considering either amplify-and-forward (AF) or decode-and-forward (DF) relaying, closed-form expressions for outage probability and throughput were derived for both NU and FU. In spite of the fact that DF relaying allows for different choice of power allocation at the relay, performance with simpler AF relays is comparable to that with DF relays. We demonstrate that careful choice of power allocations coefficient and the target rates is important to maximize throughput and energy efficiency.

APPENDIX A

Proof of Theorem 1: Substituting $\zeta_{F_1}(w)$ from line following (13) into (17), I_1 can be expressed as

$$I_1 = \int_0^{\chi_F} \lambda_w \exp\left(\frac{-\xi_1(w - \chi_F)(\lambda_x \rho_R + \lambda_y \rho_B)}{\rho_R(w - \xi_1)}\right) \frac{-2\xi_1(w - \chi_F)\lambda_x \lambda_y}{\rho_R(w - \xi_1)} K_1\left(\frac{-2\xi_1(w - \chi_F)\lambda_x \lambda_y}{\rho_R(w - \xi_1)}\right) e^{-\lambda_w w} dw. \quad (49)$$

For large values of ρ_B and ρ_R , the argument inside the Bessel function becomes very small throughout the range of SNRs (as verified in Section V). We therefore use $K_1(\theta) \simeq \frac{1}{2}\Gamma(1)(\frac{\theta}{2})^{-1}$ [32, 9.6.9]. Using $\lambda_w(w - \xi_1) = w$ we obtain

$$I_1 \simeq e^{-\xi_1(\lambda_w + \frac{1}{\rho_R})} \int_{-\xi_1 \lambda_w}^{\lambda_w(\chi_F - \xi_1)} \exp\left(-w - \frac{\lambda_w \xi_1(\xi_1 - \chi_F)(\lambda_x \rho_R + \lambda_y \rho_B)}{\rho_R w}\right) dw. \quad (50)$$

Using $\int_x^\infty t^{a-1} e^{-t-\frac{b}{t}} dt = \Gamma(a, x; b)$ [33, eq. (4)] in the above, we obtain

$$I_1 \simeq e^{-\xi_1(\lambda_w + \frac{1}{\rho_R})} \left[\Gamma(1, \xi_1 \lambda_w; \vartheta_6) - \Gamma(1, \lambda_w(\chi_F - \xi_1); \vartheta_6) \right], \quad (51)$$

where $\vartheta_6 = \frac{\lambda_w \xi_1 (\xi_1 - \chi_F) (\lambda_x \rho_R + \lambda_y \rho_B)}{\rho_R}$. Similarly, we solve for I_2 as

$$I_2 \simeq e^{-\xi_1(\lambda_w + \frac{1}{\rho_R})} \int_0^\infty \exp\left(-w - \frac{\lambda_w \xi_1 (\xi_1 - \chi_F) (\lambda_x \rho_R + \lambda_y \rho_B)}{\rho_R w}\right) dw. \quad (52)$$

Using $\int_0^\infty \exp(-\frac{b}{4t} - at) dt = \sqrt{\frac{b}{a}} K_1(\sqrt{ab})$ [31, 3.324.1] in the above, we obtain

$$I_2 \simeq e^{-\xi_1(\lambda_w + \frac{1}{\rho_R})} \sqrt{4\vartheta_6} K_1(\sqrt{4\vartheta_6}). \quad (53)$$

After substituting I_1 and I_2 from (51) and (53) into (14), we obtain (19).

APPENDIX B

Proof of Lemma 1: After some mathematical arrangements, (16) can be expressed as

$$I_1 = \int_0^{\chi_F} \exp\left(-\zeta_{F_1}(w) \left(\lambda_x + \frac{\lambda_y \rho_B}{\rho_R}\right)\right) \int_0^\infty \lambda_x \frac{\exp(-\lambda_x x)}{\exp\left(\frac{\lambda_y \zeta_{F_1}(w) [1 + \rho_B \zeta_{F_1}(w)]}{x \rho_R}\right)} \lambda_w e^{-\lambda_w w} dx dw. \quad (54)$$

For large ρ_B and ρ_R , we use the linear approximation to the exponential term in denominator to obtain

$$I_1 \simeq \int_0^{\chi_F} \exp\left(-\zeta_{F_1}(w) \left(\lambda_x + \frac{\lambda_y \rho_B}{\rho_R}\right)\right) \left[\int_0^\infty \lambda_x e^{-\lambda_x x} dx - \xi_2(w) \int_0^\infty \frac{\lambda_x e^{-\lambda_x x}}{x + \xi_2(w)} dx \right] \lambda_w e^{-\lambda_w w} dw, \quad (55)$$

where $\xi_2(w) \simeq \frac{\lambda_y \zeta_{F_1}(w) [1 + \rho_B \zeta_{F_1}(w)]}{\rho_B}$. Using [31, 3.352.6] in the above, we obtain

$$I_1 \simeq \int_0^{\chi_F} \exp\left(-\zeta_{F_1}(w) \left(\lambda_x + \frac{\lambda_y \rho_B}{\rho_R}\right)\right) \left(1 - \xi_2(w) \lambda_x e^{\xi_2(w) \lambda_x} \mathcal{E}_1(\xi_2(w) \lambda_x)\right) \lambda_w e^{-\lambda_w w} dw. \quad (56)$$

Further using $e^{\xi_2(w) \lambda_x} \mathcal{E}_1(\xi_2(w) \lambda_x) = \frac{1}{1 + \xi_2(w) \lambda_x}$ [32, 5.1.19] and simplifying, we obtain

$$I_1 \simeq \int_0^{\chi_F} \frac{\lambda_w e^{-\lambda_w w}}{1 + \xi_2(w) \lambda_x} \exp\left(-\zeta_{F_1}(w) \left(\lambda_x + \frac{\lambda_y \rho_B}{\rho_R}\right)\right) dw. \quad (57)$$

Substituting for $\zeta_{F_1}(w)$ from the line following (18) and neglecting higher-order terms of $1/\rho_B$, we get

$$I_1 \simeq 1 - e^{-\lambda_w \chi_F} - \left(\lambda_x + \frac{\lambda_y \rho_B}{\rho_R}\right) \xi_1 \left[\int_0^{\chi_F} \lambda_w e^{-\lambda_w w} dw + (\xi_1 - \chi_F) \lambda_w \int_0^{\chi_F} \frac{e^{-\lambda_w w}}{w - \xi_1} dw \right]. \quad (58)$$

We use [31, 3.352.1] to obtain

$$I_1 \simeq 1 - e^{-\lambda_w \chi_F} - \left(\lambda_x + \frac{\lambda_y \rho_B}{\rho_R}\right) \xi_1 \left[1 - e^{-\lambda_w \chi_F} + (\xi_1 - \chi_F) \lambda_w e^{-\lambda_w \xi_1} [\mathcal{E}_1(-\lambda_w \xi_1) - \mathcal{E}_1(-\lambda_w \xi_1 + \lambda_w \chi_F)] \right]. \quad (59)$$

Similarly, the expression for I_2 is given by

$$I_2 \simeq \frac{e^{-\lambda_w \xi_1}}{1 + \xi_1(\lambda_x + \lambda_y)}. \quad (60)$$

After substituting for I_1 and I_2 from (59) and (60) into (14), the high SNR approximation to p_F^{AF} is given by (20).

APPENDIX C

Proof of Theorem 3: Using (25), $p_N^{AF} = 1 - (I_5 + I_3)U(\phi) - I_4U(\phi - \alpha_B) - I_6[U(\phi) - U(\phi - \alpha_B)] - I_7U(-\phi)$. We first solve for $I_4 = \Pr\left\{V > \zeta_{N_1}(X, Z), X > \chi_{N_1}(Z), \chi_F < Z < \chi_N\right\}$ as follows:

$$I_4 = \int_{\chi_F}^{\chi_N} \int_{\chi_{N_1}(z)}^{\infty} \int_{\zeta_{N_1}(x, z)}^{\infty} f_V(v) f_X(x) f_Z(z) dv dx dz = \int_{\chi_F}^{\chi_N} \int_{\chi_{N_1}(z)}^{\infty} \lambda_x \lambda_z \exp\left(-\lambda_v \zeta_{N_1}(x, z) - \lambda_x x - \lambda_z z\right) dx dz. \quad (61)$$

Substituting $x = x - \chi_{N_1}(z)$ in the above and rearranging, we obtain

$$I_4 = \int_{\chi_F}^{\chi_N} \int_0^{\infty} \exp\left(-\chi_{N_1}(z) \left[\lambda_x + \frac{\lambda_v \rho_B}{\rho_R}\right]\right) \frac{\lambda_x e^{-\lambda_x x} \lambda_z e^{-\lambda_z z}}{\exp\left(\frac{\mu_{N_1}(z)}{\lambda_x x}\right)} dx dz, \quad (62)$$

where $\mu_{N_1}(z) = \frac{\lambda_x \lambda_v \chi_{N_1}(z) [\rho_B \chi_{N_1}(z) + 1]}{\rho_R}$. Solving the above using [31, 3.324.1], we obtain

$$I_4 = \int_{\chi_F}^{\chi_N} e^{-\chi_{N_1}(z) \left(\lambda_x + \frac{\lambda_v \rho_B}{\rho_R}\right)} 2 \mu_{N_1}(z) K_1(2 \mu_{N_1}(z)) \lambda_z e^{-\lambda_z z} dz. \quad (63)$$

The above integral does not admit a closed form. To derive a highly accurate closed-form expression (as with I_1) we use $K_1(\theta) \simeq \frac{1}{2} \Gamma(1) \left(\frac{\theta}{2}\right)^{-1}$ [32, 9.6.9] to obtain

$$I_4 \simeq \lambda_z e^{-C_\ell \chi_N} \int_{\chi_F - \chi_N}^0 e^{Cz} dz, \quad (64)$$

where $C = C_\ell - \lambda_z$ with $C_\ell = \lambda_x + \frac{\lambda_v \rho_B}{\rho_R}$. After solving the above integral, I_4 is obtained as

$$I_4 \simeq \lambda_z e^{-\chi_N C_\ell} \left[\frac{1 - e^{C(\chi_F - \chi_N)}}{C} \right]. \quad (65)$$

Using a similar approach, I_5 , I_6 and I_7 are obtained as

$$I_5 \simeq \frac{\lambda_z e^{-\chi_N C_\ell}}{C} \left[e^{C(\min(\chi_F, \chi_N) - \chi_N)} - e^{-C \chi_N} \right], \quad (66)$$

$$I_6 \simeq e^{\xi_1(C_\ell + \lambda_z)} \left[e^{-(\xi_1 + \chi_N)} e^{-(\xi_1 + \chi_F)} \lambda_z \vartheta_2 e^{\lambda_z \vartheta_2} \left[\mathcal{E}_1(\lambda_z(\chi_N + \xi_1 + \vartheta_2)) - \mathcal{E}_1(\lambda_z(\chi_F + \xi_1 + \vartheta_2)) \right] \right], \quad (67)$$

$$\text{and } I_7 \simeq e^{\xi_1(C_\ell + \lambda_z)} \left[e^{-2\xi_1} - \lambda_z \vartheta_2 e^{\lambda_z \vartheta_2} \left[\mathcal{E}_1(\lambda_z(2\xi_1 + \vartheta_2)) \right] \right], \quad (68)$$

respectively, where $\vartheta_2 = \xi_1 \left(\xi_1 + \frac{\gamma_F}{\rho_B \phi} \right) C_\ell$. Finally, I_3 can be derived as

$$I_3 = \Pr\{Z > \max(\chi_F, \chi_N)\} = e^{-\lambda_z \max(\chi_F, \chi_N)}. \quad (69)$$

After substituting the derived expressions of I_3 , I_4 , I_5 , I_6 and I_7 from (69), (65), (66), (67) and (68), respectively, into (25), p_N^{AF} is expressed as in (28).

APPENDIX D

Proof of Theorem 4: Using (3), (8) and (10) into (31), $\Pr\{A_1\}$ can be expressed as

$$\begin{aligned} \Pr\{A_1\} &= \Pr\left\{\frac{(1-\alpha_B)\rho_B X}{\alpha_B \rho_B X + 1} \geq \gamma_F, \alpha_B \rho_B X \geq \gamma_N, \frac{(1-\alpha_R)\rho_R Y}{\alpha_R \rho_R Y + 1} + \frac{(1-\alpha_B)\rho_B W}{\alpha_B \rho_B W + 1} \geq \gamma_F\right\} \\ &= \Pr\{X \geq \max(\chi_F, \chi_N)\} \left[\Pr\{W \geq \chi_F\} + \Pr\left\{\chi_F > W > \max\left(0, \frac{-\phi_R}{\rho_B(\alpha_B(1-\alpha_R) + \alpha_R \phi)}\right)\right. \right. \\ &\quad \left. \left. Y \geq \frac{(\gamma_F - \phi \rho_B W)}{\rho_R(\phi_R + (1-\alpha_R)\alpha_B \rho_B W + \alpha_R \phi \rho_B W)}\right\} \right]. \quad (70) \end{aligned}$$

Since the RVs W , X and Y are independent and have an exponential distribution, we obtain

$$\Pr\{A_1\} = e^{-\lambda_x \max(\chi_F, \chi_N)} \left[e^{-\lambda_w \chi_F} + \int_{\max(0, -\frac{\phi_R}{\vartheta_1})}^{\chi_F} \lambda_w \exp\left(\frac{\lambda_y \phi \rho_B (w - \chi_F)}{\rho_R \vartheta_1 (w + \frac{\phi_R}{\vartheta_1})}\right) e^{-\lambda_w w} dw \right]. \quad (71)$$

After substituting $\lambda_w(w + \frac{\phi_R}{\vartheta_1}) = w$ and solving, we obtain

$$\Pr\{A_1\} = e^{-\lambda_x \max(\chi_F, \chi_N)} \left[e^{-\lambda_w \chi_F} + \exp\left(\frac{\lambda_w \phi_R}{\vartheta_1} + \frac{\lambda_y \phi \rho_B}{\rho_R \vartheta_1}\right) \int_{\max(\frac{\lambda_w \phi_R}{\vartheta_1}, 0)}^{\lambda_w(\chi_F + \frac{\phi_R}{\vartheta_1})} \exp\left(\frac{-\lambda_y \lambda_w \phi \rho_B (\frac{\phi_R}{\vartheta_1} + \chi_F)}{\rho_R \vartheta_1 w}\right) e^{-w} dw \right]. \quad (72)$$

Using an approach similar to that used to derive (50), we obtain

$$\begin{aligned} \Pr\{A_1\} &= e^{-\lambda_x \max(\chi_F, \chi_N)} \left[\exp\left(\frac{\lambda_w \phi_R}{\vartheta_1} + \frac{\lambda_y \phi \rho_B}{\rho_R \vartheta_1}\right) \left\{ \Gamma\left(1, \max\left(\frac{\lambda_w \phi_R}{\vartheta_1}, 0\right); \frac{-\lambda_y \lambda_w \phi \rho_B (\frac{\phi_R}{\vartheta_1} + \chi_F)}{\rho_R \vartheta_1}\right) \right. \right. \\ &\quad \left. \left. - \Gamma\left(1, \lambda_w(\chi_F + \frac{\phi_R}{\vartheta_1}); \frac{-\lambda_y \lambda_w \phi \rho_B (\frac{\phi_R}{\vartheta_1} + \chi_F)}{\rho_R \vartheta_1}\right) \right\} + e^{-\lambda_w \chi_F} \right]. \quad (73) \end{aligned}$$

Similarly, the expression for $\Pr\{A_2\}$ is obtained as

$$\begin{aligned} \Pr\{A_2\} &= (e^{-\lambda_x \chi_F} - e^{-\lambda_x \chi_N}) \left[\exp\left(-\frac{\lambda_y \gamma_F}{\rho_R}\right) + \exp\left(\frac{\lambda_w}{\alpha_B \rho_B} + \frac{\lambda_y \phi}{\alpha_B \rho_R}\right) \left\{ \Gamma\left(1, \frac{\lambda_y \phi}{\alpha_B \rho_R}; \frac{\lambda_y \lambda_w (\phi + \alpha_B \gamma_F)}{\alpha_B^2 \rho_B \rho_R}\right) \right. \right. \\ &\quad \left. \left. - \Gamma\left(1, \frac{\lambda_y}{\rho_R}(\gamma_F + \frac{\phi}{\alpha_B}); \frac{\lambda_y \lambda_w (\phi + \alpha_B \gamma_F)}{\alpha_B^2 \rho_B \rho_R}\right) \right\} \right]. \quad (74) \end{aligned}$$

Further, using (3) and (8), $\Pr\{A_3\}$ of (31) can be expressed as

$$\Pr\{A_3\} = e^{-\lambda_w \chi_F} (1 - e^{-\lambda_x \chi_F}). \quad (75)$$

Using (73), (74) and (75) in (31), p_F^{DF} is obtained as in (32).

APPENDIX E

Proof of Lemma 3: For higher values of ρ_B and ρ_R , we use $e^{-\frac{a}{x}} = \frac{1}{1 + \frac{a}{x}}$ in (72) to obtain

$$\Pr\{A_1\} \simeq e^{-\lambda_x \max(\chi_F, \chi_N)} \left[e^{-\lambda_w \chi_F} + \exp\left(\frac{\lambda_w \phi_R}{\vartheta_1} + \frac{\lambda_y \phi \rho_B}{\rho_R \vartheta_1}\right) \int_{\max\left(\frac{\lambda_w \phi_R}{\vartheta_1}, 0\right)}^{\lambda_w(\chi_F + \frac{\phi_R}{\vartheta_1})} \left(1 - \frac{\kappa}{w + \kappa}\right) e^{-w} dw \right], \quad (76)$$

where $\kappa = \frac{\lambda_y \lambda_w \phi \rho_B}{\rho_R \vartheta_1} \left(\frac{\phi_R}{\vartheta_1} + \chi_F\right)$. Solving the above integral using [31, 3.352.3], we obtain

$$\Pr\{A_1\} \simeq e^{-\lambda_x \max(\chi_F, \chi_N)} \left[\exp\left(\frac{\lambda_w \phi_R}{\vartheta_1} + \frac{\lambda_y \phi \rho_B}{\rho_R \vartheta_1}\right) \left\{ \exp\left(-\max\left(\frac{\lambda_w \phi_R}{\vartheta_1}, 0\right)\right) - \exp\left(-\lambda_w\left(\chi_F + \frac{\phi_R}{\vartheta_1}\right)\right) \right. \right. \\ \left. \left. - \kappa e^{\kappa} \left[\mathcal{E}_1\left(\max\left(\frac{\lambda_w \phi_R}{\vartheta_1}, 0\right) + \kappa\right) - \mathcal{E}_1\left(\lambda_w\left(\chi_F + \frac{\phi_R}{\vartheta_1}\right) + \kappa\right) \right] \right\} + e^{-\lambda_w \chi_F} \right]. \quad (77)$$

At high SNRs, $\kappa \propto \frac{1}{\rho_B^2}$. We therefore neglect the product term of κe^{κ} . Now $\Pr\{A_1\}$ is expressed as

$$\Pr\{A_1\} \simeq e^{-\lambda_x \max(\chi_F, \chi_N)} \left[\exp\left(\frac{\lambda_w \phi_R}{\vartheta_1} + \frac{\lambda_y \phi \rho_B}{\rho_R \vartheta_1} - \max\left(\frac{\lambda_w \phi_R}{\vartheta_1}, 0\right)\right) + e^{-\lambda_w \chi_F} \left(1 - \exp\left(\frac{\lambda_y \phi \rho_B}{\rho_R \vartheta_1}\right)\right) \right]. \quad (78)$$

Similarly, solving for A_2 , we obtain

$$\Pr\{A_2\} \simeq (e^{-\lambda_x \chi_F} - e^{-\lambda_x \chi_N}) \left[\exp\left(\frac{\lambda_w}{\alpha_B \rho_B}\right) + \exp\left(-\frac{\lambda_y \gamma_F}{\rho_R}\right) \left(1 - \exp\left(\frac{\lambda_w}{\alpha_B \rho_B}\right)\right) \right]. \quad (79)$$

Substituting for $\Pr\{A_1\}$ and $\Pr\{A_2\}$ from (77) and (79) into (31), we obtain p_F^{DF} given by (33).

APPENDIX F

Proof of Theorem 5: Using (2) and (8) in (37), $\Pr\{B_1\}$ can be expressed as

$$\Pr\{B_1\} = \Pr\left\{\Gamma_{RF}^I < \gamma_F, \Gamma_{NF}^I \geq \gamma_F, \Gamma_{NN}^I \geq \gamma_N\right\} = \Pr\left\{\frac{(1-\alpha_B)\rho_B X}{\alpha_B \rho_B X + 1} < \gamma_F, \frac{(1-\alpha_B)\rho_B Z}{\alpha_B \rho_B Z + 1} \geq \gamma_F, \alpha_B \rho_B Z \geq \gamma_N\right\} \\ = \Pr\{X < \chi_F, Z \geq \max(\chi_F, \chi_N)\} U(\phi) = e^{-\lambda_z \max(\chi_F, \chi_N)} [1 - e^{-\lambda_x \chi_F}] U(\phi). \quad (80)$$

Using (37), $\Pr\{B_2\}$ is given by

$$\Pr\{B_2\} = \Pr\left\{\Gamma_{RF}^I \geq \gamma_F, \Gamma_{RN}^I < \gamma_N, \Gamma_{NF-F}^{COM} \geq \gamma_F, \Gamma_{NN}^I \geq \gamma_N\right\} \\ = \Pr\left\{\chi_F < X < \chi_N, \rho_B Z \left(\underbrace{(1-\alpha_B) - \alpha_B(\gamma_F - \rho_R V)}_{(\phi + \alpha_B \rho_R V)}\right) > (\gamma_F - \rho_R V), Z \geq \chi_N\right\} U(\phi) \\ = \Pr\left\{\chi_F < X < \chi_N, V > \frac{\gamma_F}{\rho_R}, Z \geq \chi_N\right\} + \Pr\left\{\chi_F < X < \chi_N, V < \frac{\gamma_F}{\rho_R}, Z \geq \max\left(\chi_N, \frac{\gamma_F - \rho_R V}{\phi + \alpha_B \rho_R V}\right)\right\}. \quad (81)$$

To satisfy $\chi_N > \chi_F$, $\max\left(\chi_N, \frac{\gamma_F - \rho_R V}{\phi + \alpha_B \rho_R V}\right) = \chi_N$. Using this fact, the above can be simplified as

$$\Pr\{B_2\} = \Pr\{\chi_F < X < \chi_N, Z \geq \chi_N\} = e^{-\lambda_z \chi_N} (e^{-\lambda_x \chi_F} - e^{-\lambda_x \chi_N}) U\left(\phi - \frac{\alpha_B \gamma_F}{\gamma_N}\right). \quad (82)$$

Further, the expression for $\Pr\{B_3\}$ is calculated as

TABLE III: B_3 - ranges of V , X and Z for different values of α_B

$\alpha_B < \frac{1}{1+\gamma_F}$	$Z > \max(\chi_F, \chi_N)$	$V > 0, X > \max(\chi_F, \chi_N)$
	$\chi_N > Z > \chi_F$	$V > \tilde{\chi}_{N_2}(Z), X > \max(\chi_F, \chi_N)$
	$\chi_F > Z > \max(\chi_N, \vartheta_5)$	$V > \tilde{\chi}_{N_1}(Z), X > \max(\chi_F, \chi_N)$
	$\min(\chi_F, \chi_N) > Z > \vartheta_5$	$V > \max(\tilde{\chi}_{N_1}(Z), \tilde{\chi}_{N_2}(Z)), X > \max(\chi_F, \chi_N)$ if $\tilde{\chi}_{N_1}(Z) \geq \tilde{\chi}_{N_2}(Z) \Rightarrow Z \geq \vartheta_4$
$\alpha_B \geq \frac{1}{1+\gamma_F}$	No range of X , V and Z is valid	

$$\begin{aligned} \Pr\{B_3\} &= \Pr\left\{\Gamma_{RF}^I \geq \gamma_F, \Gamma_{RN}^I \geq \gamma_N, \Gamma_{NF-S}^{COM} \geq \gamma_F, \Gamma_{NN-S}^{COM} \geq \gamma_N\right\} \\ &= \Pr\left\{X > \max(\chi_F, \chi_N), \rho_R V \left((1-\alpha_R)(1+\alpha_B \rho_B Z) - \alpha_R(\gamma_F - \phi \rho_B Z)\right) > (\gamma_F - \phi \rho_B Z), V \geq \frac{\gamma_N - \alpha_B \rho_B Z}{\alpha_R \rho_B}, Z < \chi_N\right\}. \end{aligned} \quad (83)$$

Using the above, the conditions for outage on ranges of X , V and Z with $\phi > 0$ and $\phi < 0$ are listed in Table III. Now, $\Pr\{B_3\}$ can be expressed as

$$\begin{aligned} \Pr\{B_3\} &= \left[\Pr\left\{Z > \max(\chi_F, \chi_N)\right\} + \Pr\left\{\chi_F < z < \chi_N, V \geq \tilde{\chi}_{N_1}(z)\right\} + \Pr\left\{\chi_F > Z > \max(\chi_N, \vartheta_5), V \geq \tilde{\chi}_{N_1}\right\} \right. \\ &\quad \left. + \Pr\left\{V \geq \tilde{\chi}_{N_1}(Z), \min(\chi_F, \chi_N) > Z > \max(\vartheta_5, \vartheta_4)\right\} + \Pr\left\{\min(\chi_F, \chi_N, \vartheta_4) > Z > \max(0, \vartheta_5), \right. \right. \\ &\quad \left. \left. V \geq \tilde{\chi}_{N_2}(Z)\right\} \right] \Pr\left\{X > \max(\chi_F, \chi_N)\right\}, \end{aligned} \quad (84)$$

where $\tilde{\chi}_{N_1}(Z) = \frac{\gamma_F - \phi \rho_B Z}{\rho_R(\phi_R + \rho_B Z(\phi \alpha_R + \alpha_B(1-\alpha_R)))}$ and $\tilde{\chi}_{N_2}(Z) = \frac{\gamma_N - \alpha_B \rho_B Z}{\alpha_R \rho_B}$. After averaging over the PDFs of RVs V , X and Z , we obtain

$$\begin{aligned} \Pr\{B_3\} &= e^{-\lambda_x \max(\chi_F, \chi_N)} \left[e^{-\lambda_z \max(\chi_F, \chi_N)} + \int_{\chi_F}^{\chi_N} \exp\left(\frac{-\lambda_v(\gamma_N - \alpha_B \rho_B z)}{\alpha_R \rho_R}\right) \lambda_z e^{-\lambda_z z} dz + \int_{\max(0, \frac{-\phi_R}{\vartheta_1})}^{\tilde{\chi}_{N_1}} \exp\left(\frac{-\lambda_v(\gamma_N - \alpha_B \rho_B z)}{\alpha_R \rho_R}\right) \lambda_z e^{-\lambda_z z} dz \right. \\ &\quad \left. + \int_{\max(\chi_N, \frac{-\phi_R}{\vartheta_1})}^{\chi_F} \exp\left(\frac{-\lambda_v(\gamma_F - \phi \rho_B z)}{\rho_R(\phi_R + \rho_B z \omega)}\right) \lambda_z e^{-\lambda_z z} dz + \int_{\max(\frac{-\phi_R}{\vartheta_1}, \vartheta_4)}^{\min(\chi_F, \chi_N)} \exp\left(\frac{-\lambda_v(\gamma_F - \phi \rho_B z)}{\rho_R(\phi_R + \rho_B z \omega)}\right) \lambda_z e^{-\lambda_z z} dz \right], \end{aligned} \quad (85)$$

where $\omega = \frac{-\lambda_v(\gamma_F - \phi \rho_B z)}{\rho_R(\phi_R + \rho_B z(\phi \alpha_R + \alpha_B(1-\alpha_R)))}$, $\tilde{\chi}_N = \min(\chi_F, \chi_N, \vartheta_4)$. After some mathematical rearrangement, the above can be expressed as

$$\begin{aligned} \Pr\{B_3\} &= e^{-\lambda_x \max(\chi_F, \chi_N)} \left[e^{-\lambda_z \max(\chi_F, \chi_N)} + \frac{\lambda_z e^{-\varepsilon_3 \gamma_N}}{\varepsilon_3 \alpha_B \rho_B - \lambda_z} \left[\exp(\varepsilon_3 \alpha_B \rho_B \chi_N - \lambda_z \chi_N) - \exp(\varepsilon_3 \alpha_B \rho_B \chi_F - \lambda_z \chi_F) \right] \right. \\ &\quad \left. + \frac{\lambda_z e^{-\varepsilon_3 \gamma_N}}{\varepsilon_3 \alpha_B \rho_B - \lambda_z} \left[\exp(\varepsilon_3 \alpha_B \rho_B \tilde{\chi}_N - \lambda_z \tilde{\chi}_N) - \exp\left(\varepsilon_3 \alpha_B \rho_B \max(0, \frac{-\phi_R}{\vartheta_1}) - \lambda_z \max(0, \frac{-\phi_R}{\vartheta_1})\right) \right] \right. \\ &\quad \left. + \underbrace{\int_{\max(\chi_N, \frac{-\phi_R}{\vartheta_1})}^{\chi_F} \exp\left(\frac{\lambda_v \phi \rho_B}{\rho_R \vartheta_1} \times \frac{z - \chi_F}{z + \frac{\phi_R}{\vartheta_1}}\right) \lambda_z e^{-\lambda_z z} dz}_{I_8} + \underbrace{\int_{\max(\frac{-\phi_R}{\vartheta_1}, \vartheta_4)}^{\min(\chi_F, \chi_N)} \exp\left(\frac{\lambda_v \phi \rho_B}{\rho_R \vartheta_1} \times \frac{z - \chi_F}{z + \frac{\phi_R}{\vartheta_1}}\right) \lambda_z e^{-\lambda_z z} dz}_{I_9} \right]. \end{aligned} \quad (86)$$

where $\varepsilon_3 = \frac{\lambda_v}{\alpha_R \rho_R}$. Substituting $\lambda_z(z + \frac{\phi_R}{\vartheta_1}) = z$, and using [33, eq. (4)], I_8 and I_9 are obtained as

$$I_8 = \exp\left(\frac{\phi_R \lambda_z \rho_R + \lambda_v \phi \rho_B}{\rho_R \vartheta_1}\right) \left[\Gamma\left(1, \lambda_z \max(0, \chi_N + \frac{\phi_R}{\vartheta_1}); \tilde{\chi}_F\right) - \Gamma\left(1, \lambda_z(\chi_F + \frac{\phi_R}{\vartheta_1}); \tilde{\chi}_F\right) \right], \quad (87)$$

$$I_9 = \exp\left(\frac{\phi_R \lambda_z \rho_R + \lambda_v \phi \rho_B}{\rho_R \vartheta_1}\right) \left[\Gamma\left(1, \lambda_z \max(0, \vartheta_4 + \frac{\phi_R}{\vartheta_1}); \tilde{\chi}_F\right) - \Gamma\left(1, \lambda_z \min(\chi_N, \chi_F) + \frac{\phi_R \lambda_z}{\vartheta_1}; \tilde{\chi}_F\right) \right], \quad (88)$$

where $\tilde{\chi}_F = \frac{\lambda_z \lambda_v \phi \rho_B (\chi_F + \frac{\phi_R}{\vartheta_1})}{\vartheta_1}$. Using I_8 and I_9 from (87) and (88) into (86), $\Pr\{B_3\}$ is expressed as

$$\begin{aligned} \Pr\{B_3\} = & e^{-\lambda_x \max(\chi_F, \chi_N)} \left[e^{-\lambda_z \max(\chi_F, \chi_N)} + \frac{\lambda_z e^{-\varepsilon_3 \gamma_N}}{\varepsilon_3 \alpha_B \rho_B - \lambda_z} \left[\exp(\varepsilon_3 \alpha_B \rho_B \chi_N - \lambda_z \chi_N) - \exp(\varepsilon_3 \alpha_B \rho_B \chi_F - \lambda_z \chi_F) \right. \right. \\ & + \frac{\exp(\varepsilon_3 \alpha_B \rho_B \tilde{\chi}_N)}{\exp(\tilde{\chi}_N \lambda_z)} - \frac{\exp\left(\varepsilon_3 \alpha_B \rho_B \max(0, \frac{-\phi_R}{\vartheta_1})\right)}{\exp(\max(0, \frac{-\phi_R}{\vartheta_1}) \lambda_z)} \left. \right] + \exp\left(\frac{\phi_R \lambda_z \rho_R + \lambda_v \phi \rho_B}{\rho_R \vartheta_1}\right) \left[\Gamma\left(1, \lambda_z \max(0, \chi_N + \frac{\phi_R}{\vartheta_1}); \tilde{\chi}_F\right) \right. \\ & \left. - \Gamma\left(1, \lambda_z(\chi_F + \frac{\phi_R}{\vartheta_1}); \tilde{\chi}_F\right) + \Gamma\left(1, \lambda_z \max(0, \vartheta_4 + \frac{\phi_R}{\vartheta_1}); \tilde{\chi}_F\right) - \Gamma\left(1, \lambda_z \min(\chi_N, \chi_F) + \frac{\phi_R \lambda_z}{\vartheta_1}; \tilde{\chi}_F\right) \right] \right]. \quad (89) \end{aligned}$$

Substituting (80), (82) and (89) into (37), p_N^{DF} is obtained as in (38).

REFERENCES

- [1] Y. Liu, Z. Qin, M. ElKashlan, Z. Ding, A. Nallanathan, and L. Hanzo, "Nonorthogonal multiple access for 5G and beyond," *Proc. IEEE*, vol. 105, no. 12, pp. 2347–2381, 2017.
- [2] L. Dai, B. Wang, Y. Yuan, S. Han, I. Chih-lin, and Z. Wang, "Non-orthogonal multiple access for 5G: solutions, challenges, opportunities, and future research trends," *IEEE Commun. Mag.*, vol. 53, no. 9, pp. 74–81, 2015.
- [3] M. Vaezi, Z. Ding, and H. V. Poor, *Multiple Access Techniques for 5G Wireless Networks and Beyond*. Springer, 2019.
- [4] O. Maraqa, A. S. Rajasekaran, S. Al-Ahmadi, H. Yanikomeroglu, and S. M. Sait, "A survey of rate-optimal power domain NOMA with enabling technologies of future wireless networks," *IEEE Commun. Surv. Tuts.*, vol. 22, no. 4, pp. 2192–2235, 2020.
- [5] Z. Ding, X. Lei, G. K. Karagiannidis, R. Schober, J. Yuan, and V. K. Bhargava, "A survey on non-orthogonal multiple access for 5G networks: Research challenges and future trends," *IEEE J. Sel. Areas Commun.*, vol. 35, no. 10, pp. 2181–2195, 2017.
- [6] W. Shin, M. Vaezi, B. Lee, D. J. Love, J. Lee, and H. V. Poor, "Non-orthogonal multiple access in multi-cell networks: Theory, performance, and practical challenges," *IEEE Commun. Mag.*, vol. 55, no. 10, pp. 176–183, 2017.
- [7] Z. Ding, Y. Liu, J. Choi, Q. Sun, M. ElKashlan, I. Chih-Lin, and H. V. Poor, "Application of non-orthogonal multiple access in LTE and 5G networks," *IEEE Commun. Mag.*, vol. 55, no. 2, pp. 185–191, 2017.
- [8] Z. Yang, Z. Ding, P. Fan, and N. Al-Dhahir, "A general power allocation scheme to guarantee quality of service in downlink and uplink NOMA systems," *IEEE Trans. Wireless Commun.*, vol. 15, no. 11, pp. 7244–7257, Nov 2016.
- [9] Y. Xiao, L. Hao, Z. Ma, Z. Ding, Z. Zhang, and P. Fan, "Forwarding strategy selection in dual-hop NOMA relaying systems," *IEEE Commun. Lett.*, vol. 22, no. 8, pp. 1644–1647, 2018.
- [10] Z. Ding, H. Dai, and H. V. Poor, "Relay selection for cooperative NOMA," *IEEE Wireless Commun. Lett.*, vol. 5, no. 4, pp. 416–419, 2016.
- [11] K. Agrawal, M. F. Flanagan, and S. Prakriya, "NOMA with battery-assisted energy harvesting full-duplex relay," *IEEE Trans. Veh. Technol.*, vol. 69, no. 11, pp. 13 952–13 957, 2020.
- [12] T.-T. Nguyen, T.-H. Vu, T.-V. Nguyen, D. B. d. Costa, and C. D. Ho, "Underlay cognitive NOMA-based coordinated direct and relay transmission," *IEEE Wireless Commun. Lett.*, vol. 10, no. 4, pp. 854–858, 2021.

- [13] Z. Ding, M. Peng, and H. V. Poor, "Cooperative non-orthogonal multiple access in 5G systems," *IEEE Commun. Lett.*, vol. 19, no. 8, pp. 1462–1465, Aug 2015.
- [14] X. Yue, Y. Liu, S. Kang, A. Nallanathan, and Z. Ding, "Exploiting full/half-duplex user relaying in NOMA systems," *IEEE Trans. Commun.*, vol. 66, no. 2, pp. 560–575, 2018.
- [15] K. Janghel and S. Prakriya, "Performance of adaptive OMA/cooperative-NOMA scheme with user selection," *IEEE Commun. Lett.*, vol. 22, no. 10, pp. 2092–2095, Oct 2018.
- [16] K. Janghel, A. Jee, and S. Prakriya, "Performance of underlay cooperative hybrid OMA/NOMA scheme with user selection," in *IEEE PIMRC*, 2020, pp. 1–6.
- [17] Z. Yang, Z. Ding, Y. Wu, and P. Fan, "Novel relay selection strategies for cooperative NOMA," *IEEE Trans. Veh. Technol.*, vol. 66, no. 11, pp. 10 114–10 123, 2017.
- [18] X. Lai, Q. Zhang, and J. Qin, "Cooperative NOMA short-packet communications in flat rayleigh fading channels," *IEEE Trans. Veh. Technol.*, vol. 68, no. 6, pp. 6182–6186, 2019.
- [19] Q. Y. Liau and C. Y. Leow, "Successive user relaying in cooperative NOMA system," *IEEE Wireless Commun. Lett.*, vol. 8, no. 3, pp. 921–924, 2019.
- [20] J. Kim and I. Lee, "Capacity analysis of cooperative relaying systems using non-orthogonal multiple access," *IEEE Commun. Lett.*, vol. 19, no. 11, pp. 1949–1952, 2015.
- [21] —, "Non-orthogonal multiple access in coordinated direct and relay transmission," *IEEE Commun. Lett.*, vol. 19, no. 11, pp. 2037–2040, 2015.
- [22] J. Kim, I. Lee, and J. Lee, "Capacity scaling for D2D aided cooperative relaying systems using noma," *IEEE Wireless Commun. Letters*, vol. 7, no. 1, pp. 42–45, 2018.
- [23] T. T. Nguyen, T. V. Nguyen, T. H. Vu, D. B. d. Costa, and C. D. Ho, "IoT-based coordinated direct and relay transmission with non-orthogonal multiple access," *IEEE Wireless Commun. Letters*, vol. 10, no. 3, pp. 503–507, 2021.
- [24] Y. Xu, B. Li, N. Zhao, Y. Chen, G. Wang, Z. Ding, and X. Wang, "Coordinated direct and relay transmission with NOMA and network coding in nakagami-m fading channels," *IEEE Trans. Commun.*, vol. 69, no. 1, pp. 207–222, 2021.
- [25] H. Liu, N. I. Miridakis, T. A. Tsiftsis, K. J. Kim, and K. S. Kwak, "Coordinated uplink transmission for cooperative NOMA systems," in *2018 IEEE Global Commun. Conf. (GLOBECOM)*, 2018, pp. 1–6.
- [26] H. Liu, Z. Ding, K. J. Kim, K. S. Kwak, and H. V. Poor, "Decode-and-forward relaying for cooperative NOMA systems with direct links," *IEEE Trans. Wireless Commun.*, vol. 17, no. 12, pp. 8077–8093, 2018.
- [27] X. Yue, Y. Liu, S. Kang, and A. Nallanathan, "Performance analysis of NOMA with fixed gain relaying over nakagami- m fading channels," *IEEE Access*, vol. 5, pp. 5445–5454, 2017.
- [28] K. Cao, B. Wang, H. Ding, T. Li, and F. Gong, "Optimal relay selection for secure NOMA systems under untrusted users," *IEEE Trans. Veh. Technol.*, vol. 69, no. 2, pp. 1942–1955, 2020.
- [29] X. Li, J. Li, Y. Liu, Z. Ding, and A. Nallanathan, "Residual transceiver hardware impairments on cooperative NOMA networks," *IEEE Trans. Wireless Commun.*, vol. 19, no. 1, pp. 680–695, 2020.
- [30] Z. Ding, R. Schober, and H. V. Poor, "Unveiling the importance of SIC in NOMA systemsâpart 1: State of the art and recent findings," *IEEE Commun. Lett.*, vol. 24, no. 11, pp. 2373–2377, 2020.
- [31] I. S. Gradshteyn and I. M. Ryzhik, *Table of Integrals, Series, and Products*, 7th ed. San Diego, CA, USA: Academic, 2007.
- [32] M. Abramowitz and I. A. Stegun, *Handbook of Mathematical Functions: with Formulas, Graphs, and Mathematical Tables*. New York, NY, USA: Dover, 1965.
- [33] M. Chaudhry and S. Zubair, "Generalized incomplete gamma functions with applications," *Journal of Computational and Applied Mathematics*, vol. 55, no. 1, pp. 99 – 123, 1994.