Active Disturbance Rejection Control with Low Power Extended State Observer and its Application to Speed Control of Single Shaft Gas Turbines

Hao Xia, Xiaoyang Li

Abstract—We propose a low power extended state observer (LPESO) based active disturbance rejection control (ADRC) strategy with outstanding high frequency noise attenuation and low power (or exponent of parameter). We prove the performance recovery property of the proposed scheme with perturbation theory. By performance recovery, we mean that the performance could be sufficiently close to the performance of the nominal system. Moreover, Bode-magnitude plot was used to demonstrate the better high-frequency noise attenuation compared to linear active disturbance rejection control (LADRC). Beyond theory, in the current paper, we apply this new controller to speed control of gas turbines, which usually suffers from unknown model uncertainties and external disturbances such as changes in ambient temperature, pressure, and electric load. Other handicaps include high-frequency measurement noise from the angular velocity sensor. The effectiveness of the proposed gas turbine control scheme is verified using NASA's TMATS toolbox with a high-fidelity model.

Index Terms—Low power extended state observer, Active disturbance rejection control, Performance recovery, High frequency measurement noise, Single shaft gas turbine

I. INTRODUCTION

In contrast to some model-based control methods, active disturbance rejection control (ADRC), due to its low dependence on model accuracy, satisfactory disturbance rejection performance and simple tuning procedure [1], has developed rapidly in recent years and has been applied to a wide range of industrial applications successfully in [2] [3] [4].

On the other hand, gas turbines have been widely used as prime movers in power systems due to its large ratio of power-volume and fast start-up [5]. Among all types of gas turbines, single shaft gas turbines (SSGT) are most suited for power plant field [6]. During its operation, SSGT often suffers from external disturbance such as change in electric load and

ambient temperature. Furthermore, due to the use of rotational speed sensor, there always exists high-frequency measurement noise. To endow the system with a satisfactory operation, the industry needs a powerful and easy to be tuned controller to replace the most deployed PID controller.

For model-based control methods, modeling the plant is the first step. However, for complex machines with high non-linearity and model-uncertainty like gas turbines, it would be too complicated to model them accurately [7]. But for ADRC, both model-uncertainty and external disturbance are considered as the matched "total disturbance", and their effects could be estimated and then canceled via some appropriate feedback laws [1] [8]. If the disturbance was canceled absolutely, the system would be transformed into a multi-integrator series, which we will call it the "nominal model". It might sounds too ideal, but at least we can make the performance of the system infinitely close to that of the nominal system under certain conditions; that is called "performance recovery". So far, only LADRC has been rigorously proved to have that property in [9](not the same name but identical structure); other variants of ADRC lack or have not been proved to have it. This superiority and the simple tuning procedure together make LADRC widely deployed in the past few years. However, there exists three problems for applying LADRC directly to the SSGT control: 1) A trade off between the fast convergence rate and the high sensitivity to high-frequency measurement noise. When the measurement output is contaminated seriously by high-frequency noise, the bandwidth (or gain) of linear extended state observer (LESO), which is the core component of LADRC, is severely limited. 2) The size of the observer parameter w_o^{n+1} is too large for some digital implementation. 3) There are unmatched total disturbances in the original nonlinear model of SSGT.

To solve the former two problems, we replace the LESO with LPESO, which was proposed separately in [10] and [11]. The LPESO, with whose parameters no more large than w_o^2 , possesses outstanding high-frequency measurement noise attenuation while preserves the same exponential error-decay speed of LESO [11]. In the current paper, we will push it forward by analyzing the performance recovery of it via perturbation theory.

More interestingly, we found a simple and fast parameter

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configuration method (compared to the comprehensive but rather complex procedure proposed in Astolfi [12]) for any $2n$ -th order high gain observers with this "low power" structure; this method places all poles of observer at -1 , because it is usually the value of *gain* that matters, rather than value of other parameters [13]. Based on this observer parameter configuration method, we proposed a useful parameter-tuning procedure for LPADRC with only four parameters to be tuned; that makes LPADRC distinguished with other strategies such as LADRC with a low-pass filter [14] and other nonlinear ADRC [15]. These features together make LPADRC an appealing choice for the speed control of SSGT.

Before designing LPADRC according to the order of the original SSGT model, we first decompose the model into an internal and an external dynamic to obtain its relative degree, which solves the third problem aforementioned, i.e. the unmatched disturbance in the original SSGT model. Then a 2nd-order, instead of 3rd-order LPADRC, will be designed.

To summarize, the contributions of this paper are:

- 1) Low power active disturbance rejection control was proposed and the performance recovery was proved.
- 2) This new ADRC possesses two merits, i.e. better high-frequency noise attenuation and lower power of *gain* compared to LADRC.
- 3) A simple while useful parameter-tuning method was given, in conjunction with a flow-chart.
- 4) By introducing the concept of relative degree, we bridged the gap between ADRC and speed control of single shaft gas turbines.

The rest of this paper is organized as follows. The design of LPADRC, performance recovery analysis and tuning procedure are given in Section II. The general nonlinear transient model of SSGT is given in Subsection III-A. A linearized model for a specific SSGT, 2nd-order LPADRC design, and a Bode-magnitude plot analyzing noise-sensitivity of both LPESO and LESO, are given in Subsection III-B. Numerical simulations using TMATS toolbox on MATLAB/Simulink are given in Subsection III-C to compare the performance of PI, LADRC and LPADRC. Conclusions are discussed in Section IV.

II. LOW POWER EXTENDED STATE OBSERVER BASED ADRC

A. Notations

The notation $\mathbb{R}$ stands for the set of real numbers and $\mathbb{D}$, Ω denote compact sets. States $\mathbf{x}(t)$, $\mathbf{w}(t)$, $\xi(t)$, $\zeta(t)$, $\mathbf{z}(t)$ are functions of time t ; so do output $y(t)$, $y^*(t)$, which represent the output of the actual system and the ideal linear system, respectively. For the sake of convenience, we will write them in the form of $\mathbf{x}, \mathbf{w}, \xi, \zeta, \mathbf{z}, y, y^*$. Capital letter V are used to denote Lyapunov function in conjunction with some scripts $0, 1, 2, 3, \dots$.

B. Observer and controller design

Given the n th-order nonlinear SISO system in the form of (1).

$$\begin{aligned}\dot{\mathbf{x}} &= f(\mathbf{x}, \mathbf{w}) + g(\mathbf{x}, \mathbf{w})u \\ y &= h(\mathbf{x})\end{aligned}\quad (1)$$

where $\mathbf{x} \in \mathbb{R}^n, u \in \mathbb{R}, y \in \mathbb{R}, \mathbf{w} \in \mathbb{D}_{\mathbf{w}} \subset \mathbb{R}^m$ where $\mathbb{D}_{\mathbf{w}}$ is a compact set. Functions $f(\mathbf{x}, \mathbf{w}), g(\mathbf{x}, \mathbf{w}), h(\mathbf{x})$ are continuously differentiable in $(\mathbf{x}, \mathbf{w})$ over $\mathbb{R}^n \times \mathbb{D}_{\mathbf{w}}$. Moreover, $f(0, 0) = 0, g(0, 0) = 0$ and $\mathbf{w}$ is continuously differentiable in t on $\mathbb{D}_{\mathbf{w}}$.

Let the p -th derivative of $y(t)$ along $\mathbf{x}(t)$ (the solution of equation (1)) be represented by:

$$y^{(p)}(t) = \frac{d^p y}{dt^p}(t) = f_{part,p}(\mathbf{x}, \mathbf{w}) + g_{part,p}(\mathbf{x}, \mathbf{w})u \quad (2)$$

where $f_{part,p}$ and $g_{part,p}$ are continuously differentiable functions defined on $\mathbb{R}^n \times \mathbb{D}_{\mathbf{w}}$.

Definition 1: [16] The nonlinear system (1) is said to have relative degree $\varrho \in (0, n]$ if $g_{part,i}(\mathbf{x}, \mathbf{w}) = 0$ for $i = 1, \dots, \varrho - 2$ and $g_{part,\varrho-1} \neq 0$.

Lemma 1: [17] If system (1) has relative degree $\rho < n$, there exists a diffeomorphism $\mathcal{T}(\mathbf{x})$ over $\mathbb{R}^n$ such that

$$\bar{\mathbf{x}} = \mathcal{T}(\mathbf{x}) \stackrel{\text{def}}{=} \begin{pmatrix} \phi(\mathbf{x}) \\ - \\ \psi(\mathbf{x}) \end{pmatrix} \stackrel{\text{def}}{=} \begin{pmatrix} \zeta \\ - \\ \xi \end{pmatrix} \quad (3)$$

In this way we obtain what is known as the "normal form" of system. *Definition 1* will latter be used to obtain the relative degree of nonlinear transient model of single shaft gas turbine.

Consider the normal system (4), which has an isolated equilibrium point at the origin.

$$\begin{aligned}\dot{\zeta} &= f_0(\zeta, \xi, \mathbf{w}) \\ \dot{\xi}_i &= \xi_{i+1}, \quad i = 1, \dots, \varrho - 1 \\ \dot{\xi}_{\varrho} &= \phi_1(\zeta, \xi, \mathbf{w}) + \phi_2(\zeta, \xi, \mathbf{w})u\end{aligned}\quad (4)$$

where $\zeta \in \mathbb{R}^{n-\varrho}, \xi = (\xi_1, \dots, \xi_{\varrho}) \in \mathbb{R}^{\varrho}$. Vector $\mathbf{w}$ is the external disturbance. Unknown functions $f_0(\cdot), \phi_1(\cdot), \phi_2(\cdot)$ are continuously differentiable in $(\zeta, \xi, \mathbf{w})$ over $\mathbb{R}_{n-\varrho} \times \mathbb{R}_{\varrho} \times \mathbb{D}_{\mathbf{w}}$ with $\phi_2(\cdot)$ satisfying $\phi_2(\cdot) > \text{constant}_1 > 0$ where constant_1 is a positive constant.

We impose a restriction on the system being minimum-phase. Hence, we need make some assumptions about the internal dynamic ζ . For linear system, *Assumption 1* indicates that there is no zeros with a positive real part.

Assumption 1: There exists a radially unbounded positive definite function $V_0(\zeta)$ such that for all $\zeta \in \mathbb{R}^{n-\varrho}, \xi \in \mathbb{R}^{\varrho}$ and $\mathbf{w} \in \mathbb{D}_{\mathbf{w}}$, V_0 satisfies

$$\frac{\partial V_0}{\partial \zeta} f_0((\zeta, \xi, \mathbf{w})) \leq 0, \quad \forall \|\zeta\| \geq \chi(\xi, \mathbf{w})$$

where $\chi(\cdot)$ is a nonnegative continuous function.

According to Assumption 1, there exists a non-decrease nonnegative function $\alpha(\cdot)$ satisfying $\lim_{x \rightarrow \infty} \alpha(x) = \infty$ and a nonnegative constant μ_0 such that for any positive μ , $\Omega_{\zeta} = \{V_0(\zeta) \leq \mu_0 + \alpha(\mu)\}$ is a positively invariant set for all $\xi \in \Omega_{\xi} = \{V'(\xi) \leq \mu\}$ where $V'(\xi) = \xi^T \mathcal{P} \xi$; matrix $\mathcal{P}$ is the positive solution of Lyapunov equation

$$\mathcal{P}(A - BK_c) + (A - BK_c)^T \mathcal{P} = -I$$

where

$$K_c = [\beta_1 w_c^{\varrho} \quad \beta_2 w_c^{\varrho-1} \quad \dots \quad \beta_{\varrho} w_c]_{1 \times \varrho}$$

with $w_c > 0$ and $\beta_i, 1 \leq i \leq \varrho$ are chosen such that

$$s^\varrho + \beta_\varrho s^{\varrho-1} + \dots + \beta_1$$

is a Hurwitz polynomial.

Now define a new state $\xi_{\varrho+1}$ with a constant $b_0 > 0$

$$\xi_{\varrho+1} \stackrel{\text{def}}{=} \phi_1(\zeta, \xi, \mathbf{w}) + (\phi_2(\zeta, \xi, \mathbf{w}) - b_0)u \quad (5)$$

With the new extended state $\xi_{\varrho+1}$, we can transform system(4) into (6)

$$\begin{aligned} \dot{\zeta} &= f_0(\zeta, \xi, \mathbf{w}) \\ \dot{\xi} &= A\xi + Bb_0u + B\xi_{\varrho+1} \end{aligned} \quad (6)$$

where

$$A = \begin{bmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \ddots & 0 \\ \vdots & \ddots & 0 & 1 & \vdots \\ 0 & \ddots & \ddots & 0 & 1 \\ 0 & 0 & \dots & 0 & 0 \end{bmatrix}_{\varrho \times \varrho}, \quad B = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}_{\varrho \times 1}$$

Then, a low power extended state observer is given as (7)

$$\begin{aligned} \dot{\hat{\xi}}_{i,1} &= \hat{\xi}_{i,2} + k_{i,1}w_o e_i, & i &= 1, \dots, \varrho-2 \\ \dot{\hat{\xi}}_{i,2} &= \hat{\xi}_{i+1,2} + k_{i,2}w_o^2 e_i, & i &= 1, \dots, \varrho-2 \\ \dot{\hat{\xi}}_{\varrho-1,1} &= \hat{\xi}_{\varrho-1,2} + k_{\varrho-1,1}w_o e_{\varrho-1} \\ \dot{\hat{\xi}}_{\varrho-1,2} &= \hat{\xi}_{\varrho,2} + k_{\varrho-1,2}w_o^2 e_{\varrho-1} + b_0u \\ \dot{\hat{\xi}}_{\varrho,1} &= \hat{\xi}_{\varrho,2} + k_{\varrho,1}w_o e_{\varrho} + b_0u \\ \dot{\hat{\xi}}_{\varrho,2} &= k_{\varrho,2}w_o^2 e_{\varrho} \end{aligned} \quad (7)$$

where $e_1 = y - \hat{\xi}_{1,1}$ and $e_i = \hat{\xi}_{i-1,2} - \hat{\xi}_{i,1}, i = 2, 3, \dots, \varrho$ and $K_i = (k_{i,1}, k_{i,2})^T, i = 1, 2, \dots, \varrho$ are observer parameters.

Extract $\varrho + 1$ states as the approximation of $\xi, \xi_{\varrho+1}$ by

$$\bar{\xi} = L\hat{\xi}$$

where $\hat{\xi} = (\hat{\xi}_{1,1}, \hat{\xi}_{1,2}, \dots, \hat{\xi}_{2\varrho,1}, \hat{\xi}_{2\varrho,2})^T \in R^{2\varrho}$ and L is a block diagonal matrix defined as

$$L = \text{diag}([1 \ 0], \dots, [1 \ 0])_{\varrho \times 2\varrho}$$

The estimate of "total disturbance" can be canceled by choosing the feedback control law as

$$\Gamma(\bar{\xi}, \hat{\xi}_{\varrho,2}) = \frac{v - \hat{\xi}_{\varrho,2}}{b_0} \quad (8)$$

where v is the feedback law which could be LQR, PD, or slide mode control.

In this paper we choose the feedback law to be:

$$\Gamma(\bar{\xi}, \hat{\xi}_{\varrho,2}) = \frac{-[K_c \ 1] \begin{bmatrix} \bar{\xi} \\ \hat{\xi}_{\varrho,2} \end{bmatrix}}{b_0} \quad (9)$$

If the estimation error satisfies (10)

$$\left\| \begin{bmatrix} \xi \\ \xi_{\varrho+1} \end{bmatrix} - \begin{bmatrix} \bar{\xi} \\ \hat{\xi}_{\varrho,2} \end{bmatrix} \right\| = 0 \quad (10)$$

for all $t \geq 0$ with $\|\cdot\|$ denoting the 2-norm, then substituting (9) into (6) leads to the "target system" (11)

$$\begin{aligned} \dot{\zeta} &= f_0(\zeta, \xi, \mathbf{w}) \\ \dot{\xi} &= (A - BK_c)\xi \\ y &= \xi_1 \end{aligned} \quad (11)$$

In the following, we will consider the output of (11) as the "target output" which is decided by K_C and denoted by $y^*(t)$.

In practice, due to the exist of peaking-phenomenon of LPESO [11], we need an analogous-saturation function to avoid needless large control signal by

$$u = Mg \left(\frac{\Gamma(\bar{\xi})}{M} \right) \quad (12)$$

where [9]

$$g(x) = \begin{cases} x & 0 \leq |x| \leq 1 \\ \text{sgn}(x) \left[|x| + w_o(|x| - 1) - \frac{w_o(x^2 - 1)}{2} \right] & 1 \leq |x| \leq 1 + w_o^{-1} \\ \text{sgn}(x) \left(1 + \frac{1}{2w_o} \right) & 1 + w_o^{-1} \leq |x| \end{cases} \quad (13)$$

Constant M could be any positive constant satisfying

$$M > \max_{(\zeta, \xi, \mathbf{w}) \in \Omega_\zeta \times \Omega_\xi \times \mathbb{D}_w} \left| \frac{-\phi_1(\zeta, \xi, \mathbf{w}) - K_c \xi}{\phi_2(\zeta, \xi, \mathbf{w})} \right|$$

Generally, when the estimation error is small enough, the feedback control will satisfy

$$\left| \Gamma(\bar{\xi}, \hat{\xi}_{\varrho,2}) \right| \leq M \quad (14)$$

The normal saturation function $\text{sat}(\cdot)$ has the similar function as $g(\cdot)$ while it would lead to a more complicated stability proof [9].

For the ease of analysis, we take the estimation error as

$$\tilde{\xi}_i = (\xi_i, \xi_{i+1})^T - \hat{\xi}_i, \quad \forall i = 1, \dots, \varrho - 1$$

$$\tilde{\xi}_{\varrho,1} = \xi_{\varrho} - \hat{\xi}_{\varrho,1}$$

$$\tilde{\xi}_{\varrho,2} = \phi_1(\cdot) + (\phi_2(\cdot) - b_0)Mg_{w_o} \left(\frac{\Gamma(\xi, \hat{\xi}_{\varrho,2})}{M} \right) - \hat{\xi}_{\varrho,2}$$

and scale it by

$$z_i = w_o^{2-i} D_2^{-1} \tilde{\xi}_i \quad (15)$$

This scaling leads to a new representation given as

$$\dot{z} = w_o J z + B_{2\varrho} w_o^{-\varrho} \Phi(t, \zeta, \xi, \mathbf{w}) \quad (16)$$

where $\Phi(\cdot)$ is a scalar function which will be defined explicitly in Section II-B.

$$J = \begin{bmatrix} E_1 & N & 0 & \dots & 0 \\ Q_2 & E_2 & N & \ddots & \vdots \\ 0 & Q_i & E_i & N & 0 \\ \vdots & \ddots & \ddots & \ddots & N \\ 0 & \dots & 0 & Q_{\varrho} & E_{\varrho} \end{bmatrix}, B_{2\varrho} = \begin{pmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{pmatrix}$$

and E_i, Q_i , and N are matrices defined as

$$E_i = A_2 - K_i C_2, \quad Q_i = \begin{bmatrix} 0 & k_{i,1} \\ 0 & k_{i,2} \end{bmatrix}, \quad N = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

with

$$A_2 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} B_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix} C_2 = (1 \quad 0) D_2 = \begin{bmatrix} w_o & 0 \\ 0 & w_o^2 \end{bmatrix}$$

C. Performance recovery

In this subsection, we are to show that the output y of the whole system composed of (4)(7)(12), could be infinitely close to the output y^* of the target system (11).

Theorem 1: For closed-loop system consisting of the plant (4), LPESO (7), and feedback control law (12), if b_0 is chosen such that

$$\max_{(\zeta, \xi, \mathbf{w}) \in \Omega_\zeta \times \Omega_\xi \times \mathbb{D}_\mathbf{w}} \left| \frac{\phi_2(\zeta, \xi, \mathbf{w}) - b_0}{b_0} \right| \leq \frac{1}{\|G(s)\|_\infty}$$

where

$$G\left(\frac{s}{w_o}\right) = C^1 \left(\frac{s}{w_o} I - J \right)^{-1} B$$

$$\|G(s)\|_\infty = \left\| G\left(\frac{s}{w_o}\right) \right\|_\infty = \max_{\omega \in R} |G(j\omega)|$$

and J is a Hurwitz matrix, then there exists $w_o^* > 0$ such that for every $w_o > w_o^*$:

- $\lim_{w_o \rightarrow \infty} |y(t) - y^*(t)| = 0, \forall t > 0$
- $|y(t)| \leq O(w_o^{-1}), \forall t > T_1(w_c) > 0$

where $y^*(t)$ is the "target output" of system (11), $T_1(w_c)$ is a non-increasing function of w_c , and $O(w_o^{-1})$ satisfies

$$\lim_{w_o \rightarrow \infty} O(w_o^{-1}) = 0$$

Remark 1: *Theorem 1* suggests that, we can achieve not only stability but also the "performance recovery". If M is chosen appropriately and w_o is large enough, we can obtain an output trajectory sufficiently close to the target trajectory determined by w_c .

Proof: The proof is along the spirit of Khalil [9] but with modifications due to the replacement of observer.

Now, we represent the closed-loop system by

$$\dot{\zeta} = f_0(\zeta, \xi, \mathbf{w}) \quad (17)$$

$$\dot{\xi} = A\xi + B(\phi_1(\cdot) + \phi_2(\cdot)u) \quad (18)$$

$$\dot{\mathbf{z}} = w_o J \mathbf{z} + B_{2g} w_o^{-g} \Phi(t, \zeta, \xi, \mathbf{w}) \quad (19)$$

where

$$\Phi = k_{g,2} w_o^{g+1} (z_{g-1,2} - z_{g,1}) \left(-1 + \frac{(\phi_2(\cdot) - b_0) g'_{w_o}(\cdot)}{b_0} \right)$$

$$+ \left[\dot{\phi}_1 + \dot{\phi}_2 M g \left(\frac{\Gamma(\xi, \hat{\xi}_{g,2})}{M} \right) + \frac{(\phi_2(\cdot) - b_0) g'(\cdot) (-K_c) \dot{\xi}}{b_0} \right]$$

$$\stackrel{def}{=} w_o^{g+1} \Lambda_1 B_{2g} C^1 z + B_{2g}^T \Delta_1(\cdot)$$

where

$$\Lambda_1 = \left(-1 + \frac{(\phi_2(\cdot) - b_0) g'_{w_o}(\cdot)}{b_0} \right)$$

$$C^1 = \begin{pmatrix} 1 & 0 & \cdots & 0 & 1 & -1 & 0 \end{pmatrix}$$

and Δ_1 satisfies

$$|\Delta_1| \leq \left\| \frac{\partial \phi_1}{\partial \xi} \dot{\xi} + \frac{\partial \phi_1}{\partial \zeta} \dot{\zeta} + \frac{\partial \phi_1}{\partial \mathbf{w}} \dot{\mathbf{w}} \right\| + \left\| \frac{\partial \phi_2}{\partial \xi} \dot{\xi} + \frac{\partial \phi_2}{\partial \zeta} \dot{\zeta} + \frac{\partial \phi_2}{\partial \mathbf{w}} \dot{\mathbf{w}} \right\| M$$

$$+ \left\| \frac{(\phi_2(\cdot) - b_0) g'(\cdot) (-K_c) \dot{\xi}}{b_0} \right\|$$

$$\leq c_1 \|\xi\| + c_2 \|\zeta\| + c_3$$

for some positive constants $c_i > 0, i = 1, 2, 3$.

By the singular perturbation theory, we decompose the system into a fast dynamic $\mathbf{z}(t)$ and a slow dynamic $\zeta(t), \xi(t)$. We are to show that, the decay speed of $\|\mathbf{z}\|$, from $\mathbf{z} = \mathbf{z}(0)$ to sufficiently small, is faster than the dynamic of ξ , and, hence, as we stated before, the dynamic of system (18) would be very close to system (11).

Now, consider system (20)

$$\dot{\mathbf{z}} = w_o J \mathbf{z} + w_o \Lambda_1 B C^1 \mathbf{z} \quad (20)$$

According to the circle criteria, the system (20) is exponentially stable, and by Kalman-Yakubovich-Popov (KYP) Lemma, there exists a Lyapunov $V_1 = z^T P_z z$ whose derivative along $z(z(0), t)$ (the solution of (20)) is bounded by $-w_o L_0 V_1$ with $L_0 > 0$ a positive constant [13]. The derivative of V_1 versus t along system 19 is derived:

$$\dot{V}_1 \leq -w_o L_0 V_1 + z^T P \Phi(t, \zeta, \xi, \mathbf{w})$$

$$\leq -w_o L_0 \lambda_1 \|\mathbf{z}\|^2 + w_o^{-g} \|\mathbf{z}\| \|P_z\| (c_1 \|\xi\| + c_2 \|\zeta\| + c_3)$$

$$\leq -w_o L_0 \lambda_1 (1 - \theta) \|\mathbf{z}\|^2$$

$$- \|\mathbf{z}\| \|P_z\| [w_o \theta L_0 \lambda_1 \|\mathbf{z}\| - w_o^{-g} (c_1 \|\xi\| + c_2 \|\zeta\| + c_3)]$$

$$\leq -w_o L_0 (1 - \theta) \|\mathbf{z}\|^2, \forall \|\mathbf{z}\| \geq \frac{\|P_z\| (c_1 \|\xi\| + c_2 \|\zeta\| + c_3)}{w_o^{g+1} \theta L_0 \lambda_1} \quad (21)$$

for some positive constant $\theta \in (0, 1)$ and $\lambda_1 = \lambda_{\min}(P_z)$. There exists $L_1 > 0$ such that

$$\left\{ \|\mathbf{z}\| \leq \frac{c_1 \|\xi\| + c_2 \|\zeta\| + c_3}{w_o^{g+1} \theta L_0 \lambda_1} \right\} \subset \Omega_{\mathbf{z}}$$

where

$$\Omega_{\mathbf{z}} = \left\{ \mathbf{z} \in \mathbb{R}^{2g} \mid \|\mathbf{z}\| \leq L_1 w_o^{-(g+1)} \right\}$$

By comparison lemma [17] and the property of quadratic function $\lambda_{\min}(P_z) \|\mathbf{z}\| \leq V_1 \leq \lambda_{\max}(P_z) \|\mathbf{z}\|$, we have

$$\dot{V}_1 \leq -w_o L_0 (1 - \theta) \|\mathbf{z}\|^2$$

$$\leq -L_0 \frac{1 - \theta}{\lambda_{\max}(P_z)} w_o V_1 \leq -w_o L_2 V_1, \forall \mathbf{z} \in R/\Omega_{\mathbf{z}}$$

for some constant $L_2 > 0$, which leads to

$$\lambda_{\min}(P_z) \|\mathbf{z}\| \leq V_1 \leq e^{-w_o L_2 t} V_1(\mathbf{z}(0))$$

$$\leq \lambda_{\max}(P_z) \|\mathbf{z}(0)\| e^{-w_o L_2 t}, \forall \mathbf{z} \in R/\Omega_{\mathbf{z}}$$

Hence, any trajectories starting outside $\Omega_{\mathbf{z}}$ will enter $\Omega_{\mathbf{z}}$ after $t = T(w_o)$ and $\lim_{w_o \rightarrow \infty} T(w_o) = 0$. We have

$$\|\tilde{\xi}\| \leq w_o^g \|\mathbf{z}\| \leq O(w_o^{-1})$$

Equation (14) indicates for sufficiently small estimation error $\tilde{\xi}$, control signal u is in the linear interval of $g(\cdot)$:

$$u = \frac{-K_c \bar{\xi} - \hat{\xi}_{\theta,2}}{b_0} \quad (22)$$

Substituting (22) in to (18) leads to

$$\begin{aligned} \dot{\xi} &= A\xi + B \left(\phi_1 - K_c \bar{\xi} - \frac{(\phi_2 - b_0) K_c \bar{\xi} - \phi_2 \hat{\xi}_{\theta,2}}{b_0} \right) \\ &= (A - BK_c) \xi + B \begin{pmatrix} \frac{\phi_2}{b_0} K_c & 1 \end{pmatrix} \begin{pmatrix} \bar{\xi} \\ \hat{\xi}_{\theta} \end{pmatrix} \\ &= (A - BK_c) \xi + B \Delta_2 \end{aligned} \quad (23)$$

with

$$\|\Delta_2\| \leq \iota_2 w_o^e \|\mathbf{z}\|$$

where $\iota_2 > 0$. Repeating the previous work for system (20) now to system (23) would lead to the result that any trajectory $\xi(t)$ started in Ω_{ξ} will remain in this set for all $t > 0$ and will enter $\{\|\xi\| \leq \frac{L_1}{w_o}\}$ within finite time $T(w_o)$. Since $y(0) = y^*(0)$ and $\dot{y}, \dot{y}^*$ are bounded, by the continuous dependence of the solution of state equation on initial states and parameters [?], we have

$$\begin{aligned} |y(t) - y^*(t)| &= O(T(w_o)), \quad \forall t \in [0, T(w_o)] \\ |y(t) - y^*(t)| &= O(T(w_o)) + O(w_o^{-1}), \quad \forall t \in [T(w_o), \infty) \end{aligned}$$

Hence, we obtain

$$\lim_{w_o \rightarrow \infty} |y(t) - y^*(t)| = 0$$

Because that

$$|y(t) - 0| \leq |y - y^*(t)| + |y^*(t) - 0| \quad (24)$$

For any given $\delta > 0$, we can choose $w_o > 0$ large enough such that

$$|y - y^*(t)| \leq \frac{\delta}{2}, \quad \forall t \in [0, \infty]$$

Due to the exponential stability of the target system, there exists $T_1(\delta, w_c) > 0$ depending on δ and w_c such that

$$|y^*(t) - 0| \leq \frac{\delta}{2}, \quad \forall t \in [T_1(\delta, w_c), \infty]$$

Hence,

$$|y(t) - 0| \leq \delta, \quad \forall t \in [T_1(\delta, w_c), \infty]$$

The proof is completed. ■

Although so far we have shown the output y could be sufficiently close to the origin with a large enough w_o , the asymptotic stability has not been proved; this is due to the continuously varying disturbance w . In order to obtain asymptotic stability, it is necessary to put some restrictions on w , and $f_0(\cdot)$.

Theorem 2: For closed-loop system (17)~(19), if the following requirements are satisfied:

- $\mathbf{w}$ is constant, which indicates $\dot{\mathbf{w}} = 0$.
- $f_0(\zeta, 0, \mathbf{w}) = 0$ has unique solution $\zeta_s(\mathbf{w})$, and ζ_s is an exponentially stable equilibrium point of $\dot{\zeta} = f_0(\zeta, 0, \mathbf{w})$.
- The system $\dot{\bar{\zeta}} = f_0(\bar{\zeta} + \zeta_s, \xi, \mathbf{w})$ is uniformly ISS [?] with respect to input ξ , where $\bar{\zeta} = \zeta - \zeta_s$.

then, there exists $w_o^* > 0$ such that for $w_o > w_o^*$, the output $y(t)$ satisfies

$$\lim_{t \rightarrow \infty} |y(t)| = 0$$

Proof: First, rewrite the system as

$$\dot{\bar{\zeta}} = f_0(\bar{\zeta} + \zeta_s, 0, \mathbf{w}) + f_0(\bar{\zeta} + \zeta_s, \xi, \mathbf{w}) - f_0(\bar{\zeta} + \zeta_s, 0, \mathbf{w}) \quad (25)$$

$$\stackrel{\text{def}}{=} f_0(\bar{\zeta} + \zeta_s, 0, \mathbf{w}) + \Delta_3(\cdot)$$

$$\dot{\xi} = (A - BK_c) \xi + \Delta_2(\cdot) \quad (26)$$

$$\dot{\mathbf{z}} = w_o J \mathbf{z} + w_o \Lambda_1 B_{2\theta} C^1 \mathbf{z} + B_{2\theta} \Delta_1(\cdot) \quad (27)$$

According to the differentiability of $f_0(\cdot)$, we have

$$\|\Delta_3\| \leq \iota_1 \|\xi\|$$

and some properties we obtained before

$$\begin{aligned} \|\Delta_2\| &\leq \iota_2 w_o^e \|\mathbf{z}\| \\ \|\Delta_1\| &\leq \iota_3 w_o^{-e} \|\bar{\zeta}\| + \iota_4 w_o^{-e} \|\xi\| + \iota_5 \|\mathbf{z}\| \end{aligned} \quad (28)$$

for some positive constants $\iota_i, i = 1, \dots, 5$. According to Lyapunov converse theorem [17] and the ISS of internal dynamic $\dot{\bar{\zeta}} = f_0(\bar{\zeta} + \zeta_s, \xi, \mathbf{w})$, there exists positive definite function $V_3(\bar{\zeta})$ of system (25) satisfying

$$\dot{V}_3 \leq -\kappa_1 \|\bar{\zeta}\|^2, \quad \left\| \frac{\partial V_3}{\partial \bar{\zeta}} \right\| \leq \kappa_2 \|\bar{\zeta}\|$$

and a positive definite function $V_2(\xi)$ whose derivative along $\xi(t, \xi(0))$ satisfies

$$\dot{V}_2 \leq -\kappa_3 \|\xi\|^2, \quad \left\| \frac{\partial V_2}{\partial \xi} \right\| \leq \kappa_4 \|\xi\|$$

$\kappa_i, i = 1, 2, 3, 4$ are some positive constants. Let

$$V = \kappa_0 V_2 + V_3 + w_o^{2\theta} \mathbf{z}^T P_z \mathbf{z}$$

with a positive constant $\kappa_0 > 0$. The time-derivative of V along (25)~(27) satisfies

$$\begin{aligned} \dot{V} &\leq -\kappa_1 \|\bar{\zeta}\|^2 - \kappa_0 \kappa_3 \|\xi\|^2 - (w_o^{2\theta+1} \iota_5 \|P_z\| w_o^{2\theta}) \|\mathbf{z}\|^2 \\ &\quad + \kappa_0 \kappa_4 \|\mathbf{z}\| \|\xi\| + \kappa_2 \|\xi\| \|\bar{\zeta}\| + w_o^e \iota_4 \|P_z\| \|\mathbf{z}\| \|\xi\| + w_o^e \iota_3 \|P_z\| \|\mathbf{z}\| \|\bar{\zeta}\| \\ &\leq - \begin{pmatrix} \|\bar{\zeta}\| \\ \|\xi\| \\ \|\mathbf{z}\| \end{pmatrix}^T \begin{bmatrix} \kappa_1 & -\iota_1 \kappa_2 / 2 & O(w_o^e) \\ -\iota_1 \kappa_2 / 2 & \kappa_0 \kappa_3 & O(w_o^e) \\ O(w_o^e) & O(w_o^e) & w_o^{2\theta+1} + * \end{bmatrix} \begin{pmatrix} \|\bar{\zeta}\| \\ \|\xi\| \\ \|\mathbf{z}\| \end{pmatrix} \end{aligned}$$

If κ_0 are chosen such that $\kappa_0 \kappa_1 \kappa_3 \geq \iota_1^2 \kappa_2^2$, the determinant will be

$$w_o^{2\theta+1} \begin{vmatrix} \kappa_1 & -\iota_1 \kappa_2 / 2 \\ -\iota_1 \kappa_2 / 2 & \kappa_0 \kappa_3 \end{vmatrix} - O(w_o^e)$$

Hence, by Sylvester criterion, there exists $w_o^* > 0$ such that for every $w_o > w_o^*$, $\dot{V}$ satisfies

$$\dot{V} \leq -\tau \left(\|\bar{\zeta}\|^2 + \|\xi\|^2 + \|\mathbf{z}\|^2 \right)$$

for some $\tau > 0$. The proof is completed. ■

Remark 2: Theorem 2 is a complement to Theorem 1. It suggests that LPADRC has the "integral part" which is similar to PID controller.

D. Tuning procedure

Considering the complexity of plants and variety of performance requirements, it seems that no universal criteria for selecting parameters can be adopted. But we do have some useful principles for parameter-tuning. The first parameters to be set are $K_i, i = 1, \dots, \varrho$ because we need ensure that J is a Hurwitz matrix; we give a simple law to assign all the eigenvalues of J at $\lambda_j^j = -1, 1 \leq j \leq 2\varrho$ (the same as LESO [18]):

$$\begin{aligned} k_{i,1} &= 2, & 1 \leq i \leq \varrho \\ k_{i,2} &= \frac{\varrho+1-i}{i}, & 1 \leq i \leq \varrho \end{aligned} \quad (29)$$

Next, we set $\beta_i, 1 \leq i \leq \varrho$ according to the rule:

$$\beta_i = C_{\varrho}^{i-1}, 1 \leq i \leq \varrho$$

where C_{ϱ}^{i-1} denotes the binomial coefficient. The third to be set is b_0 , whose effect has not been decided yet, but generally b_0 should satisfy

$$b_0 \geq 2 \max_{(\eta, \xi, \mathbf{w}) \in \Omega_{\zeta} \times \Omega_{\xi} \times \mathbb{D}_{\mathbf{w}}} |\phi_2(\zeta, \xi, \mathbf{w})| \quad (30)$$

Recall that LPADRC can recover the performance of state-feedback control; accordingly, w_c has little affect on stability if w_o is large enough. Therefore, we shall first set the w_o , which decides the estimate-speed of LPESO. Then we choose an appropriate w_c to satisfy requirements. Now, there is only M to be set. There is an interplay between w_c and M . If M is too small for a fixed w_c , the system may oscillate. To summarize, the overall procedure of parameter-tuning is given in Figure 1:

Fig. 1. The tuning procedure of LPADRC

Remark 3: There is an interplay between w_o and b_0 . Fix w_o , as b_0 increases, the decay speed of estimation-error will decrease, which indicates worse disturbance rejection. The increase of b_0 also affect the selection of w_c by attenuating the control signal u .

TABLE I
NOMENCLATURE

Symbol	Denotation	Sub	Denotation
P	pressure(bar)	0	nominal value
V	volume of chamber(m^3)	1	compressor inlet
T	temperature(K)	2	chamber inlet
R	gas constant($J/(kg \cdot K)$)	3	turbine inlet
π	pressure ratio	4	turbine outlet
σ	pressure loss	c	compressor
L	torque($N \cdot m$)	t	turbine
I	shaft inertia($kg \cdot m^2$)	V	constant volume
γ	adiabatic index	f	fuel
C	specific heat ($KJ/(kg^\circ C)$)	I	inlet duct
n	speed (rpm/min)	N	nozzle
ρ	density (kg/m^3)	P	constant pressure
η	efficiency	comb	chamber
v	gas flow rate(kg/s)	e	electrical torque
LHV	lower heating value of fuel(J/Kg)		

III. SINGLE SHAFT GAS TURBINE CONTROL

A. Nonlinear model of SSGT

Figure 2 shows the layout of a common SSGT consisting of inlet tube, compressor, combustion, turbine and nozzle. The compressor inhales air and increases its pressure. Then the high-pressure air enters the combustion chamber, where the fuel is added and then ignited to produce heat. The power generated by expanding hot and high-pressure gas through the turbine will be exhausted by both compressor and electric load.

Fig. 2. Layout of a common single shaft gas turbine used for electric generation consisting of a compressor, a combustion chamber and a turbine.

Generally, when gas turbine works in the steady state, the power generated by the turbine is equal to the sum of power absorbed by the compressor and exhausted by the electric load. If power balance is disturbed by change in electric load or ambient condition, there is a dynamic process in the chamber. Consequently, it will lead to pressure and temperature change.

According to A.M.Y.Razak [5], differential equations (31), which derived from the conversation law of mass and momen-

tum, represents the SSGT model.

$$\begin{aligned} \frac{dP_3}{dt} &= \frac{RT_3}{V} (v_c + v_f - v_t) \\ \frac{dT_3}{dt} &= \{ v_c c_P T_1 \left[\left(\pi_c^{\frac{\gamma-1}{\gamma}} - 1 \right) / \eta_c + 1 \right] - v_t c_P T_3 + v_f LHV \\ &\quad - C_V T_3 (v_c - v_t) \} / (\rho_2 V C_V) \\ \frac{dn}{dt} &= \frac{900}{\pi^2 I n} \{ v_t c_P T_3 \left(1 - \pi_t^{\frac{1-\gamma}{\gamma}} \right) \eta_t \\ &\quad - v_c c_P T_1 \left(\pi_c^{\frac{\gamma-1}{\gamma}} - 1 \right) / \eta_c - \frac{2\pi}{60} n L_e \} \end{aligned} \quad (31)$$

where π_c and π_t are the pressure ratio given as

$$\pi_c = \frac{P_2}{P_1} = \frac{P_3}{\sigma_{comb} P_1}, \quad \pi_t = \frac{P_3}{P_4} = \frac{P_3 \sigma_I \sigma_N}{P_1} \quad (32)$$

The gas flow rate v_c and v_t are computed as the functions of pressure ratio and rotational speed, and we need to derive them from the experimental data. Figure 2 shows a typical compressor characteristic map derived from experimental data, which consists of two sets of curves. The first set describes the relation between stagnation pressure ratio π_c and non-dimensional air flow rate $\frac{v_c \sqrt{T_1}}{P_1}$ under the same speed. The second set describes the relation between efficiency η_c and $\frac{v_c \sqrt{T_1}}{P_1}$ under same speed. Similarly, the turbine characteristic map describes same relations for turbine. Gas flow rate of compressor and turbine can be represented as functions as:

$$\begin{aligned} v_c &= g_1 \left(\frac{n}{\sqrt{T_1}}, \pi_c \right) \cdot \frac{P_1}{\sqrt{T_1}} = f_1(n, P_3, P_1, T_1) \\ v_t &= g_2 \left(\frac{n}{\sqrt{T_3}}, \pi_t \right) \cdot \frac{P_3}{\sqrt{T_3}} = f_2(n, P_3, P_1, T_3) \end{aligned} \quad (33)$$

where $g_1(\cdot)$ and $g_2(\cdot)$ are continuously differentiable functions. It needs to be noticed that η_c and η_t varies slowly and slightly during the transition process compared to the gas flow rate. For simplicity, we treat them as constants [6].

Let $x = (x_1 \ x_2 \ x_3)^T = (P_3 \ T_3 \ n)^T$, $u = v_f$, $y(t) = n$. Substituting (33) into (31) leads to the state-space form:

$$\begin{aligned} \dot{x} &= \begin{pmatrix} \psi_1(x, \mathbf{w}) \\ \psi_2(x, \mathbf{w}) \\ \psi_3(x, \mathbf{w}) \end{pmatrix} + \begin{pmatrix} b_1 x_2 \\ b_2 \\ 0 \end{pmatrix} u \\ y &= (0 \ 0 \ 1) x \end{aligned}$$

where $\mathbf{w} = (P_1 \ T_1 \ L_e)^T$ represents the external disturbance and b_1, b_2 are positive constants given by :

$$b_1 = \frac{R}{V} \quad b_2 = \frac{LHV}{\rho_2 V c_V}$$

$\psi_i(x, w)$, $i = 1, 2, 3$ are nonlinear continuously differentiable

functions in the form of:

$$\begin{aligned} \psi_1(x, w) &= \frac{R}{V} x_2 [f_1(x, w) - f_2(x, w)] \\ \psi_2(x, w) &= \frac{1}{\rho_2 V c_V} \{ f_1(x, w) c_P T_1 \left[1 - \frac{1}{\eta_c} + \frac{\left(\frac{x_1}{P_1 \sigma_{comb}} \right)^{\frac{\gamma-1}{\gamma}}}{\eta_c} \right] \right. \\ &\quad \left. - f_2(x, w) c_P x_2 - c_V (f_1(x, w) - f_2(x, w)) \right\} \\ \psi_3(x, w) &= \frac{900}{\pi^2 I n} \{ f_2(x, w) c_P x_2 \left[1 - \left(\frac{x_1 \sigma_I \sigma_N}{P_1} \right)^{\frac{1-\gamma}{\gamma}} \right] \eta_t \right. \\ &\quad \left. - f_1(x, w) c_P T_1 \left[\left(\frac{x_1}{P_1 \sigma_{comb}} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right] / \eta_c - \frac{2\pi}{60} x_3 L_e \right\} \end{aligned}$$

where $f_1(\cdot)$ and $f_2(\cdot)$ are aforementioned.

It can be seen that the exact dynamic mathematical model is unavailable due to some unavoidable approximation and unknown external disturbance. Now, by Lemma 1, we can easily verify that the transient SSGT model (??) has relative degree $\rho = 2$.

B. Linear model of a specific SSGT and 2nd-LPADRC design

To investigate the influence of measurement noise on control signal u , we linearize a specific SSGT model at steady state and derive the transfer function $\frac{u(s)}{v(s)}$. This SSGT data is given by National Aeronautics and Space Administration (NASA) in its open-source project. The first step would be the interpolating the gas flow function v_c and v_t based on the characteristic maps. We choose a polynomial function (34)

$$f(n, \pi) = cont_1 + cont_2 * \pi + cont_3 * n + cont_4 * \pi n \quad (34)$$

where $cont_i, i = 1, \dots, 4$ are constants. Recall that $x_1 = P_1, x_2 = T_3, x_3 = n$. The interpolation results are listed as below:

$$v_c = -19.8 + 0.0003632x_1x_3 - 3.09x_1 + 0.005x_3$$

$$v_t = 101x_1x_2^{-\frac{1}{2}} - 6.04 \times 10^{-6}x_1x_3 - 0.073x_1^{\frac{3}{2}}x_2^{-\frac{1}{2}} + 3.6 \times 10^{-5}x_1^2x_3$$

The other parameters of SSGT at the steady state are given in Table 2.

TABLE II

THE PARAMETER VALUES OF A SPECIFIC SINGLE SHAFT GAS TURBINE

symbol	value	symbol	value	symbol	value
n(rpm)	10000	T_0	288.15	P_0	1
T_3	1710	P_3	19.4	c_P	1004
c_V	717	L_e	0	I	30
γ	1.4	R	287	LHV	4.08×10^7
σ_{comb}	0.95	σ_I	0.98	σ_N	0.99
η_c	0.8043	η_t	0.9013		

Through the linearization and shifting equilibrium point $x = (19.4, 1710, 10000)$ to $(0, 0, 0)$, the linear model is developed :

$$\dot{x} = \begin{bmatrix} -1466 & -5.76 & 0.0055 \\ -59000 & -328.4 & 16.05 \\ -4400 & -22 & 2.35 \end{bmatrix} x + \begin{bmatrix} 98 \\ 95000 \\ 0 \end{bmatrix} u = Ax + Bu \quad (35)$$

Fig. 3. Compressor characteristic map: The left figure is the map about non-dimensional air flow rate and Pressure ratio. The right figure is the map about non-dimensional air flow rate and efficiency. n_0 is the nominal rotational speed.

The LPESO and the feedback law are given as:

$$\begin{bmatrix} \dot{\hat{\xi}}_{1,1} \\ \dot{\hat{\xi}}_{1,2} \\ \dot{\hat{\xi}}_{2,1} \\ \dot{\hat{\xi}}_{2,2} \end{bmatrix} = \begin{bmatrix} -2w_o & 1 & 0 & 0 \\ -2w_o^2 & 0 & 0 & 1 \\ 0 & 2w_o & -2w_o & 1 \\ 0 & \frac{1}{2}w_o^2 & -\frac{1}{2}w_o^2 & 0 \end{bmatrix} \hat{\xi} + \begin{bmatrix} 0 \\ b_0 \\ b_0 \\ 0 \end{bmatrix} u + \begin{bmatrix} 2w_o \\ 2w_o^2 \\ 0 \\ 0 \end{bmatrix} (x_3 + \nu(t)) \quad (36)$$

$$\dot{\hat{\xi}} = F\hat{\xi} + B_e u + B_\nu y + B_\nu \nu(t) \quad (37)$$

$$u = \frac{1}{b_0} [-w_c^2 \quad 0 \quad -2w_c \quad -1] \hat{\xi} = C_c \hat{\xi} \quad (38)$$

Notice that, in steady state, control signal u satisfies $|u| < M$. Hence, we omitted the analogous saturation function in (36). Now, the whole closed-loop system can be represented as

$$\begin{bmatrix} \dot{x} \\ \dot{\hat{\xi}} \end{bmatrix} = \begin{bmatrix} A & B * C_c \\ B_\nu * [0 \quad 0 \quad 1] & F + B_e * C_c \end{bmatrix} \begin{bmatrix} x \\ \hat{\xi} \end{bmatrix} + \begin{bmatrix} 0_{3 \times 1} \\ B_\nu \end{bmatrix} \nu(t) = \mathcal{A} \begin{bmatrix} x \\ \hat{\xi} \end{bmatrix} + \mathcal{B} \nu(t)$$

$$u = [0_{1 \times 3} \quad C_c] \begin{bmatrix} x \\ \hat{\xi} \end{bmatrix} = \mathcal{C} \begin{bmatrix} x \\ \hat{\xi} \end{bmatrix}$$

Hence, the transfer function $\frac{u(s)}{\nu(s)}$ will be

$$T_{LPESO} = \frac{u(s)}{\nu(s)} = \mathcal{C} (sI - \mathcal{A})^{-1} \mathcal{B} \quad (39)$$

For comparison, a LESO designed for this SSGT is given as (40)

$$\dot{z} = \begin{bmatrix} -3w_o & 1 & 0 \\ -3w_o^2 & 0 & 1 \\ -w_o^3 & 0 & 0 \end{bmatrix} z + \begin{bmatrix} 3w_o \\ 3w_o^2 \\ w_o^3 \end{bmatrix} (x_3 + \nu(t)) \quad (40)$$

and the feedback law is

$$u = \frac{1}{b_0} [-w_c^2 \quad -2w_c \quad -1] z \quad (41)$$

By repeating the same procedure to LESO, we obtain the transfer function $T_{LESO} = \frac{u(s)}{\nu(s)}$ for the LESO based strategy.

Fig. 4. The Bode-magnitude plot of $\frac{u(s)}{\nu(s)}$ of LPESO-based strategy and LESO-based strategy, respectively. Both strategy has the same w_o, w_c, b_0 . It is seen clearly that within the high-frequency band, LPESO has much smaller noise amplification gain than LESO.

Now, with the same w_o, w_c, b_0 for both two control scheme, we give a Bode-magnitude plot to visually compare the frequency noise attenuation feature of LPADRC and linear ADRC. It is seen from Figure 4 that the amplification of high frequency noise $\nu(t)$ through LPESO is much lower than LESO.

C. Numerical simulations for transient process

To illustrate the effectiveness of the proposed scheme, some simulations are carried out using the "T-MATS" toolbox written by NASA in MATLAB/Simulink. We compare the performance of PID, 2nd-order ADRC, 2nd-order LPADRC

(the proposed scheme). Figure 5 shows the layout of the closed-loop system.

Fig. 5. Schematic diagram of the gas turbine fuel flow rate control system. The SSGT graphic is from Neilson [19]

The main restriction in practice is the trade-off between disturbance rejection and control signal noisy level. Too noisy signal will damage actuators. Hence, our criteria is: 1) better disturbance rejection with the same noisy u , or 2) less noisy u with the same disturbance rejection performance.

The specifications of the SSGT and parameter values of controllers are given in Table III and Table IV. The sample time is $T_s = 0.0001$. High-Frequency noise is generated by the 'band-limited -white noise' block together with a high-pass filter $G(s) = \frac{s}{s+500}$. We simulate 'change in step load' and 'change in gradual altitude' experiments, respectively.

TABLE III

SPECIFICATIONS OF THE GAS TURBINE USED FOR THIS SIMULATION.

Description	Specification
Type	single shaft
Nominal speed	10000(rpm/min)
Nominal power	100(horsepower)
Compression ratio	20

TABLE IV

PARAMETERS OF PID, ADRC, LPADRC.

Controller	Parameters
PI	$P = 0.01, I = 0.05$
LADRC	$w_o = 50, w_c = 20, b_0 = 50000, M = 3$
LPADRC	$w_o = 100, w_c = 20, b_0 = 50000, M = 3$

1) *Simulation for 'altitude change'*: The altitude of environment affects the ambient temperature T_0 and ambient pressure P_0 . Consequently, change in altitude will trigger a transient process. Hence, we must assure an effective control when it suffers an gradual altitude changing with time :

$$Altitude(ft) = \begin{cases} 0 & t \leq 4s \\ 5000(t-4) & 4s \leq t \leq 6s \\ 10000 & 6s \leq t \end{cases}$$

According to the criteria we set before, Figure 6 shows that proposed scheme achieved better disturbance rejection with the same noisy level of control signal u .

2) *Simulation for 'step load change'*: The changes in electric load are frequently encountered in practice, which could be caused by sudden transmission line fault or decrease in

electricity consumption, etc. The load change with time:

$$Load(hp) = \begin{cases} 100 & t \leq 5s \\ 50 & 5s < t \end{cases}$$

Figure 7 indicates the same conclusion as Figure 6.

IV. CONCLUSIONS

This study set out to provide an alternative to LADRC by the Low Power Active Disturbance Rejection Control (LPADRC). The performance recovery of LPADRC has been proved via Lyapunov method, which makes it the second variants of ADRC possessing this property. Then, to facilitate the utilization of LPADRC, we provided a simple observer parameter-configuration method and then a whole parameter-tuning procedure of the controller; the observer parameter-configuration method could also be adopted to other standard low power high gain observer. Furthermore, the gap between ADRC and SSGT model caused by unmatched disturbances has been bridged by introducing the concept of relative degree. Finally, the effectiveness and superiority of the proposed strategy were verified by utilizing NASA's TMATS toolbox with a high-fidelity gas turbines model. The simulation results, in both graphical and numerical form, showed that LPADRC improved the disturbance rejection performance without de-generating the control signal when measured output suffering from high-frequency noise.

However, the peaking-phenomenon problem, which is also a classic problem of high gain observer, has not been solved. We adopted the conventional method by restricting the magnitude of the control signal. In the future, we will focus on solving that problem. Besides that, the merit "low power" is not distinct for low-order system; so, it would be a quite interesting topic to apply LPADRC to some high-order systems.

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Fig. 6. Transient process of output y and control signal u when suffering an altitude change for PI, LADRC and LPADRC based control strategy. When suffers from a gradual altitude change disturbance, in all three control strategy, LPADRC based strategy has the smallest overshoot and the best undisturbed control signal. The integral of steady state error $Int = \int_0^{3.5} |u - u_{average}| dt$ are: $Int_{PI} = 0.00470$, $Int_{LADRC} = 0.0191$, $Int_{LPADRC} = 0.00175$

Fig. 7. Transient process of output y and control signal u when suffering an step load change for PI, LADRC and LPADRC based control scheme. This result leads to the same conclusion with the first simulation. The integral of steady state error $Int = \int_0^{4.5} |u - u_{average}| dt$ are: $Int_{PI} = 0.00561$, $Int_{LADRC} = 0.0229$, $Int_{LPADRC} = 0.00212$

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