A Modified General Diode Equation

This paper was downloaded from TechRxiv (<https://www.techrxiv.org>).

LICENSE

CC BY-NC-SA 4.0

SUBMISSION DATE / POSTED DATE

01-02-2021 / 30-04-2021

CITATION

Chakravorty, Pragnan (2021): A Modified General Diode Equation. TechRxiv. Preprint.
<https://doi.org/10.36227/techrxiv.13678774.v2>

DOI

[10.36227/techrxiv.13678774.v2](https://doi.org/10.36227/techrxiv.13678774.v2)

A Modified General Diode Equation

Pragnan Chakravorty, *Senior Member, IEEE*

Abstract— The general diode equation or the non-ideal diode equation is the foundation of circuit models of active devices for the past several decades. Apart from the effect of p-n junction, this equation also accounts for the series bulk resistance of a diode. Despite a reasonable agreement of the equation with measured IV characteristics, it is shown here that the equation is incompatible with basic theories of circuits and systems. Therefore, a modification in the equation is proposed to remove this incompatibility. This modified equation has an excellent agreement with the measured IV characteristics.

Index Terms— Diode circuit model, device modeling, compact model, semi-analytical model, ohmic transform, modified general diode equation

I. INTRODUCTION

EVER since the publication of Shockley's seminal research on p-n junction diodes [1], general diode equation (GDE) has become the de-facto basis of device modeling and circuit simulations [2] enabled over computer simulation program with integrated circuit emphasis (SPICE). Unfortunately, the proposed solutions of this equation are mostly approximate [3-5]. Since an exact solution of the GDE has remained a challenge, most of the research on diode circuit modeling have focused on finding a solution of this equation. So, research on verifying or improving the accuracy of the equation in prediction of practical IV characteristics have been very limited. Though the IV characteristics from GDE provide a fair estimate of the practical results, the need for higher accuracy has either led to the use of abstract mathematics [6] or use of multiple diodes [7] (i.e. multiple exponentiations [8-9]) to model the behavior of a single diode. Consequently, device modeling has become more of an art than science as proclaimed in [10]. More appropriately, this art is mostly the use of empirical factors or relations that are qualitatively based on science and become quantitative only when matched with measured results. For example, the average velocity is a kind of empirical factor for a body moving with variable velocities; it becomes quantitative only after the total distance and total time of travel are measured. Such empirical factors are inevitable in many cases where the dynamics of science is obscure. The ideality factor η is one such case which empirically predicts the combined effect of diffusion current and recombination current on the IV characteristics of a diode

[11]. Since it is presently impossible to determine the exact distribution of the total current in these two categories, the empirical nature of η is inevitable. Therefore, such empirical factors or relations are an integral part of modern science and may not be deemed merely as art. However, many problems like those of radio propagation models [12] are so complex that their solutions are obtained purely using curve fitting, giving rise to only empirical formulations that are not even qualitatively connected with scientific understanding. Irrespective of whether such formulations are connected with science or not, it is important to ascertain that those are never in disagreement with established science. In the upcoming sections it is shown that GDE has disagreements with established theories of circuits and systems. Thereafter, a modified general diode equation (MGDE) is proposed which is fully compatible with the established theories and, unlike GDE, has an excellent agreement with measured results.

II. THE LIMITATIONS OF GDE

A. In Context of Exact Solution

An exact solution should be expressible in the form of $i = f(v)$ without any deliberation; therefore, as the exact solution claimed in [6] is not expressible in this form, it is not exact solution with respect to the sense mentioned here; [6], also, deliberately uses a Lambert W-function such that the basic structure of the GDE gets changed. Since, so far, the solution of GDE takes the form of $i = f(v, i)$, a true solution of GDE still remains approximate in nature [3-5]. However, if i and v are current and voltage respectively, GDE does have an exact solution in terms of an unknown voltage, i.e. $v = f(i)$ where voltage across a diode may be obtained using an estimated current through it. Using this approach, IV curves of three different diodes are calculated and subsequently plotted in Fig.1 comparing them with measurements obtained from publ-

Fig.1. Comparison of calculated and measured IV characteristics of Diodes: D1 (IN5622), D2 (IN5420) and D3 (IN4001) using SPICE TABLE I.

Manuscript received February 08, 2021

Pragnan Chakravorty is currently with Clique for Applied Research in Electronic Technology, Bhilai, India. (e-mail: pciitkgp@ieee.org)

-ished datasheets [13-15]. From the view point of rationality, what is known across a diode is the voltage and not the current. Therefore, the approach of a known current and unknown voltage to solve the GDE is again an approximation. The GDE is expressed in (1); irrespective of whether the solution of it is approximate or exact, it is always iterative due to the presence empirical factors like ideality factor η and series bulk resistance R_{bs} . In (1), I_o is the reverse saturation current and $V_T = kT/q$ is the thermal voltage related to physical parameters specified in standard references [7,11]. In (2), V_d is the voltage across the diode, V is the total voltage across the series circuit and I is the total current through it. Usually, to closely match the solution of GDE with measurements, I_o is also taken as an empirical factor in most of the SPICE parameter sheets; this practice is inappropriate because of the fact that I_o , in forward bias, is the measurable minimum reverse saturation current at a given temperature. Thus, the forward bias I_o should at most be equal to the minimum I_o obtained from measurements. Surprisingly, the values of these currents in SPICE sheets are over 100 times more than their measured values [16-17] indicating a disagreement with realistic conditions. Another unrealistic approximation in SPICE sheets is that of the values of η . These should be between 1 and 2 as recommended in the academic references [11]; however, the value of η is more than 2 in many SPICE parameter sheets [16-17]. In the calculations adopted here (see Fig.1), I_o is directly taken from measurements (i.e. not empirically adjusted) and η is estimated between 1 and 2 to obtain the closest match with measurements and to conform with rational estimates. These newly extracted SPICE parameters for the three different diodes are enlisted in TABLE I. The same set of diodes and their SPICE parameters is used in all the examples shown ahead.

$$I = I_o \left(e^{\frac{V - IR_{bs}}{\eta V_T}} - 1 \right) \quad (1)$$

where $V = V_d + IR_{bs}$ (2)

TABLE I
EXTRACTED SPICE PARAMETERS FOR 25⁰ CELSIUS

Sr. No.	Name of the Diode	Model No.	I_o (Amperes)	η	R_{bs} (Ohms)
1	D1	IN5622	1.25×10^{-9}	1.66	48.5×10^{-3}
2	D2	IN5420	1.25×10^{-9}	1.35	55.0×10^{-3}
3	D3	IN4001	14.0×10^{-9}	1.78	18.5×10^{-3}

Accordingly, the objective of the presented calculations is not only to obtain an exact solution of the GDE but also to keep the empirical parameters within the rational estimates.

B. In Context of Circuit Theory

The difference between the ideal diode equation (IDE), i.e. $I = I_o(e^{V/\eta V_T} - 1)$, and the GDE must be clearly stated at this moment to appreciate the issues discussed ahead. While the IDE may be seen as a device equation, GDE is essentially a circuit equation formed by the series connection between the

ideal diode and the bulk resistance. This is why Kirchhoff's voltage law (KVL) has been used to formulate the GDE. In other words, the general diode is a circuit rather than a single device. From Fig.1, it is amply clear that the solution of GDE provides a reasonable estimate of the actual forward IV characteristics of a diode for a large range of currents which the IDE cannot. Therefore, GDE leads to a large signal model whereas IDE is useful only for small signal approximations using $R_d = (dI/dV)^{-1}$, a small signal dynamic resistance of the junction. Perhaps, this is the reason why validity of the GDE has never been questioned. However, a close examination of it in terms of circuit theory needs to be done in view of the fact that irrational estimates of the SPICE parameters bring more accuracy to the solution. It is obvious that the relation $V_d = V - IR_{bs}$, used in the exponentiation of the GDE (1), is a KVL equation where current I multiplied by R_{bs} is the voltage drop across the bulk resistance as per ohms law. So far, (1) seems to be compliant with circuit theory. However, the equation which is based on KVL must be expressible in terms of voltages on either of its sides. Also, both the sides must be linearly related. In the larger picture of (1), none of these conditions are satisfied. Another important aspect is that in a KVL equation the voltage drops must be expressible in form of ohms law. However, the voltage drop across the diode junction V_d remains indeterminate in the GDE. A deliberate attempt to write V_d in terms ohms law, i.e. $V_d = I R_d$, leads to (3) which, unfortunately, doesn't match with the practical results as shown in Fig.2. All these discrepancies clearly indicate that, small signal or large signal, the GDE doesn't comply with the basic requirements of KVL.

$$I = \frac{V}{R_d + R_{bs}} \quad (3)$$

Since R_d is the only defined resistance of the ideal diode and V_d can't be express as $I R_d$ within the GDE, it remains one of the bottlenecks in the formation of a GDE compliant with circuit theory and measured results.

Fig. 2 The incompatibility of GDE with circuit theory.

C. In Context of System Implementation

The subtracted portion of I_o in the right hand side (RHS) of (1) is negligibly small for all practical diodes and may be removed without any loss of accuracy. Subsequently, (1) may be expressed as (4) where $I_o = I_{o1} \times I_{o2}$. I_2 characterizes the

systemic behavior of R_{bs} alone within the GDE. This is depicted in. Fig.3. Usually, a resistance is the simplest possible electrical system [18] where different voltages fed at different times are input signals [19] and the corresponding currents are the output signals. In that sense, current I_2 and the bulk resistance R_{bs} must come under the purview of a defined signal through a defined system.

$$I \cong \left(I_{o1} e^{\frac{V}{\eta V_T}} \right) \times \left(I_{o2} e^{-\frac{R_{bs}}{\eta V_T}} \right) = (I_1) \times (I_2) \quad (4)$$

Fig. 3 The incompatibility of GDE from the perspective of a system.

Unfortunately, as obvious from Fig.3, the representation of the bulk resistance in the GDE does not fit into the definition of any system. Because, for a system to exist, both input and output must exist. i.e. no system can be defined with an output alone; which is why solution of an equation of the form $I = e^{-IR}$ is single valued. In terms of physics, a current cannot exist through a resistor if a voltage is not applied across it.

III. THE MODIFIED GDE

Now, it is sufficiently clear that the GDE should not be acceptable in its present form, not just because it is approximate but also because it doesn't comply with general laws of circuits and systems. So, the aim of this article is to propose a modified general diode equation (MGDE) that is fully compliant with prevalent theories, laws and rational estimations and the one that achieves a very good agreement with the measured results. It is commonly believed that circuit laws like KVL are not applicable to non-linear devices. Fundamentally, this is because these devices don't follow ohms law. For example, as mentioned earlier, the ideal diode current (IDC) expressed in the form of $I_d = V_d / R_d$ doesn't agree with the IDE as shown in Fig.4. This form may be called IDC with resistive transform (IDC-RT).

Any resistance, as generally interpreted from ohms law, is an empirically derived function of proportionality that relates voltage and current of an electrical device. This function, in most cases, is a constant because it is so observed. Therefore, when the same process of observation finds it variable in nature, it should not be deemed as non-ohmic. From another point of view, if a rheostat, i.e. a variable resistance, is believed to be compliant with all circuit theories including ohms law then there is no reason to believe that the variable resistances of non linear devices are non-ohmic, i.e. practically, resistances like R_d are electrically controlled rheostats. To be precise, if I_d , expressible in the form of $I_d = V_d / R_{do}$, is in agreement with IDE, where R_{do} is a voltage dependent resistance, then it should be accepted to be compliant with the generalized interpretation of ohms law. In this way, the incompatibility of non-linear devices with ohms law may be removed. The ohms law compliant R_{do} may then

be related to R_d as $R_{do} = \Theta R_d$ where Θ is defined as the ohmic transform factor. In another sense, R_{do} is the large signal equivalent of the small signal resistance R_d . The factor Θ is quite accurately calculated from (5) by neglecting only the subtracted portion of I_o in the RHS of (5). The derived value of this factor is expressed in (6). The total ohmic diode resistance R_{to} including the series bulk resistance R_{bs} may now be expressed as (7). Fig.5 plots the general diode current with ohmic transform (GDC-OT), which is $I = V / R_{to}$, and compares it with the measurements.

Fig. 4 Ideal diode current (IDC) and with resistive transform (IDC-RT).

$$\frac{V}{\Theta R_d} = I_o \left(e^{\frac{V}{\eta V_T}} - 1 \right) \quad (5)$$

$$\Theta \cong \frac{V}{\eta V_T} \quad (6)$$

$$R_{to} = \Theta (R_d + R_{bs}) \quad (7)$$

Fig. 5 The results after ohmic transform and addition of R_{bs} .

The mismatch in the calculated and measured results of Fig.5 is attributed to the use of constant bulk resistance. The bulk resistance R_{bs} should be a variable like R_d [10, 20-21]. Practically, any resistance that is forced to process a non linear current through itself cannot remain fixed. This may be termed as an induced variability in an otherwise constant resistance. Since the non-linear current, generated by the junction, is forced through R_{bs} connected in series with it, R_{bs} must be a variable resistance that decreases with increasing voltage such

that the IV characteristics match with measurements. Consequently, a general scaling factor S_G is formulated in (8), in accord with the propositions of [20], which when divided with the bulk resistance, provides the desired result. The S_G may be composed of a static part S_s and a dynamic part dependent on the voltage. the factor m in (8) may be called the scaling order. Since, no resistance should reduce down to zero, a damping factor σ is introduced to retard this process so that the final value of the bulk resistance settles near its initial static value. The factor $I_o R_{bs}$ may be interpreted as the minimum voltage required to establish a non zero current through the diode. Practically, this non zero current is the leakage current I_o . The minimum voltage also ensures that that the variable bulk resistance is not mathematically infinite at zero forward bias voltage.

$$S_G = S_s (I_o R_{bs} + V)^{m/(1+\sigma V_N)} \quad (8)$$

S_G may also be seen as a current amplification factor of the bulk because it nonlinearly increases the current through the circuit by reducing the effective resistance. It may be noted that all the newly introduced empirical factors, m , S_s and σ in S_G are unitless quantities. The other entities used in it are made unitless by the process of normalization against their unit quantities as indicated by their subscript N . Now we may write the MGDE in its compact form as in (9)

$$I = \frac{V}{\Theta \left(R_d + \frac{R_{bs}}{S_G} \right)} \quad (9)$$

This MGDE is fully compliant with the established theories of circuits and systems. Moreover, it has an exact solution having excellent agreement with measured results as shown in Fig.6.

Fig. 6 The IV characteristics from MGDE compared to those of measurements

TABLE II shows the three new empirical factors, for the present set of diodes, which may be adopted as ancillary

TABLE II

EXTRACTED ANCILLARY SPICE PARAMETERS FOR 25⁰ CELSIUS

Sr. No.	Name of the Diode	Model No.	S_s	m	σ
1	D1	IN5622	1.176	6.3	0.01
2	D2	IN5420	2.041	8.8	0.20
3	D3	IN4001	1.428	9.1	0.01

SPICE parameters in standard modeling practices.

IV. CONCLUSION

A generalized view of the ohms law leads to the interpretation of resistance as an electrical ratio of voltage upon current which may not always be a positive constant. This interpretation puts the category of non-ohmic resistances in disuse. i.e. non-linear resistances such as R_d or R_{bs} of general diodes or negative resistances of devices like tunnel diodes may all be now assumed as ohmic resistances. A modified general diode equation is proposed whereby a non ideal diode may be modeled as a combination of the ideal diode resistance R_d and the series bulk resistance R_{bs} . This could be achieved due to the application of a novel analytical transform, Θ , named as ohmic transform. The transform enables the diode resistances to behave like normal resistors compliant with Ohm's law and circuit theories. Conforming with realistic conjectures [20-21], the series bulk resistance is made variable like the existing ideal diode resistance. This results in a MGDE whose solution is the simplest, most fundamental, most reasonable, most compatible with established scientific theories and most accurate among all the outcomes in the past 70 years of research in the area of semiconductor circuit modeling. The formulations presented here are expected to positively disrupt the electrical modeling of every semiconductor device. The empirical parameters extracted in the given examples are obtained using manual trials. The accuracy of these extractions may be greatly improved using modern optimization techniques [22]. All the advantages of this new model could be achieved at the cost of one empirical parameter more than those of the DC SPICE model based on GDE. Therefore, the computational effort in extracting the parameters of MGDE is insignificantly more than that of usual GDE-SPICE model.

REFERENCES

- [1] W. Shockley, "The theory of p-n junctions in semiconductors and p-n junction transistors," *The Bell System Technical Journal*, vol. 28, no. 3, pp. 435-489, July 1949.
- [2] L. W Nagel and D. O Pederson, "Simulation Program with Integrated Circuit Emphasis (SPICE)," *University of California, Berkeley Technical Report No. UCB/ERL M382, April 1973.*
- [3] T. A. Fjeldly, B. -. Moon and M. Shur, "Approximate analytical solution of generalized diode equation," *IEEE Transactions on Electron Devices*, vol. 38, no. 8, pp. 1976-1977, Aug. 1991.
- [4] J. M. Pimbley, T. A. Fjeldly, B. Moon and M. Shur, "Iterative solutions of the generalized diode equation," *IEEE Transactions on Electron Devices*, vol. 39, no. 5, pp. 1268-1269, May 1992
- [5] Jin He et al., "A new analytic approximation to general diode equation", *Solid-State Electronics*, vol. 50, no. 7-8, , pp.1371-1374, July-August 2006
- [6] T. C. Banwell and A. Jayakumar, "Exact analytical solution for current flow through diode with series resistance," *Electronics Letters*, vol. 36, no. 4, pp. 291-292, 17 Feb. 2000.
- [7] C.G. Fonstad, *Microelectronic Devices and Circuits*, MA: Copyright © Fonstad, 2006, pp.157
- [8] B.V. Zeghbroeck, *Principles of Semiconductor Devices*, CO: Copyright © Zeghbroeck, 2011, ch.4.4.4
- [9] C. Cane et al., "An easy technique for determining diffusion and generation-recombination components of the current of pn junctions for better modelling," *ICMTS 93 Proceedings of the 1993 Int. Conference on Microelectronic Test Structures, Sitges,1993*, pp. 167-170

- [10] L.O Chua, "Device Modeling Via Basic Nonlinear Circuit Elements," *IEEE Transactions on Circuits and Systems*, vol. cas-27, no. 11, pp. 1014-1044, Nov. 1980
- [11] K. Kano, *Semiconductor Devices*, Prentice Hall, New Jersey, 1998, ch.7
- [12] T. K. Sarkar et al., "A survey of various propagation models for mobile communication," *IEEE Antennas and Propagation Magazine*, vol. 45, no. 3, pp. 51-82, June 2003
- [13] 1N5614-1N5622 Data Sheet, Microsemi, AZ: ©2007 1-15-2007, 2007
- [14] 1N5415-1N5420 Data Sheet, Microsemi Corp, CA
- [15] 1N4001-1N4007 Data Sheet, ON Semiconductor, AZ: ©2012
- [16] 1N5622 SPICE Model, Voltage Multipliers Inc., CA
- [17] 1N5420 SPICE Model, Voltage Multipliers Inc., CA
- [18] S. S. Haykin and B. V. Veen, *Signals & Systems*, Hoboken, NJ: Wiley, 2003
- [19] P. Chakravorty, "What Is a Signal? [Lecture Notes]," *IEEE Signal Processing Magazine*, vol. 35, no. 5, pp. 175-177, Sept. 2018
- [20] P. Chakravorty, "A DC Circuit Model of Distributed Diodes on Active Substrate Boards for CAD Tools," IEEE TechRxiv, 2020, Preprint. <https://doi.org/10.36227/techrxiv.13369751.v2>
- [21] A. Barna and D. Horelick, "A simple diode model including conductivity modulation," *IEEE Transactions on Circuit Theory*, vol. 18, no. 2, pp. 233-240, March 1971.
- [22] P. Chakravorty and D. Mandal, "Role of Boundary Dynamics in Improving Efficiency of Particle Swarm Optimization on Antenna Problems," 2015 IEEE Symposium Series on Computational Intelligence, Cape Town, 2015, pp. 1157-1163