A Sensitivity-Based Three-Phase Weather-Dependent Power Flow Algorithm for Networks with Local Controllers— Supplementary

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Abstract: This supplementary document is part of the original 2-Part manuscript titled “*A Sensitivity-Based Three-Phase Weather-Dependent Power Flow Algorithm for Networks with Local Controllers.*” This research focuses on proposing a novel sensitivity-based three-phase weather-dependent power flow algorithm for distribution networks with local voltage controllers (LVCs). The proposed algorithm has four distinct characteristics: a) it considers the three-phase unbalanced nature of distribution systems, b) the operating state of LVCs is calculated using sensitivity parameters, which accelerates the convergence speed of the algorithm, c) it considers the precise switching sequence of LVCs based on their reaction time delays, and d) the nonlinear influence of weather variations in the power flow is also taken into consideration. In this supplementary document, the relevant derivations of the sensitivity parameters are presented to complement the original 2-Part manuscript.

In this supplementary document, we present the derivation of the sensitivity parameters. Let us reconsider the simplified network with LVCs presented in Fig. S1 below. It includes an OLTC connected between the buses h - w , three single-phase capacitors connected to the three phases of bus q , and a DG connected to bus g . Furthermore, an SVR is also connected through the 3-bus equivalent circuit to buses p , m , s .

Fig. S1. Simplified Network including a three-phase SVR, capacitor, DG, OLTC. All control variables are included in the current sources.

As mentioned in Part I, the sensitivity matrix $S_{V,x}$, relates the variation of controlled voltages dV_{LVC} with the variation of control variables dx_{LVC} . This relationship is given as follows:

$$dV_{LVC} = S_{V,x} \cdot dx_{LVC} \quad (S1)$$

where

$$\mathbf{S}_{V,x} = \begin{bmatrix} \frac{dV_{oltc}}{dx_{oltc}} & \frac{dV_{oltc}}{dx_{dg1}} & \frac{dV_{oltc}}{dx_{dg02}} & \frac{dV_{oltc}}{dx_{svr}} & \frac{dV_{oltc}}{dx_{cap}} \\ \frac{dV_{dg1}}{dx_{oltc}} & \frac{dV_{dg1}}{dx_{dg1}} & \frac{dV_{dg1}}{dx_{dg02}} & \frac{dV_{dg1}}{dx_{svr}} & \frac{dV_{dg1}}{dx_{cap}} \\ \frac{dV_{dg02}}{dx_{oltc}} & \frac{dV_{dg02}}{dx_{dg1}} & \frac{dV_{dg02}}{dx_{dg02}} & \frac{dV_{dg02}}{dx_{svr}} & \frac{dV_{dg02}}{dx_{cap}} \\ \frac{dV_{svr}}{dx_{oltc}} & \frac{dV_{svr}}{dx_{dg1}} & \frac{dV_{svr}}{dx_{dg02}} & \frac{dV_{svr}}{dx_{svr}} & \frac{dV_{svr}}{dx_{cap}} \\ \frac{dV_{cap}}{dx_{oltc}} & \frac{dV_{cap}}{dx_{dg1}} & \frac{dV_{cap}}{dx_{dg02}} & \frac{dV_{cap}}{dx_{svr}} & \frac{dV_{cap}}{dx_{cap}} \end{bmatrix} \quad (S2)$$

The vector of LVC control variables ($\mathbf{x}_{LVC}$) and voltages ($\mathbf{V}_{LVC}$) are defined as:

$$\mathbf{x}_{LVC} = [\mathbf{x}_{oltc} \quad \mathbf{x}_{dg1} \quad \mathbf{x}_{dg02} \quad \mathbf{x}_{svr} \quad \mathbf{x}_{cap}]^T \quad (S3)$$

$$\mathbf{V}_{LVC} = [\mathbf{V}_{oltc} \quad \mathbf{V}_{dg1} \quad \mathbf{V}_{dg02} \quad \mathbf{V}_{svr} \quad \mathbf{V}_{cap}]^T \quad (S4)$$

where

- $\mathbf{x}_{oltc} = [Tap_{oltc}^a \quad Tap_{oltc}^b \quad Tap_{oltc}^c]^T$ includes the tap variables of each phase of the OLTC transformer. It is a 3×1 vector, which directly regulates the vector $\mathbf{V}_{oltc} = [|V_w^a| \quad |V_w^b| \quad |V_w^c|]^T$ that includes the magnitude of the three-phase voltages of bus w .
- $\mathbf{x}_{dg1} = [Q_{dg1}]^T$ includes the positive-sequence reactive power of the DG. It is a 1×1 vector that regulates the vector $\mathbf{V}_{dg1} = [|V_{g1}|]^T$, which includes the magnitude of positive-sequence voltage of bus g .
- $\mathbf{x}_{dg02} = [I_{g2}^{real} \quad I_{g2}^{im} \quad I_{g0}^{real} \quad I_{g0}^{im}]^T$ includes the real and imaginary components of the negative- and zero-sequence current of the DG. The vector $\mathbf{x}_{dg02}$ is of dimension 4×1 , which regulates the vector $\mathbf{V}_{dg02} = [V_{g2}^{real} \quad V_{g2}^{im} \quad V_{g0}^{real} \quad V_{g0}^{im}]^T$ that includes the real and imaginary components of negative- and zero-sequence voltages of bus g .
- $\mathbf{x}_{svr} = [Tap_{svr}^a \quad Tap_{svr}^b \quad Tap_{svr}^c]^T$ includes the tap variables of each phase of the SVR having a dimension 3×1 . The vector $\mathbf{x}_{svr}$ regulates the vector $\mathbf{V}_{svr} = [|V_s^a| \quad |V_s^b| \quad |V_s^c|]^T$ that includes the three-phase voltage magnitudes of bus s .
- $\mathbf{x}_{cap} = [C_a \quad C_b \quad C_c]^T$ includes the capacitors of each phase of bus q . It is of dimension 3×1 . The vector $\mathbf{x}_{cap}$ regulates the vector $\mathbf{V}_{cap} = [|V_q^a| \quad |V_q^b| \quad |V_q^c|]^T$ that includes the three-phase voltage magnitudes of bus q .

I. RELATION BETWEEN CONTROL VARIABLES AND CURRENT SOURCES OF LVCs

We consider the relation of how the control variables (Taps of SVR and OLTC, reactive power of DGs, capacitors) perturb the current of LVCs. More specifically, based on Fig. S1, we explain:

- 1) how the SVR taps ($Tap_{svr}^a, Tap_{svr}^b, Tap_{svr}^c$) perturb the current sources $\mathbf{I}_p^{abc}, \mathbf{I}_m^{abc}, \mathbf{I}_s^{abc}$,
- 2) how the OLTC taps ($Tap_{oltc}^a, Tap_{oltc}^b, Tap_{oltc}^c$) perturb the current sources $\mathbf{I}_h^{abc}, \mathbf{I}_w^{abc}$,
- 3) how the capacitors C_a, C_b, C_c perturb the current source $\mathbf{I}_q^{abc}$, and
- 4) how the reactive power of DG (Q_{dg1}) perturb the positive-sequence current source $\mathbf{I}_{g1}^{abc}$.

A. Relation between the perturbations of $\mathbf{I}_p^{abc}, \mathbf{I}_m^{abc}, \mathbf{I}_s^{abc}$ and $Tap_{svr}^a, Tap_{svr}^b, Tap_{svr}^c$ of the SVR

Let us assume, we have an SVR of Type A in Open Delta configuration. Based on reference [27] of the manuscript (Part I), the current sources $\mathbf{I}_p^{abc}, \mathbf{I}_m^{abc}, \mathbf{I}_s^{abc}$ are expressed, as follows:

$$\mathbf{I}_p^{abc} = \mathbf{A} \cdot \mathbf{Y}_{pm} \cdot (-\mathbf{V}_m + (\mathbf{I}_d + \mathbf{K}) \cdot \mathbf{V}_p) + \mathbf{Y}_{pm} \cdot \mathbf{K} \cdot \mathbf{V}_p \quad (S5)$$

$$\mathbf{I}_m^{abc} = -\mathbf{Y}_{pm} \cdot \mathbf{K} \cdot \mathbf{V}_p \quad (S6)$$

$$\mathbf{I}_s^{abc} = \mathbf{0} \quad (S7)$$

The matrices $\mathbf{K}$ and $\mathbf{A}$ are given in [27, Table I] as follows:

$$\mathbf{K} = \begin{bmatrix} \frac{n_1}{N_1} & 0 & -\frac{n_1}{N_1} \\ 0 & \frac{n_2}{N_2} & -\frac{n_2}{N_2} \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0.00625 \cdot Tap_{svr}^a & 0 & -0.00625 \cdot Tap_{svr}^a \\ 0 & 0.00625 \cdot Tap_{svr}^b & -0.00625 \cdot Tap_{svr}^b \\ 0 & 0 & 0 \end{bmatrix} \quad (S8)$$

$$\mathbf{A} = \begin{bmatrix} \frac{n_1}{N_1} & 0 & 0 \\ 0 & \frac{n_2}{N_2} & 0 \\ -\frac{n_1}{N_1} & -\frac{n_2}{N_2} & 0 \end{bmatrix} = \begin{bmatrix} 0.00625 \cdot Tap_{svr}^a & 0 & 0 \\ 0 & 0.00625 \cdot Tap_{svr}^b & 0 \\ -0.00625 \cdot Tap_{svr}^a & -0.00625 \cdot Tap_{svr}^b & 0 \end{bmatrix} \quad (S9)$$

where n_1, n_2, n_3 are the series windings of phases a, b, c , respectively, while N_1, N_2, N_3 are the shunt windings of phases a, b, c , respectively. The winding ratios $\frac{n_1}{N_1}, \frac{n_2}{N_2}, \frac{n_3}{N_3}$ are related to the tap ratios as follows [27]:

$$Tap_{svr}^a = \frac{1}{0.00625} \cdot \frac{n_1}{N_1} \quad (S10)$$

$$Tap_{svr}^b = \frac{1}{0.00625} \cdot \frac{n_2}{N_2} \quad (S11)$$

$$Tap_{svr}^c = \frac{1}{0.00625} \cdot \frac{n_3}{N_3} \quad (S12)$$

A perturbation ($dTap$) at the tap of phase a (Tap_{svr}^a) of the SVR will cause a perturbation ($d\mathbf{I}_p^{Tap_{svr}^a}$) of the current $\mathbf{I}_p^{abc}$ according to (S13):

$$d\mathbf{I}_p^{Tap_{svr}^a} = \mathbf{I}_{p(perturbed)}^{abc} - \mathbf{I}_{p(unperturbed)}^{abc} \quad (S13)$$

In Equation (S13), $\mathbf{I}_{p(unperturbed)}^{abc}$ is calculated in Equation (S14) below:

$$\mathbf{I}_{p(unperturbed)}^{abc} = \mathbf{A}_{unperturbed} \cdot \mathbf{Y}_{pm} \cdot (-V_m + (\mathbf{I}_d + \mathbf{K}_{unperturbed}) \cdot V_p) + \mathbf{Y}_{pm} \cdot \mathbf{K}_{unperturbed} \cdot V_p \quad (S14)$$

where $\mathbf{K}_{unperturbed}$ and $\mathbf{A}_{unperturbed}$ are the $\mathbf{A}$ and $\mathbf{K}$ matrices with unperturbed taps, as shown in Equations (S15) - (S16).

$$\mathbf{K}_{unperturbed} = \begin{bmatrix} 0.00625 \cdot Tap_{svr}^a & 0 & -0.00625 \cdot Tap_{svr}^a \\ 0 & 0.00625 \cdot Tap_{svr}^b & -0.00625 \cdot Tap_{svr}^b \\ 0 & 0 & 0 \end{bmatrix} \quad (S15)$$

$$\mathbf{A}_{unperturbed} = \begin{bmatrix} 0.00625 \cdot Tap_{svr}^a & 0 & 0 \\ 0 & 0.00625 \cdot Tap_{svr}^b & 0 \\ -0.00625 \cdot Tap_{svr}^a & -0.00625 \cdot Tap_{svr}^b & 0 \end{bmatrix} \quad (S16)$$

Moreover, in Equation (S13), $\mathbf{I}_{p(perturbed)}^{abc}$ is calculated in Equation (S17) below:

$$\mathbf{I}_{p(perturbed)}^{abc} = \mathbf{A}_{perturbed} \cdot \mathbf{Y}_{pm} \cdot (-V_m + (\mathbf{I}_d + \mathbf{K}_{perturbed}) \cdot V_p) + \mathbf{Y}_{pm} \cdot \mathbf{K}_{perturbed} \cdot V_p \quad (S17)$$

where $\mathbf{K}_{perturbed}$ and $\mathbf{A}_{perturbed}$ are the $\mathbf{A}$ and $\mathbf{K}$ matrices with perturbed tap ratios, as follows:

$$\mathbf{K}_{perturbed} = \begin{bmatrix} 0.00625 \cdot (Tap_{svr}^a + dTap) & 0 & -0.00625 \cdot (Tap_{svr}^a + dTap) \\ 0 & 0.00625 \cdot Tap_{svr}^b & -0.00625 \cdot Tap_{svr}^b \\ 0 & 0 & 0 \end{bmatrix} \quad (S18)$$

$$\mathbf{A}_{\text{perturbed}} = \begin{bmatrix} 0.00625 \cdot (\text{Tap}_{\text{svr}}^a + d\text{Tap}) & 0 & 0 \\ 0 & 0.00625 \cdot \text{Tap}_{\text{svr}}^b & 0 \\ -0.00625 \cdot (\text{Tap}_{\text{svr}}^a + d\text{Tap}) & -0.00625 \cdot \text{Tap}_{\text{svr}}^b & 0 \end{bmatrix} \quad (\text{S19})$$

In a similar manner, we calculate the perturbations of all current sources, due to the tap perturbation of each phase e.g. $d\mathbf{I}_p^{\text{Tap}_{\text{svr}}^b}$, $d\mathbf{I}_p^{\text{Tap}_{\text{svr}}^c}$, $d\mathbf{I}_m^{\text{Tap}_{\text{svr}}^a}$, $d\mathbf{I}_m^{\text{Tap}_{\text{svr}}^b}$, $d\mathbf{I}_m^{\text{Tap}_{\text{svr}}^c}$, $d\mathbf{I}_s^{\text{Tap}_{\text{svr}}^a}$, $d\mathbf{I}_s^{\text{Tap}_{\text{svr}}^b}$, $d\mathbf{I}_s^{\text{Tap}_{\text{svr}}^c}$. Each of these are 3×1 in dimension.

B. Relation between the perturbations of $\mathbf{I}_h^{abc}$, $\mathbf{I}_w^{abc}$ and $\text{Tap}_{\text{oltc}}^a$, $\text{Tap}_{\text{oltc}}^b$, $\text{Tap}_{\text{oltc}}^c$ of the OLTC

The perturbations of $\mathbf{I}_h^{abc}$, $\mathbf{I}_w^{abc}$ due to the perturbations of $\text{Tap}_{\text{oltc}}^a$, $\text{Tap}_{\text{oltc}}^b$, $\text{Tap}_{\text{oltc}}^c$ are derived in a similar manner as in the case of SVR described above, using the current sources of the OLTC model proposed in reference [28] of the manuscript. Therefore, a detailed derivation is not deemed necessary here for the OLTC.

C. Relation between the perturbations of $\mathbf{I}_q^{abc}$ and C_a, C_b, C_c of the capacitor

The current $\mathbf{I}_q^{abc}$ represents the three-phase currents flowing through the capacitors. It is expressed as follows:

$$\mathbf{I}_q^{abc} = [j\omega \cdot V_{qa} \cdot C_a \quad j\omega \cdot V_{qb} \cdot C_b \quad j\omega \cdot V_{qc} \cdot C_c]^T \quad (\text{S20})$$

where V_{qf} is the voltage of phase $f = \{a, b, c\}$ of bus q .

A small perturbation dC_a of capacitance of phase a of bus q will perturb the current $\mathbf{I}_q^{abc}$, as shown in Equation (S21):

$$d\mathbf{I}_q^{C_a} = [j\omega \cdot V_{qa} \cdot dC_a \quad 0 \quad 0]^T \quad (\text{S21})$$

The perturbation of $\mathbf{I}_q^{abc}$ due to the perturbation of dC_b is equal to $d\mathbf{I}_q^{C_b} = [0 \quad j\omega \cdot V_{qb} \cdot dC_b \quad 0]^T$. Similarly, $d\mathbf{I}_q^{C_c}$ could be calculated in a similar manner.

D. Relation between the perturbations of $\mathbf{I}_{dg1}^{abc}$ and Q_{dg1}

The three-phase positive-sequence apparent power of DG of bus g ($S_{3ph-pos}$) is calculated, as shown in Equation (S22):

$$S_{3ph-pos} = P_{3ph-pos} + jQ_{3ph-pos} = 3 \cdot V_{pos} \cdot I_{pos} \quad (\text{S22})$$

where V_{pos} and I_{pos} are the complex positive-sequence voltage and current of DG, respectively.

Based on Equation (S22), a small perturbation at the positive sequence reactive power of DG ($dQ_{3ph-pos}$) will cause a perturbation at the positive-sequence current, as follows:

$$dI_{pos} = \frac{j \cdot dQ_{3ph-pos}}{3 \cdot V_{pos}} \quad (\text{S23})$$

The perturbation of Equation (S23) will cause a perturbation at the positive sequence components of three-phase currents (I_{a1}, I_{b1}, I_{c1}) of DG, as shown in Equation (S24):

$$[dI_{a1} \quad dI_{b1} \quad dI_{c1}]^T = [dI_{pos} \quad \alpha^2 \cdot dI_{pos} \quad \alpha \cdot dI_{pos}]^T \quad (\text{S24})$$

where α is the phasor rotation operator $\alpha = e^{j\frac{2}{3}\pi}$.

Finally, $d\mathbf{I}_{dg1}$ is defined in Equation (S25).

$$d\mathbf{I}_{dg1} = [dI_{a1} \quad dI_{b1} \quad dI_{c1}]^T \quad (\text{S25})$$

II. DERIVATION OF THE ELEMENTS OF $\mathbf{S}_{V,x}$ MATRIX

The derivation of each element of the $\mathbf{S}_{V,x}$ is explained below:

1. Derivation of $\frac{dV_{oltc}}{dX_{oltc}}$

Initially, a small tap perturbation $dTap$ is assumed at each phase of OLTC, which causes a perturbation at the three-phase current sources of OLTC by $d\mathbf{I}_\gamma$, as follows:

$$d\mathbf{I}_\gamma = \begin{bmatrix} d\mathbf{I}_\gamma^{Tap^a_{oltc}} & d\mathbf{I}_\gamma^{Tap^b_{oltc}} & d\mathbf{I}_\gamma^{Tap^c_{oltc}} \end{bmatrix} \quad (S26)$$

where $\gamma = \{h, w\}$ and $d\mathbf{I}_\gamma^{Tap^f_{oltc}}$ is the perturbation of current of bus γ due to the perturbation of OLTC tap of phase $f = \{a, b, c\}$, calculated according to Section I. B of the supplementary material. In Equation (S26), $d\mathbf{I}_\gamma$ is a 3×3 matrix, because $d\mathbf{I}_\gamma^{Tap^f_{oltc}}$ is a 3×1 vector.

The perturbation of the three-phase voltages of OLTC bus w due to $d\mathbf{I}_h$ and $d\mathbf{I}_w$ is given by Equation (S27).

$$\begin{bmatrix} dV_{w-Tap^a_{oltc}}^a & dV_{w-Tap^b_{oltc}}^a & dV_{w-Tap^c_{oltc}}^a \\ dV_{w-Tap^a_{oltc}}^b & dV_{w-Tap^b_{oltc}}^b & dV_{w-Tap^c_{oltc}}^b \\ dV_{w-Tap^a_{oltc}}^c & dV_{w-Tap^b_{oltc}}^c & dV_{w-Tap^c_{oltc}}^c \end{bmatrix} = \mathbf{Z}_{wh} \cdot d\mathbf{I}_h + \mathbf{Z}_{ww} \cdot d\mathbf{I}_w \quad (S27)$$

where $dV_{w-Tap^r_{oltc}}^l$ is the perturbation of the voltage (complex value) of phase $l = \{a, b, c\}$ of bus w , due to the perturbation of OLTC tap of phase $r = \{a, b, c\}$. $\mathbf{Z}_{wh}$ is the 3×3 submatrix of the impedance matrix of the network (see Equation (14) of Part I) that corresponds to the w and h buses. $\mathbf{Z}_{ww}$ is the 3×3 submatrix of the impedance matrix of the network that corresponds to the w bus.

The sensitivity of the absolute value of phase voltage of OLTC bus w with respect to the tap of each phase of OLTC transformer is given in Equation (S28).

$$\begin{bmatrix} \frac{d|V_w^a|}{dT_{ap^a_{oltc}}} & \frac{d|V_w^a|}{dT_{ap^b_{oltc}}} & \frac{d|V_w^a|}{dT_{ap^c_{oltc}}} \\ \frac{d|V_w^b|}{dT_{ap^a_{oltc}}} & \frac{d|V_w^b|}{dT_{ap^b_{oltc}}} & \frac{d|V_w^b|}{dT_{ap^c_{oltc}}} \\ \frac{d|V_w^c|}{dT_{ap^a_{oltc}}} & \frac{d|V_w^c|}{dT_{ap^b_{oltc}}} & \frac{d|V_w^c|}{dT_{ap^c_{oltc}}} \end{bmatrix} = \frac{\begin{bmatrix} dV_{w-Tap^a_{oltc}}^a & dV_{w-Tap^b_{oltc}}^a & dV_{w-Tap^c_{oltc}}^a \\ dV_{w-Tap^a_{oltc}}^b & dV_{w-Tap^b_{oltc}}^b & dV_{w-Tap^c_{oltc}}^b \\ dV_{w-Tap^a_{oltc}}^c & dV_{w-Tap^b_{oltc}}^c & dV_{w-Tap^c_{oltc}}^c \end{bmatrix} + \begin{bmatrix} V_w^a & V_w^a & V_w^a \\ V_w^b & V_w^b & V_w^b \\ V_w^c & V_w^c & V_w^c \end{bmatrix}}{dT_{ap}} - \begin{bmatrix} V_w^a & V_w^a & V_w^a \\ V_w^b & V_w^b & V_w^b \\ V_w^c & V_w^c & V_w^c \end{bmatrix} \quad (S28)$$

where V_w^a, V_w^b, V_w^c are complex voltages.

The sensitivity parameter $\frac{dV_{oltc}}{dX_{oltc}}$ of Equation (S2) is given in Equation (S29).

$$\frac{dV_{oltc}}{dX_{oltc}} = \begin{bmatrix} \frac{d|V_w^a|}{dT_{ap^a_{oltc}}} & \frac{d|V_w^a|}{dT_{ap^b_{oltc}}} & \frac{d|V_w^a|}{dT_{ap^c_{oltc}}} \\ \frac{d|V_w^b|}{dT_{ap^a_{oltc}}} & \frac{d|V_w^b|}{dT_{ap^b_{oltc}}} & \frac{d|V_w^b|}{dT_{ap^c_{oltc}}} \\ \frac{d|V_w^c|}{dT_{ap^a_{oltc}}} & \frac{d|V_w^c|}{dT_{ap^b_{oltc}}} & \frac{d|V_w^c|}{dT_{ap^c_{oltc}}} \end{bmatrix} \quad (S29)$$

2. Derivation of $\frac{dV_{oltc}}{dX_{dg1}}$

The perturbation of the three-phase currents of DG, due to the perturbation of positive sequence reactive power (dQ_{dg1}) is given by Equation (S30):

$$d\mathbf{I}_{dg1} = [dI_{a1} \quad dI_{b1} \quad dI_{c1}]^T \quad (\text{S30})$$

where dI_{a1} , dI_{b1} , dI_{c1} are derived, as explained in Equations (S22) - (S24).

The perturbation of the three-phase voltages of OLTC bus w due to dQ_{dg1} is given by Equation (S31).

$$\begin{bmatrix} dV_{w-Q_{dg1}}^a \\ dV_{w-Q_{dg1}}^b \\ dV_{w-Q_{dg1}}^c \end{bmatrix} = \mathbf{Z}_{wg} \cdot d\mathbf{I}_{dg1} \quad (\text{S31})$$

where $dV_{w-Q_{dg1}}^l$ is the perturbation of the voltage of phase $l = \{a, b, c\}$ of bus w , due to the perturbation of Q_{dg1} .

The sensitivity of absolute value of phase voltage of OLTC bus w due to the perturbation of Q_{dg1} is given in Equation (S32).

$$\begin{bmatrix} \frac{d|V_w^a|}{dQ_{dg1}} \\ \frac{d|V_w^b|}{dQ_{dg1}} \\ \frac{d|V_w^c|}{dQ_{dg1}} \end{bmatrix} = \frac{\begin{bmatrix} dV_{w-Q_{dg1}}^a \\ dV_{w-Q_{dg1}}^b \\ dV_{w-Q_{dg1}}^c \end{bmatrix} + \begin{bmatrix} V_w^a \\ V_w^b \\ V_w^c \end{bmatrix} - \begin{bmatrix} V_w^a \\ V_w^b \\ V_w^c \end{bmatrix}}{dQ_{dg1}} \quad (\text{S32})$$

The sensitivity parameter $\frac{dV_{oltc}}{dX_{dg1}}$ is given in Equation (S33).

$$\frac{dV_{oltc}}{dX_{dg1}} = \begin{bmatrix} \frac{d|V_w^a|}{dQ_{dg1}} \\ \frac{d|V_w^b|}{dQ_{dg1}} \\ \frac{d|V_w^c|}{dQ_{dg1}} \end{bmatrix} \quad (\text{S33})$$

3. Derivation of $\frac{dV_{oltc}}{dX_{dg02}}$

Initially, a small perturbation equal to dI is assumed at the real and imaginary components of negative- and zero-sequence currents of bus g such that $dI_2^{real} = dI_2^{im} = dI_0^{real} = dI_0^{im} = dI$, where $I_2^{real}, I_2^{im}, I_0^{real}, I_0^{im}$ is the real component of negative sequence current, the imaginary component of negative sequence current, the real component of zero-sequence current, the imaginary component of zero-sequence current, respectively.

The perturbation caused at the currents of phases a, b, c ($dI_a^{dI_2^{real}}, dI_b^{dI_2^{real}}, dI_c^{dI_2^{real}}$) due to the perturbation dI_2^{real} is calculated by Equation (S34).

$$\begin{bmatrix} dI_a^{dI_2^{real}} & dI_b^{dI_2^{real}} & dI_c^{dI_2^{real}} \end{bmatrix}^T = [dI_2^{real} \quad \alpha \cdot dI_2^{real} \quad \alpha^2 \cdot dI_2^{real}]^T \quad (\text{S34})$$

where for example $dI_a^{dI_2^{real}}$ is the perturbation of current of phase a due to the perturbation of I_2^{real} . Similarly, for the other elements.

The perturbation caused at the currents of phases a, b, c ($dI_a^{dI_2^{im}}, dI_b^{dI_2^{im}}, dI_c^{dI_2^{im}}$) due to the perturbation of dI_2^{im} is calculated by Equation (S35).

$$\begin{bmatrix} dI_a^{dI_2^{im}} & dI_b^{dI_2^{im}} & dI_c^{dI_2^{im}} \end{bmatrix}^T = [j \cdot dI_2^{im} \quad j \cdot \alpha \cdot dI_2^{im} \quad j \cdot \alpha^2 \cdot dI_2^{im}]^T \quad (\text{S35})$$

The perturbation caused at the currents of phases a, b, c ($dI_a^{dI_0^{real}}, dI_b^{dI_0^{real}}, dI_c^{dI_0^{real}}$) due to the perturbation of dI_0^{real} is calculated by Equation (S36).

$$\begin{bmatrix} dI_a^{dI_0^{real}} & dI_b^{dI_0^{real}} & dI_c^{dI_0^{real}} \end{bmatrix}^T = [dI_0^{real} \quad dI_0^{real} \quad dI_0^{real}]^T \quad (S36)$$

The perturbation caused at the currents of phases a, b, c ($dI_a^{dI_0^{im}}, dI_b^{dI_0^{im}}, dI_c^{dI_0^{im}}$) due to the perturbation of dI_0^{im} is calculated by Equation (S37).

$$\begin{bmatrix} dI_a^{dI_0^{im}} & dI_b^{dI_0^{im}} & dI_c^{dI_0^{im}} \end{bmatrix}^T = [j \cdot dI_0^{im} \quad j \cdot dI_0^{im} \quad j \cdot dI_0^{im}]^T \quad (S37)$$

By writing Equations (S34)-(S37) in a more compact form, Equation (S38) is derived, where each column of Equation (S38) represents one of the Equations (S34)-(S37).

$$\mathbf{dI}_{DG02} = \begin{bmatrix} dI_a^{dI_2^{real}} & dI_a^{dI_2^{im}} & dI_a^{dI_0^{real}} & dI_a^{dI_0^{im}} \\ dI_b^{dI_2^{real}} & dI_b^{dI_2^{im}} & dI_b^{dI_0^{real}} & dI_b^{dI_0^{im}} \\ dI_c^{dI_2^{real}} & dI_c^{dI_2^{im}} & dI_c^{dI_0^{real}} & dI_c^{dI_0^{im}} \end{bmatrix} \quad (S38)$$

The perturbation of the three-phase voltages of OLTC bus w due to $\mathbf{dI}_{DG02}$ is given by Equation (S39).

$$\begin{bmatrix} dV_{w-I_2^{real}}^a & dV_{w-I_2^{im}}^a & dV_{w-I_0^{real}}^a & dV_{w-I_0^{im}}^a \\ dV_{w-I_2^{real}}^b & dV_{w-I_2^{im}}^b & dV_{w-I_0^{real}}^b & dV_{w-I_0^{im}}^b \\ dV_{w-I_2^{real}}^c & dV_{w-I_2^{im}}^c & dV_{w-I_0^{real}}^c & dV_{w-I_0^{im}}^c \end{bmatrix} = \mathbf{Z}_{wg} \cdot \mathbf{dI}_{DG02} \quad (S39)$$

where for example $dV_{w-I_2^{real}}^f$ is the perturbation of phase $f = \{a, b, c\}$ of bus w due to the perturbation of I_2^{real} . Similarly for the other elements.

The sensitivities of absolute value of phase voltages of OLTC bus w due to the perturbation of negative- and zero-sequence current components of DG are given in Equation (S40).

$$\begin{bmatrix} \frac{d|V_w^a|}{dI_2^{real}} & \frac{d|V_w^a|}{dI_2^{im}} & \frac{d|V_w^a|}{dI_0^{real}} & \frac{d|V_w^a|}{dI_0^{im}} \\ \frac{d|V_w^b|}{dI_2^{real}} & \frac{d|V_w^b|}{dI_2^{im}} & \frac{d|V_w^b|}{dI_0^{real}} & \frac{d|V_w^b|}{dI_0^{im}} \\ \frac{d|V_w^c|}{dI_2^{real}} & \frac{d|V_w^c|}{dI_2^{im}} & \frac{d|V_w^c|}{dI_0^{real}} & \frac{d|V_w^c|}{dI_0^{im}} \end{bmatrix} = \frac{\begin{bmatrix} dV_{w-I_2^{real}}^a & dV_{w-I_2^{im}}^a & dV_{w-I_0^{real}}^a & dV_{w-I_0^{im}}^a \\ dV_{w-I_2^{real}}^b & dV_{w-I_2^{im}}^b & dV_{w-I_0^{real}}^b & dV_{w-I_0^{im}}^b \\ dV_{w-I_2^{real}}^c & dV_{w-I_2^{im}}^c & dV_{w-I_0^{real}}^c & dV_{w-I_0^{im}}^c \end{bmatrix} + \begin{bmatrix} V_w^a & V_w^a & V_w^a & V_w^a \\ V_w^b & V_w^b & V_w^b & V_w^b \\ V_w^c & V_w^c & V_w^c & V_w^c \end{bmatrix} - \begin{bmatrix} V_w^a & V_w^a & V_w^a & V_w^a \\ V_w^b & V_w^b & V_w^b & V_w^b \\ V_w^c & V_w^c & V_w^c & V_w^c \end{bmatrix}}{dI} \quad (S40)$$

The sensitivity parameter $\frac{dV_{oltc}}{dX_{dg02}}$ is given in Equation (S41).

$$\frac{dV_{oltc}}{dX_{dg02}} = \begin{bmatrix} \frac{d|V_w^a|}{dI_2^{real}} & \frac{d|V_w^a|}{dI_2^{im}} & \frac{d|V_w^a|}{dI_0^{real}} & \frac{d|V_w^a|}{dI_0^{im}} \\ \frac{d|V_w^b|}{dI_2^{real}} & \frac{d|V_w^b|}{dI_2^{im}} & \frac{d|V_w^b|}{dI_0^{real}} & \frac{d|V_w^b|}{dI_0^{im}} \\ \frac{d|V_w^c|}{dI_2^{real}} & \frac{d|V_w^c|}{dI_2^{im}} & \frac{d|V_w^c|}{dI_0^{real}} & \frac{d|V_w^c|}{dI_0^{im}} \end{bmatrix} \quad (S41)$$

4. Derivation of $\frac{dV_{oltc}}{dX_{svr}}$

Initially, a small perturbation at each tap of SVR equal to $dTap$ is assumed. This perturbation perturbs the SVR current sources of Fig. S1 by $\mathbf{dI}_\gamma$ as follows:

$$\mathbf{dI}_\gamma = [\mathbf{dI}_\gamma^{Tap_{svr}^a} \quad \mathbf{dI}_\gamma^{Tap_{svr}^b} \quad \mathbf{dI}_\gamma^{Tap_{svr}^c}] \quad (S42)$$

where $\gamma = \{p, m, s\}$ and $\mathbf{dI}_\gamma^{Tap_{svr}^f}$ is calculated (for $\gamma = p$ and $f = a$) according to in Equation (S13).

Equation (S43) calculates the perturbation of phase voltages of bus w , due to the perturbation of the SVR taps. More specifically, $dV_{w-Tap_{svr}^f}^\xi$ expresses the perturbation of the voltage of phase $\xi = \{a, b, c\}$ of bus w , due to the tap perturbation $dTap_{svr}^f$ of phase $f = \{a, b, c\}$.

$$\begin{bmatrix} dV_{w-Tap_{svr}^a}^a & dV_{w-Tap_{svr}^b}^a & dV_{w-Tap_{svr}^c}^a \\ dV_{w-Tap_{svr}^a}^b & dV_{w-Tap_{svr}^b}^b & dV_{w-Tap_{svr}^c}^b \\ dV_{w-Tap_{svr}^a}^c & dV_{w-Tap_{svr}^b}^c & dV_{w-Tap_{svr}^c}^c \end{bmatrix} = \mathbf{Z}_{wp} \cdot d\mathbf{I}_p + \mathbf{Z}_{wm} \cdot d\mathbf{I}_m + \mathbf{Z}_{ws} \cdot d\mathbf{I}_s \quad (S43)$$

The sensitivity of absolute value of phase voltage of bus w due to the perturbation of SVR taps is given in Equation (S44).

$$\begin{bmatrix} \frac{d|V_w^a|}{dT_{ap_{svr}^a}} & \frac{d|V_w^a|}{dT_{ap_{svr}^b}} & \frac{d|V_w^a|}{dT_{ap_{svr}^c}} \\ \frac{d|V_w^b|}{dT_{ap_{svr}^a}} & \frac{d|V_w^b|}{dT_{ap_{svr}^b}} & \frac{d|V_w^b|}{dT_{ap_{svr}^c}} \\ \frac{d|V_w^c|}{dT_{ap_{svr}^a}} & \frac{d|V_w^c|}{dT_{ap_{svr}^b}} & \frac{d|V_w^c|}{dT_{ap_{svr}^c}} \end{bmatrix} = \frac{\begin{bmatrix} dV_{w-Tap_{svr}^a}^a & dV_{w-Tap_{svr}^b}^a & dV_{w-Tap_{svr}^c}^a \\ dV_{w-Tap_{svr}^a}^b & dV_{w-Tap_{svr}^b}^b & dV_{w-Tap_{svr}^c}^b \\ dV_{w-Tap_{svr}^a}^c & dV_{w-Tap_{svr}^b}^c & dV_{w-Tap_{svr}^c}^c \end{bmatrix} \begin{bmatrix} V_w^a & V_w^a & V_w^a \\ V_w^b & V_w^b & V_w^b \\ V_w^c & V_w^c & V_w^c \end{bmatrix} - \begin{bmatrix} V_w^a & V_w^a & V_w^a \\ V_w^b & V_w^b & V_w^b \\ V_w^c & V_w^c & V_w^c \end{bmatrix}}{dT_{ap}} \quad (S44)$$

The sensitivity parameter $\frac{dV_{oltc}}{dX_{svr}}$ is given in Equation (S45).

$$\frac{dV_{oltc}}{dX_{svr}} = \begin{bmatrix} \frac{d|V_w^a|}{dT_{ap_{svr}^a}} & \frac{d|V_w^a|}{dT_{ap_{svr}^b}} & \frac{d|V_w^a|}{dT_{ap_{svr}^c}} \\ \frac{d|V_w^b|}{dT_{ap_{svr}^a}} & \frac{d|V_w^b|}{dT_{ap_{svr}^b}} & \frac{d|V_w^b|}{dT_{ap_{svr}^c}} \\ \frac{d|V_w^c|}{dT_{ap_{svr}^a}} & \frac{d|V_w^c|}{dT_{ap_{svr}^b}} & \frac{d|V_w^c|}{dT_{ap_{svr}^c}} \end{bmatrix} \quad (S45)$$

5. Derivation of $\frac{dV_{oltc}}{dX_{cap}}$

Initially, a small perturbation at each phase capacitor equal to dC is assumed. This perturbation perturbs the current of each phase of bus q by $d\mathbf{I}_{cap}$ as follows:

$$d\mathbf{I}_{cap} = [d\mathbf{I}_q^{c_a} \quad d\mathbf{I}_q^{c_b} \quad d\mathbf{I}_q^{c_c}] \quad (S46)$$

where $d\mathbf{I}_q^{c_a}$, $d\mathbf{I}_q^{c_b}$, $d\mathbf{I}_q^{c_c}$ are calculated as explained in Equation (S21).

The perturbation of the three-phase voltages of OLTC bus w due to $d\mathbf{I}_{cap}$ is given by Equation (S47).

$$\begin{bmatrix} dV_{w-Ca}^a & dV_{w-Cb}^a & dV_{w-Cc}^a \\ dV_{w-Ca}^b & dV_{w-Cb}^b & dV_{w-Cc}^b \\ dV_{w-Ca}^c & dV_{w-Cb}^c & dV_{w-Cc}^c \end{bmatrix} = \mathbf{Z}_{wq} \cdot d\mathbf{I}_{cap} \quad (S47)$$

where dV_{w-Cr}^l is the perturbation of the voltage of phase $l = \{a, b, c\}$ of bus w , due to the perturbation of capacitor of phase r .

The sensitivity of absolute value of phase voltage of OLTC bus w with respect to each phase capacitor of bus q are given in Equation (S48).

$$\begin{bmatrix} \frac{d|V_w^a|}{dC_a} & \frac{d|V_w^a|}{dC_b} & \frac{d|V_w^a|}{dC_c} \\ \frac{d|V_w^b|}{dC_a} & \frac{d|V_w^b|}{dC_b} & \frac{d|V_w^b|}{dC_c} \\ \frac{d|V_w^c|}{dC_a} & \frac{d|V_w^c|}{dC_b} & \frac{d|V_w^c|}{dC_c} \end{bmatrix} = \frac{\begin{bmatrix} dV_{w-Ca}^a & dV_{w-Cb}^a & dV_{w-Cc}^a \\ dV_{w-Ca}^b & dV_{w-Cb}^b & dV_{w-Cc}^b \\ dV_{w-Ca}^c & dV_{w-Cb}^c & dV_{w-Cc}^c \end{bmatrix} \begin{bmatrix} V_w^a & V_w^a & V_w^a \\ V_w^b & V_w^b & V_w^b \\ V_w^c & V_w^c & V_w^c \end{bmatrix} - \begin{bmatrix} V_w^a & V_w^a & V_w^a \\ V_w^b & V_w^b & V_w^b \\ V_w^c & V_w^c & V_w^c \end{bmatrix}}{dC} \quad (S48)$$

The sensitivity parameter $\frac{dV_{oltc}}{dX_{cap}}$ is given in Equation (S49).

$$\frac{dV_{oltc}}{dX_{cap}} = \begin{bmatrix} \frac{d|V_w^a|}{dC_a} & \frac{d|V_w^a|}{dC_b} & \frac{d|V_w^a|}{dC_c} \\ \frac{d|V_w^b|}{dC_a} & \frac{d|V_w^b|}{dC_b} & \frac{d|V_w^b|}{dC_c} \\ \frac{d|V_w^c|}{dC_a} & \frac{d|V_w^c|}{dC_b} & \frac{d|V_w^c|}{dC_c} \end{bmatrix} \quad (S49)$$

6. Derivation of $\frac{dV_{dg1}}{dX_{oltc}}$

Initially, a small tap perturbation $dTap$ is assumed at each phase of OLTC, which causes a perturbation at the current sources of Fig. 6 by dI_γ as follows:

$$dI_\gamma = \begin{bmatrix} dI_\gamma^{Tap_{oltc}^a} & dI_\gamma^{Tap_{oltc}^b} & dI_\gamma^{Tap_{oltc}^c} \end{bmatrix} \quad (S50)$$

where $\gamma = \{h, w\}$ and $dI_\gamma^{Tap_{oltc}^f}$ is calculated according to Equation (S13).

The perturbation of the three-phase voltages of DG bus g due to dI_γ is given by Equation (S51).

$$\begin{bmatrix} dV_{g-Tap_{oltc}^a}^a & dV_{g-Tap_{oltc}^b}^a & dV_{g-Tap_{oltc}^c}^a \\ dV_{g-Tap_{oltc}^a}^b & dV_{g-Tap_{oltc}^b}^b & dV_{g-Tap_{oltc}^c}^b \\ dV_{g-Tap_{oltc}^a}^c & dV_{g-Tap_{oltc}^b}^c & dV_{g-Tap_{oltc}^c}^c \end{bmatrix} = Z_{gh} \cdot dI_h + Z_{gw} \cdot dI_w \quad (S51)$$

The perturbation of the positive-sequence voltage of DG bus g is given by Equation (S52).

$$\begin{bmatrix} dV_{g1-Tap_{oltc}^a} & dV_{g1-Tap_{oltc}^b} & dV_{g1-Tap_{oltc}^c} \end{bmatrix} = \begin{bmatrix} \frac{1}{3} & \frac{\alpha}{3} & \frac{\alpha^2}{3} \end{bmatrix} \cdot \begin{bmatrix} dV_{g-Tap_{oltc}^a}^a & dV_{g-Tap_{oltc}^b}^a & dV_{g-Tap_{oltc}^c}^a \\ dV_{g-Tap_{oltc}^a}^b & dV_{g-Tap_{oltc}^b}^b & dV_{g-Tap_{oltc}^c}^b \\ dV_{g-Tap_{oltc}^a}^c & dV_{g-Tap_{oltc}^b}^c & dV_{g-Tap_{oltc}^c}^c \end{bmatrix} \quad (S52)$$

where for example $dV_{g1-Tap_{oltc}^a}$ is the perturbation of the positive-sequence voltage of DG bus g due to the Tap_{oltc}^a . Similarly, for the other elements.

The sensitivity of absolute value of positive-sequence voltage of bus g with respect to each tap of OLTC are given in Equation (S53).

$$\begin{bmatrix} \frac{dV_{dg1}}{dT_{ap_{oltc}^a}} & \frac{dV_{dg1}}{dT_{ap_{oltc}^b}} & \frac{dV_{dg1}}{dT_{ap_{oltc}^c}} \end{bmatrix} = \frac{\left[\left[\frac{dV_{g1-Tap_{oltc}^a}}{dT_{ap_{oltc}^a}} \quad \frac{dV_{g1-Tap_{oltc}^b}}{dT_{ap_{oltc}^b}} \quad \frac{dV_{g1-Tap_{oltc}^c}}{dT_{ap_{oltc}^c}} \right] + [V_{g1} \quad V_{g1} \quad V_{g1}] \right] - |[V_{g1} \quad V_{g1} \quad V_{g1}]|}{dT_{ap}} \quad (S53)$$

The sensitivity parameter $\frac{dV_{dg1}}{dX_{oltc}}$ is given in Equation (S54).

$$\frac{dV_{dg1}}{dX_{oltc}} = \begin{bmatrix} \frac{dV_{dg1}}{dT_{ap_{oltc}^a}} & \frac{dV_{dg1}}{dT_{ap_{oltc}^b}} & \frac{dV_{dg1}}{dT_{ap_{oltc}^c}} \end{bmatrix} \quad (S54)$$

7. Derivation of $\frac{dV_{dg1}}{dX_{dgi}}$

The perturbation of the three-phase currents of DG, due to the perturbation of positive sequence reactive power (dQ_{dg1}) is given by Equation (S55):

$$dI_{dg1} = [dI_{a1} \quad dI_{b1} \quad dI_{c1}]^T \quad (S55)$$

where $dI_{a1}, dI_{b1}, dI_{c1}$ are derived, as explained in Equations (S22) - (S24).

The perturbation of the three-phase voltages of bus g due to dI_{dg1} is given by Equation (S56).

$$\begin{bmatrix} dV_{g-Q_{dg1}}^a & dV_{g-Q_{dg1}}^b & dV_{g-Q_{dg1}}^c \end{bmatrix}^T = Z_{gg} \cdot dI_{dg1} \quad (S56)$$

where Z_{gg} is the 3x3 sub-matrix of impedance matrix of Equation (13) of Part I, corresponding to the voltage of g^{th} bus and current of g^{th} bus.

The perturbation of the positive sequence voltage of DG bus g due to the perturbation of $dV_{g-Q_{dg1}}^a, dV_{g-Q_{dg1}}^b, dV_{g-Q_{dg1}}^c$ is given by Equation (S57).

$$dV_{g1} = \begin{bmatrix} \frac{1}{3} & \frac{\alpha}{3} & \frac{\alpha^2}{3} \end{bmatrix} \cdot \begin{bmatrix} dV_{g-Q_{dg1}}^a & dV_{g-Q_{dg1}}^b & dV_{g-Q_{dg1}}^c \end{bmatrix}^T \quad (S57)$$

The variation of the magnitude of the positive-sequence voltage is calculated by Equation (S58).

$$dV_{dg1} = |V_{g1} + dV_{g1}| - |V_{g1}| \quad (S58)$$

Finally, the sensitivity parameter $\frac{dV_{dg1}}{dX_{dg1}}$ is calculated by Equation (S59).

$$\frac{dV_{dg1}}{dX_{dg1}} = \frac{|V_{g1} + dV_{g1}| - |V_{g1}|}{dQ_{dg1}} \quad (S59)$$

8. Derivation of $\frac{dV_{dg1}}{dX_{dg02}}$

Initially, a small perturbation equal to dI is assumed at the real and imaginary components of negative- and zero-sequence currents such that $dI_2^{real} = dI_2^{im} = dI_0^{real} = dI_0^{im} = dI$, where $dI_2^{real}, dI_2^{im}, dI_0^{real}, dI_0^{im}$ is the real component of negative sequence current, the imaginary component of negative sequence current, the real component of zero-sequence current, the imaginary component of zero-sequence current, respectively.

The perturbation caused at the currents of phases a, b, c ($dI_a^{dI_2^{real}}, dI_b^{dI_2^{real}}, dI_c^{dI_2^{real}}$) due to the perturbation dI_2^{real} is calculated by Equation (S60).

$$\begin{bmatrix} dI_a^{dI_2^{real}} & dI_b^{dI_2^{real}} & dI_c^{dI_2^{real}} \end{bmatrix}^T = [dI_2^{real} \quad \alpha \cdot dI_2^{real} \quad \alpha^2 \cdot dI_2^{real}]^T \quad (S60)$$

where for example $dI_a^{dI_2^{real}}$ is the perturbation of current of phase a due to the perturbation of I_2^{real} . Similarly for the other elements.

The perturbation caused at the currents of phases a, b, c ($dI_a^{dI_2^{im}}, dI_b^{dI_2^{im}}, dI_c^{dI_2^{im}}$) due to the perturbation of dI_2^{im} is calculated by Equation (S61).

$$\begin{bmatrix} dI_a^{dI_2^{im}} & dI_b^{dI_2^{im}} & dI_c^{dI_2^{im}} \end{bmatrix}^T = [j \cdot dI_2^{im} \quad j \cdot \alpha \cdot dI_2^{im} \quad j \cdot \alpha^2 \cdot dI_2^{im}]^T \quad (S61)$$

The perturbation caused at the currents of phases a, b, c ($dI_a^{dI_0^{real}}, dI_b^{dI_0^{real}}, dI_c^{dI_0^{real}}$) due to the perturbation of dI_0^{real} is calculated by Equation (S62).

$$\begin{bmatrix} dI_a^{dI_0^{real}} & dI_b^{dI_0^{real}} & dI_c^{dI_0^{real}} \end{bmatrix}^T = [dI_0^{real} \quad dI_0^{real} \quad dI_0^{real}]^T \quad (S62)$$

The perturbation caused at the currents of phases a, b, c ($dI_a^{dI_0^{im}}, dI_b^{dI_0^{im}}, dI_c^{dI_0^{im}}$) due to the perturbation of dI_0^{im} is calculated by Equation (S63).

$$\begin{bmatrix} dI_a^{dI_0^{im}} & dI_b^{dI_0^{im}} & dI_c^{dI_0^{im}} \end{bmatrix}^T = [j \cdot dI_0^{im} \quad j \cdot dI_0^{im} \quad j \cdot dI_0^{im}]^T \quad (S63)$$

By writing Equations (S60) - (S63) in a more compact form, Equation (S64) is derived, where each column of Equation (S64) represents one of the Equations (S60) - (S63).

$$dI_{DG02} = \begin{bmatrix} dI_a^{dI_2^{real}} & dI_a^{dI_2^{im}} & dI_a^{dI_0^{real}} & dI_a^{dI_0^{im}} \\ dI_b^{dI_2^{real}} & dI_b^{dI_2^{im}} & dI_b^{dI_0^{real}} & dI_b^{dI_0^{im}} \\ dI_c^{dI_2^{real}} & dI_c^{dI_2^{im}} & dI_c^{dI_0^{real}} & dI_c^{dI_0^{im}} \end{bmatrix} \quad (S64)$$

The perturbation of the three-phase voltages of DG bus g due to dI_{DG02} is given by Equation (S65):

$$\begin{bmatrix} dV_{g-I_2^{real}}^a & dV_{g-I_2^{im}}^a & dV_{g-I_0^{real}}^a & dV_{g-I_0^{im}}^a \\ dV_{g-I_2^{real}}^b & dV_{g-I_2^{im}}^b & dV_{g-I_0^{real}}^b & dV_{g-I_0^{im}}^b \\ dV_{g-I_2^{real}}^c & dV_{g-I_2^{im}}^c & dV_{g-I_0^{real}}^c & dV_{g-I_0^{im}}^c \end{bmatrix}^T = Z_{gg} \cdot dI_{DG02} \quad (S65)$$

where for example $dV_{g-I_2^{real}}^a$ is the perturbation of the voltage of phase a of bus g , due to the perturbation of I_2^{real} . Similarly, for the other elements.

The perturbation of the positive-sequence voltage of DG bus g with respect to the current components is given by Equation (S66).

$$\begin{bmatrix} dV_{g1-I_2^{real}} & dV_{g1-I_2^{im}} & dV_{g1-I_0^{real}} & dV_{g1-I_0^{im}} \end{bmatrix} = \begin{bmatrix} \frac{1}{3} & \frac{\alpha}{3} & \frac{\alpha^2}{3} \end{bmatrix} \cdot \begin{bmatrix} dV_{g-I_2^{real}}^a & dV_{g-I_2^{im}}^a & dV_{g-I_0^{real}}^a & dV_{g-I_0^{im}}^a \\ dV_{g-I_2^{real}}^b & dV_{g-I_2^{im}}^b & dV_{g-I_0^{real}}^b & dV_{g-I_0^{im}}^b \\ dV_{g-I_2^{real}}^c & dV_{g-I_2^{im}}^c & dV_{g-I_0^{real}}^c & dV_{g-I_0^{im}}^c \end{bmatrix} \quad (S66)$$

where for example $dV_{g1-I_2^{real}}$ is the perturbation of the positive-sequence voltage of DG bus g due to the perturbation of I_2^{real} .

The sensitivity of absolute value of positive-sequence voltage of DG bus g with respect to each current component is given in Equation (S67).

$$\left[\frac{dV_{dg1}}{dI_2^{real}} \quad \frac{dV_{dg1}}{dI_2^{im}} \quad \frac{dV_{dg1}}{dI_0^{real}} \quad \frac{dV_{dg1}}{dI_0^{im}} \right] = \frac{[dV_{g1-I_2^{real}} \quad dV_{g1-I_2^{im}} \quad dV_{g1-I_0^{real}} \quad dV_{g1-I_0^{im}}] + [V_{g1} \quad V_{g1} \quad V_{g1} \quad V_{g1}]}{dI} - [V_{g1} \quad V_{g1} \quad V_{g1} \quad V_{g1}] \quad (S67)$$

The sensitivity parameter $\frac{dV_{dg1}}{dX_{dg02}}$ is given in Equation (S68).

$$\frac{dV_{dg1}}{dX_{oltc}} = \left[\frac{dV_{dg1}}{dI_2^{real}} \quad \frac{dV_{dg1}}{dI_2^{im}} \quad \frac{dV_{dg1}}{dI_0^{real}} \quad \frac{dV_{dg1}}{dI_0^{im}} \right] \quad (S68)$$

9. Derivation of $\frac{dV_{dg1}}{dX_{svr}}$

Initially, a small perturbation at each winding ratio of SVR equal to $dTap$ is assumed. This perturbation perturbs the current sources of Fig. S1 by dI_γ as follows:

$$dI_\gamma = \begin{bmatrix} dI_\gamma^{Tap_{svr}^a} & dI_\gamma^{Tap_{svr}^b} & dI_\gamma^{Tap_{svr}^c} \end{bmatrix} \quad (S69)$$

where $\gamma = \{p, m, s\}$ and $dI_\gamma^{Tap_{svr}^f}$ is calculated (for $\gamma = p$ and $f = a$) in Equation (S13).

Equation (S70) calculates the perturbation of phase voltages of bus g , due to the tap perturbation of each phase of SVR. More specifically, $dV_{g-Tap_{svr}^f}^\xi$ expresses the perturbation of the voltage of phase $\xi = \{a, b, c\}$ of bus g , due to the tap perturbation $dTap_{svr}^f$ of phase $f = \{a, b, c\}$.

$$\begin{bmatrix} dV_{g-Tap_{svr}^a}^a & dV_{g-Tap_{svr}^b}^a & dV_{g-Tap_{svr}^c}^a \\ dV_{g-Tap_{svr}^a}^b & dV_{g-Tap_{svr}^b}^b & dV_{g-Tap_{svr}^c}^b \\ dV_{g-Tap_{svr}^a}^c & dV_{g-Tap_{svr}^b}^c & dV_{g-Tap_{svr}^c}^c \end{bmatrix} = \mathbf{Z}_{gp} \cdot d\mathbf{I}_p + \mathbf{Z}_{gm} \cdot d\mathbf{I}_m + \mathbf{Z}_{gs} \cdot d\mathbf{I}_s \quad (S70)$$

The perturbation of the positive-sequence voltage of DG bus g due to the tap perturbation is given in Equation (S71).

$$\begin{bmatrix} dV_{g1-Tap_{svr}^a} & dV_{g1-Tap_{svr}^b} & dV_{g1-Tap_{svr}^c} \end{bmatrix} = \begin{bmatrix} \frac{1}{3} & \frac{\alpha}{3} & \frac{\alpha^2}{3} \end{bmatrix} \cdot \begin{bmatrix} dV_{g-Tap_{svr}^a}^a & dV_{g-Tap_{svr}^b}^a & dV_{g-Tap_{svr}^c}^a \\ dV_{g-Tap_{svr}^a}^b & dV_{g-Tap_{svr}^b}^b & dV_{g-Tap_{svr}^c}^b \\ dV_{g-Tap_{svr}^a}^c & dV_{g-Tap_{svr}^b}^c & dV_{g-Tap_{svr}^c}^c \end{bmatrix} \quad (S71)$$

The sensitivity of absolute value of positive-sequence voltage of bus g with respect to each tap of SVR is given in Equation (S72).

$$\left[\frac{dV_{dg1}}{dT_{ap_{svr}^a}} \quad \frac{dV_{dg1}}{dT_{ap_{svr}^b}} \quad \frac{dV_{dg1}}{dT_{ap_{svr}^c}} \right] = \frac{[dV_{g1-Tap_{svr}^a} \quad dV_{g1-Tap_{svr}^b} \quad dV_{g1-Tap_{svr}^c}] + [V_{g1} \quad V_{g1} \quad V_{g1}]}{dT_{ap}} - [V_{g1} \quad V_{g1} \quad V_{g1}] \quad (S72)$$

The sensitivity parameter $\frac{dV_{dg1}}{dX_{svr}}$ is given in Equation (S73).

$$\frac{dV_{dg1}}{dX_{svr}} = \left[\frac{dV_{dg1}}{dT_{ap_{svr}^a}} \quad \frac{dV_{dg1}}{dT_{ap_{svr}^b}} \quad \frac{dV_{dg1}}{dT_{ap_{svr}^c}} \right] \quad (S73)$$

10. Derivation of $\frac{dV_{dg1}}{dX_{cap}}$

Initially, a small perturbation at the capacitor of each phase equal to dC is assumed. This perturbation perturbs the current of capacitors by $d\mathbf{I}_{cap}$ as follows:

$$d\mathbf{I}_{cap} = \begin{bmatrix} d\mathbf{I}_q^{c_a} & d\mathbf{I}_q^{c_b} & d\mathbf{I}_q^{c_c} \end{bmatrix} \quad (S74)$$

where $d\mathbf{I}_q^{c_a}, d\mathbf{I}_q^{c_b}, d\mathbf{I}_q^{c_c}$ are calculated as explained in Equation (S21).

Equation (S75) calculates the perturbation of phase voltages, due to the capacitor perturbation. More specifically, $dV_{g-\zeta}^{\xi}$ expresses the perturbation of the voltage of phase $\zeta = \{a, b, c\}$ of bus g , due to the perturbation of capacitor of phase $f = \{a, b, c\}$.

$$\begin{bmatrix} dV_{g-Ca}^a & dV_{g-Cb}^a & dV_{g-Cc}^a \\ dV_{g-Ca}^b & dV_{g-Cb}^b & dV_{g-Cc}^b \\ dV_{g-Ca}^c & dV_{g-Cb}^c & dV_{g-Cc}^c \end{bmatrix} = \mathbf{Z}_{gq} \cdot d\mathbf{I}_{cap} \quad (S75)$$

The perturbation of the positive-sequence voltage of bus g with respect to capacitor's perturbation is given by Equation (S76).

$$\begin{bmatrix} dV_{g1-Ca} & dV_{g1-Cb} & dV_{g1-Cc} \end{bmatrix} = \begin{bmatrix} \frac{1}{3} & \frac{\alpha}{3} & \frac{\alpha^2}{3} \end{bmatrix} \cdot \begin{bmatrix} dV_{g-Ca}^a & dV_{g-Cb}^a & dV_{g-Cc}^a \\ dV_{g-Ca}^b & dV_{g-Cb}^b & dV_{g-Cc}^b \\ dV_{g-Ca}^c & dV_{g-Cb}^c & dV_{g-Cc}^c \end{bmatrix} \quad (S76)$$

The sensitivity of absolute value of positive-sequence voltage of DG bus g with respect to each capacitor is given in Equation (S77).

$$\begin{bmatrix} \frac{dV_{dg1}}{dC_a} & \frac{dV_{dg1}}{dC_b} & \frac{dV_{dg1}}{dC_c} \end{bmatrix} = \frac{||dV_{g1-Ca} \quad dV_{g1-Cb} \quad dV_{g1-Cc}|| + |V_{g1} \quad V_{g1} \quad V_{g1}| - ||V_{g1} \quad V_{g1} \quad V_{g1}||}{dC} \quad (S77)$$

The sensitivity parameter $\frac{dV_{dg1}}{dX_{cap}}$ is given in Equation (S78).

$$\frac{dV_{dg1}}{dX_{cap}} = \begin{bmatrix} \frac{dV_{dg1}}{dC_a} & \frac{dV_{dg1}}{dC_b} & \frac{dV_{dg1}}{dC_c} \end{bmatrix} \quad (S78)$$

11. Derivation of $\frac{dV_{dg02}}{dX_{oltc}}$

Initially, a small perturbation $dTap$ is assumed at each phase of OLTC, which causes a perturbation at the current sources of Fig. S1 by $d\mathbf{I}_\gamma$ as follows:

$$d\mathbf{I}_\gamma = \begin{bmatrix} d\mathbf{I}_\gamma^{Tap_{oltc}^a} & d\mathbf{I}_\gamma^{Tap_{oltc}^b} & d\mathbf{I}_\gamma^{Tap_{oltc}^c} \end{bmatrix} \quad (S79)$$

where $\gamma = \{h, w\}$ and $d\mathbf{I}_\gamma^{Tap_{oltc}^f}$ is calculated according to Equation (S13).

The perturbation of the three-phase voltages of bus g due to $d\mathbf{I}_\gamma$ is given by Equation (S80).

$$\begin{bmatrix} dV_{g-Tap_{oltc}^a}^a & dV_{g-Tap_{oltc}^b}^a & dV_{g-Tap_{oltc}^c}^a \\ dV_{g-Tap_{oltc}^a}^b & dV_{g-Tap_{oltc}^b}^b & dV_{g-Tap_{oltc}^c}^b \\ dV_{g-Tap_{oltc}^a}^c & dV_{g-Tap_{oltc}^b}^c & dV_{g-Tap_{oltc}^c}^c \end{bmatrix} = \mathbf{Z}_{gh} \cdot d\mathbf{I}_h + \mathbf{Z}_{gw} \cdot d\mathbf{I}_w \quad (S80)$$

The perturbation of the negative- and zero sequence-voltage of bus g due to the tap perturbation is given in Equation (S81):

$$\begin{bmatrix} dV_{g2-Tap_{oltc}^a} & dV_{g2-Tap_{oltc}^b} & dV_{g2-Tap_{oltc}^c} \\ dV_{g2-Tap_{oltc}^a} & dV_{g2-Tap_{oltc}^b} & dV_{g2-Tap_{oltc}^c} \\ dV_{g0-Tap_{oltc}^a} & dV_{g0-Tap_{oltc}^b} & dV_{g0-Tap_{oltc}^c} \\ dV_{g0-Tap_{oltc}^a} & dV_{g0-Tap_{oltc}^b} & dV_{g0-Tap_{oltc}^c} \end{bmatrix} = \frac{1}{3} \cdot \begin{bmatrix} 1 & \alpha^2 & \alpha \\ 1 & \alpha^2 & \alpha \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} dV_{g-Tap_{oltc}^a}^a & dV_{g-Tap_{oltc}^b}^a & dV_{g-Tap_{oltc}^c}^a \\ dV_{g-Tap_{oltc}^a}^b & dV_{g-Tap_{oltc}^b}^b & dV_{g-Tap_{oltc}^c}^b \\ dV_{g-Tap_{oltc}^a}^c & dV_{g-Tap_{oltc}^b}^c & dV_{g-Tap_{oltc}^c}^c \end{bmatrix} \quad (S81)$$

where for example $dV_{g2-Tap_{oltc}^a}$ is the perturbation of negative-sequence voltage of bus g (V_{g2}), due to the tap perturbation of phase a (Tap_{oltc}^a). Similarly for the other elements.

Finally, the sensitivity parameters $\frac{dV_{dg02}}{dX_{oltc}}$ are calculated by dividing the real and imaginary components of Equation (S81) by the tap variation, as shown in Equation (S82).

$$\frac{dV_{dg02}}{dX_{oltc}} = \begin{bmatrix} \frac{dV_{g2-Tap_{oltc}^a}^{real}}{dT_{ap}} & \frac{dV_{g2-Tap_{oltc}^b}^{real}}{dT_{ap}} & \frac{dV_{g2-Tap_{oltc}^c}^{real}}{dT_{ap}} \\ \frac{dV_{g2-Tap_{oltc}^a}^{im}}{dT_{ap}} & \frac{dV_{g2-Tap_{oltc}^b}^{im}}{dT_{ap}} & \frac{dV_{g2-Tap_{oltc}^c}^{im}}{dT_{ap}} \\ \frac{dV_{g0-Tap_{oltc}^a}^{real}}{dT_{ap}} & \frac{dV_{g0-Tap_{oltc}^b}^{real}}{dT_{ap}} & \frac{dV_{g0-Tap_{oltc}^c}^{real}}{dT_{ap}} \\ \frac{dV_{g0-Tap_{oltc}^a}^{im}}{dT_{ap}} & \frac{dV_{g0-Tap_{oltc}^b}^{im}}{dT_{ap}} & \frac{dV_{g0-Tap_{oltc}^c}^{im}}{dT_{ap}} \end{bmatrix} \quad (S82)$$

where for example $dV_{g2-Tap_{oltc}^a}^{real}$ is the perturbation caused at the real component of negative-sequence voltage of bus g due to the perturbation of Tap_{oltc}^a . Similarly, for the other elements.

12. Derivation of $\frac{dV_{dg02}}{dX_{dg1}}$

The perturbation of the three-phase currents of DG, due to the perturbation of positive sequence reactive power (dQ_{dg1}) is given by Equation (S83):

$$d\mathbf{I}_{dg1} = [dI_{a1} \quad dI_{b1} \quad dI_{c1}]^T \quad (S83)$$

where $dI_{a1}, dI_{b1}, dI_{c1}$ are derived, as explained in Equations (S22) - (S24).

The perturbation of the three-phase voltages of bus g due to $d\mathbf{I}_{dg1}$ is given by Equation (S84).

$$\begin{bmatrix} dV_{g-Q_{dg1}}^a & dV_{g-Q_{dg1}}^b & dV_{g-Q_{dg1}}^c \end{bmatrix}^T = \mathbf{Z}_{gg} \cdot d\mathbf{I}_{dg1} \quad (S84)$$

The perturbation of the negative- and zero sequence-voltage of bus g due to the reactive power perturbation is given in Equation (S85):

$$\begin{bmatrix} dV_{g2-Q_{dg1}} \\ dV_{g2-Q_{dg1}} \\ dV_{g0-Q_{dg1}} \\ dV_{g0-Q_{dg1}} \end{bmatrix} = \frac{1}{3} \cdot \begin{bmatrix} 1 & \alpha^2 & \alpha \\ 1 & \alpha^2 & \alpha \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} dV_{g-Q_{dg1}}^a & dV_{g-Q_{dg1}}^b & dV_{g-Q_{dg1}}^c \end{bmatrix}^T \quad (S85)$$

Finally, the sensitivity parameter $\frac{dV_{dg02}}{dX_{dg1}}$ is calculated by dividing the real and imaginary components of Equation (S85) by the reactive power variation, as shown in Equation (S86):

$$\frac{dV_{dg02}}{dX_{dg1}} = \begin{bmatrix} \frac{dV_{g2-Q_{dg1}}^{real}}{dQ_{dg1}} \\ \frac{dV_{g2-Q_{dg1}}^{im}}{dQ_{dg1}} \\ \frac{dV_{g0-Q_{dg1}}^{real}}{dQ_{dg1}} \\ \frac{dV_{g0-Q_{dg1}}^{im}}{dQ_{dg1}} \end{bmatrix} \quad (S86)$$

13. Derivation of $\frac{dV_{dg02}}{dX_{dg02}}$

Initially, a small perturbation equal to dI is assumed at the real and imaginary components of negative- and zero-sequence currents such that $dI_2^{real} = dI_2^{im} = dI_0^{real} = dI_0^{im} = dI$, where $dI_2^{real}, dI_2^{im}, dI_0^{real}, dI_0^{im}$ is the real component of negative sequence current, the imaginary component of negative sequence current, the real component of zero-sequence current, the imaginary component of zero-sequence current, respectively.

The perturbation caused at the currents of phases a, b, c ($dI_a^{dI_2^{real}}, dI_b^{dI_2^{real}}, dI_c^{dI_2^{real}}$) due to the perturbation dI_2^{real} is calculated by Equation (S87).

$$\begin{bmatrix} dI_a^{dI_2^{real}} & dI_b^{dI_2^{real}} & dI_c^{dI_2^{real}} \end{bmatrix}^T = [dI_2^{real} \quad \alpha \cdot dI_2^{real} \quad \alpha^2 \cdot dI_2^{real}]^T \quad (S87)$$

where for example $dI_a^{dI_2^{real}}$ is the perturbation of current of phase a due to the perturbation of I_2^{real} . Similarly for the other elements.

The perturbation caused at the currents of phases a, b, c ($dI_a^{dI_2^{im}}, dI_b^{dI_2^{im}}, dI_c^{dI_2^{im}}$) due to the perturbation of dI_2^{im} is calculated by Equation (S88).

$$\begin{bmatrix} dI_a^{dI_2^{im}} & dI_b^{dI_2^{im}} & dI_c^{dI_2^{im}} \end{bmatrix}^T = [j \cdot dI_2^{im} \quad j \cdot \alpha \cdot dI_2^{im} \quad j \cdot \alpha^2 \cdot dI_2^{im}]^T \quad (S88)$$

The perturbation caused at the currents of phases a, b, c ($dI_a^{dI_0^{real}}, dI_b^{dI_0^{real}}, dI_c^{dI_0^{real}}$) due to the perturbation of dI_0^{real} is calculated by Equation (S89).

$$\begin{bmatrix} dI_a^{dI_0^{real}} & dI_b^{dI_0^{real}} & dI_c^{dI_0^{real}} \end{bmatrix}^T = [dI_0^{real} \quad dI_0^{real} \quad dI_0^{real}]^T \quad (S89)$$

The perturbation caused at the currents of phases a, b, c ($dI_a^{dI_0^{im}}, dI_b^{dI_0^{im}}, dI_c^{dI_0^{im}}$) due to the perturbation of dI_0^{im} is calculated by Equation (S90).

$$\begin{bmatrix} dI_a^{dI_0^{im}} & dI_b^{dI_0^{im}} & dI_c^{dI_0^{im}} \end{bmatrix}^T = [j \cdot dI_0^{im} \quad j \cdot dI_0^{im} \quad j \cdot dI_0^{im}]^T \quad (S90)$$

By writing Equations (S87) - (S90) in a more compact form, Equation (S91) is derived, where each column of Equation (S91) represents one of the Equations (S87) - (S90).

$$\mathbf{dI}_{DG02} = \begin{bmatrix} dI_a^{dI_2^{real}} & dI_a^{dI_2^{im}} & dI_a^{dI_0^{real}} & dI_a^{dI_0^{im}} \\ dI_b^{dI_2^{real}} & dI_b^{dI_2^{im}} & dI_b^{dI_0^{real}} & dI_b^{dI_0^{im}} \\ dI_c^{dI_2^{real}} & dI_c^{dI_2^{im}} & dI_c^{dI_0^{real}} & dI_c^{dI_0^{im}} \end{bmatrix} \quad (S91)$$

The perturbation of the three-phase voltages of DG bus g due to $\mathbf{dI}_{DG02}$ is given by Equation (S92).

$$\begin{bmatrix} dV_{g-I_2^{real}}^a & dV_{g-I_2^{im}}^a & dV_{g-I_0^{real}}^a & dV_{g-I_0^{im}}^a \\ dV_{g-I_2^{real}}^b & dV_{g-I_2^{im}}^b & dV_{g-I_0^{real}}^b & dV_{g-I_0^{im}}^b \\ dV_{g-I_2^{real}}^c & dV_{g-I_2^{im}}^c & dV_{g-I_0^{real}}^c & dV_{g-I_0^{im}}^c \end{bmatrix} = \mathbf{Z}_{gg} \cdot \mathbf{dI}_{DG02} \quad (S92)$$

The perturbation of the negative- and zero sequence-voltage of bus g due to the perturbation of negative- and zero-sequence current component of DG is given in Equation (S93).

$$\begin{bmatrix} dV_{g2-I_2^{real}} & dV_{g2-I_2^{im}} & dV_{g2-I_0^{real}} & dV_{g2-I_0^{im}} \\ dV_{g2-I_2^{real}} & dV_{g2-I_2^{im}} & dV_{g2-I_0^{real}} & dV_{g2-I_0^{im}} \\ dV_{g0-I_2^{real}} & dV_{g0-I_2^{im}} & dV_{g0-I_0^{real}} & dV_{g0-I_0^{im}} \\ dV_{g0-I_2^{real}} & dV_{g0-I_2^{im}} & dV_{g0-I_0^{real}} & dV_{g0-I_0^{im}} \end{bmatrix} = \frac{1}{3} \cdot \begin{bmatrix} 1 & \alpha^2 & \alpha \\ 1 & \alpha^2 & \alpha \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} dV_{g-I_2^{real}}^a & dV_{g-I_2^{im}}^a & dV_{g-I_0^{real}}^a & dV_{g-I_0^{im}}^a \\ dV_{g-I_2^{real}}^b & dV_{g-I_2^{im}}^b & dV_{g-I_0^{real}}^b & dV_{g-I_0^{im}}^b \\ dV_{g-I_2^{real}}^c & dV_{g-I_2^{im}}^c & dV_{g-I_0^{real}}^c & dV_{g-I_0^{im}}^c \end{bmatrix} \quad (S93)$$

Finally, the sensitivity parameter $\frac{dV_{dg02}}{dX_{dg02}}$ is calculated by separating the real and imaginary components of Equation (S93) and dividing by the current perturbation, as shown in Equation (S94):

$$\frac{dV_{dg02}}{dX_{dg02}} = \begin{bmatrix} \frac{dV_{g2-I_2}^{real}}{dI_2^{real}} & \frac{dV_{g2-I_2}^{real}}{dI_2^{im}} & \frac{dV_{g2-I_0}^{real}}{dI_0^{real}} & \frac{dV_{g2-I_0}^{real}}{dI_0^{im}} \\ \frac{dV_{g2-I_2}^{im}}{dI_2^{real}} & \frac{dV_{g2-I_2}^{im}}{dI_2^{im}} & \frac{dV_{g2-I_0}^{im}}{dI_0^{real}} & \frac{dV_{g2-I_0}^{im}}{dI_0^{im}} \\ \frac{dV_{g0-I_2}^{real}}{dI_2^{real}} & \frac{dV_{g0-I_2}^{real}}{dI_2^{im}} & \frac{dV_{g0-I_0}^{real}}{dI_0^{real}} & \frac{dV_{g0-I_0}^{real}}{dI_0^{im}} \\ \frac{dV_{g0-I_2}^{im}}{dI_2^{real}} & \frac{dV_{g0-I_2}^{im}}{dI_2^{im}} & \frac{dV_{g0-I_0}^{im}}{dI_0^{real}} & \frac{dV_{g0-I_0}^{im}}{dI_0^{im}} \end{bmatrix} \quad (S94)$$

where for example $dV_{g2-I_2}^{real}$ is the variation of the real component of negative sequence voltage of bus g (V_{g2}^{real}), due to the variation of the real component of negative sequence current of bus g (I_2^{real}).

14. Derivation of $\frac{dV_{dg02}}{dX_{svr}}$

Initially, a small perturbation is assumed at the tap of each phase of SVR equal to $dTap$. This perturbation perturbs the SVR current sources of Fig. S1 by dI_γ as follows:

$$dI_\gamma = \begin{bmatrix} dI_\gamma^{Tap^a_{svr}} & dI_\gamma^{Tap^b_{svr}} & dI_\gamma^{Tap^c_{svr}} \end{bmatrix} \quad (S95)$$

where $\gamma = \{p, m, s\}$ and $dI_\gamma^{Tap^f_{svr}}$ is calculated for $\gamma = p$ and $f = a$ in Equation (S13).

Equation (S96) calculates the perturbation of the phase voltages of bus g , due to the tap perturbation of SVR. More specifically, $dV_{g-Tap^f_{svr}}^\xi$ expresses the perturbation of the voltage of phase $\xi = \{a, b, c\}$ of bus g , due to the tap perturbation $dTap_{svr}^f$ of phase $f = \{a, b, c\}$.

$$\begin{bmatrix} dV_{g-Tap^a_{svr}}^a & dV_{g-Tap^b_{svr}}^a & dV_{g-Tap^c_{svr}}^a \\ dV_{g-Tap^a_{svr}}^b & dV_{g-Tap^b_{svr}}^b & dV_{g-Tap^c_{svr}}^b \\ dV_{g-Tap^a_{svr}}^c & dV_{g-Tap^b_{svr}}^c & dV_{g-Tap^c_{svr}}^c \end{bmatrix} = Z_{gp} \cdot dI_p + Z_{gm} \cdot dI_m + Z_{gs} \cdot dI_s \quad (S96)$$

The perturbation of the negative- and zero sequence-voltage of bus g due to the SVR tap perturbation is given in Equation (S97):

$$\begin{bmatrix} dV_{g2-Tap^a_{svr}} & dV_{g2-Tap^b_{svr}} & dV_{g2-Tap^c_{svr}} \\ dV_{g2-Tap^a_{svr}} & dV_{g2-Tap^b_{svr}} & dV_{g2-Tap^c_{svr}} \\ dV_{g0-Tap^a_{svr}} & dV_{g0-Tap^b_{svr}} & dV_{g0-Tap^c_{svr}} \\ dV_{g0-Tap^a_{svr}} & dV_{g0-Tap^b_{svr}} & dV_{g0-Tap^c_{svr}} \end{bmatrix} = \frac{1}{3} \cdot \begin{bmatrix} 1 & \alpha^2 & \alpha \\ 1 & \alpha^2 & \alpha \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} dV_{g-Tap^a_{svr}}^a & dV_{g-Tap^b_{svr}}^a & dV_{g-Tap^c_{svr}}^a \\ dV_{g-Tap^a_{svr}}^b & dV_{g-Tap^b_{svr}}^b & dV_{g-Tap^c_{svr}}^b \\ dV_{g-Tap^a_{svr}}^c & dV_{g-Tap^b_{svr}}^c & dV_{g-Tap^c_{svr}}^c \end{bmatrix} \quad (S97)$$

where for example $dV_{g2-Tap^a_{svr}}$ denotes the perturbation of the negative-sequence voltage of DG bus g (V_{g2}), due to the SVR tap perturbation of phase a (Tap_{svr}^a).

Finally, the sensitivity parameters $\frac{dV_{dg02}}{dX_{svr}}$ are calculated by dividing the real and imaginary components of Equation (S97) by the tap perturbation, as shown in Equation (S98).

$$\frac{dV_{dg02}}{dX_{svr}} = \begin{bmatrix} \frac{dV_{g2-Tap^a_{svr}}^{real}}{dT_{ap}} & \frac{dV_{g2-Tap^b_{svr}}^{real}}{dT_{ap}} & \frac{dV_{g2-Tap^c_{svr}}^{real}}{dT_{ap}} \\ \frac{dV_{g2-Tap^a_{svr}}^{im}}{dT_{ap}} & \frac{dV_{g2-Tap^b_{svr}}^{im}}{dT_{ap}} & \frac{dV_{g2-Tap^c_{svr}}^{im}}{dT_{ap}} \\ \frac{dV_{g0-Tap^a_{svr}}^{real}}{dT_{ap}} & \frac{dV_{g0-Tap^b_{svr}}^{real}}{dT_{ap}} & \frac{dV_{g0-Tap^c_{svr}}^{real}}{dT_{ap}} \\ \frac{dV_{g0-Tap^a_{svr}}^{im}}{dT_{ap}} & \frac{dV_{g0-Tap^b_{svr}}^{im}}{dT_{ap}} & \frac{dV_{g0-Tap^c_{svr}}^{im}}{dT_{ap}} \end{bmatrix} \quad (S98)$$

15. Derivation of $\frac{dV_{dg02}}{dX_{cap}}$

Initially, a small perturbation at each capacitor of each phase equal to dC is assumed. This perturbation perturbs the phase current of capacitors by $d\mathbf{I}_{cap}$ as follows:

$$d\mathbf{I}_{cap} = \begin{bmatrix} d\mathbf{I}_q^{C_a} & d\mathbf{I}_q^{C_b} & d\mathbf{I}_q^{C_c} \end{bmatrix} \quad (S99)$$

where $d\mathbf{I}_q^{C_a}$, $d\mathbf{I}_q^{C_b}$, $d\mathbf{I}_q^{C_c}$ are calculated as explained in Equation (S21).

Equation (S100) calculates the perturbation of phase voltages of bus g , due to the capacitor perturbations:

$$\begin{bmatrix} dV_{g-Ca}^a & dV_{g-Cb}^a & dV_{g-Cc}^a \\ dV_{g-Ca}^b & dV_{g-Cb}^b & dV_{g-Cc}^b \\ dV_{g-Ca}^c & dV_{g-Cb}^c & dV_{g-Cc}^c \end{bmatrix} = \mathbf{Z}_{gq} \cdot d\mathbf{I}_{cap} \quad (S100)$$

where dV_{g-Cr}^l is the perturbation of the voltage of phase l of bus g , due to the perturbation of capacitor of phase r .

The perturbation of the negative- and zero sequence-voltage of bus g due to the capacitor perturbation is given in Equation (S101):

$$\begin{bmatrix} dV_{g2-Ca} & dV_{g2-Cb} & dV_{g2-Cc} \\ dV_{g2-Ca} & dV_{g2-Cb} & dV_{g2-Cc} \\ dV_{g0-Ca} & dV_{g0-Cb} & dV_{g0-Cc} \\ dV_{g0-Ca} & dV_{g0-Cb} & dV_{g0-Cc} \end{bmatrix} = \frac{1}{3} \cdot \begin{bmatrix} 1 & \alpha^2 & \alpha \\ 1 & \alpha^2 & \alpha \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} dV_{g-Ca}^a & dV_{g-Cb}^a & dV_{g-Cc}^a \\ dV_{g-Ca}^b & dV_{g-Cb}^b & dV_{g-Cc}^b \\ dV_{g-Ca}^c & dV_{g-Cb}^c & dV_{g-Cc}^c \end{bmatrix} \quad (S101)$$

where for example dV_{g2-Ca} is the perturbation of negative-sequence voltage of DG bus g (V_{g2}), due to the perturbation of capacitor (C_a). Similarly for the other elements.

Finally, the sensitivity parameters $\frac{dV_{dg02}}{dX_{cap}}$ are calculated by dividing the real and imaginary components of Equation (S101) by the capacitor's perturbation, as shown in Equation (S102):

$$\frac{dV_{dg02}}{dX_{cap}} = \begin{bmatrix} \frac{dV_{g2-Ca}^{real}}{dC} & \frac{dV_{g2-Cb}^{real}}{dC} & \frac{dV_{g2-Cc}^{real}}{dC} \\ \frac{dV_{g2-Ca}^{im}}{dC} & \frac{dV_{g2-Cb}^{im}}{dC} & \frac{dV_{g2-Cc}^{im}}{dC} \\ \frac{dV_{g0-Ca}^{real}}{dC} & \frac{dV_{g0-Cb}^{real}}{dC} & \frac{dV_{g0-Cc}^{real}}{dC} \\ \frac{dV_{g0-Ca}^{im}}{dC} & \frac{dV_{g0-Cb}^{im}}{dC} & \frac{dV_{g0-Cc}^{im}}{dC} \end{bmatrix} \quad (S102)$$

16. Derivation of $\frac{dV_{svr}}{dX_{oltc}}$

Similar to $\frac{dV_{oltc}}{dX_{oltc}}$

17. Derivation of $\frac{dV_{svr}}{dX_{dg1}}$

Similar to $\frac{dV_{oltc}}{dX_{dg1}}$

18. Derivation of $\frac{dV_{svr}}{dX_{dg02}}$

Similar to $\frac{dV_{oltc}}{dX_{dg02}}$

19. Derivation of $\frac{dV_{svr}}{dX_{svr}}$

Similar to $\frac{dV_{oltc}}{dX_{svr}}$

20. Derivation of $\frac{dV_{svr}}{dX_{cap}}$

Similar to $\frac{dV_{oltc}}{dX_{cap}}$

21. Derivation of $\frac{dV_{cap}}{dX_{oltc}}$

Similar to $\frac{dV_{oltc}}{dX_{oltc}}$

22. Derivation of $\frac{dV_{cap}}{dX_{dg1}}$

Similar to $\frac{dV_{oltc}}{dX_{dg1}}$

23. Derivation of $\frac{dV_{cap}}{dX_{dg02}}$

Similar to $\frac{dV_{oltc}}{dX_{dg02}}$

24. Derivation of $\frac{dV_{cap}}{dX_{svr}}$

Similar to $\frac{dV_{oltc}}{dX_{svr}}$

25. Derivation of $\frac{dV_{cap}}{dX_{cap}}$

Similar to $\frac{dV_{oltc}}{dX_{cap}}$