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An Equivalent Surface Dipoles based Electric Field to Magnetic Field Transformations for Estimating Radiated Emissions

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Abstract—The new generation of communication devices operating in higher frequency bands is constantly pushing the complexity of field measurements for electromagnetic compliance and the design of field probes. These compliance are to be met for both electric and magnetic fields, of which if one set is available, the other can be estimated computationally using source reconstruction algorithms. The main objective of this paper is to refine the dipole-based source reconstruction algorithms using surface equivalence principle. Compared to earlier works, the algorithm is based on a novel way of placing a single dipole per point tangentially to a fictitious surface based on surface equivalence theorem instead of multiple orthogonal dipoles per point. We also rigorously prove the existence, uniqueness, and error bounds involved in the inverse problem. The numerical results are compared with the state-of-art to demonstrate the effectiveness in obtaining accurate fields and locating the regions of the Device Under Test responsible for overshooting the interference limits.

Index Terms—EMI, EMC, Field Measurements, Radiated Emissions, DUT, Surface Equivalence Theorem.

I. INTRODUCTION

THE ever-increasing demand for bandwidth of new generation communication devices poses a new challenge to the well-established electromagnetic interference and electromagnetic compatibility (EMI-EMC) measurements and testing [1]. These EMC measurement standards are regularly updated for higher operating frequency range [2], [3]. The military standard MIL-STD 461G prescribes strict limits for the radiated electromagnetic fields compared to the commercial ones. The example of two Radiated Emissions tests under this standard is RE101 and RE102 for the radiated magnetic and electric fields by a Device Under Test (DUT). The electric and magnetic fields are measured using electric and magnetic probes connected to a spectrum analyzer or a vector network analyzer. The objective behind EMI-EMC measurements is not to find accurate radiation field patterns (as in the case of antennas). Instead, it is to find the maximum value of the field at a spherical measurement surface enclosing the DUT accurately. This objective is effectively done by performing near-field measurements of the DUT and obtaining the field computationally at the location specified by the EMI-EMC

standard using source reconstruction algorithm to estimate the radiated field.

The near field measurement-based source reconstruction algorithms broadly can be grouped under dipoles based, integral equations, Finite Element Method (FEM), and Finite Difference Method (FDM). The integral equations and dipole-based method give greater accuracy as compared to the other methods [4], [5]. There also have been recent developments where these methods are combined with optimization techniques [6] and regularization for broadband frequency range [7]. The source reconstruction based on FDM [8] require simultaneous near field measurements on three different planes and a Perfectly Electrical Conductor (PEC) enclosure to predict the radiated emissions correctly. Moreover, the usage of FDM inherits its shortcomings in terms of numerical errors [9]. The source reconstruction algorithm in [10] based on the Finite Element Method (FEM) has also been used to predict the far-field in the context of EMI and identify the location of interference from multiple sources [11] based on the estimation of s-parameters. The dipole-based method has limitations in terms of position and density of the dipoles to be fixed a priori [5]. However, the method is flexible enough to use new measurement devices [12].

The dipoles-based source reconstruction algorithms' advantage over other techniques is a faster and more straightforward implementation. The primary objective behind these algorithms is to build a mathematical model without the internal details of the DUT [13]. In these methods, multiple orthogonal electric and magnetic dipoles are placed on each point in three-dimensional space [14]. Although there have been dipole placements where its alignment is also treated as an unknown [13], this is equivalent to placing two orthogonal dipoles on the surface. In [13], the authors used both electric and magnetic fields for the source reconstruction. A similar treatment [15] uses only magnetic field measurement data for the source reconstruction with either a pair of orthogonal electric or magnetic dipoles per point. The effectiveness of the source reconstruction in this work remained limited to near-field regions. A regularized dipole-based method was proposed in [16] where two orthogonal magnetic dipoles and one electric dipole moment were placed per point. The dipole-based source reconstruction applied to the cellphone suggested using six dipoles (three electric and three magnetic dipoles) per point [14]. It was further enhanced using machine learning [17] and using the dipole configuration of [16]. The work in only [13],

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and [15] report the quantified error or accuracy in the above cited methods. The other issue is that these approaches have limited applicability to non-planar structures (refer to Table 1 in [18]).

This work addresses some of the limitations highlighted above with the proposed equivalent surface dipoles-based field transformation from the computational perspective. The approach presented here assumes that the near field measurement data is readily available and therefore does not venture out in the intricacies of the measurement of phase and magnitude in practice (interested reader may refer to the recent work [19] and the references therein). The proposed method places only one dipole per point compared to multiple dipoles placed by the above state-of-art methods. As the novel contribution of this work, we have refined the dipole source reconstruction method using the equivalent surface principle. To the best of our knowledge, the presented method has not been applied before to the discussed field transformations. Unlike the state-of-the-art methods, this work is also applied to a nonplanar radiating structure in high frequency. The predominant ones are also compared in terms of the relative error with the proposed technique. Analyzing the proposed procedure, we rigorously prove the existence and uniqueness of the solution and error bounds in the transformed field. The effectiveness of the proposed method in locating the potential source of EMI in the DUT is also demonstrated.

The paper is outlined as: the theoretical background and notation related to the electromagnetic fields of a dipole are set in the section II. The section III presents the proposed procedure with the mathematical analysis in terms of existence, uniqueness and error bounds. The results validating the proposed procedure are presented in section IV followed by the conclusion in section V.

II. BACKGROUND AND NOTATION

An infinitesimally small electric dipole, in a high-frequency regime, can be considered as a localized oscillating source where $\mathbf{p}$ is the electric dipole moment(also captures the alignment). The generalized expressions (independent of a coordinate system) of the radiated electric and magnetic field intensity of an electric dipole [20] are

$$\mathbf{E}(\mathbf{r}) = \frac{1}{4\pi\epsilon_0} \left\{ k^2 (\hat{\mathbf{r}} \times \mathbf{p}) \times \hat{\mathbf{r}} \frac{e^{ikr}}{r} + [3\hat{\mathbf{r}}(\hat{\mathbf{r}} \cdot \mathbf{p}) - \mathbf{p}] \left(\frac{1}{r^3} - \frac{ik}{r^2} \right) e^{ikr} \right\} \quad (1)$$

$$\mathbf{H}(\mathbf{r}) = \frac{ck^2}{4\pi} (\hat{\mathbf{r}} \times \mathbf{p}) \frac{e^{ikr}}{r} \left(1 - \frac{1}{ikr} \right) \quad (2)$$

where k is the wavenumber and c is the speed of light. The $r = |\mathbf{r} - \mathbf{r}'|$ is based on the observation point $\mathbf{r}'$ and the position of the dipole $\mathbf{r}$. Although the proposed transformation uses only the electric dipoles, the magnetic dipole moments

are indispensable for implementing the state-of-art methods. Therefore the generalized expressions [20] are as follows:

$$\mathbf{E}(\mathbf{r}) = -\frac{Z_0 k^2}{4\pi} (\hat{\mathbf{r}} \times \mathbf{m}) \frac{e^{ikr}}{r} \left(1 - \frac{1}{ikr} \right), \quad (3)$$

$$\mathbf{H}(\mathbf{r}) = \frac{1}{4\pi} \left\{ k^2 (\hat{\mathbf{r}} \times \mathbf{m}) \times \hat{\mathbf{r}} \frac{e^{ikr}}{r} + [3\hat{\mathbf{r}}(\hat{\mathbf{r}} \cdot \mathbf{m}) - \mathbf{m}] \left(\frac{1}{r^3} - \frac{ik}{r^2} \right) e^{ikr} \right\}. \quad (4)$$

where Z_0 is the characteristic impedance and $\mathbf{m}$ is the magnetic dipole moment.

The expressions for the field of electric dipole are used to define a set of dipoles on fictitious surface based on surface equivalence theorem. Consider a source S which radiates the electromagnetic field $(\mathbf{E}(\mathbf{r}), \mathbf{H}(\mathbf{r}))$ in the three-dimensional space $\mathbb{R}^3$. The space $\mathbb{R}^3$ is divided into two domains such that $S \in \Omega$ and $\bar{\Omega} \cup \Omega_e \in \mathbb{R}^3$. If we replace the original field in Ω as $(\mathbf{E}_i(\mathbf{r}), \mathbf{H}_i(\mathbf{r}))$ then the surface equivalence surface allows us to replace the original source S with equivalent surface currents $(\mathbf{J}(\mathbf{r}), \mathbf{M}(\mathbf{r}))$ on the boundary surface $\Gamma_s = \partial\Omega$ which radiate $(\mathbf{E}(\mathbf{r}), \mathbf{H}(\mathbf{r}))$ in the Ω_e and the electromagnetic fields different from $(\mathbf{E}(\mathbf{r}), \mathbf{H}(\mathbf{r}))$ (usually set to zero according to Love's condition) in the region Ω . The objective of reconstructing $(\mathbf{E}(\mathbf{r}), \mathbf{H}(\mathbf{r}))$ can be achieved using a single current density function (either $\mathbf{J}(\mathbf{r})$ or $\mathbf{M}(\mathbf{r})$). These current densities satisfy

$$\begin{aligned} \hat{\mathbf{n}} \times [\mathbf{H}(\mathbf{r}) - \mathbf{H}_i(\mathbf{r})] &= \mathbf{J}(\mathbf{r}) \\ -\hat{\mathbf{n}} \times [\mathbf{E}(\mathbf{r}) - \mathbf{E}_i(\mathbf{r})] &= \mathbf{M}(\mathbf{r}) \end{aligned} \quad (5)$$

for $\mathbf{r} \in \Gamma_s$.

III. PROPOSED FIELD TRANSFORMATIONS

Consider a typical measurement setup for estimating the high frequency radiation emissions from a DUT as shown in Figure 1. The DUT is represented by the volume region Ω . The boundary of the DUT is defined by $\Gamma = \partial\Omega$. The area of interest where the electromagnetic fields are assessed or computed is the region exterior to Ω . The region is represented by $\Omega_e = \mathbb{R}^3/\Omega$ where $\bar{\Omega} = \Omega \cup \Gamma$ is a closed region.

The reconstruction of radiated electromagnetic fields by the DUT requires the flow of either electric or magnetic currents or both on an equivalent surface. These currents are proposed to be represented by a set of electric dipoles placed on the equivalent surface Γ_d . The mathematical justification of source reconstruction using a single type of dipoles (either electric or magnetic) can be found in [21]. The electric and magnetic field due to each electric dipole in this set can be easily computed using equations (1) and (2). Each dipole is placed tangentially to the surface Γ_d which ensures the applicability of the surface equivalence principle. The density of electric dipoles required to discretize the equivalent surface currents is dictated by the accuracy requirement and the available computational resources. The convex hull of the DUT is chosen as the equivalent surface Γ_d (a spherical surface for point sources such as dipoles). The equivalent surface Γ_d is discretized into a triangular mesh with the average edge length

Fig. 1: Measurement setup.

specified as one-tenth of the wavelength for larger DUTs or equal to the spatial resolution of the near field measurement data. We define an electric dipole placed at the midpoint and align its moment $\mathbf{p}$ along the edge. The set of electric dipoles represents the equivalent surface dipoles decomposing the electromagnetic field for $\mathbf{r} \in \Omega_e$ as

$$\mathbf{E}(\mathbf{r}) = \sum_{j=1}^{N_d} p_j \mathbf{E}_j(\mathbf{r}) \quad (6)$$

$$\mathbf{H}(\mathbf{r}) = \sum_{j=1}^{N_d} p_j \mathbf{H}_j(\mathbf{r}) \quad (7)$$

where N_d is the total number of dipoles, $\mathbf{E}_j(\mathbf{r}) = E_{jx}(\mathbf{r})\hat{x} + E_{jy}(\mathbf{r})\hat{y} + E_{jz}(\mathbf{r})\hat{z}$ and $\mathbf{H}_j(\mathbf{r}) = H_{jx}(\mathbf{r})\hat{x} + H_{jy}(\mathbf{r})\hat{y} + H_{jz}(\mathbf{r})\hat{z}$ are the electric and magnetic field of the j^{th} dipole respectively computed using equation (1) and (2). The next step is to define a set of measurement points $\{\mathbf{r}_i\}_{i=1}^{N_m}$ lying on the measurement surface Γ_m . This naturally transforms equations (6) and (7) into the rectangular system of linear equations as

$$\mathbf{T}\mathbf{p} = \mathbf{e}, \quad (8)$$

$$\mathbf{K}\mathbf{p} = \mathbf{h}, \quad (9)$$

where element wise $\mathbf{p}_j = p_j$ are the unknown coefficients, the element of the electric field system matrix is defined as

$$\mathbf{T}_{ij} = \begin{cases} E_{jx}(\mathbf{r}_i), & \text{if } 1 \leq i \leq N_m \\ E_{jy}(\mathbf{r}_i), & \text{if } N_m + 1 \leq i \leq 2N_m \\ E_{jz}(\mathbf{r}_i), & \text{if } 2N_m + 1 \leq i \leq 3N_m \end{cases}, \quad (10)$$

and the magnetic field system matrix

$$\mathbf{K}_{ij} = \begin{cases} H_{jx}(\mathbf{r}_i), & \text{if } 1 \leq i \leq N_m \\ H_{jy}(\mathbf{r}_i), & \text{if } N_m + 1 \leq i \leq 2N_m \\ H_{jz}(\mathbf{r}_i), & \text{if } 2N_m + 1 \leq i \leq 3N_m \end{cases}. \quad (11)$$

The right hand side vectors element wise are given by

$$\mathbf{e}_i = \begin{cases} E_x(\mathbf{r}_i), & \text{if } 1 \leq i \leq N_m \\ E_y(\mathbf{r}_i), & \text{if } N_m + 1 \leq i \leq 2N_m \\ E_z(\mathbf{r}_i), & \text{if } 2N_m + 1 \leq i \leq 3N_m \end{cases}, \quad (12)$$

and

$$\mathbf{h}_i = \begin{cases} H_x(\mathbf{r}_i), & \text{if } 1 \leq i \leq N_m \\ H_y(\mathbf{r}_i), & \text{if } N_m + 1 \leq i \leq 2N_m \\ H_z(\mathbf{r}_i), & \text{if } 2N_m + 1 \leq i \leq 3N_m \end{cases}, \quad (13)$$

where $\mathbf{E}(\mathbf{r}_i) = E_x(\mathbf{r}_i)\hat{x} + E_y(\mathbf{r}_i)\hat{y} + E_z(\mathbf{r}_i)\hat{z}$ and $\mathbf{H}(\mathbf{r}_i) = H_x(\mathbf{r}_i)\hat{x} + H_y(\mathbf{r}_i)\hat{y} + H_z(\mathbf{r}_i)\hat{z}$ are the measured electromagnetic fields of the DUT at the observation point $\mathbf{r}_i$.

A. Procedure for Computing the required Field

For a DUT, whose electromagnetic field data is of interest can be obtained as follows:

- i) Use as an input, the measured electric field $\mathbf{E}(\mathbf{r})$ (a complex number which includes both phase and magnitude) at a predefined set of measurement points $\{\mathbf{r}_i\}_{i=1}^{N_m}$ and set $\mathbf{e}$ vector according to equation (12). If the magnetic field measurements are chosen as an initial step then set $\mathbf{h}$ vector according to the equation (13).
- ii) Obtain the unknown coefficients i.e the dipole moments for the set of dipoles using the equation (8) (or equation (9) if magnetic field is being measured in the initial step) using the inverse method outlined in the following subsection.
- iii) Compute the magnetic field at the measurement points set $\{\mathbf{r}_i\}_{i=1}^{N_m}$ using equation (9) (or equation (8) if magnetic field measurements are chosen initially).

B. Existence & Uniqueness of the Solution

The existence of the surface current density $\mathbf{J}$ replacing the existing DUT is proved using the surface equivalence principle. Assume that the DUT is replaced with an equivalent surface Γ_d that encloses a volume region Ω_d . The solution provides a surface current density $\mathbf{J}(\mathbf{r})$ on Γ_d such that it radiates the original electromagnetic field ($\mathbf{E}(\mathbf{r}), \mathbf{H}(\mathbf{r})$) in the region $\mathbb{R}^3/\Omega_d$. Let us assume that such a $\mathbf{J}(\mathbf{r})$ does not exist. The absence of $\mathbf{J}(\mathbf{r})$ implies the existence of the original electromagnetic field across the surface Γ_d . This contradicts the assumption that the DUT is replaced with an equivalent surface. Hence, there exists a $\mathbf{J}(\mathbf{r})$ on Γ_d that will radiate ($\mathbf{E}(\mathbf{r}), \mathbf{H}(\mathbf{r})$) in the region $\mathbb{R}^3/\Omega_d$.

The uniqueness of the solution current density is ensured due to the Surface Equivalence Principle. Consider the DUT replaced with the equivalent surface Γ_d . The Γ_d splits $\mathbb{R}^3$ into two regions $\mathbb{R}^3/\Omega_d$ and Ω_d . Let us assume that there are two surface current densities $\mathbf{J}_1(\mathbf{r})$ and $\mathbf{J}_2(\mathbf{r})$ which radiate ($\mathbf{E}(\mathbf{r}), \mathbf{H}(\mathbf{r})$) in $\mathbb{R}^3/\Omega_d$ and ($\mathbf{E}_d(\mathbf{r}), \mathbf{H}_d(\mathbf{r})$) in the region Ω_d . If a new current density $\mathbf{J}(\mathbf{r}) = \mathbf{J}_1(\mathbf{r}) - \mathbf{J}_2(\mathbf{r})$ is placed on Γ_d then it radiates zero electromagnetic fields by principle of superposition. This is possible only if $\mathbf{J}(\mathbf{r}) = 0$ which implies $\mathbf{J}_1(\mathbf{r}) = \mathbf{J}_2(\mathbf{r})$. Hence, the uniqueness of $\mathbf{J}(\mathbf{r})$ is ensured if $\mathbf{E}_d(\mathbf{r}), \mathbf{H}_d(\mathbf{r})$ is specified in the Ω_d . The reader should note that this uniqueness is related to the error free specifications of $\mathbf{E}_d(\mathbf{r}), \mathbf{H}_d(\mathbf{r})$. If there is error in the measured field itself the solution will have the variation which is bounded as is demonstrated in next subsection.

C. Error Bounds

The proposed procedure takes the input as the measured electric or magnetic field data and computes the second one at the predefined measurement points. In the analysis presented here we prove that the error of the computed field is upper bounded by the error of the measurement system.

Let the measured electric field $\mathbf{e} + \delta\mathbf{e}$ has an uncertainty $\delta\mathbf{e}$ causing deviation in the solution $\mathbf{p}$. Due to $\mathbf{T}$ being a linear operator, we get

$$\mathbf{T}(\mathbf{p} + \delta\mathbf{p}) = \mathbf{e} + \delta\mathbf{e}. \quad (14)$$

In the next step, the magnetic field is obtained as

$$\mathbf{h} + \delta\mathbf{h} = \mathbf{K}(\mathbf{p} + \delta\mathbf{p}) \quad (15)$$

corrupted with the error. Subtracting (8) from (14) and equation (9) from (15) gives

$$\mathbf{T}\delta\mathbf{p} = \delta\mathbf{e}, \quad (16)$$

$$\delta\mathbf{h} = \mathbf{K}\delta\mathbf{p}. \quad (17)$$

Combining the above two equations, we get

$$\delta\mathbf{h} = \mathbf{K}\mathbf{T}^{-1}\delta\mathbf{e}. \quad (18)$$

Using the Cauchy-Schwarz inequality [22] we get

$$\|\delta\mathbf{h}\|_2 \leq \|\mathbf{K}\|_2 \|\mathbf{T}^{-1}\|_2 \|\delta\mathbf{e}\|_2 \quad (19)$$

when the electric field was chosen in the initial step. If magnetic field is measured in first step the inequality is

$$\|\delta\mathbf{e}\|_2 \leq \|\mathbf{T}\|_2 \|\mathbf{K}^{-1}\|_2 \|\delta\mathbf{h}\|_2. \quad (20)$$

This clearly demonstrates that the computed field is bounded by the error in the input field measurement data. The interested reader may refer [23] and the references therein for more rigorous proofs based on Sobolev spaces.

D. Solution of the Inverse Source Reconstruction

The DUT replaced with equivalent surface dipoles results in the discretization of the inverse problem of source reconstruction. We demonstrate the method here using the electric field measurements taken as initial step. The arguments remain same for the magnetic field measurements. The linear system of equations is given by

$$\mathbf{T}\mathbf{p} = \mathbf{e} \quad (21)$$

The inverse problem finds the solution vector $\mathbf{p}$ by minimizing $\|\mathbf{T}\mathbf{p} - \mathbf{e}\|_2 / \|\mathbf{e}\|_2$. The number of unknowns N_d is the number of dipoles placed on the equivalent surface Γ_d . The matrix $\mathbf{T}$ has the dimension of $3N_m$ rows and N_d columns.

Since the measurement surface Γ_m encloses Γ_d , usually the discretization criteria will result in the $N_m > N_d$. The strict adherence to the criteria will result in the oversampling of the field at Γ_m . The existence of non-radiating and minimum energy modes when the field is modeled at Γ_d causes these fields to be evanescent at the Γ_m [24]. This is the cause of the ill-posedness when the equation (21) is solved. These evanescent modes are responsible for the existence of the null

space of $\mathbf{T}$. The equation (21) should be solved in the least square sense with Tikhonov's regularization [25]. The other techniques such as Truncated SVD or pseudo-inverse can also be used. These techniques, however, become computationally prohibitive whenever the size of the problem increases. In such scenarios, we need to resort to iterative solvers which are also well suited for fast algorithms [25].

To use iterative solvers, we need to convert the linear system of equation (21) to a normal residual system of equations

$$\mathbf{T}^H \mathbf{T} \mathbf{p} = \mathbf{T}^H \mathbf{e} \quad (22)$$

where $\mathbf{T}^H$ is the Hermitian transpose of $\mathbf{T}$. In the case of an underdetermined system, the normal error system of the equations are employed. The choice of the iterative solver, does not effect the solution accuracy significantly for the proposed algorithm from implementation point of view, the standard libraries for solving the least square problem can be used [26].

IV. RESULTS AND VALIDATION

The proposed transformations for estimating the radiated emissions are being validated using two planar structures: a microwave low-pass filter and a patch antenna followed by a horn antenna as a specimen of non planar structure. The initial two examples are chosen for providing realistic scenarios covering one example of low and high-frequency regime respectively. The other crucial difference is that the microwave low pass filter is non-radiating, and the patch antenna works as a radiating structure in the far-field region. The electric and magnetic field data of the examples are generated synthetically using a commercial integral equations based solver [27]. The accuracy of the proposed procedure is analyzed in terms of relative error in the measured field. The relative error at a measurement point $\mathbf{r}_i$ is defined as $\delta\mathbf{H}(\mathbf{r}_i) = \frac{\|\mathbf{H}(\mathbf{r}_i) - \mathbf{H}_m(\mathbf{r}_i)\|_2}{\|\mathbf{H}(\mathbf{r}_i)\|_2}$. The overall error (similar to [13]) in the obtained magnetic field is defined as

$$\epsilon_H = \sqrt{\frac{\sum_{i=1}^{N_m} \|\mathbf{H}(\mathbf{r}_i) - \mathbf{H}_m(\mathbf{r}_i)\|_2^2}{\sum_{i=1}^{N_m} \|\mathbf{H}(\mathbf{r}_i)\|_2^2}}. \quad (23)$$

The effectiveness of the iterative solver in finding the solution of the inverse source reconstruction problem is determined based on the residual defined as

$$\delta r = \frac{\|\mathbf{A}\mathbf{x} - \mathbf{b}\|_2}{\|\mathbf{b}\|_2}, \quad (24)$$

where $\mathbf{A}$ can be $\mathbf{T}$ or $\mathbf{K}$ based on the field transformation.

The proposed transformation is compared with the state-of-the-art methods. In order to eradicate any inequity, the measurement data is obtained on a planar surface (instead of a spherical surface as proposed here). Secondly, the number of measurement points is kept the same for all the approaches, including the proposed one. The number of dipoles used for source reconstruction is almost same for all the approaches however, the number, type and orientation of dipoles placed on each point in the 3D space vary according to the technique employed as shown in Fig. 2 and 3. Third, the state of the art methods use the field measurement data on scanning plane

Fig. 2: Source reconstruction setup of the proposed transformation for comparing it with the state-of-the-art methods.

Fig. 3: Source reconstruction setup of the state-of-the-art methods with different dipole orientations.

to obtain the field on a validation plane. This work obtains the magnetic field on the same scanning plane as that of the electric field measurement data. The comparison is made using three scenarios: first, original measurement data is used; the second scenario (measurement error) deals with a random noise added to the original data resulting in 3% relative error in the field values, and the last scenario (observation error) introduces 3% relative error in the coordinates of the observation points on the measurement surface. The relative error of 3% brings the performance of the proposed method compared to the other methods; otherwise, it still outperforms till 1% of error. The comparison presented in the tables also provides the dipole orientations of the methods.

A. Planar Structure: Microwave Low Pass Filter

The design of microwave low pass filter (cut-off frequency of 800MHz) operating at 500MHz is specified in Fig. 4. The measurement scanning plane is of size 46mm $\times$ 140mm with 2400 measurement points. The reactive near field for the structure corresponds to a distance less than $h = 23.85$ mm.

In the first experiment, the input electric field data at a $h = 5$ mm (please refer Fig. 2 and 3 for the setup) corre-

Fig. 4: Microwave Low Pass Filter: Design specifications in mm (operating at 500 MHz).

TABLE I: Microwave Low Pass Filter: Measurement scanning plane at $h = 5$ mm operating at 500 MHz

Scenarios	Reference	δr	ϵ_H	Number of Dipoles
No Error	This Work	0.03885	0.62869	1044
	$p_x p_y p_z m_x m_y m_z$ [14]	0.11674	0.99986	1218
	$p_z m_x m_y$ [16]	0.69470	0.91711	1056
	$p_x p_y$ [15]	0.11605	1.07214	1056
	$m_x m_y$ [15]	0.04123	0.56436	1056
	$p_x p_y m_x m_y$ [13]	0.11929	1.04279	1120
3% Measurement Error	This Work	0.04201	0.64173	1044
	$p_x p_y p_z m_x m_y m_z$ [14]	0.11686	0.99794	1218
	$p_z m_x m_y$ [16]	0.69471	0.91710	1056
	$p_x p_y$ [15]	0.11635	1.06619	1056
	$m_x m_y$ [15]	0.04476	0.70268	1056
	$p_x p_y m_x m_y$ [13]	0.11946	1.03158	1120
3% Observation Error	This Work	0.15932	0.85319	1044
	$p_x p_y p_z m_x m_y m_z$ [14]	0.22351	0.99112	1218
	$p_z m_x m_y$ [16]	0.69332	0.92100	1056
	$p_x p_y$ [15]	0.19785	1.14061	1056
	$m_x m_y$ [15]	0.14067	14.96741	1056
	$p_x p_y m_x m_y$ [13]	0.21314	0.99653	1120

sponding to the reactive near field is presented in TABLE I. In terms of the effectiveness of the inverse solver w.r.t. parameter δr , the proposed transformation shows better results for the first and second scenarios. In case of observation errors, we obtain comparable results. The proposed transformation compared to other methods performs better in terms of error ϵ_H except the method based on placing surface magnetic dipoles, which gives a comparable performance. In the reactive near field region, the dipoles operate in quasi statics region due to the decoupling of the radiated electric and magnetic fields. The high values of ϵ_H for all the techniques (including the proposed one) confirm that in this region, it becomes challenging to obtain the field transformations accurately.

The electric field data corresponds to the near field region where $h = 23.85$ mm for the second experiment is shown in TABLE II. The inverse solver's effectiveness of the proposed transformation in terms of δr is demonstrated over other techniques. The error ϵ_H in the obtained magnetic field clearly shows that the proposed transformation outperforms the other compared methods. The original and reconstructed magnetic field along with the relative error in dB for both of $h = 5$ mm and $h = 23.85$ mm is shown in Fig.5.

B. Planar Structure: Microstrip Patch Antenna

The design specification of the microstrip patch antenna operating at 5.6GHz is depicted in Fig. 6. The dimension of the measurement scanning plane is 40mm $\times$ 85mm with 2500 measurement points. The reactive near field for this structure ends at $h = 27.34$ mm.

The results of the first experiment (measurement scanning plane at $h = 5$ mm) in the reactive field region is shown in TABLE III. The effectiveness of the inverse solver δr and the low error in the obtained magnetic field ϵ_H exhibits the improved performance over the existing methods for all three scenarios.

The highlights of the second experiment where the transformation is performed in the near field region (measurement scanning plane at $h = 27.34$ mm) are demonstrated in TABLE

Fig. 5: Microwave Low Pass Filter: Magnetic field (dBA/m) and relative error (dB) in the field using proposed technique at 500 MHz.

Fig. 6: Microstrip Patch Antenna: Design specifications in mm (operating at 5.6 GHz).

TABLE II: Microwave Low Pass Filter: Measurement scanning plane at $h = 23.85\text{mm}$ operating at 500 MHz

Scenarios	Reference	δr	ϵ_H	Number of Dipoles
No Error	This Work	0.00083	0.03473	1044
	$p_x p_y p_z m_x m_y m_z$ [14]	0.06863	0.27347	1218
	$p_z m_x m_y$ [16]	0.47874	0.72088	1056
	$p_x p_y$ [15]	0.06539	0.27337	1056
	$m_x m_y$ [15]	0.01113	0.10679	1056
	$p_x p_y m_x m_y$ [13]	0.06847	0.28169	1120
	This Work	0.02591	0.07502	1044
3% Measurement Error	$p_x p_y p_z m_x m_y m_z$ [14]	0.07174	0.38794	1218
	$p_z m_x m_y$ [16]	0.47890	0.72243	1056
	$p_x p_y$ [15]	0.06914	0.38390	1056
	$m_x m_y$ [15]	0.02723	0.14343	1056
	$p_x p_y m_x m_y$ [13]	0.07174	0.39302	1120
3% Observation Error	This Work	0.07038	0.10351	1044
	$p_x p_y p_z m_x m_y m_z$ [14]	0.10600	0.33807	1218
	$p_z m_x m_y$ [16]	0.48816	0.72259	1056
	$p_x p_y$ [15]	0.10022	0.36251	1056
	$m_x m_y$ [15]	0.06938	0.33972	1056
	$p_x p_y m_x m_y$ [13]	0.10710	0.36100	1120

IV. The effectiveness of the solver and the error in the obtained magnetic field show improved performance for the first scenario and are almost similar for the second and third scenarios where there is error in measurement or observation. These results shown visually in Fig. 7 are obtained using lesser number of dipoles compared to the existing methods.

TABLE III: Microstrip Patch Antenna: Measurement scanning plane at $h = 5\text{mm}$ operating at 5.6 GHz

Scenarios	Reference	δr	ϵ_H	Number of Dipoles
No Error	This Work	0.01389	0.05676	726
	$p_x p_y p_z m_x m_y m_z$ [14]	0.16793	0.26970	864
	$p_z m_x m_y$ [16]	0.24671	0.43527	900
	$p_x p_y$ [15]	0.15504	0.29761	812
	$m_x m_y$ [15]	0.19691	0.54529	812
	$p_x p_y m_x m_y$ [13]	0.16782	0.30359	880
	This Work	0.02750	0.06686	726
3% Measurement Error	$p_x p_y p_z m_x m_y m_z$ [14]	0.16865	0.27195	864
	$p_z m_x m_y$ [16]	0.24698	0.43540	900
	$p_x p_y$ [15]	0.15588	0.29827	812
	$m_x m_y$ [15]	0.19768	0.54531	812
	$p_x p_y m_x m_y$ [13]	0.16853	0.30441	880
3% Observation Error	This Work	0.05472	0.11450	726
	$p_x p_y p_z m_x m_y m_z$ [14]	0.19497	0.32348	864
	$p_z m_x m_y$ [16]	0.26265	0.44427	900
	$p_x p_y$ [15]	0.17997	0.35419	812
	$m_x m_y$ [15]	0.22673	0.79466	812
	$p_x p_y m_x m_y$ [13]	0.19367	0.36850	880

C. Non-Planar Structure: Horn Antenna

As a non-planar structure, the magnetic field of a horn antenna is obtained using the proposed transformation. The compared state-of-the-art methods only describe the procedure for planar structures (Table 1 in [18]). The critical steps, such as the placements of the dipoles and the measurement surface, remain at the practitioner's discretion. These critical steps can be efficiently handled using the proposed transformation for a non planar structure.

The operating frequency of the horn antenna shown in Figure 8a is 15 GHz. The equivalent surface for this horn antenna is the convex hull Γ_d shown in Figure 8b. The number of dipoles placed on Γ_d is 1191. A voltage source of 1 Volts is applied at one of the edge at the input wave-guide as a voltage gap. The number of measurement points (placed equispaced along θ and ϕ) on a spherical surface of radius 60mm is 2400. The linear system of equations obtained in this scenario are

Fig. 7: Microstrip Patch Antenna: Magnetic field (dBA/m) and relative error (dB) in the field using proposed technique at 5.6 GHz.

Fig. 8: Horn Antenna: Geometry, measurement surface and reconstructed dipole Moment.

solved using the LSMR iterative scheme with tolerance of 10^{-6} .

As a first step, the electric field data is given as an input to the proposed transformation. The magnetic field obtained in this case matches with the original field data as shown in Figure 9a and 9b. The maximum relative error is -30 dB which exists in the minimal regions of the original magnetic field as can be seen in Figure 9c. The maximal regions of magnetic field has a lower error of order of -80 dB.

The advantage of the proposed procedure is seen in Figure 8b where the high radiating regions of the DUT can be distinctly identified. This demonstrate that the proposed procedure potentially can serve as a tool for practicing EMC-EMI engineers for diagnostics purpose. This will help in addressing the electromagnetic compliance issues of the DUTs by identifying and rectifying the high radiating regions.

V. CONCLUSION

The paper presents a dipole-based source reconstruction algorithm using the equivalent surface principle. The source reconstruction is different from previously reported approaches

TABLE IV: Microstrip Patch Antenna: Measurement scanning plane at $h = 27.34$ mm operating at 5.6 GHz

Scenarios	Reference	δr	ϵ_H	Number of Dipoles
No Error	This Work	0.00024	0.00090	726
	$p_x p_y p_z m_x m_y m_z$ [14]	0.00658	0.00630	864
	$p_z m_x m_y$ [16]	0.04267	0.06125	900
	$p_x p_y$ [15]	0.00679	0.00663	812
	$m_x m_y$ [15]	0.00650	0.01552	812
	$p_x p_y m_x m_y$ [13]	0.00692	0.00646	880
3% Measurement Error	This Work	0.02416	0.02454	726
	$p_x p_y p_z m_x m_y m_z$ [14]	0.02451	0.02509	864
	$p_z m_x m_y$ [16]	0.04566	0.06350	900
	$p_x p_y$ [15]	0.02486	0.02525	812
	$m_x m_y$ [15]	0.02433	0.02752	812
	$p_x p_y m_x m_y$ [13]	0.02484	0.02515	880
3% Observation Error	This Work	0.03153	0.03210	726
	$p_x p_y p_z m_x m_y m_z$ [14]	0.03146	0.03227	864
	$p_z m_x m_y$ [16]	0.05259	0.06914	900
	$p_x p_y$ [15]	0.03253	0.03305	812
	$m_x m_y$ [15]	0.03187	0.03643	812
	$p_x p_y m_x m_y$ [13]	0.03250	0.03272	880

as it places the dipoles tangentially on a closed equivalent surface enclosing a DUT. The theoretical analysis of error bounds on the obtained field is also presented. The comparison with the state-of-art methods shows a clear edge over the obtained magnetic field accuracy for noise free measurement data and comparable performance for noise corrupted measurement data. The limitations of the proposed method are seen in the low-frequency region based on the condition $kh \rightarrow 0$, which keeps the error in the transformed field very high. However, the pattern of the field is reliably recovered. This high error is due to quasi-static effects inherent whenever the operating frequency and the measurement distance are low. Nevertheless, this high error in this reactive near field region agrees with previous observations [13]. Although the numerical results are presented for the electric field to the magnetic field, the implementation of the vice-versa case remains straightforward. The range of numerical examples validates the effectiveness

Fig. 9: Horn Antenna: Magnetic field (dBA/m) and relative error (dB) in the field using proposed technique at 15 GHz for the antenna aligned along z axis.

of the proposed procedure in obtaining the fields for radiated emissions and identifying the radiating regions for resolving the electromagnetic compliance issues of a DUT. The further extension to this work will involve improving the error in the electromagnetic field's estimation in the reactive near field region.

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