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A Modified General Diode Equation

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Abstract— The general diode equation or the non-ideal diode equation is the foundation of circuit models of active devices for the past several decades. Apart from the effect of p-n junction, this equation also accounts for the series bulk resistance of a diode. Despite a reasonable agreement of the equation with measured VI characteristics, it is shown here that the equation is incompatible with basic theories of circuits and systems. Therefore, a modification in the equation is proposed to remove this incompatibility. This modified equation has an excellent agreement with the measured VI characteristics.

Index Terms— Diode circuit model, device modeling, compact model, semi-analytical model, ohmic transform, modified general diode equation

I. INTRODUCTION

EVER since the publication of Shockley's seminal research on p-n junction diodes [1], general diode equation (GDE) has become the de-facto basis of device modeling and circuit simulations [2] enabled over computer simulation program with integrated circuit emphasis (SPICE). Unfortunately, the proposed solutions of this equation are mostly approximate [3-5]. Since an exact solution of the GDE has remained a challenge, most of the research on diode circuit modeling have focused on finding a solution of this equation. So, research on verifying or improving the accuracy of the equation in prediction of practical VI characteristics have been very limited. Though the VI characteristics from GDE provide a fair estimate of the practical results, the need for higher accuracy has either led to the use of abstract mathematics [6] or use of multiple diodes [7] (i.e. multiple exponentiations [8-9]) to model the behavior of a single diode. Consequently, device modeling has become more of an art than science as proclaimed in [10]. More appropriately, this art is mostly the use of empirical factors or relations that are qualitatively based on science and become quantitative only when matched with measured results. For example, the average velocity is a kind of empirical factor for a body moving with variable velocities; it becomes quantitative only after the total distance and total time of travel are measured. Such empirical factors are inevitable in many cases where the dynamics of science is obscure. The ideality factor η is one such case which empirically predicts the combined effect of diffusion current and recombination current on the VI characteristics of a diode

[11]. Since it is impossible to determine the exact distribution of the total current in these two categories, the empirical nature of η is inevitable. Therefore, such empirical factors or relations are an integral part of modern science and may not be deemed merely as art. However, many problems like those of radio propagation models [12] are so complex that their solutions are obtained purely using curve fitting, giving rise to only empirical formulations that are not even qualitatively connected with scientific understanding. Irrespective of whether such formulations are connected with science or not, it is important to ascertain that those are never in disagreement with established science. In the upcoming sections it is shown that GDE has disagreements with established theories of circuits and systems. Thereafter, a modified general diode equation (MGDE) is proposed which is fully compatible with the established theories and, unlike GDE, has an excellent agreement with measured results.

II. THE LIMITATIONS OF GDE

A. In Context of Exact Solution

An exact solution should be expressible in the form of $i = f(v)$ without any deliberation; therefore, as the exact solution claimed in [6] is not expressible in this form, it is not exact solution with respect to the sense mentioned here; [6], also, deliberately uses a Lambert W-function such that the basic structure of the GDE gets changed. Since, so far, the solution of GDE takes the form of $i = f(v).f(i)$, a true solution of GDE still remains approximate in nature [3-5]. However, if i and v are current and voltage respectively, GDE does have an exact solution in terms of an unknown voltage, i.e. $v = f(i)$ where voltage across a diode may be obtained using an estimated current through it. Using this approach, VI curves of three different diodes are plotted in Fig.1 and compared with their measurements extracted from published datasheets [13-15].

Fig.1. Comparison of calculated and measured VI characteristics of Diodes: D1 (IN5622), D2 (IN5420) and D3 (IN4001) using SPICE TABLE I.

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From the view point of rationality, what is known across a diode is the voltage and not the current. Therefore, the approach of a known current and unknown voltage to solve the GDE is again an approximation. Irrespective of whether the solution of GDE, expressed in (1), is approximate or exact, it is always iterative due to the presence empirical factors like ideality factor η and series bulk resistance R_{bs} . In (1), I_o is the reverse saturation current and V_T is the thermal voltage related to physical parameters specified in standard references [7,11]. In (2), V_d is the voltage across the diode, V is the total voltage across the series circuit and I is the total current through it. Usually, to closely match the solution of GDE with measurements, I_o is also taken as an empirical factor in most of the SPICE parameter sheets; this practice is inappropriate because of the fact that I_o , in forward bias, is the measurable, minimum reverse saturation current at a given temperature. Thus, the forward bias I_o should at most be equal to the minimum I_o obtained from measurements. Surprisingly, the values of these currents in SPICE sheets are over 100 times more than their measured values [16-17] indicating an incompatibility with realistic conditions. Another unrealistic approximation in SPICE sheets is that of the values of η . These should be between 1 and 2 as recommended in the academic references [11]; however, the value of η is more than 2 in many SPICE parameter sheets [16-17]. In the calculations adopted here (see Fig.1), I_o is directly taken from measurements (i.e. not empirically adjusted) and η is estimated between 1 and 2 to obtain the closest match with measurements and to conform with rational estimates. These newly extracted SPICE parameters for the three different diodes are enlisted in TABLE I. The same set of SPICE parameters is used in all the examples shown ahead.

$$I = I_o \left(e^{\frac{V_d}{\eta V_T}} - 1 \right) \quad (1)$$

$$V_d = V - IR_{bs} \quad (2)$$

TABLE I
EXTRACTED SPICE PARAMETERS FOR 25⁰ CELSIUS

Sr. No.	Name of the Diode	Model No.	I_o (Amperes)	η	R_{bs} (Ohms)
1	D1	IN5622	1.25×10^{-9}	1.66	48.5×10^{-3}
2	D2	IN5420	1.25×10^{-9}	1.35	55.0×10^{-3}
3	D3	IN4001	14.0×10^{-9}	1.78	18.5×10^{-3}

Accordingly, the objective of the present research is not only to obtain an exact solution of the GDE but also to keep the empirical parameters within the rational estimates.

B. In Context of Circuit Theory

From Fig.1, it is amply clear that the solution to GDE provides a reasonable estimate of the actual forward VI characteristics of a diode. Perhaps, this is the reason why validity of the GDE has never been questioned. However, a close examination of its in terms of circuit theory needs to be done in view of the fact that irrational estimates of the SPICE

parameters bring more accuracy to the solution. It is obvious that Kirchhoff's voltage law (KVL) has been used to formulate the GDE. i.e. the relation $V_d = V - IR_{bs}$, used in the exponentiation of (1), is a KVL equation. Also, the total current I multiplied by R_{bs} is the voltage drop across the bulk resistance as per ohms law. So far, (1) seems to be perfectly compliant with circuit theory. However, the equation which is based on KVL must have only voltages in its left hand side (LHS) and the right hand side (RHS). Moreover, the LHS and RHS must be linearly related. In the larger picture of (1), none of these conditions are satisfied. To be doubly sure about the incongruity of (1) with circuit theory, another test may be done where if (1) is correct then I in its RHS must satisfy (3); here, $R_d = (dI/dV)^{-1}$ is the dynamic resistance of the diode obtained from differentiating (1) with respect to V when $R_{bs} = 0$.

$$I = \frac{V}{R_d + R_{bs}} \quad (3)$$

I in RHS of (2) is replaced by the RHS of (3) and the following results are plotted in Fig.2. Due to the mismatch between the calculated and measured results of Fig.2, it is evident that (1) doesn't comply with circuit theory. Therefore, reformulating (1) in compliance with circuit theory is imperative.

Fig. 2 The incompatibility of GDE with circuit theory.

C. In Context of System Implementation

The subtracted I_o in RHS of (1) is negligibly small for all practical diodes and may be removed without any loss of accuracy. Subsequently, (1) may be expressed as (4) where $I = I_2$ is the independent current through the bulk resistance. From the perspective of electrical systems, as in Fig.3, a resistor is the simplest possible system [18] where different voltages fed

$$I \cong \left(I_o e^{\frac{V}{\eta V_T}} \right) \times \left(e^{-IR_{bs}} \right) = (I_1) \times (I_2) \quad (4)$$

Fig. 3 The incompatibility of GDE with the perspective of a system.

at different times are input signals [19] and the corresponding currents are the output signals. In that sense, current I_2 and resistance R_{bs} must come under the purview of a defined signal through a defined system. Unfortunately, as obvious from Fig.1, the representation of the bulk resistance in the GDE does not fit into the definition of any system. Because, for a system to exist, both input and output must exist. i.e. no system can be defined with an output alone; which is why solution of an equation of the form $I = e^{-IR}$ is single valued. In terms of physics, a current cannot exist through a resistor if a voltage is not applied across it.

III. THE MODIFIED GDE

Now, it is sufficiently clear that the GDE should not be acceptable in its present form, not just because it is approximate but also because it doesn't comply with general laws of physics. So, the aim of this article is to propose a modified general diode equation (MGDE) that is fully compliant with prevalent scientific theories, laws and rational estimations and the one that achieves a very good agreement with the measured results. Though the equations of Kirchhoff's voltage and current laws are linear, they are applicable to non linear devices as well; because, nonlinearities of devices don't affect the linearity of energy equations [20]. In fact, nonlinearity may be brought in circuit equations using variable resistances [10, 20]. To begin with, an attempt to formulate a variable resistance is made that may replicate the behavior of an ideal diode. This is not as simple as it seems, because, when the ideal diode current is expressed in the form of $I_d = V_d/R_d$, it is unable to replicate its actual behavior as shown in Fig.4. Here, R_d is the dynamic or variable resistance of the ideal diode obtained from the preceding section. As done earlier, the ideal diode current may be obtained from (1) by keeping $R_{bs} = 0$. When I_d is written in the form of $I_d = V_d/R_d$, it may be called as ideal diode current with resistive transform (IDC-RT). The IDC-RT looks to be compliant with circuit theories and the Ohm's law.

Fig. 4 Ideal diode current (IDC) and with resistive transform (IDC-RT).

Nevertheless, this compliance is of no use if IDC-RT doesn't match with IDC. The mismatch may be attributed to the fact that IDC and its resistance R_d are intrinsically non-ohmic; which is why an ohmic transform expressed in (5) may be necessary to make both the versions of current match with each other. Then the ideal diode resistance may be put in its ohmic form as $R_{do} = \Theta R_d$. The ohmic transform factor Θ may

be quite accurately calculated from (5) by neglecting the subtracted I_o in the RHS. Θ is expressed in (6).

$$\frac{V}{\Theta R_d} = I_o \left(e^{\frac{V}{\eta V_T}} - 1 \right) \quad (5)$$

$$\Theta \cong \frac{V}{V_T} \quad (6)$$

The total ohmic diode resistance R_{to} including the series bulk resistance R_{bs} may now be expressed as (7). Fig.5 plots the general diode current with ohmic transform (GDC-OT), which is $I_{gd} = V/R_{to}$, and compares it with the measurements.

$$R_{to} = \Theta (R_d + R_{bs}) \quad (7)$$

Fig. 5 The results after ohmic transform and addition of R_{bs} .

The mismatch in the calculated and measured results of Fig.5 may not be unreasonable; because, a bulk resistance R_{bs} is supposed to be a variable like R_d [10, 20-21], whereas, here it is considered as a fixed value; this is why, the calculated curves of Fig.5 resemble those of [6] where fixed series resistances are used. A dominant fixed resistance is expected to produce a flat logarithmic current response in the saturation region as is observed.

In reality, the modeled bulk resistance should decrease its value with increasing voltage such that the VI characteristics match with measurements. Consequently, a general scaling factor S_G is formulated in (8), in accord with the propositions of [20], which when divided with the bulk resistance, provides the desired result. The S_G may be composed of a static part S_s and a dynamic part dependent on the voltage. the factor m in (8) may be called the scaling order. Since, no resistance should practically reduce down to zero, a damping factor σ is introduced to retard this process so that the final value of the bulk resistance settles near its initial static value. The factor $I_o R_{bs}$ may be interpreted as a residual voltage in forward bias due to the presence of leakage current I_o .

$$S_G = S_s (I_o R_{bs} + V)_N^{m/(1+\sigma V_N)} \quad (8)$$

S_G may also be seen as a current amplification factor because it nonlinearly increases the current through the circuit by reducing the effective resistance. It may be noted that all the newly introduced empirical factors, m , S_s and σ in S_G are unitless quantities. The other entities used in it are made unitless by the process of normalization against their unit quantities as indicated by their subscript N . Now we may write the MGDE in its compact form revealed in (9)

$$I = \frac{V}{\Theta \left(R_d + \frac{R_{bs}}{S_G} \right)} \quad (9)$$

This MGDE is fully compliant with the established theories of circuits and systems. Moreover, it has an exact solution having excellent agreement with measured results as shown in Fig.6.

Fig. 6 The VI characteristics from MGDE compared to those of measurements

The three new empirical factors may be adopted as ancillary SPICE parameters in standard modeling practices. For the present set of diodes, these parameters are extracted and given in TABLE II.

TABLE II
EXTRACTED ANCILLARY SPICE PARAMETERS FOR 25⁰ CELSIUS

Sr. No.	Name of the Diode	Model No.	S_s (Static Scaling Factor)	m (Scaling Order)	σ (Damping Factor)
1	D1	IN5622	1.176	6.3	0.01
2	D2	IN5420	2.041	8.8	0.20
3	D3	IN4001	1.428	9.1	0.01

IV. CONCLUSION

A modified general diode equation is proposed whereby a non ideal diode may be modeled as a combination of the ideal diode resistance R_d and the series bulk resistance R_{bs} . This could be achieved due to the application of a novel analytical transform, Θ , named as ohmic transform. The transform enables the diode resistances to behave like normal resistors compliant with Ohm's law and circuit theories. Conforming with realistic conjectures [20-21], the series bulk resistance is made variable like the existing ideal diode resistance. This results in a MGDE whose solution is the simplest, most fundamental, most reasonable, most compatible with

established scientific theories and most accurate among all the outcomes in the past 70 years of research in the area of semiconductor circuit modeling. The formulations presented here are expected to positively disrupt the electrical modeling of every semiconductor device. The empirical parameters extracted in the given examples are obtained using manual trials. The accuracy of these extractions may be greatly improved using modern optimization techniques [22]. All the advantages of this new model could be achieved at the cost of two empirical parameters more than those of the basic GDE-SPICE model.

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