A novel Dynamic Pareto bi-level Multi-Objective Particle Swarm Optimization (DPb-MOPSO) algorithm

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Abstract—Distributed architecture-based Particle Swarm Optimization is very useful for static optimization and not yet explored to solve complex dynamic multi-objective optimization problems. This study proposes a novel Dynamic Pareto bi-level Multi-Objective Particle Swarm Optimization (DPb-MOPSO) algorithm with two optimization levels. In the first level, all solutions are optimized in the same search space and the second level is based on a distributed architecture using the Pareto ranking operator for dynamic multi-swarm subdivision. The proposed approach adopts a dynamic handling strategy using a set of detectors to keep track of change in the objective function that is impacted by the problem's time-varying parameters at each level. To ensure timely adaptation during the optimization process, a dynamic response strategy is considered for the re-evaluation of all non-improved solutions, while the worst particles are replaced with a newly generated one. The convergence and diversity performance of the DPb-MOPSO algorithm are proven through Friedman Analysis of Variance, and the Lyapunov theorem is used to prove stability analysis over the Inverted Generational Distance (IGD) and Hypervolume Difference (HVD) metrics. Compared to other evolutionary algorithms, the novel DPb-MOPSO is shown to be most robust for solving complex problems over a range of changes in both the Pareto Optimal Set and Pareto Optimal Front.

Index Terms—Dynamic Multi-Objective Optimization, Dynamic Particle Swarm Optimization, Dynamic Problems, Detectors, Dynamic Response, Friedman Analysis of Variance, Lyapunov Theorem.

I. INTRODUCTION

RECENTLY, the main issue facing optimization has focused on the challenging field of Dynamic Multi-Objective Problems (DMOPs) with two or three conflicting functions characterized by dynamic objectives, parameters and/or constraints. A great deal of research conducts on Multi-Objective Evolutionary Algorithms (MOEAs) for solving static single and multi-objective problems referred to as SOPs and MOPs respectively are adapted for DMOPs [9], [32], [33] using a powerful range of bio-inspired intelligent techniques like the Genetic Algorithm (GA) [15] and the Particle Swarm Optimization (PSO) [6], [28]–[30]. These

algorithms are designed to resolve evolutionary stagnation issues. In dynamic optimization, any MOEA should be able not only to find the best set of compromised solutions over Pareto Optimal Set (POS) and Pareto Optimal Front (POF) but also to track the changes that can affect. Several variants of PSO algorithm are modified for solving dynamic SOP (DSOP) when the standard PSO cannot be used for DMOP without modification because of the dual issues of outdated memory and the loss of diversity that needs crucial actions to be undertaken like change detection, memory update and diversity enhancement. Four types of DMOPs have classified by Farina *et al.* [22], in type I the POS change and the POF remains the same, but for type II both POS and POF change over time. However the POF change and the POS remains constant for DMOPs in type III. Despite the environmental changes both POS and POF still unchanged in type IV.

Several detection techniques are developed to tackle time-dependent issues. The re-evaluation of objectives and constraints' values of certain percentage or randomly selected outdated population are very useful. Different response strategies using explicit actions like the re-initialization or hypermutation of populations is considered for ensuring fast convergence and diversity [1], [26]. PSO-based approaches are very considered for solving DSOP and a few methods are designed to solve DMOPs [2]. Nonetheless, distributed methods using genetic and population-based strategies have been shown to be important for static optimization [7] which has not been treated for handling the DMOPs.

To the best of our knowledge, does not exist any developed approach using the multi-objective particle swarm optimization (MOPSO) with distributed architecture for solving DMOPs. For that reason, the main motivation underlying this study presents the proposed Dynamic Pareto bi-level Multi-Objective Particle Swarm Optimization (DPb-MOPSO) based on a distributed architecture for handling DMOPs with various types of changes on POS and POF. The distributed architecture of DPb-MOPSO is presented over upper and lower level for distribution and diversity amelioration. For the upper level, all individuals are optimized in the same research space. Nevertheless, the second lower level provides a distributed architecture using a dynamic multi-swarm subdivision based on the Pareto Ranking operator. Filling the contradictory gaps of convergence and diversity that leads to the algorithm losing its ability to react efficiently to changes. The DPb-MOPSO presents a dynamic handling strategy to effectively detect and react to the change. The detection strategy has carried out

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using a ten percent of solutions as detectors. The response strategy aims to re-evaluate all solutions with negative change to be replaced with new generated ones and the solutions with positive progress are adapted to accelerate optimization after each change.

So far, the state of the art involves a wealth of comparable algorithms to our novel DPb-MOPSO system over two performance metrics [26] namely the Inverted Generational Distance (IGD) and the Hypervolume Difference (HVD). These include the Multi-objective Evolutionary Algorithm Based on Decomposition (MOEAD) [31], the Dual multi-objective particle swarm optimization (dMOPSO) [18], the Dynamic Non-dominated Sorting Genetic Algorithm II (DNSGA-II) [14], the distributed Multi-Objective Particles Swarms Optimization, based on dynamic subdivision of the population using Pareto fronts (pbMOPSO) [7], the Dynamic Multi-objective Particle Swarm Optimization (Dynamic-MOPSO) [2], the dynamic Competitive-cooperative co-evolutionary Algorithm (dCOEA) [8], the Population Prediction Strategy (PPS) [34], the Steady-State and Generational Evolutionary Algorithm (SGEA) [13] and the Multi-Objective Particles Swarms Optimization (MOPSO) [5].

The rest of this paper has organized as follows. Section II presents an overview of dynamic multi-objective optimization methods. Section III details the proposed DPb-MOPSO. Section IV introduced preliminaries of the experimental study. Section V, describes the experimental results through Friedman Analysis of Variance overall quantitative results, furthermore a stability analysis has presented using Lyapunov theorem in Section VI. Finally, Section VII concluded this paper and suggested some perspectives for future works.

II. OVERVIEW OF DYNAMIC MULTI-OBJECTIVE OPTIMIZATION METHODS

In dynamic optimization, we aim to optimize a dynamic problem f_t with m objective functions by an algorithm G at a given period $[t_{begin}, t_{end}]$ such as presented in the following mathematical definition.

$$\begin{aligned} \text{Min } F(x, t) &= (f_1(x, t), f_2(x, t), \dots, f_{n_m}(x, t)) \\ \text{Subject to } &g_i(x, t) \leq 0, i = 1, \dots, n_g \\ &h_j(x, t) = 0, j = 1, \dots, n_h \\ &x \in [x_{min}, x_{max}]^{n_x}, t \in [t_{begin}, t_{end}] \end{aligned} \quad (1)$$

Several existing studies [26] have focused on three research levels, including the development of an exhaustive set of challenging dynamic benchmarks, the development of new algorithms for handling various dynamic problems and the development of efficient quality indicators to measure the performance of MOEAs. Many investigations have been carried out to encourage researchers to develop DMOPs instead of DSOPs. For example, the continuous FDA functions [22] with dynamic shift of POF and POS, the five ZJZ (F5 to F10) [34] with a non-linear correlation between the decision variables and the nine Unconstrained Dynamic Functions (UDFs) [27]. The challenge of dynamic optimization does not limit to the development of generic benchmarks, but it has interested also for solving dynamic real-world problems [23]. In literature, a powerful set of MOEAs is designed to address the static

MOPs [11], [15], [16] are used for solving DMOPs. During the last decade, several challenging approaches are developed for solving a powerful ranges of DMOPs. Those approaches are classified into four categories as resumed in Table I and detailed as follow; Firstly, the diversity-based methods are used to preserve population diversity through the re-initialization or hyper-mutation behavior after each transition or throughout the execution time like the variations Dynamic NSGA-II (DNSGA-II-A and DNSGA-II-B) [14] that aims to identify a random solution to be re-evaluated to monitor environmental change or to replace a percentage of the new population with new randomly generated solutions. Also, the DC-NSGA-II [24] algorithm presents a self-adaptive penalty function to deal with time-varying constraint problems. Thus, the Dynamic-MOPSO system [2] adapts a dynamic handling strategy to detect changes by controlling the evolution of the objective function over time and re-initialize all solutions with negative changes to maintain diversity.

In addition, several memory-based techniques [8], [13], [25], [27] are considered over an extra memory mechanism to implicitly or explicitly store the useful information and outdated solutions for potential uses. Furthermore, change prediction-based approaches were achieved to exploit past information and anticipate the new position of POS using a prediction model to reduce the number of function evaluations [3]. In [10] the D-QMOO algorithm was proposed based on the Feed-forward Prediction Strategy (FPS) to study the size and the distribution of the anticipatory solutions over time. Also, the Population Prediction Strategy (PPS) [17] is developed to predict a totality of solutions instead of using some isolated point. Finally, the parallel methods are implemented using multiple sub-populations mechanisms like the master-slave method in [20], [21], the independent runs, the island model and the cellular EAs. The DMOEA [33] adapts the sub-population technique to solve a single objective problem. In this case study, a dynamic version of the MOPSO method is considered over a distributed architecture. The standard MOPSO algorithm safer from the stagnation at local optima and loss the diversity over time. Generally, the re-initialization strategy is the most useful technique to address the issues of population diversity after environmental changes. In addition, there is no approach using MOPSO with distributed architecture for solving DMOPs.

For the performance assessment metrics [26] four categories are classified as follows: the first deals with the accuracy measurement like Generational Distance (GD), variable space generational distance (VD) and Success Ratio (SR). The second aims to measure diversity such as Maximum Spread (MS), Coverage Scope (CS) and Path Length measure (PL). The third presents the robustness indicators like stability measure (stb) and reactivity measure (react). Ultimately, the last one shown a combined performance for convergence and diversity measurement like the accuracy, Inverted Generational Distance (IGD), Hypervolum (HV), Hypervolume difference (HVD) and HV ratio.

TABLE I: Dynamic Multi-Objective Optimization Methods

Algorithms	Tested Benchmark Function
MOO algorithms adapted for DMOOP	
SPEA2 , Kim M.(2004) [16]	DSW1, DSW2, DTF, FDA DMOPs
Multiple Single Objective Pareto Sampling (MSOPS), E. J. Hughes (2003) [11]	
NSGA-II, Deb <i>et al.</i> , (2002) [15]	
Diversity-Based Approaches	
Modified NSGA-II : (DNSGA-II)-A and (DNSGA-II)-B, Deb <i>et al.</i> , (2007) [14]	Modified FDA2 and hydro-thermal scheduling problem
Dynamic Constrained NSGA-II (DC-NSGA-II), Azzouzet <i>et.</i> , (2015) [24]	DCTPs test problems
Dynamic-MOPSO Aboud <i>et al.</i> , (2017) [2]	FDA1, DIMP2 and dMOP3
Memory-Based Approaches	
Steady-state and Generational EA (SGEA) Jiang <i>et al.</i> , (2016) [13]	five FDA [22], three dMOP [8], six ZJZ [34] and seven UDF [27].
Adaptive Dynamic NSGA-II (A-Dy-NSGA-II), Azzouz <i>et al.</i> , (2017) [25]	FDA1, FDA2, DMZDT test functions and WYL
Dynamic Competitive Cooperative CO-EA (dCOEA), Gohet <i>et al.</i> , (2009) [8]	FDA1, dMOP1, dMOP2 and dMOP3
Change Prediction-Based Approaches	
Population Prediction Strategy (PPS), Koo <i>et al.</i> , (2010) [17]	FDA1, FDA4, dMOP1, dMOP2 and F5-F8
Dynamic MOEA with Predicted Re-Initialization (DMOEA/PRI), Zhou <i>et al.</i> , (2007) [3]	FDA1 and ZJZ
Dynamic Queuing MO Optimizer (D-QMOO), Hatzakis and Wallace (2006) [10]	FDA1
Parallel Approaches	
Work of Cámara <i>et al.</i> , (2008) [21]	FDA1, modified FDA2 and FDA3
Dynamic Multi-objective Optimization EA (DMOEA), Zheng (2007) [33]	FDA1, modified FDA2 and FDA3, FDA4 and FDA5
Dynamic Version of Parallel Single Front Genetic Algorithm (PSFGA), Cámara <i>et al.</i> , (2007) [20]	FDA1 and FDA2

III. THE PROPOSED DYNAMIC PARETO BI-LEVEL MULTI-OBJECTIVE PARTICLE SWARM OPTIMIZATION

The proposed Dynamic Pareto bi-level Multi-Objective Particle Swarm Optimization (DPb-MOPSO) approach is shown in Figure 1 presenting two optimization levels called the upper (L_1) and the lower (L_2) levels described in the following sub-sections A and B and detailed in Algorithms 1 and 2 respectively. The two levels have included a dynamism handling strategy to detect and react effectively to the changes. The utility of bi-level optimization is proved over a distributed architecture to give more opportunity for the system to enhance convergence with better distribution over time. The proposed system is executed until a maximum number of iterations T_{max} subdivided as follows: 50% for L_1 , 25% for L_2 and 25% for the re-execution step on L_1 .

A. DPb-MOPSO: Upper Level (L_1)

The upper level (L_1) is detailed in Algorithm 1 and started with random initialization of all particles p_i in the swarm S and the leaders archive A_1 . Each particle is a candidate solution x_i^* in n dimensional search space. For each particle p_i , we select a leader A_i based on the crowding distance operator between two random particles. The particle with the lowest value is selected as leader, then we calculate the fitness function F_i of each particle and update its position (X) and velocity (V) based on their self-experience (p_{best}) as a local component and the experience of their neighbors (g_{best}) as a global component using 2 and 3 respectively. This level is executed until 50% of T_{max} .

$$X_{t+1} = X_t + V_{t+1} \quad (2)$$

$$V_{t+1} = wV_t + c_1r_1(p_{best}(t) - X_t) + c_2r_2(g_{best}(t) - X_t) \quad (3)$$

Where; w is the inertia weights, c_1 and c_2 are acceleration coefficients, r_1 and r_2 are two parameters aiming to influence of the cognitive and social components of each particle. In Addition, all the changes are known for the algorithm and checked according to the frequency τ_t assuming time-dependent parameters t for each problem, where $t = \frac{\tau}{n_t} \left\lfloor \frac{\tau}{\tau_t} \right\rfloor$, $\forall t \in T$, with; n_t , τ and τ_t are the severity, the iteration

Algorithm 1 : DPb-MOPSO algorithm-Upper Level (L_1)

Input: P_0 (swarm of particles), k (maximum size of P_0), M (number of objective functions), T_{max} (maximum number of iterations);
Output: A_1 (Intermediate external archive), P' (set of intermediate population);
1: Initialize $t = 0$, P_0 , A_1 ;
2: **while** $t \leq T_{max}/2$ **do**
3: **for** $i = 1$ to k **do**
4: Select the leader;
5: Evaluate the fitness function F_i of each P_i ;
6: Update X_i and V_i according to 2 and 3 respectively;
7: **if** Change is detected **then**
8: Apply the dynamism handling strategy (Algorithm 3);
9: **end if**
10: **end for**
11: Update A_1 ;
12: $t++$;
13: **end while**
14: Select P' (final population of L_1);
15: Apply the Ranking Operator to subdivide P' into a set of fronts ranked from 0 to $N-1$ to construct multiple sub-swarms F_n ;
16: Store the non-dominated solutions on A_1 ;
17: Return A_1 , F_n (multiple sub-swarms)

counter and the frequency of change respectively. If a change is detected on all the objective values, the dynamism handling strategy detailed in the subsection C and the algorithm 3 is executed. Furthermore, all the optimal solutions X_i^* are saved into the archive A_1 using the ϵ operator to measure the best approximation for the true POF at the end of (L_1).

B. DPb-MOPSO: Lower Level (L_2)

At half the maximum number of iterations $T_{max}/2$ of (L_1), the switched level (L_2) over a distributed architecture is started with the extraction of the intermediate population $P'(POS)$. Then, the Pareto Ranking Operator is applied to subdivide $P'(POS)$ into N sub-swarms presenting the set of fronts denoted by $F_0, F_1, \dots, F_n$. In most of the cases, the front F_0 has the maximum number of solutions. So, for ensuring equality between fronts size, all extra-individuals from F_0 are injected into other fronts. For each front F_i ; where $i = 0 \dots N-1$, the DPb-MOPSO algorithm in (L_1) is re-executed in a parallel execution until 25% of T_{max} and all the non-dominated solutions are stored in A_2 . In addition, all the non-dominated solutions of (L_2) are regrouped to construct a second intermediate population P'' and re-executed in (L_1)

Fig. 1: The Proposed Dynamic Pareto bi-level Multi-Objective Particle Swarm Optimization (DPb-MOPSO)

Algorithm 2: DPb-MOPSO algorithm-Lower Level (L_2)

Input: F_n (set of fronts) $\forall n=0 \dots N-1$, K (size of the front), T_{max} (maximum number of iterations of (L_1)), A_2 (intermediate external archive), POS_n (Final population of all sub-swarms);

Output: A (global external archive), P'' (set of intermediate population);

- 1: Adjust the size of each sub-swarm F_n ;
- 2: Initialize the leaders archive A_2 ;
- 3: **while** $t \leq T_{max}/4$ **do**
- 4: **for** $i=1$ to k **do**
- 5: Select the leader;
- 6: Evaluate the fitness function f_i of $p_i \in POS_n$;
- 7: Update X_i and V_i according to 2 and 3 respectively;
- 8: **if** Change is detected **then**
- 9: Apply the dynamism handling strategy (Algorithm 3);
- 10: **end if**
- 11: **end for**
- 12: Update the leaders archive A_2 ;
- 13: $t++$;
- 14: **end while**
- 15: Store the non-dominated solutions on A_2 ;
- 16: Regroups all the sub-swarms POS_n into P'' ;
- 17: Re-execute P'' on L_1 until $T_{max}/4$;
- 18: Update the leaders archive A_1 ;
- 19: Best compromised POF in $A = A_1 \cup A_2$;
- 20: Return A ;

until 25% of T_{max} . When achieving T_{max} , all the non-dominated solutions from both A_1 and A_2 are regrouped in the global archive A using Epsilon dominance operator.

C. Dynamism Handling Strategy

Filling the gap for ensuring convergence during the run that may cause a lack of diversity in a dynamic environment. Two steps for dynamism handling strategy are considered in Algorithm 3 and detailed as follows:

• Step 1: Environment detection strategy

The environmental change is checked at the frequency τ_t , which is equal to 5, 10, and 20 for severe, moderate, and slight environmental changes respectively. The comparison procedure is started after the evaluation of fitness function F_i by selecting 10% of previous $POF(t-1)$ and current $POF(t)$ as detectors; then, we select 10% of their mapped $POS(t-1)$ and $POS(t)$ for controlling their evolution over time. The strong dominance operator of Epsilon-Dominance method is used to compare dynamic behavior of particles between

Algorithm 3 :Dynamism Handling Strategy

Input: POS_{t-1} (mapped solutions of POF_t of previous iteration), POF_t (population of current iteration);
Output: POS_{t+1} (new population of iteration t+1); **Detection Strategy**

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1: if  $t = \tau_t$  then
2:   Extract detectors: 10% of best particles from POF(t);
3:   Extract 10% of their images from POS(t);
4:   Extract 10% of their projection from POS(t-1);
5:   for  $i = 0$  to  $|POS_{(t)} - 1|$  do
6:     Compare the Fitness of POS(t) and POS(t-1) using strong dominance operator
       of Epsilon-Dominance method;
7:     if  $f_i(t) \succ f_i(t-1)$  then
8:       Negative change= true;
9:       Select the set of solutions with negative changes  $\overline{POS}_-(t-1)$ ;
10:      Select the set of solutions with positive changes  $\overline{POS}_+(t-1)$ ;
11:    end if
12:  end for
13:  Response Strategy
14:  Re-initialize  $\overline{POS}_-(t)$ ;
15:  Construct population  $POS_{t+1}$  including  $\overline{POS}_+(t)$ ;
16:  Re-evaluate the archives  $A_1$  and  $A_2$ ;
17: end if
18: return Set of new population  $POS_{t+1}$ ;

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$POS(t-1)$ and $POS(t)$. A solution X_t strongly dominates the solution X_{t-1} if X_t is strictly better ($X_t \succ X_{t-1}$) than X_{t-1} for all M objectives.

• **Step 2: Response change strategy**

If a change is identified successfully, the DPb-MOPSO adapts all solutions with negative changes $POS_-(t)$ to be re-initialized and all detectors with positive changes $POS_+(t)$ are integrated in the new population to ensure good diversity and convergence using only solutions with positive effects. Also, the leader's archive is re-evaluated to update all non-dominated solutions and overcome the deterioration of the research ability.

IV. EXPERIMENTAL STUDY: PRELIMINARIES

In this section, we aim to present the evolutionary algorithms, the DMOPs and the quality indicators considered for this case study. The empirical study was discussed referring to the contribution of Jiang *et al.* [13].

A. Evolutionary and Swarm Intelligence Multi-Objective Algorithms

Our main case of study consists of the comparison of performance between the proposed DPb-MOPSO and several algorithms regrouped as follow:

- The five static optimization methods considered for DMOPs namely;
 - The dCOEA [8] defined as a hybrid competitive and cooperative method adapts an additional archive to save the outdated individuals to assess the new optimization step.
 - The MOEA/D [31] presenting a decomposition-based approach aiming to replace a limited number of individuals with any new created solutions.
 - The dMOPSO [18] is a decomposition PSO-based approach used for static MOPs.
 - The MOPSO [5] is the standard version of PSO designed for static MOPs.
 - The pbMOPSO [7] is the MOPSO-based approach using a distributed architecture for static MOPs.

- The two diversity based approaches:
 - The DNSGA-II [14] using a randomly created or mutated individuals after the change.
 - The Dynamic-MOPSO [2] presenting a dynamic version of the standard MOPSO approach with a dynamism handling strategy for solving DMOPs in type I.
- The prediction-based approach PPS [34] using a univariate auto-regression model to predict the new location of POS.
- The memory-based approach SGEA [13] aiming to reuse some outdated solutions with good distribution to relocate solutions closer to the new POF.

All algorithms are executed during 30 independent runs for each DMOPs. Each run is stopped when the maximum number of evaluations T_{max} is achieved using $(3 \times n_t \times \tau_t + 50)$. Where, τ_t is the frequency of changes fixed to 5, 10 and 20 respectively, and n_t is the severity of changes fixed to 10. All experimental settings are summarized in Table II.

TABLE II: Parameters Settings of MOEAs

Common parameters of MOPSO-based Algorithms	
Perturbation index	20
Mutation probability	1.0/number of problems' variables
Acceleration Coefficients (c1, c2)	c1 = c2 = rand (1.5, 2.0)
Inertia weight (w)	rand (0.1, 0.4)
Common parameters of dCOEA, PPS, MOEA/D, SGEA and DNSGA-II	
Crossover Probability	1.0
Mutation Distribution Index	20
Crossover Distribution Index	20
Common parameters for all algorithms	
Swarm size	100
Archive size	100
Max iteration	$3 \times n_t \times \tau_t + 50$
Independent runs	30

B. Dynamic Multi Objective Problems

Twenty-one DMOPs [13], [27] are used for algorithms testing. Table III presenting all parameter settings and the changed nature of POS and POF.

C. Quality Indicators

The Inverted Generational Distance (IGD) and the Hypervolume Difference (HVD) are used to measure both of convergence and diversity considering the smaller values as the best. The IGD is computed using 4 based on the minimum euclidean distance d_i between i th points in the generated $nPOF$ ($|POF|$) and the true POF^* . In addition, the HVD is calculated using 5 to measure the difference between the obtained POF to the true POF^* .

$$IGD = \frac{1}{nPOF} \sum_{i=1}^{nPOF} d_i \quad (4)$$

$$HVD = HV(POF) - HV(POF^*) \quad (5)$$

V. EXPERIMENTAL RESULTS AND STATISTICAL ANALYSIS

The concepts of inferential statistics over Friedman two-way Analysis of Variance (Friedman two-way ANOVA) is devoted to discussing all Mean (M.) and Standard Deviation

TABLE III: Parameter settings and dynamic characteristics of DMOPs

DMOPs	Nb. Obj.	Nb. Var.	Type	POS Change Nature	POF Change Nature	
					Shift	Curvature
FDA problems Farina <i>et al.</i> [22]	FDA1	2	20	I	No change	No change +Convex
	FDA2	2	15	II	Vertical + Density of solutions change	Change from convex to concave
	FDA3	2	30	II	Vertical + Density of solutions change	Change + Convex
	FDA4	3	12	I	No change + Spread of solutions change	No change + Concave
	FDA5	3	12	II	Radius variations + Density of solutions change	Change + Concave
dMOP problems Goh <i>et al.</i> [8]	dMOP1	2	10	III	No change	Change from convex to concave
	dMOP2	2	10	II	Sinusoidal change + Vertical shift	Change from convex to concave
	dMOP3	2	10	I	Sinusoidal change + Vertical shift	No change + Convex
ZJZ problems Zhou <i>et al.</i> [34]	F5	2	20	II	Vertical shift	Change from convex to concave
	F6	2	20	II		
	F7	2	20	II		
	F8	3	20	II		
	F9	2	20	II		
UDF problems Biswas <i>et al.</i> [27]	F10	2	20	II		
	UDF1	2	10	I	Vertical shift	No change
	UDF2	2	10	I	Vertical shift + curvature change	No change
	UDF3	2	10	III	No variation	No change
	UDF4	2	10	II	Horizontal shift	Angular shift + Change from convex to concave
	UDF5	2	10	II	Polynomial +Vertical shift	Angular shift + Change from convex to concave
	UDF6	2	10	III	No variation	Angular shift
	UDF7	3	10	III	No variation	No change

(Std.) values showed in Tables VIII, IX, XII and XV. Previous analysis using only IGD and HVD metrics is insufficient to evaluate the performance of a swarm intelligence algorithms and highlight the significant level [12]. In this study, the ANOVA procedure is used and does not assume a normal distribution to compare multiple treatments and decide the significance level between algorithms.

A. Procedure of Friedman two-way Analysis of Variance

The Friedman two-way ANOVA test is done using the following steps :

- Step 1: the normality distribution test is done through the Q-Q plot for all algorithms over IGD and HVD metrics with both τ_t and n_t are equal to 10. Taking the example of Q-Q plot of DPb-MOPSO in Figure 2, the data-points does not fall the straight line of the ideal normal distribution curve and does not fit the normal distribution. However, due to the non-normality of data the Friedman two-way ANOVA by ranks is used as a non-parametric statistical procedure to decide the significance level between algorithms.

Fig. 2: DPb-MOPSO: Q-Q plot and histogram.

- Step 2: prepare the observation's matrix for all algorithms/problems pair.
- Step 3: define both null hypothesis H_0 (no difference between the mean values), and the alternative hypothesis H_1 (presents the difference).
- Step 4: state the significance level Alpha (α) to determine at which level the null hypothesis may be rejected. The p -value helps us to estimate how significant the

results of algorithms, and we reject the null hypothesis if p -value $\leq \alpha$; where $\alpha = 0.05$. $H_0: \mu_1 = \mu_i$ (all treatment effects are zero) and $H_1: \mu_1 \neq \mu_i$ (not all treatment effects are zero).

- Step 5: determine the degree of freedom (df= k-1), where k is the number of algorithms equal to 10, the df is equal to 9.
- Step 6: state the decision rule, according to nine degree of freedom and $\alpha=0.05$ is defined following to the Chi-Square (χ^2) table¹. The critical value (CV) is equal to 16.92, if the computed χ^2 greater than CV we're going to reject the null hypothesis H_0 .
- Step 7: Friedman two-way ANOVA by ranks is considered as follows:
 - 1) Assign the rank values (r_{ij}) for each problem i while; 1 is the best result to k which is the worst result (where; $1 \leq i \leq P$ and $1 \leq j \leq k$) and for the ties cases, we assign the average of ranks.
 - 2) Compute the average of ranks obtained for each algorithm/problem pair using 6.

$$R_j = \frac{1}{n} \sum_i r_j^i. \quad (6)$$

- 3) Hypothesis testing, to retain null hypothesis the Friedman statistic F_f is computed according to 7 based on the probability distribution χ^2 with nine degree of freedom.

$$F_f = \frac{12n}{k(k+1)} \left[\sum_i R_j^2 - \frac{k(k+1)^2}{4} \right]. \quad (7)$$

In most of the cases, the Friedman ranking test assumed the assumption of ignoring the null hypothesis implies that at least two algorithms differ in significance. The Friedman ranks and their p -value can not define where the algorithm variance is which leads to the Friedman $1 \times N$ ANOVA multiple comparisons with the DPb-MOPSO is the control method using different Post-hoc procedures [12] described in step 8.

¹<http://kisi.deu.edu.tr/joshua.cowley/Chi-square-table.pdf>

- Step 8: The Post-hoc procedures are classified in Table IV aiming to compute the Adjusted *P-Value* (APV) and identify the significant level of variation between algorithms.

TABLE IV: Post-hoc procedures for $1 \times N$ comparisons

Classes	Procedures	Adjusted P-Value (APV)
One-step	Bonferroni_Dunn	$\min\{v, 1\}$; where $v=(K-1)p_i$ k: number of tested algorithms and $i \in 1, \dots, K$.
Step-down	Holm	$\min\{v, 1\}$; where $v = \max(k-1)p_j : 1 \leq j \leq i$
Step-up	Hochberg	$\max(k-j)p_j : k-1 \geq j \geq i$.
Two-step	Li	$p_i/(p_i + 1 - p_{k-1})$.

B. Results Analysis through Friedman two-way ANOVA Test

Different configurations are considered on MOEAs to demonstrate the impact of the frequency of the change for solving problems with the different nature of changes in the POS and POF. Those configurations are achieved according to the constant value of the severity of change n_t set to 10 and the variation of the frequency τ_t from 5, 10 to 20 representing severe, moderate and slight environmental changes respectively. All significant results are reported in Tables VIII, IX, XII and XV and highlighted in bold faces for twenty one problems ($P=21$) considering five FDA, three dMOP, six F(ZJZ) and seven UDF functions per algorithm K is equal to ten ($K=10$). We can conclude that the five MOPSO-based algorithms (dMOPSO, MOPSO, pbMOPSO, Dynamic-MOPSO and DPb-MOPSO) are the most competitive methods over both IGD and HVD metrics. The mean values are not very useful to confirm whether the new proposed DPb-MOPSO has significant improvement compared to other algorithms. For that reason, three groups of analysis are considered for the Friedman two-way ANOVA ranking test and the Post-hoc procedures:

- Friedman two-way ANOVA test for moderate environmental changes ($\tau_t = n_t = 10$).
- Friedman two-way ANOVA test for severe environmental changes ($\tau_t = 5$ and $n_t = 10$).
- Friedman two-way ANOVA test for slight environmental changes ($\tau_t = 20$ and $n_t = 10$).

1) **Friedman rankings test:** let's first consider the Friedman two-way ANOVA by ranks over a group of twenty one problems with moderate environmental changes. The average rankings are obtained over IGD and HVD metrics assigning to the best algorithm the ranking one and to the worst the ranking k . Under the null hypothesis, the test statistics of the Chi-Square (χ^2) distribution with nine degrees of freedom and the *p-value* are computed. Based on Table V, we can conclude that the DPb-MOPSO has the best ranks compared to other algorithms. In addition, the proposed DPb-MOPSO is the winner for solving the group of eight problems (5 FDA and 3 dMOP functions) with severe environmental changes such as demonstrated on Table VI. Besides, Table VII presents the average rankings for slight environmental changes while the DPb-MOPSO has the best rank over IGD and fail over HVD compared to MOPSO algorithm. Whereas, all computed $\chi^2 \geq 16.92$ assuming the existence of significant difference between all algorithms. Friedman's ranking test detects the

significant difference over the whole multiple comparison and cannot be able to detect when one method performed better than the other. This issue leads to the Friedman $1 \times N$ ANOVA multiple comparisons with the DPb-MOPSO is the control method using four Post-hoc comparisons procedures such as resuming in Table IV and detailed in the following subsection.

TABLE V: Friedman average rankings over HVD and IGD metrics with $\tau_t = n_t = 10$ for FDA, dMOP, F(ZJZ) and UDF.

Algorithms	Ranks	
	IGD	HVD
DNSGA-II	6.86	7.6
dCOEA	7.5	7.86
PPS	6.67	7.31
MOEA/D	7.31	7.98
SGEA	5.67	5.93
MOPSO	5.29	3.83
dMOPSO	6.21	3.57
pbMOPSO	4.38	4.21
Dynamic-MOPSO	3.6	4.1
DPb-MOPSO	1.52	2.62
χ^2	73.09	87.26
p-value ($\alpha=0.05$)	3.78E-12	5.76E-15

TABLE VI: Friedman average rankings over HVD and IGD metrics with $\tau_t=5$ and $n_t=10$ for FDA and dMOP.

Algorithms	Ranks	
	IGD	HVD
DNSGA-II	8.81	8.63
dCOEA	6.44	7.25
PPS	7.75	8.63
MOEA/D	8	9.13
SGEA	5.75	6.38
MOPSO	3.75	2.75
dMOPSO	5.38	2.75
pbMOPSO	3.38	3.38
Dynamic-MOPSO	3.13	3.5
DPb-MOPSO	2.63	2.36
χ^2	39.06	59.70
p-value ($\alpha=0.05$)	0.000011	1.53E-9

TABLE VII: Friedman average rankings over HVD and IGD metrics with $\tau_t=20$ and $n_t=10$ for FDA and dMOP.

Algorithms	Ranks	
	IGD	HVD
DNSGA-II	8.5	8.25
dCOEA	7.13	8.25
PPS	8	8.5
MOEA/D	7.5	8.5
SGEA	5.38	6.5
MOPSO	4.06	2.31
dMOPSO	5.81	2.56
pbMOPSO	4	3
Dynamic-MOPSO	2.5	4.13
DPb-MOPSO	2.13	3
χ^2	40.79	58.78
p-value ($\alpha=0.05$)	0.000005	2.30E-9

2) **Friedman $1 \times N$ ANOVA Multiple Comparisons:** The $1 \times N$ comparisons are made to evaluate the studies of $k-1$ algorithms and calculate the unadjusted and the adjusted *P-Value* (APV) for each hypothesis except DPb-MOPSO is the control method. The average rankings of all methods are transformed to the unadjusted *p-value* which is computed using the normal approximation. The test statistics are considered to compare the i th algorithm with the j th algorithm which are ordered from the smallest to the largest ranks (i). Applying the four Post-hoc procedures can then lead to the APV determining the

TABLE VIII: Mean and Standard Deviation Values of all MOEAs over IGD and HVD Metrics for UDF with ($\tau_t = n_t = 10$).

Prob.	QI	Val.	Tested Algorithms of Jiang <i>et al.</i> [13]					Additional Tested MOPSO based Algorithms				
			DNSGA-II	dCOEA	PPS	MOEA/D	SGEA	MOPSO	dMOPSO	pbMOPSO	Dynamic-MOPSO	DPb-MOPSO
UDF1	HVD	M.	5.14E-1	7.47E-1	7.97E-1	6.12E-1	5.18E-1	-2.99E+0	-2.99E+0	4.83E-3	5.24E-3	3.14E-4
		Std.	3.2E-2	3.8E-2	5.2E-2	9.4E-2	5.0E-2	1.2E-5	1.1E-7	4.5E-4	1.1E-4	6.1E-6
	IGD	M.	1.07E-1	2.91E-1	2.67E-1	1.70E-1	1.24E-1	5.52E-3	5.52E-3	1.70E-5	1.72E-5	1.14E-6
		Std.	2.4E-2	2.3E-2	2.2E-2	5.1E-2	3.3E-2	8.2E-11	1.0E-10	1.2E-6	4.4E-7	1.8E-8
UDF2	HVD	M.	5.51E-1	6.13E-1	4.32E-1	5.42E-1	5.10E-1	-2.99E+0	-2.99E+0	4.84E-3	5.21E-3	3.16E-4
		Std.	2.4E-2	2.8E-2	1.9E-2	1.7E-2	2.5E-2	9.5E-6	1.1E-7	3.2E-4	7.2E-5	8.2E-6
	IGD	M.	1.12E-1	1.83E-1	2.54E-2	1.16E-1	8.95E-2	5.53E-3	5.53E-3	1.71E-5	1.70E-5	1.14E-6
		Std.	1.0E-2	2.0E-2	5.0E-3	9.5E-3	1.3E-2	1.6E-10	0.0E+0	8.1E-7	2.1E-7	2.2E-8
UDF3	HVD	M.	1.22E+0	1.23E+0	1.73E+0	1.22E+0	1.22E+0	5.05E-3	5.04E-3	4.85E-3	5.05E-3	3.08E-4
		Std.	1.9E-3	7.0E-2	3.1E-4	2.4E-3	2.4E-3	1.4E-5	1.1E-7	3.0E-4	8.9E-6	5.5E-6
	IGD	M.	6.06E-1	6.51E-1	4.55E+0	6.06E-1	6.06E-1	1.52E-5	1.48E-5	1.59E-5	1.51E-5	1.04E-6
		Std.	3.3E-6	7.7E-2	1.1E+0	6.3E-5	7.4E-6	8.1E-8	5.6E-10	1.0E-6	6.9E-8	1.7E-8
UDF4	HVD	M.	3.47E-1	5.06E-1	3.77E-1	6.41E-1	3.32E-1	-2.44E-2	-2.39E-2	8.65E-4	7.10E-3	2.92E-4
		Std.	8.3E-2	3.7E-2	2.1E-2	1.9E-1	7.1E-2	1.8E-5	1.6E-7	5.8E-4	2.3E-4	9.0E-6
	IGD	M.	1.70E-1	2.87E-1	1.85E-1	3.19E-1	1.68E-1	8.18E-5	8.18E-5	1.35E-5	1.42E-5	1.01E-6
		Std.	4.7E-2	2.8E-2	8.2E-3	1.3E-1	4.4E-2	2.1E-8	1.8E-10	6.2E-7	1.8E-7	2.1E-8
UDF5	HVD	M.	2.78E-1	3.98E-1	2.70E-1	3.65E-1	2.72E-1	-2.44E-2	-2.39E-2	8.53E-4	7.12E-3	2.92E-4
		Std.	2.5E-2	2.3E-2	1.5E-2	2.7E-2	1.8E-2	1.7E-5	1.5E-7	3.9E-4	1.6E-4	1.1E-5
	IGD	M.	1.18E-1	2.05E-1	2.89E-2	1.61E-1	1.00E-1	8.21E-5	8.21E-5	1.37E-5	1.42E-5	1.02E-6
		Std.	1.2E-2	3.5E-2	1.3E-2	1.4E-2	1.1E-2	1.6E-8	2.0E-10	6.3E-7	1.4E-7	2.3E-8
UDF6	HVD	M.	9.34E-1	1.26E+0	1.83E+0	1.21E+0	9.77E-1	-3.96E+2	-3.99E+2	-2.07E-2	-7.85E-3	-2.68E-2
		Std.	1.5E-1	7.2E-2	1.0E-2	1.4E-1	2.0E-1	6.7E-3	3.6E-2	2.3E-2	3.6E-3	2.6E-2
	IGD	M.	4.57E-1	8.04E-1	1.34E+0	5.31E-1	6.68E-1	2.88E+0	2.88E+0	2.05E-3	4.41E-4	2.19E-3
		Std.	8.7E-2	1.0E-1	7.1E-2	1.6E-1	2.0E-1	2.3E-6	1.8E-6	4.8E-4	9.6E-5	4.3E-4
UDF7	HVD	M.	2.40E+0	1.91E+0	2.06E+0	2.32E+0	2.06E+0	4.62E+0	-1.57E+1	-8.36E-1	-2.97E-1	-2.08E+0
		Std.	7.4E-2	1.7E-1	5.4E-2	2.4E-1	1.2E-1	3.0E-2	5.6E-1	2.1E+0	3.5E-1	2.4E+0
	IGD	M.	5.24E-1	8.40E-1	6.68E-1	5.08E-1	5.08E-1	3.73E-1	3.81E-1	6.94E-4	7.42E-4	3.59E-4
		Std.	2.2E-2	6.4E-2	4.4E-2	1.4E-1	4.2E-2	2.3E-6	7.7E-3	7.5E-5	5.9E-5	1.9E-5

TABLE IX: Mean and Standard Deviation Values of all MOEAs over IGD and HVD Metrics for F (ZJZ) with ($\tau_t = n_t = 10$).

Prob.	QI	Val.	Tested Algorithms of Jiang <i>et al.</i> [13]					MOPSO-based Algorithms				
			DNSGA-II	dCOEA	PPS	MOEA/D	SGEA	MOPSO	dMOPSO	pbMOPSO	Dynamic-MOPSO	DPb-MOPSO
F5	HVD	M.	1.25E+0	1.10E+0	4.01E-1	1.19E+0	7.16E-1	-4.73E+3	-5.23E+3	-1.31E+3	-1.84E+3	-9.79E+5
		Std.	2.5E-2	1.6E-1	9.9E-2	2.9E-2	8.2E-2	9.7E+29	9.7E+2	6.9E+3	4.5E+3	2.6E+6
	IGD	M.	7.82E-1	8.01E-1	2.69E-1	6.88E-1	4.41E-1	4.80E+0	6.62E+0	4.74E+0	4.78E+0	1.24E-1
		Std.	3.9E-2	2.2E-1	4.3E-2	4.1E-2	4.5E-2	8.2E-2	0.0E+0	2.0E-2	7.5E-2	5.5E-2
F6	HVD	M.	4.76E-1	9.22E-1	4.92E-1	5.75E-1	3.60E-1	-9.86E+3	1.53E+1	-3.21E+3	-3.84E+3	-2.22E+6
		Std.	3.7E-2	1.0E-1	1.5E-1	7.5E-2	2.5E-2	1.4E+30	1.2E+2	6.3E+3	9.4E+3	8.6E+6
	IGD	M.	3.02E-1	6.57E-1	2.60E-1	3.44E-1	2.90E-1	6.73E-1	6.95E-1	6.71E-1	6.72E-1	1.56E-1
		Std.	2.1E-2	1.3E-1	6.5E-2	5.6E-2	1.3E-2	3.6E-3	1.8E-3	4.4E-3	2.5E-3	1.4E-1
F7	HVD	M.	6.49E-1	1.22E+0	4.49E-1	6.50E-1	6.05E-1	7.07E+3	1.13E+3	1.13E+3	8.51E+3	-8.69E+5
		Std.	1.0E-2	1.5E-1	1.4E-1	2.8E-2	1.5E-2	6.0E+3	0.0E+0	0.0E+0	5.5E+3	3.0E+6
	IGD	M.	4.19E-1	1.56E+0	2.63E-1	4.18E-1	4.47E-1	4.00E-1	4.26E-1	4.00E-1	4.00E-1	2.72E-1
		Std.	6.9E-3	6.0E-1	7.1E-2	6.0E-2	1.0E-2	1.5E-4	1.3E-8	4.1E-4	1.5E-4	1.5E-1
F8	HVD	M.	1.06E+0	8.85E-1	1.34E+0	1.06E+0	4.57E-1	-1.67E+4	-1.34E+4	-1.33E+4	-8.90E+4	-1.02E+4
		Std.	4.6E-2	1.2E-1	1.0E-1	6.6E-2	3.2E-2	1.7E+4	1.1E+4	1.4E+4	1.7E+4	1.1E+4
	IGD	M.	4.86E-1	4.00E-1	4.56E-1	5.49E-1	2.51E-1	1.15E-3	6.27E-3	4.91E-4	1.13E-3	3.55E-4
		Std.	1.3E-2	6.7E-2	3.1E-2	2.3E-2	1.4E-1	8.1E-5	7.2E-4	3.0E-5	1.2E-4	1.1E-5
F9	HVD	M.	8.87E-1	1.07E+0	6.88E-1	8.58E-1	5.76E-1	-4.98E+3	-2.62E+3	-1.03E+3	-3.15E+3	-1.94E+6
		Std.	3.4E-2	1.9E-1	7.7E-2	4.6E-2	7.0E-2	8.4E+29	9.7E-5	3.7E+3	7.5E+3	3.7E+6
	IGD	M.	4.74E-1	8.87E-1	3.59E-1	4.29E-1	3.65E-1	1.46E+0	1.99E+0	1.42E+0	1.43E+0	4.82E-2
		Std.	2.1E-2	3.3E-1	4.4E-2	2.4E-2	3.4E-2	1.1E-1	0.0E+0	6.3E-3	1.7E-2	1.6E-2
F10	HVD	M.	1.22E+0	8.58E-1	5.38E-1	1.05E+0	5.77E-1	1.23E+3	1.98E+3	1.94E+3	1.23E+3	-1.77E+6
		Std.	5.0E-2	8.8E-2	1.2E-1	5.9E-2	2.3E-2	1.2E+3	0.0E+0	1.7E+3	1.2E+3	4.3E+6
	IGD	M.	1.05E+0	5.76E-1	3.79E-1	6.39E-1	3.80E-1	1.29E+0	1.76E+0	1.27E+0	1.29E+0	3.23E-2
		Std.	1.5E-1	8.1E-2	8.7E-2	8.6E-2	1.3E-2	1.9E-2	5.2E-8	4.8E-3	2.6E-2	1.9E-2

TABLE X: Adjusted p-values over IGD for twenty one problems with τ_t and n_t are equal to 10 while DPb-MOPSO is the control method, $\alpha = 0.05$.

i	DPb-MOPSO vs.	unadjusted p	p_{Bonf}	p_{Holm}	$p_{Hochberg}$	p_{Li}
1	dCOEA	0	0	0	0	0
2	MOEAD	0	0	0	0	0
3	DNSGAII	0	0	0	0	0
4	PPS	0	0	0	0	0
5	dMOPSO	0.000001	0.000005	0.000003	0.000003	0.000001
6	SGEA	0.000009	0.000083	0.000037	0.000037	0.00001
7	MOPSO	0.000057	0.00051	0.00017	0.00017	0.000058
8	pbMOPSO	0.002229	0.020062	0.004458	0.004458	0.002285
9	Dynamic-MOPSO	0.026625	0.239629	0.026625	0.026625	0.026625

TABLE XI: Adjusted p-values over HVD for twenty one problems with τ_t and n_t are equal to 10 while DPb-MOPSO is the control method, $\alpha = 0.05$.

i	DPb-MOPSO vs.	unadjusted p	p_{Bonf}	p_{Holm}	$p_{Hochberg}$	p_{Li}
1	MOEAD	0	0	0	0	0
2	dCOEA	0	0	0	0	0
3	DNSGAII	0	0.000001	0.000001	0.000001	0
4	PPS	0.000001	0.000005	0.000003	0.000003	0.000001
5	SGEA	0.000397	0.003573	0.001985	0.001985	0.000573
6	pbMOPSO	0.087763	0.789867	0.351052	0.308063	0.11256
7	Dynamic-MOPSO	0.114128	1.027155	0.351052	0.308063	0.141587
8	MOPSO	0.193738	1.743642	0.387476	0.308063	0.218746
9	dMOPSO	0.308063	2.772569	0.387476	0.308063	0.308063

degree of rejection of the null hypothesis at 0.05 significance level taking into account the issues of Type I error (rejecting

a true null hypothesis) [12].

Table X presents the APV over IGD metric for all tested

TABLE XII: Mean and Standard Deviation Values of all MOEAs over IGD Metric for FDA and dMOP.

Prob.	(τ_t, n_t)	Values	Tested Algorithms of Jiang <i>et al.</i> [13]					MOPSO-based Algorithms				
			DNSGA-II	dCOEA	PPS	MOEA/D	SGEA	MOPSO	dMOPSO	pbMOPSO	Dynamic-MOPSO	DPb-MOPSO
FDA1	(5, 10)	M.	6.40E-1	6.36E-2	2.08E-1	3.56E-1	3.41E-2	6.05E-5	1.68E-4	3.75E-3	2.67E-3	1.78E-3
		Std.	9.8E-2	1.1E-2	8.4E-2	4.9E-2	8.0E-3	2.0E-6	8.8E-6	4.0E-3	2.4E-3	4.3E-4
	(10, 10)	M.	5.82E-2	4.13E-2	4.27E-2	1.21E-1	1.48E-2	5.19E-5	1.45E-4	3.15E-3	5.30E-4	3.21E-4
		Std.	3.8E-3	6.5E-3	1.9E-2	1.1E-2	2.0E-3	1.1E-6	5.1E-6	4.4E-3	8.4E-4	4.0E-4
	(20, 10)	M.	4.14E-2	2.39E-2	1.62E-2	4.04E-2	7.55E-3	3.90E-5	1.11E-4	6.06E-5	4.20E-5	2.31E-5
		Std.	4.2E-3	2.2E-3	7.9E-3	2.2E-3	1.4E-3	4.8E-7	2.4E-6	9.8E-6	8.8E-7	5.2E-5
FDA2	(5, 10)	M.	2.85E-2	7.28E-2	8.13E-2	8.40E-2	1.50E-2	7.00E-3	4.18E-2	6.93E-3	3.55E-3	3.25E-3
		Std.	2.4E-3	3.8E-2	3.0E-2	1.3E-2	1.6E-3	1.2E-3	1.7E-3	1.9E-3	1.4E-3	7.5E-4
	(10, 10)	M.	1.68E-3	4.73E-2	6.35E-2	3.38E-2	9.11E-3	4.16E-3	2.32E-2	3.52E-3	1.26E-3	1.37E-3
		Std.	9.0E-4	3.3E-2	1.0E-2	8.8E-3	6.3E-4	4.7E-4	1.4E-3	6.2E-4	5.4E-4	4.8E-4
	(20, 10)	M.	6.51E-3	3.24E-2	6.27E-2	1.64E-2	6.32E-3	1.43E-3	6.46E-3	1.20E-3	3.17E-4	4.55E-4
		Std.	5.26E-4	4.60E-2	9.07E-3	4.99E-3	4.07E-4	2.0E-5	3.1E-4	3.2E-4	1.5E-4	1.4E-4
FDA3	(5, 10)	M.	2.63E-1	2.63E-1	4.43E-1	2.47E-1	6.25E-2	4.23E-2	5.19E-2	1.28E-1	3.08E-2	1.05E-2
		Std.	6.0E-2	3.5E-2	1.1E-1	2.3E-2	3.8E-2	1.3E-4	3.6E-2	5.0E-2	9.7E-3	5.3E-3
	(10, 10)	M.	1.08E-1	1.95E-1	2.19E-1	1.30E-1	4.03E-2	3.97E-2	5.68E-2	2.14E-1	1.80E-2	9.71E-3
		Std.	3.3E-2	3.2E-2	1.8E-2	2.5E-2	2.9E-2	3.6E-5	4.4E-2	7.2E-2	4.5E-3	9.4E-3
	(20, 10)	M.	9.03E-2	1.26E-1	1.92E-1	5.45E-2	3.52E-2	3.47E-2	4.84E-2	4.63E-2	1.94E-2	1.03E-2
		Std.	2.8E-3	3.1E-2	2.4E-2	8.3E-3	2.9E-2	5.2E-6	3.5E-2	3.6E-2	7.2E-3	6.2E-3
FDA4	(5, 10)	M.	1.49E+0	1.62E-1	3.07E-1	1.36E+0	4.60E-1	6.19E-4	1.28E-2	3.53E-4	6.54E-4	2.16E-4
		Std.	1.2E-1	6.1E-3	1.9E-2	1.6E-1	6.6E-2	1.8E-5	1.7E-3	1.4E-5	2.9E-5	1.7E-5
	(10, 10)	M.	7.63E-1	1.24E-1	2.11E-1	5.77E-1	1.83E-1	6.20E-4	1.25E-2	7.35E-4	6.58E-4	2.16E-4
		Std.	4.4E-2	4.5E-3	2.0E-2	5.4E-2	6.6E-3	1.6E-5	1.8E-3	6.4E-5	2.3E-5	6.9E-6
	(20, 10)	M.	2.62E-1	1.03E-1	1.79E-1	2.22E-1	1.26E-1	3.82E-4	8.17E-3	2.12E-4	3.77E-4	7.40E-5
		Std.	1.6E-2	1.7E-3	3.0E-3	1.3E-2	1.5E-3	1.4E-5	1.2E-3	9.8E-6	1.4E-5	1.8E-6
FDA5	(5, 10)	M.	1.76E+0	4.33E-1	6.55E-1	1.57E+0	5.23E-1	2.50E-2	7.21E-2	1.81E-3	3.93E-3	4.73E-3
		Std.	1.0E-1	4.6E-2	3.1E-2	1.3E-1	3.3E-2	1.8E-4	6.0E-3	2.1E-4	8.1E-4	9.7E-4
	(10, 10)	M.	1.02E+0	3.62E-1	4.80E-1	8.19E-1	3.62E-1	2.16E-2	6.08E-2	7.19E-3	1.92E-3	1.33E-03
		Std.	5.4E-2	4.0E-2	3.5E-2	6.0E-2	8.5E-3	2.0E-4	3.6E-3	5.0E-4	2.1E-4	1.8E-4
	(20, 10)	M.	4.88E-1	3.10E-1	3.71E-1	4.07E-1	3.09E-1	1.42E-2	3.89E-2	3.84E-3	1.03E-3	5.74E-4
		Std.	1.2E-2	2.7E-2	1.2E-2	1.4E-2	2.2E-3	1.3E-4	3.5E-3	1.6E-4	6.2E-5	4.6E-5
dMOP1	(5, 10)	M.	1.31E-1	6.95E-2	4.15E-1	1.36E-2	1.12E-2	4.06E-3	4.39E-3	1.48E-4	1.10E-4	6.58E-5
		Std.	1.1E-2	1.4E-2	7.4E-1	9.0E-3	8.1E-3	2.8E-7	1.8E-3	1.8E-5	8.3E-6	3.5E-6
	(10, 10)	M.	8.83E-3	3.93E-2	5.09E-2	9.39E-3	8.24E-3	2.02E-3	2.29E-3	1.28E-3	9.96E-5	5.30E-5
		Std.	5.0E-3	6.2E-3	9.3E-2	4.3E-3	5.3E-3	2.2E-7	1.4E-3	2.8E-5	1.2E-5	4.7E-6
	(20, 10)	M.	7.39E-3	1.88E-2	4.39E-2	7.17E-3	6.54E-3	2.71E-3	2.71E-3	2.03E-3	5.02E-5	1.98E-5
		Std.	3.2E-3	2.3E-3	8.4E-2	2.7E-3	3.0E-3	1.3E-7	1.0E-7	6.3E-6	9.1E-7	7.4E-7
dMOP2	(5, 10)	M.	6.87E-1	1.20E-1	1.56E-1	4.91E-1	3.02E-2	1.59E-3	4.19E-3	6.98E-4	1.87E-3	1.70E-3
		Std.	7.5E-2	2.0E-2	1.8E-2	4.1E-2	3.4E-3	6.0E-7	6.6E-3	2.6E-4	1.5E-3	4.3E-4
	(10, 10)	M.	1.18E-1	7.32E-2	4.28E-1	1.88E-1	1.21E-2	6.33E-4	2.95E-3	1.08E-3	6.10E-4	6.06E-4
		Std.	9.4E-3	8.9E-3	1.7E-2	1.9E-2	5.7E-4	2.0E-6	8.7E-3	3.0E-4	6.4E-5	8.3E-5
	(20, 10)	M.	1.57E-1	3.46E-2	2.02E-2	5.63E-2	6.32E-3	1.92E-3	2.70E-3	1.88E-3	1.42E-3	1.61E-3
		Std.	6.70E-4	4.32E-3	2.49E-3	3.91E-3	1.74E-4	3.7E-7	4.2E-3	5.8E-5	5.2E-4	5.8E-4
dMOP3	(5, 10)	M.	5.62E-1	4.95E-2	1.76E-1	3.42E-1	1.81E-1	9.13E-1	1.19E+0	5.40E-1	5.02E-1	6.67E-1
		Std.	3.9E-2	4.8E-3	8.0E-2	1.9E-2	9.6E-2	2.9E-1	5.2E-2	4.0E-2	1.1E-2	9.7E-2
	(10, 10)	M.	2.00E-1	2.95E-2	1.13E-1	1.68E-1	1.32E-1	2.67E-1	2.95E-1	2.51E-1	1.03E-1	1.93E-1
		Std.	1.5E-2	2.4E-3	1.2E-2	1.0E-2	1.3E-2	2.0E-2	4.2E-3	1.4E-2	2.7E-2	2.8E-2
	(20, 10)	M.	1.07E-1	1.63E-2	8.99E-2	6.27E-2	8.15E-2	5.04E+0	8.88E+0	7.44E+0	3.27E+0	5.11E+0
		Std.	8.50E-3	1.71E-3	6.74E-3	4.37E-3	1.25E-2	2.7E+0	1.9E-1	1.7E+0	6.8E-1	1.0E+0

TABLE XIII: Adjusted p-values over IGD for FDA and dMOP problems with $\tau_t = 5$ and $n_t = 10$ while DPb-MOPSO is the control method, $\alpha = 0.05$.

i	DPb-MOPSO vs.	unadjusted p	p_{Bonf}	p_{Holm}	$p_{Hochberg}$	p_{Li}
1	DNSGAII	0.000044	0.000393	0.000393	0.000393	0.000169
2	MOEAD	0.000384	0.003459	0.003075	0.003075	0.001483
3	PPS	0.000711	0.006395	0.004974	0.004974	0.002738
4	dCOEA	0.011787	0.106084	0.070723	0.070723	0.043558
5	SGEA	0.038989	0.350898	0.194943	0.194943	0.130919
6	dMOPSO	0.06928	0.623519	0.27712	0.27712	0.211156
7	MOPSO	0.457391	4.116515	1.372172	0.741182	0.638627
8	pbMOPSO	0.620294	5.582646	1.372172	0.741182	0.705591
9	Dynamic-MOPSO	0.741182	6.670634	1.372172	0.741182	0.741182

TABLE XIV: Adjusted p-values over HVD metric for FDA and dMOP problems with $\tau_t = 5$ and $n_t = 10$ while DPb-MOPSO is the control method, $\alpha = 0.05$.

i	DPb-MOPSO vs.	unadjusted p	p_{Bonf}	p_{Holm}	$p_{Hochberg}$	p_{Li}
1	MOEAD	0.000018	0.000158	0.000158	0.000158	0.000267
2	DNSGAII	0.000074	0.000665	0.000591	0.000517	0.001121
3	PPS	0.000074	0.000665	0.000591	0.000517	0.001121
4	dCOEA	0.002249	0.020244	0.013496	0.013496	0.03305
5	SGEA	0.013243	0.119186	0.066215	0.066215	0.167523
6	Dynamic-MOPSO	0.56326	5.069341	2.25304	0.934192	0.895388
7	pbMOPSO	0.620294	5.582646	2.25304	0.934192	0.904084
8	MOPSO	0.934192	8.407725	2.25304	0.934192	0.934192
9	dMOPSO	0.934192	8.407725	2.25304	0.934192	0.934192

problems with moderate changes. We can see the significant improvement of DPb-MOPSO compared to dCOEA, MOEAD, DNSGA-II, PPS, dMOPSO, SGEA, MOPSO, pbMOPSO and Dynamic-MOPSO for all post-hoc procedures. Except for the Bonferroni–Dunn procedure, the DPb-MOPSO has the same significance level compared to Dynamic-MOPSO with a APV > 0.05 . Also, it has the best significance as the MOEAD, dCOEA, DNSGA-II, PPS and SGEA while it is the same for the pbMOPSO, Dynamic-MOPSO, MOPSO and dMOPSO

over HVD metric such as reported on Table XI.

For severe environmental changes, Table XIII presents the significant difference between the DPb-MOPSO and DNSGAII, MOEAD and PPS for all pot-hoc procedures and dCOEA using the Li procedure with a $p\text{-value} \leq 0.05$ for the IGD metric. In addition, we can infer that there is no difference with $p\text{-value} > 0.05$ likewise SGEA, dMOPSO, MOPSO, pbMOPSO, Dynamic-MOPSO and dCOEA using Bonferroni–Dunn, Holm and Hochberg procedures. All APV

TABLE XV: Mean and Standard Deviation Values of all MOEAs over HVD Metric for FDA and dMOP.

Prob.	(τ_t, n_t)	Val	Tested Algorithms of Jiang <i>et al.</i> [13]					MOPSO-based Algorithms				
			DNSGA-II	dCOEA	PPS	MOEA/D	SGEA	MOPSO	dMOPSO	pbMOPSO	Dynamic-MOPSO	DPb-MOPSO
FDA1	(5, 10)	M.	8.70E-1	1.25E-1	3.87E-1	7.70E-1	8.14E-2	3.83E-3	4.48E-3	-1.82E-1	-1.52E-1	-1.69E-1
		Std.	7.5E-2	2.4E-2	1.0E-1	9.4E-2	2.0E-2	1.6E-4	6.3E-5	2.5E-1	1.7E-1	4.6E-2
	(10, 10)	M.	1.36E-1	8.52E-2	2.97E-1	2.88E-1	3.81E-2	4.14E-3	4.56E-3	-1.52E-3	-4.77E-2	-9.20E-3
		Std.	1.7E-2	2.0E-2	1.6E-2	2.9E-2	1.4E-2	6.4E-5	4.0E-5	4.4E-2	8.1E-2	6.3E-2
	(20, 10)	M.	3.55E-2	5.46E-2	2.84E-1	1.34E-1	2.02E-2	4.30E-3	4.65E-3	2.32E-3	3.41E-3	-1.62E-3
		Std.	1.3E-2	1.6E-2	1.5E-2	9.2E-3	1.2E-2	3.6E-5	1.7E-5	6.9E-4	1.4E-4	7.8E-3
FDA2	(5, 10)	M.	4.71E-2	1.85E-1	3.21E-1	1.30E-1	2.54E-2	-8.99E+0	-9.96E+0	-1.30E+0	-3.94E+0	-3.74E+0
		Std.	1.4E-2	6.4E-2	6.7E-2	2.5E-2	1.3E-2	8.5E-1	1.4E+0	6.4E-1	1.1E+1	1.4E+0
	(10, 10)	M.	2.05E-2	1.24E-1	2.66E-1	6.29E-2	1.67E-2	8.54E+0	-9.72E+0	-6.21E+0	-1.36E+0	-1.08E+0
		Std.	1.4E-2	4.6E-2	1.4E-2	1.8E-2	1.4E-2	3.2E-1	1.4E+0	1.4E+1	6.5E-1	8.9E-1
	(20, 10)	M.	1.33E-2	8.64E-2	2.55E-1	3.24E-2	1.23E-2	-8.43E+0	-8.19E+0	-3.68E+0	-3.78E-1	-2.51E+0
		Std.	1.4E-2	7.0E-2	9.4E-3	1.4E-2	1.4E-2	1.8E-2	1.1E+0	7.8E+0	4.3E-1	1.4E+0
FDA3	(5, 10)	M.	1.54E+0	1.45E+0	1.75E+0	1.66E+0	9.80E-1	-4.86E-1	-4.27E-1	-5.36E-1	-2.60E-1	-1.12E-1
		Std.	1.6E-1	8.5E-2	1.8E-1	7.8E-2	1.0E-1	8.2E-3	2.6E-1	4.9E-1	1.2E-1	6.2E-2
	(10, 10)	M.	1.09E+0	1.32E+0	1.16E+0	1.12E+0	9.24E-1	-5.62E-1	-5.27E-1	-6.57E-1	-2.68E-1	6.10E-1
		Std.	9.9E-2	7.7E-2	4.6E-2	9.3E-2	8.2E-2	6.5E-4	1.9E-1	4.6E-1	1.0E-1	3.7E-1
	(20, 10)	M.	1.04E+0	1.15E+0	1.03E+0	9.47E-1	9.11E-1	-5.54E-1	-4.13E-1	-2.95E-1	-3.09E-1	-1.66E-1
		Std.	7.9E-2	6.6E-2	7.4E-2	2.2E-2	8.1E-2	2.1E-4	3.6E-1	2.9E-1	1.4E-1	1.1E-1
FDA4	(5, 10)	M.	2.05E+0	3.80E-1	7.77E-1	3.97E+0	1.03E+0	4.17E-2	-6.51E+1	6.18E-3	2.89E-2	1.24E-4
		Std.	2.0E-1	2.6E-2	6.8E-2	1.6E+0	1.3E-1	4.6E-3	7.3E+0	3.3E-3	6.7E-3	1.9E-3
	(10, 10)	M.	1.58E+0	2.70E-1	4.34E-1	1.24E+0	2.74E-1	4.26E-2	-6.92E+1	5.67E-3	3.16E-2	7.82E-3
		Std.	6.6E-2	3.5E-2	7.2E-2	1.3E-1	2.4E-2	5.0E-3	8.2E+0	2.5E-3	5.6E-3	1.3E-3
	(20, 10)	M.	5.48E-1	1.80E-1	3.34E-1	4.34E-1	1.44E-1	4.41E-2	-7.01E+1	5.66E-3	5.00E-2	5.19E-4
		Std.	5.7E-2	2.4E-2	8.3E-3	5.0E-2	2.0E-2	4.0E-3	8.5E+0	2.2E-3	3.8E-3	6.3E-4
FDA5	(5, 10)	M.	6.75E+0	2.76E+0	3.88E+0	7.08E+0	2.70E+0	-6.38E-1	-9.97E-1	4.97E-2	-3.52E-2	-2.99E-1
		Std.	1.9E-1	2.8E-1	3.1E-1	1.0E+0	2.2E-1	6.1E-1	5.5E+0	1.5E-2	9.2E-2	1.2E-1
	(10, 10)	M.	5.41E+0	2.37E+0	2.19E+0	4.80E+0	1.88E+0	-8.66E-1	-5.97E-1	5.85E-2	4.59E-2	3.97E-2
		Std.	1.6E-1	2.7E-1	3.9E-1	2.6E-1	9.3E-2	6.7E-1	3.8E+0	6.8E-2	1.6E-2	2.4E-2
	(20, 10)	M.	2.64E+0	2.02E+0	1.04E+0	2.15E+0	1.78E+0	-1.51E+0	-1.47E+0	-4.18E-1	4.83E-2	1.25E-3
		Std.	1.1E-1	1.8E-1	1.1E-1	1.0E-1	7.1E-2	5.4E-1	5.4E+0	3.9E-2	8.8E-3	4.4E-3
dMOPI	(5, 10)	M.	3.93E-2	1.73E-1	2.86E-1	4.64E-2	3.75E-2	-3.29E-1	-3.07E-1	-9.10E-4	9.92E-4	-9.74E-4
		Std.	3.81E-2	3.3E-2	3.6E-1	3.6E-2	2.5E-2	9.9E-5	1.2E-1	1.2E-3	9.6E-4	4.0E-4
	(10, 10)	M.	2.28E-2	1.12E-1	9.27E-2	2.57E-2	1.90E-2	-3.29E-1	-3.29E-1	-1.33E-4	2.13E-3	1.46E-4
		Std.	2.0E-2	2.0E-2	1.3E-1	1.5E-2	1.4E-2	5.9E-5	3.3E-5	8.4E-4	3.2E-4	4.4E-4
	(20, 10)	M.	1.71E-2	5.65E-2	6.02E-2	1.59E-2	1.80E-2	-3.29E-1	-3.29E-1	-2.45E-1	2.79E-3	1.12E-4
		Std.	1.4E-2	8.1E-3	8.1E-2	7.9E-3	1.3E-2	3.6E-5	1.7E-5	2.6E-3	2.7E-4	1.4E-4
dMOP2	(5, 10)	M.	8.06E-1	3.03E-1	3.95E-1	9.04E-1	8.71E-2	-1.08E-1	-1.98E-2	9.71E-3	-1.05E-1	-1.16E-1
		Std.	1.1E-1	4.9E-2	3.9E-2	7.3E-2	1.9E-2	8.2E-5	2.2E-1	2.6E-2	5.6E-2	1.3E-2
	(10, 10)	M.	2.90E-1	2.07E-1	1.17E-1	4.46E-1	3.59E-2	-1.08E-1	-8.61E-2	-3.94E-3	-8.63E-2	-6.00E-2
		Std.	2.5E-2	2.4E-2	4.3E-2	4.2E-2	1.1E-2	6.1E-5	1.2E-1	3.3E-2	3.7E-2	4.4E-2
	(20, 10)	M.	4.50E-2	1.09E-1	5.65E-2	1.98E-1	1.85E-2	-1.08E-1	-8.57E-2	-1.06E-1	-6.42E-2	-9.12E-2
		Std.	1.2E-2	1.5E-2	6.2E-3	1.4E-2	1.1E-2	3.3E-5	1.2E-1	2.4E-3	4.4E-2	3.9E-2
dMOP3	(5, 10)	M.	9.51E-1	1.05E-1	4.22E-1	7.61E-1	4.07E-1	-1.13E+1	-1.10E+1	-1.40E+1	-1.44E+1	-1.45E+1
		Std.	3.4E-2	1.6E-2	1.5E-2	5.3E-2	2.4E-2	1.2E+0	1.3E-1	6.4E-1	2.2E-1	4.5E-1
	(10, 10)	M.	4.74E-1	6.57E-2	2.79E-1	4.54E-1	3.18E-1	2.49E+0	2.63E+0	1.20E+0	-8.80E-1	-6.10E-2
		Std.	2.8E-2	1.3E-2	2.7E-2	2.8E-2	2.9E-2	1.1E+0	2.8E-1	3.5E+0	2.2E-1	1.1E+0
	(20, 10)	M.	2.76E-1	3.63E-2	2.21E-1	2.87E-1	2.15E-1	-1.63E+1	-1.73E+1	-1.65E+1	-1.93E+1	-1.99E+1
		Std.	2.5E-2	1.3E-2	1.5E-2	2.0E-2	3.0E-2	1.9E+0	1.3E-1	1.6E+0	4.6E+0	3.5E+0

TABLE XVI: Adjusted p-values over IGD for FDA and dMOP problems with $\tau_t = 20$ and $n_t = 10$ while DPb-MOPSO is the control method, $\alpha = 0.05$.

i	DPb-MOPSO vs.	unadjusted p	p_{Bonf}	p_{Holm}	$p_{Hochberg}$	p_{Li}
1	DNSGAII	0.000025	0.000229	0.000229	0.000229	0.00013
2	PPS	0.000104	0.000937	0.000833	0.000833	0.000532
3	MOEAD	0.000384	0.003459	0.00269	0.00269	0.001961
4	dCOEA	0.000957	0.008612	0.005742	0.005742	0.004867
5	dMOPSO	0.014856	0.1337	0.074278	0.074278	0.070572
6	SGEA	0.031803	0.286226	0.127212	0.127212	0.139824
7	MOPSO	0.200591	1.805317	0.601772	0.430999	0.506239
8	pbMOPSO	0.2155	1.939496	0.601772	0.430999	0.524143
9	Dynamic-MOPSO	0.804353	7.239181	0.804353	0.804353	0.804353

TABLE XVII: Adjusted p-values over HVD for FDA and dMOP problems with $\tau_t = 20$ and $n_t = 10$ while MOPSO is the control method, $\alpha = 0.05$.

i	MOPSO vs.	unadjusted p	p_{Bonf}	p_{Holm}	$p_{Hochberg}$	p_{Li}
1	PPS	0.000044	0.000393	0.000393	0.000349	0.000333
2	MOEAD	0.000044	0.000393	0.000393	0.000349	0.000333
3	DNSGAII	0.000088	0.00079	0.000614	0.000527	0.000669
4	dCOEA	0.000088	0.00079	0.000614	0.000527	0.000669
5	SGEA	0.005672	0.051047	0.028359	0.028359	0.041448
6	Dynamic-MOPSO	0.23119	2.080713	0.924762	0.86883	0.638013
7	pbMOPSO	0.649723	5.847504	1.949168	0.86883	0.832026
8	DPb-MOPSO	0.649723	5.847504	1.949168	0.86883	0.832026
9	dMOPSO	0.86883	7.819472	1.949168	0.86883	0.86883

on table XIV are below 0.05 indicates the rejection of the null hypothesis using HVD metric. Clearly, we can conclude that DPb-MOPSO significantly performed the MOEAD, DNSGA-II, PPS and dCOEA for all post-hoc procedures compared to the four MOPSO-based algorithms (dMOPSO, MOPSO, pbMOPSO and Dynamic-MOPSO) and the SGEA resulting the same significance level. For slight environmental changes in Table XVI, we can conclude the best significance of the DPb-MOPSO compared to DNSGA-II, PPS, MOEAD

and dCOEA through IGD metric. While it is the same likewise the SGEA, MOPSO, pbMOPSO, Dynamic-MOPSO and dMOPSO algorithms. But, the APV values over HVD indicator reported on Table XVII highlight the significance of the MOPSO as the control method with the best rank and performed compared to PPS, MOEAD, DNSGA-II, dCOEA and SGEA over all post-hoc procedures. However, it fails compared to SGEA over Bonferroni–Dunn procedure and the four MOPSO-based algorithms for all post-hoc procedures

Fig. 3: Spectrum Lyapunov Exponent of MOPSO-based Algorithms for FDA, dMOP, F (ZJZ) and UDF Functions.

when the null hypothesis is retained with significant level $\alpha = 0.05$ assuming an equal importance between DPb-MOPSO and MOPSO over HVD.

VI. STABILITY ANALYSIS USING LYAPUNOV THEOREM

The stability analysis is an important aspect when analyzing the robustness of dynamic systems. The Lyapunov's theorem developed by Alexandr Mikhailovich Lyapunov [19] is the well-known mechanism for stability analysis. This theorem aims to measure the growth of the initial values over time while the small differences from one instance to another is called Lyapunov Exponent (LE). In addition, LE can grow into very large differences to indicate the speed with which two initially close dynamics diverge or converge. The positive value of LE indicates a divergence whereas the negative value present the convergence in phase space. In [4], the difference between two points t_i and t_j over time is denoted by $t_i, t_{i+1}, t_{i+2} \dots t_{i+n}$ and $t_j, t_{j+1}, t_{j+2} \dots t_{j+n}$. The distance $d(k)$ between two sequences after k steps is computed using: $d(k) = |t_{i+k} - t_{j+k}|$ to measure the convergence and divergence over the Lyapunov Exponent in time-varying space. If the system is chaotic, $d(k)$ will initially rise exponentially with k and we can compute and plot $\ln d(k)$ vs k to estimate the Lyapunov Exponent (LE). When LE values are very small and close over time, this state indicates that the system is stable and reaches the same solution over time. In this study, we consider a set of IGD values during 30 independent runs of the five MOPSO-based algorithms over several DMOPs with n_t and τ_t are equal to 10. The independent IGD values are computed using the minimum Euclidean distance d_i between i points in the generated $nPOF$ ($|POF|$) and the true POF^* over time t . The stability analysis through Lyapunov Exponent for MOPSO-based systems is computed using $\ln(IGD())$. Figure 3 shown the spectrum of Lyapunov Exponent over FDA, dMOP, F and UDF functions. So, we can conclude that all Lyapunov exponent for all MOPSO-based algorithms over FDA1, FDA2, FDA4, dMOP1, dMOP2 and dMOP3 are under zero with negative values assuming that the systems converge over time. For UDF problems, the spectrum of Lyapunov exponent indicates the stability of DPb-MOPSO assuming the best convergence to stability with negative values of LE. The location points for F5, F9 and F10 indicate that the Lyapunov exponent diverges to deep positive values for MOPSO, dMOPSO, pbMOPSO and Dynamic-MOPSO systems compared to the spectrum of DPb-MOPSO that converge faster for all F(ZJZ) problems. Almost, the proposed DPb-MOPSO algorithm has the best spectrum of convergence over many tested problems.

VII. CONCLUSION

This study presents a novel Dynamic Pareto bi-level Multi-Objective Particle Swarm Optimization (DPb-MOPSO) algorithm for solving different types of DMOPs (I, II and III). The proposed DPb-MOPSO presents a distributed architecture over two optimization levels, including a dynamism-handling strategy to detect and respond efficiently to changes. The Friedman

two-way ANOVA test is used to evaluate the proposed DPb-MOPSO compared to several MOEAs. Based on the statistical analysis (at 0.05 significance level), the DPb-MOPSO is shown to be a more robust algorithm in terms of its ability to ensure a good trade-off between convergence and diversity in a time-dependent environment. Further the spectrum of Lyapunov exponents has been shown to be an efficient tool for analyzing periodic motions and stability analysis in dynamical systems. The superiority of the proposed method is clearly observed when handling DMOPs with two or three objectives including different types of changes in POS and POF. In conclusion, our study has demonstrated the novelty and robustness of the DPb-MOPSO for dynamic multi-objective problems. However, these characteristics have yet to be explored for problems with more than three objectives. For future work, we will further enhance the proposed approach for Many Objective Optimization Problems (MaOOPs) as well as for pertinent feature selection in a real-world problem.

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REFERENCES

- [1] A. Aboud, R. Fdhila, and A. M. Alimi. Mopso for dynamic feature selection problem based big data fusion. In *IEEE International Conference on Systems, Man, and Cybernetics*, pages 003918–003923, Oct 2016.
- [2] A. Aboud, R. Fdhila, and A. M. Alimi. Dynamic multi objective particle swarm optimization based on a new environment change detection strategy. In Derong Liu, Shengli Xie, Yuanqing Li, Dongbin Zhao, and El-Sayed M. El-Alfy, editors, *Neural Information Processing*, pages 258–268, Cham, 2017. Springer International Publishing.
- [3] Z. Aimin, J. Yaochu, Z. Qingfu, S. Bernhard, and T. Edward. Prediction-based population re-initialization for evolutionary dynamic multi-objective optimization. In *Evolutionary Multi-Criterion Optimization*, pages 832–846, Berlin, Heidelberg, 2007. Springer.
- [4] B. Biswal and C. Dasgupta. Stochastic neural network model for spontaneous bursting in hippocampal slices. *PHYSICAL REVIEW E* 66, 051908, 2002.
- [5] C. A. Coello Coello and M. S. Lechuga. Mopso: a proposal for multiple objective particle swarm optimization. In *Proceedings of the 2002 Congress on Evolutionary Computation.*, volume 2, pages 1051–1056, May 2002.
- [6] R. Eberhart and J. Kennedy. Particle swarm optimization. In *Neural Networks*, pages 1942–1948. The 1995 IEEE International Conference on Neural Networks, Piscataway, New Jersey, IEEE Service Center, 1995.
- [7] R. Fdhila, C. Walha, T. M. Hamdani, and A. M. Alimi. Hierarchical design for distributed mopso using sub-swarms based on a population pareto fronts analysis for the grasp planning problem. In *13th International Conference on Hybrid Intelligent Systems*, pages 203–208, Dec 2013.
- [8] C. K. Goh and K. C. Tan. A competitive-cooperative coevolutionary paradigm for dynamic multiobjective optimization. *IEEE Transactions on Evolutionary Computation*, 13(1):103–127, Feb 2009.
- [9] M. Li H. Chen and X. Chen. Using diversity as an additional-objective in dynamic multi-objective optimization algorithms. In *2009 Second International Symposium on Electronic Commerce and Security*, volume 1, pages 484–487, May 2009.
- [10] L. Hatzakis and D. Wallace. Dynamic multi-objective optimization with evolutionary algorithms: a forward-looking approach. In *Proceedings of the 8th annual conference on Genetic and evolutionary computation*, pages 1201–1208. ACM, 2006.
- [11] E. J. Hughes. Multiple single objective pareto sampling. In *The 2003 Congress on Evolutionary Computation*, volume 4, pages 2678–2684 Vol.4, Dec 2003.

- [12] D. Molina J. Derrac, S. García and F. Herrera. A practical tutorial on the use of nonparametric statistical tests as a methodology for comparing evolutionary and swarm intelligence algorithms. *Swarm and Evolutionary Computation*, 1(1):3 – 18, 2011.
- [13] S. Jiang and S. Yang. A steady-state and generational evolutionary algorithm for dynamic multiobjective optimization. *IEEE Transactions on Evolutionary Computation*, 21(1):65–82, Feb 2017.
- [14] B. Udaya K. Deb, N. Rao and S. Karthik. Dynamic multi-objective optimization and decision-making using modified nsga-ii: A case study on hydro-thermal power scheduling. pages 803–817, Berlin, Heidelberg, 2007. Springer.
- [15] S. Agarwal K. Deb, A. Pratap and T. Meyarivan. A fast and elitist multi-objective genetic algorithm: Nsga-ii. *IEEE Transactions on Evolutionary Computation*, 6(2):182–197, Apr 2002.
- [16] M. Kim, T. Hiroyasu, M. Miki, and S. Watanabe. Spea2+: Improving the performance of the strength pareto evolutionary algorithm 2. pages 742–751, Berlin, Heidelberg, 2004. Springer.
- [17] W. T. Koo, C. K. Goh, and K. C. Tan. A predictive gradient strategy for multiobjective evolutionary algorithms in a fast changing environment. *Memetic Computing*, 2(2):87–110, Jun 2010.
- [18] K. B. Lee and J. H. Kim. Dmopso: Dual multi-objective particle swarm optimization. In *2014 IEEE Congress on Evolutionary Computation*, pages 3096–3102, July 2014.
- [19] A.M. Lyapunov. The general problem of the stability of motion. *Kharkov Mathematical Society, Kharkov*, 1892.
- [20] J. Ortega M. Cámara and F. Toro. Parallel processing for multi-objective optimization in dynamic environments. In *IEEE International Parallel and Distributed Processing Symposium*, pages 1–8. IEEE, 2007.
- [21] J. Ortega M. Cámara and F. Toro. Parallel multi-objective optimization evolutionary algorithms in dynamic environments. In *Proceedings of the first international workshop on parallel architectures and bioinspired algorithms*, pages 13–20, 2008.
- [22] K. Deb M. Farina and P. Amato. Dynamic multiobjective optimization problems: test cases, approximations, and applications. *IEEE Transactions on Evolutionary Computation*, 8(5):425–442, Oct 2004.
- [23] S. Nguyen, M. Zhang, M. Johnston, and K. C. Tan. Automatic design of scheduling policies for dynamic multi-objective job shop scheduling via cooperative coevolution genetic programming. *IEEE Transactions on Evolutionary Computation*, 18(2):193–208, April 2014.
- [24] S. Bechikh R. Azzouz and L. Ben Said. Multi-objective optimization with dynamic constraints and objectives: new challenges for evolutionary algorithms. In *Proceedings of the 2015 Annual Conference on Genetic and Evolutionary Computation*, pages 615–622. ACM, 2015.
- [25] S. Bechikh R. Azzouz and L. Ben Said. A dynamic multi-objective evolutionary algorithm using a change severity-based adaptive population management strategy. *Soft Computing*, 21(4):885–906, 2017.
- [26] S. Bechikh R. Azzouz and L. Ben Said. *Dynamic Multi-objective Optimization Using Evolutionary Algorithms: A Survey*, pages 31–70. Springer International Publishing, Cham, 2017.
- [27] P. N. Suganthan S. Biswas, S. Das and C. A. C. Coello. Evolutionary multiobjective optimization in dynamic environments: A set of novel benchmark functions. In *IEEE Congress on Evolutionary Computation*, pages 3192–3199, July 2014.
- [28] A. Song, W. Chen, T. Gu, H. Yuan, S. Kwong, and J. Zhang. Distributed virtual network embedding system with historical archives and set-based particle swarm optimization. *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, pages 1–16, 2019.
- [29] J. Tsai, P. Chou, and J. Chou. Color filter polishing optimization using anfis with sliding-level particle swarm optimizer. *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, 50(3):1193–1207, 2020.
- [30] Y. Wang and L. Li. Heterogeneous redundancy allocation for series-parallel multi-state systems using hybrid particle swarm optimization and local search. *IEEE Transactions on Systems, Man, and Cybernetics - Part A: Systems and Humans*, 42(2):464–474, 2012.
- [31] Q. Zhang and H. Li. Moea/d: A multiobjective evolutionary algorithm based on decomposition. *IEEE Transactions on Evolutionary Computation*, 11(6):712–731, Dec 2007.
- [32] Zhuhong Zhang. Multiobjective optimization immune algorithm in dynamic environments and its application to greenhouse control. *Applied Soft Computing*, 8(2):959 – 971, 2008.
- [33] B. Zheng. A new dynamic multi-objective optimization evolutionary algorithm. In *Third International Conference on Natural Computation*, volume 5, pages 565–570, Aug 2007.
- [34] A. Zhou, Y. Jin, and Q. Zhang. A population prediction strategy for evolutionary dynamic multiobjective optimization. *IEEE Transactions on Cybernetics*, 44(1):40–53, Jan 2014.

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