Adaptive Dynamic Programming based Control Scheme for Uncertain Two-Wheel Robots

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Abstract—The paper addresses the problem of effectively controlling a two-wheel robot given its inherent non-linearity and parameter uncertainties. In order to deal with the unknown and uncertain dynamics of the robot, it is proposed to employ the adaptive dynamic programming, a reinforcement learning based technique, to develop an optimal control law. It is interesting that the proposed algorithm does not require kinematic parameters while finding the optimal state controller is guaranteed. Moreover, convergence of the optimal control scheme is theoretically proved. The proposed approach was implemented in a synthetic two-wheel robot where the obtained results demonstrate its effectiveness.

Index Terms—Reinforcement learning, two-wheel robot, adaptive control, adaptive dynamic programming

I. INTRODUCTION

Two-wheel self-balancing vehicles have been increasingly become a popular alternative transportation method in recent years due to its flexibility during travelling in metropolitan areas [1]. Technically, the vehicle is built based on a two-wheel robot platform, which is well known in the control community due to its unstable dynamic performance and strong non-linearity. That is, effectively and robustly controlling a two-wheel robot given its uncertainty is a fundamental but still challenging problem.

In the control theory, sliding model control is widely used in systems highly requiring robustness [2], [3]. Nevertheless, the sliding model control assumes that a dynamic model of a system is deterministic and known, which is impractical [4], [5]. Recent researches show trend of adaptively learning system dynamics during controlling the system [6], [7]. One of good strategies for learning dynamic behaviours of intelligent robots is using reinforcement learning (RL) [8], which practically simulates learning ability of animals while interacting with environment to adapt and survive. RL has been used in variety of real-time optimal control methods such as adaptive dynamic programming (ADP) [8], where system dynamics are not exactly known. For instance, the authors in [9] exploited the policy iteration (PI) algorithm to eliminate internal kinematic models in optimally control an uncertain

nonlinear system. In [10], Vrabie *et al.* presented a control design using RL to solve the linear quadratic regulator problem online. Another PI based method was developed in [11] with assumption of a full model-free system, where noise signals are proposed to add to input signals during learning process.

In this paper, we propose to exploit the ADP to model uncertain and nonlinear systems such as a two-wheel robot, which allows a control scheme to eliminate kinematic components in the system dynamic models. In other words, the dynamic parameters can be adaptively learned over time. We then employ the PI to design an efficient optimal control law for those systems, where its convergence is theoretically proved. The proposed approach was implemented and verified in a two-wheel robot in a synthetic environment in which the obtained results are highly promising.

The remaining of the paper is arranged as follows. Section II introduces the adaptive dynamic programming model before the proposed algorithm is discussed in Section III. Implementation of the control law in a two-wheel robot is presented in Section IV, where the obtained results are demonstrated. Conclusions are drawn in Section V.

II. ADAPTIVE DYNAMIC PROGRAMMING MODEL

Let us consider a continuous linear system as follows,

$$\dot{x} = Ax + Bu, \quad (1)$$

where $x \in R^n$ is a state vector of the system, $u \in R^m$ is a signal control vector, $A \in R^{n \times n}$ and $B \in R^{n \times m}$ are uncertainty matrices. Assume that the system is stable, there exist a matrix K so that $A-BK$ is a Hurwitz matrix. That is, a full linear state feedback controller must be in the form

$$u = -Kx, \quad (2)$$

which holds the following cost function

$$J(x_0, u) = \int_0^\infty (x^T Q x + u^T R u) dt = x^T P x, \quad (3)$$

where $Q = Q^T \geq 0$ and $R = R^T \geq 0$. Hence, $(A, Q^{1/2})$ is observable. If both matrices A and B are completely identified, solution to a linear quadratic regulator (LQR) problem is equivalent to that of Riccati equation [12]

$$A^T P + PA + Q - PBR^{-1}B^T P = 0. \quad (4)$$

The equation has a uniquely positive symmetric solution P^* . Thus, the coefficient matrix of the optimal state feedback controller K^* is computed as follows,

$$K^* = R^{-1}B^T P^*. \quad (5)$$

Suppose that $u_K(x) = -Kx$ is an arbitrary feedback law. If u_K is applied to the system (1), the cost function (3) can be written

$$J(x_0; u_K) = x_0^T P_K x_0, \quad (6)$$

where P_K is defined as a cost matrix associated with the feedback gains K and given by

$$P_K = \int_0^\infty e^{(A-BK)^T t} (Q + K^T R K) e^{(A-BK)t} dt. \quad (7)$$

P_K is finite iff it is a Hurwitz matrix. That is, P_K is the unique (positive definite) solution of the linear equation

$$(A - BK)^T P + P(A - BK) + Q + K^T R K = 0. \quad (8)$$

Moreover, if K_1 and K_2 are gain matrices associated with the cost matrices P_1 and P_2 , it can be shown that

$$\begin{aligned} P_1 - P_2 = & \int_0^\infty e^{(A-BK_2)^T t} \left[(K_1 - K_2)^T R (K_1 - K_2) \right. \\ & - (K_1 - K_2)^T (B^T P_1 - RK_2) \\ & \left. - (B^T P_1 - RK_2)^T (K_1 - K_2) \right] e^{(A-BK_2)t} dt. \end{aligned} \quad (9)$$

or

$$\begin{aligned} P_1 - P_2 = & \int_0^\infty e^{(A-BK_1)^T t} \left[(K_1 - K_2)^T R (K_1 - K_2) \right. \\ & - (K_1 - K_2)^T (B^T P_2 - RK_2) \\ & \left. - (B^T P_2 - RK_2)^T (K_1 - K_2) \right] e^{(A-BK_1)t} dt. \end{aligned} \quad (10)$$

Theorem 1. If $K_0 \in R^{m \times n}$ is a matrix of the state feedback controller (1). The positive symmetric solution P_k of the Lyapunov equation

$$(A - BK_k)^T P_k + P_k(A - BK_k) + Q + K_k R K_k^T = 0, \quad (11)$$

can be computed as follows,

$$\begin{aligned} K_k &= R^{-1}B^T P_{k-1} \\ A_k &= A - BK_k, \end{aligned} \quad (12)$$

where K_1 is chosen such that $A_1 = A - BK_1$ is a Hurwitz matrix. And,

1. $A_k = A - BK_k$ is a Hurwitz matrix.
2. $P^* \leq P_{K+1} \leq P_k$.

3. $\lim_{n \rightarrow \infty} K_k = K^*, \lim_{n \rightarrow \infty} P_k = P^*$.

Proof: 1. Let P_1 be a cost matrix associated with K_1 . P_1 then satisfies (11) with $k=1$. Now set $K_2 = R^{-1}B^T P_1$ and let P_2 be its associated cost matrix. Using (10) we obtain

$$P_1 - P_2 = \int_0^\infty e^{A_2^T t} (K_1 - K_2)^T R (K_1 - K_2) e^{A_2 t} dt \geq 0$$

so that $P_2 \leq P_1$. In addition, By using (10),

$$P_2 - P^* = \int_0^\infty e^{A_2^T t} (K_2 - K^*)^T R (K_2 - K^*) e^{A_2 t} dt \geq 0.$$

Hence P_2 is lower bounded has a finite norm. In other words, A_2 is a Hurwitz matrix. The result can be extended for $k = 3, 4, \dots$

2. Obviously, $P^* \leq P_2 \leq P_1$ holds when $k=1$. The result can be extended for $k = 2, 3, \dots$

3. $\lim_{k \rightarrow \infty} P_k = P_\infty$ holds, presented by the theorem on the monotonic convergence of positive operators [13]. By calculating limit of (11) as $k \rightarrow \infty$ one obtains

$$A^T P_\infty + P_\infty A + Q - P_\infty B R^{-1} B^T P_\infty = 0. \quad (13)$$

Since P^* is the unique positive definite solution of (13), $P_\infty = P^*$. Hence $\lim_{k \rightarrow \infty} P_k = P^*$. Given (12), it yields $\lim_{k \rightarrow \infty} K_k = K^*$. ■

III. PROPOSED APPROACH

In order to solve (4) without using the matrix A, an approximate solution can be obtained through the policy iteration (PI) technique [10]. In that case, (3) can be rewritten as follows,

$$\begin{aligned} & x^T(t) P_k x(t) - x^T(t+T) P_k x(t+T) \\ & = \int_t^{t+T} (x^T Q_k x + u_k^T R u_k) dt, \end{aligned} \quad (14)$$

where $u_k = -K_k x$ is the state feedback control law in time domain $[t, t+T]$. From (14), P_k can be approximated without requiring A. However, persistent excitation needs to be guaranteed so that P_k is unique. To address the issue, adding probe noise to the state feedback control signal was proposed in [11]. Thus, the system model (1) is rewritten by

$$\dot{x} = A_k x + B(K_k x + u), \quad (15)$$

where $A_k = A - BK_k$, which can be used to represent (14) as follows,

$$\begin{aligned} & x^T(t+T) P_k x(t+T) - x^T(t) P_k x(t) \\ & = \int_t^{t+T} \left[x^T (A_k^T P_k + P_k A_k) x + 2(u + K_k x)^T B^T P_k x \right] dt \\ & = - \int_t^{t+T} x^T Q_k x dt + 2 \int_t^{t+T} (u + K_k x)^T R K_{k+1} x dt, \end{aligned} \quad (16)$$

where $Q_k = Q + K_k^T R K_k$. And the control signal is added by a small noise,

$$u = -K_k x + e. \quad (17)$$

Let us define $P \in R^{n \times n} \rightarrow \hat{P} \in R^{n \times (n+1)/2}$ and $x \in R^n \rightarrow \bar{x} \in R^{n \times (n+1)/2}$, where

$$\hat{P} = [p_{11}, 2p_{12}, \dots, 2p_{1n}, p_{22}, 2p_{23}, \dots, 2p_{n-1,n}, p_{nn}]^T$$

$$\bar{x} = [x_1^2, x_1x_2, \dots, x_1x_n, x_2^2, x_2x_3, \dots, x_{n-1}x_n, x_n^2]^T.$$

By using Kronecker product one has

$$x^T Q_k x = (x^T \otimes x^T) \text{vec}(Q_k),$$

$$(u + K_k x) R K_{k+1} x = [(x^T \otimes x^T)(I_n \otimes K_k^T R) + (x^T \otimes u^T)(I_n \otimes R)] \text{vec}(K_{k+1})$$

If considering (16) in some time domains $[t_0, t_1], [t_1, t_2], \dots, [t_{l-1}, t_l]$, one has l equations. We define matrices $\delta_{xx} \in R^{l \times \frac{n(n+1)}{2}}, I_{xx} \in R^{l \times n^2}, I_{xu} \in R^{l \times mn}$ as follows

$$\delta_{xx} = [\bar{x}(t_0) - \bar{x}(t_1), \bar{x}(t_1) - \bar{x}(t_2), \dots, \bar{x}(t_{l-1}) - \bar{x}(t_l)]^T$$

$$I_{xx} = \left[\int_{t_0}^{t_1} x \otimes x dt, \int_{t_1}^{t_2} x \otimes x dt, \dots, \int_{t_{l-1}}^{t_l} x \otimes x dt \right]^T$$

$$I_{xu} = \left[\int_{t_0}^{t_1} x \otimes u dt, \int_{t_1}^{t_2} x \otimes u dt, \dots, \int_{t_{l-1}}^{t_l} x \otimes u dt \right]^T.$$

l equations can be presented in a matrix format.

$$\Theta_k \begin{bmatrix} \hat{P}_k \\ \text{vec}(K_{k+1}) \end{bmatrix} = \Xi_k, \quad (18)$$

where $\Theta_k \in R^{l \times n(n+1) + mn}$ and $\Xi_k \in R^l$ are specified by

$$\Theta_k = [\delta_{xx}, 2I_{xx}(I_n \otimes K_k^T R) + 2I_{xu}(I_n \otimes R)]$$

$$\Xi_k = I_{xx} \text{vec}(Q_k)$$

When Θ_k is a full rank column matrix, equation (18) can be solved by using the least squares method

$$\begin{bmatrix} \hat{P}_k \\ \text{vec}(K_{k+1}) \end{bmatrix} = (\Theta_k^T \Theta_k)^{-1} \Theta_k^T \Xi_k \quad (19)$$

It is noted that the probe noise signal in this work is selected as sum of sine signals,

$$e = 0.1 \sum \sin(\omega_i t),$$

where ω_i is frequency randomly selected in $[-500, 500]$ Hz. Convergence of the proposed algorithm is provided in the following theorem, which is derived from Kleinman's theorem 1 [14].

Theorem 2. Given the initial stable controller $K_0 \in R^{m \times n}$ and Θ_k is full rank, the sequence matrices $\{P_i\}_{i=0}^\infty$ and $\{K_i\}_{i=0}^\infty$ obtained from the equation (19) converge to the optimal values P^* and K^* .

Proof: Given the state feedback controller K_k , if $P_k^T = P_k$ is the solution of (11) and K_{k+1} is identified by (12), then from (16) P_k and K_{k+1} satisfy (19). On the other hand, if Θ_k is full rank, the solution of (19) is unique. Hence, (19) is equivalent to (11) and (12). In other words, the optimal algorithm based on the adaptive dynamic programming is equivalent to the iterative method in the **theorem 1**. ■

TABLE I: PARAMETERS OF TWO-WHEEL ROBOTS.

Symbol	Definition
x	Position of two-wheel self-balancing vehicle
θ	Inclined angle of vehicle body
ψ	Directional angle of the vehicle
d	Distance between the center of vehicle body to the axle connecting the two wheels of the car
l	Radius of the vehicle's wheel
r	Mass of the vehicle body
m_B	Mass of left wheel (right wheel)
m_W	Mass of the vehicle body
J	Moment of inertia of vehicle wheel corresponding to the axle of wheel
K	Moment of inertia of vehicle wheel corresponding to vertical axle
K_m	Torque
i_L, i_R	Current flows through the motor of the left wheel (right wheel)
T_L, T_R	Torque of motor of left wheel (right wheel)
γ_L, γ_R	Rotational angle of left wheel (right wheel)
c_α	Coefficient of friction on wheel's axle
I_1, I_2, I_3	Moment of inertia of vehicle body corresponding to the reference system B
(L, R, B)	Left wheel, Right wheel, Vehicle Body
$\hat{w}^L, \hat{w}^R, \hat{w}^B$	Angular velocity of (L, R, B)
$\hat{v}^L, \hat{v}^R, \hat{v}^B$	Velocity of center (L, R, B)
$\hat{v}^C, \hat{w}^C$	Translational velocity and angular velocity in reference system C

Fig. 1: Model of a two-wheel robot.

IV. EVALUATION ON TWO-WHEEL ROBOTS

A. Two-Wheel Robot Model

The model of a two-wheel robot is illustrated in Fig. 2, where its parameters are explained in Table 1. It is noted that the coefficient transfer matrices between some reference

systems are given by

$$\begin{aligned} {}^N R_C(\psi) &= \begin{bmatrix} \cos \psi & -\sin \psi & 0 \\ \sin \psi & \cos \psi & 0 \\ 0 & 0 & 1 \end{bmatrix}, \\ {}^C R_B(\theta) &= \begin{bmatrix} \cos \theta & 0 & \sin \theta \\ 0 & 1 & 0 \\ -\sin \theta & 0 & \cos \theta \end{bmatrix} \end{aligned} \quad (20)$$

The coordinates of its body, left and right wheels in the reference system N are identified as follows,

$$\begin{aligned} x_B &= x_C + l \sin \theta \cos \psi, y_B = y_C + l \sin \theta \sin \psi, \\ z_B &= l \cos \theta, x_L = x_C - \frac{l}{2} \sin \psi, y_L = y_C + \frac{d}{2} \cos \psi, \\ x_R &= x_C + \frac{d}{2} \sin \psi, y_R = y_C + \frac{d}{2} \cos \psi \end{aligned} \quad (21)$$

The nonholonomic equations in the equivalent system can be presented by

$$\begin{cases} \dot{x}_L \cos \psi + \dot{y}_L \sin \psi = r \dot{\gamma}_L \\ \dot{x}_R \cos \psi + \dot{y}_R \sin \psi = r \dot{\gamma}_R \\ (\dot{x}_L + \dot{x}_R) \sin \psi = (\dot{y}_L + \dot{y}_R) \cos \psi \end{cases}, \quad (22)$$

which is equivalent to

$$\begin{cases} \dot{x}_C \cos \psi + \dot{y}_C \sin \psi - \frac{d}{2} \dot{\psi} - r \dot{\gamma}_L = 0 \\ \dot{x}_C \cos \psi + \dot{y}_C \sin \psi + \frac{d}{2} \dot{\psi} - r \dot{\gamma}_R = 0 \\ \dot{x}_C \sin \psi - \dot{y}_C \cos \psi = 0 \end{cases} \quad (23)$$

Hence, the translational velocity of the robot is

$$\dot{x} = \dot{x}_C \cos \psi + \dot{y}_C \sin \psi \quad (24)$$

$$\dot{\gamma}_L = \frac{\dot{x} - d/2 \dot{\psi}}{r}, \dot{\gamma}_R = \frac{\dot{x} + d/2 \dot{\psi}}{r}. \quad (25)$$

Taking derivatives of (23) leads to the translational velocities of the left and right wheels and the vehicle body as follows,

$$\begin{aligned} \hat{v}^L &= (\dot{x}_C - \frac{d}{2} \dot{\psi} \cos \psi) \hat{n}_1 + (\dot{y}_C - \frac{d}{2} \dot{\psi} \sin \psi) \hat{n}_2 \\ \hat{v}^R &= (\dot{x}_C + \frac{d}{2} \dot{\psi} \cos \psi) \hat{n}_1 + (\dot{y}_C + \frac{d}{2} \dot{\psi} \sin \psi) \hat{n}_2 \\ \hat{v}^B &= (\dot{x}_C + l \dot{\theta} \cos \theta \cos \psi - l \dot{\psi} \sin \theta \sin \psi) \hat{n}_1 \\ &\quad + (\dot{y}_C + l \dot{\theta} \cos \theta \sin \psi + l \dot{\psi} \sin \theta \cos \psi) \hat{n}_2 - (l \dot{\theta} \sin \theta) \hat{n}_3. \end{aligned} \quad (26)$$

From Fig. 2, angular velocities of the left and right wheels and the vehicle body can be computed by

$$\begin{aligned} \hat{w}^L &= \hat{w}^C + \dot{\gamma}_L \hat{c}_2 = \dot{\psi} \hat{c}_3 + \frac{1}{r} (\dot{x} - \frac{d}{2} \dot{\psi}) \hat{c}_2, \\ \hat{w}^R &= \hat{w}^C + \dot{\gamma}_R \hat{c}_2 = \dot{\psi} \hat{c}_3 + \frac{1}{r} (\dot{x} + \frac{d}{2} \dot{\psi}) \hat{c}_2, \\ \hat{w}^B &= \hat{w}^C + \dot{\theta} \hat{b}_2 = (-\dot{\psi} \sin \theta) \hat{b}_1 + \dot{\theta} \hat{b}_2 + (\dot{\psi} \cos \theta) \hat{b}_3. \end{aligned} \quad (27)$$

On the other hand, the translational velocities of the left and right wheels and the vehicle body can be computed by the other methods,

$$\begin{aligned} \hat{v}^L &= \dot{x} \hat{c}_1 + \hat{w}^C \frac{d}{2} \hat{c}_2 = (\dot{x} - \frac{d}{2} \dot{\psi}) \hat{c}_1, \\ \hat{v}^R &= \dot{x} \hat{c}_1 + \hat{w}^C \frac{d}{2} \hat{c}_2 = (\dot{x} + \frac{d}{2} \dot{\psi}) \hat{c}_1, \\ \hat{v}^B &= \dot{x} \hat{c}_1 + \hat{w}^B l \hat{b}_3 = \dot{x} \hat{c}_1 + (l \dot{\theta}) \hat{b}_1 + (l \dot{\psi} \sin \theta) \hat{b}_2, \end{aligned} \quad (28)$$

which leads to the translational kinetic energy of the robot

$$T_{trans} = \frac{1}{2} m_W (\hat{v}^L \hat{v}^L) + \frac{1}{2} m_W (\hat{v}^R \hat{v}^R) + \frac{1}{2} m_B (\hat{v}^B \hat{v}^B). \quad (29)$$

Moreover, the rotational kinetic energy is calculated by

$$T_{rot} = \frac{1}{2} (\hat{w}^L)^T \hat{I}_L \hat{w}^L + \frac{1}{2} (\hat{w}^R)^T \hat{I}_R \hat{w}^R + \frac{1}{2} (\hat{w}^B)^T \hat{I}_B \hat{w}^B. \quad (30)$$

Replacing (25) to (28) yields

$$T_{rot} = \frac{1}{2} ((I_1 - I_3) \sin^2 \theta + I_3 + 2K) \dot{\psi}^2 + \frac{1}{2} I_2 \dot{\theta}^2 + \frac{1}{2} J (\dot{\gamma}_L^2 + \dot{\gamma}_R^2) \quad (31)$$

It is noticed that the potential energy of the robot is given by

$$V = m_B g l \cos \theta. \quad (32)$$

Thus, the Lagrange function is formulated as follows,

$$L = T - V = T_{trans} + T_{rot} - V \quad (33)$$

If we define $q_1 = x_C, q_2 = y_C, q_3 = \theta, q_4 = \psi, q_5 = \gamma_L, q_6 = \gamma_R$, the kinetic function Lagrange II for the two-wheel robots can be given by

$$\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{q}_i} \right) - \frac{\partial L}{\partial q_i} = Q_i + \sum_j \lambda_j a_{ij} (i = 1, 2, \dots, 6, j = 1, 2, 3), \quad (34)$$

where

$$\begin{aligned} a_{1i} &= \frac{\partial}{\partial \dot{q}_i} \left(\dot{x}_C \cos \psi + \dot{y}_C \sin \psi - \frac{d}{2} \dot{\psi} - r \dot{\gamma}_L \right) \\ a_{2i} &= \frac{\partial}{\partial \dot{q}_i} \left(\dot{x}_C \cos \psi + \dot{y}_C \sin \psi + \frac{d}{2} \dot{\psi} - r \dot{\gamma}_R \right) \\ a_{3i} &= \frac{\partial}{\partial \dot{q}_i} (\dot{x}_C \sin \psi - \dot{y}_C \cos \psi) \end{aligned} \quad (35)$$

Here are steps to solve (34).

Step 1: x_C

$$\begin{aligned} m_B \left(\ddot{x}_C + l \ddot{\theta} \cos \theta \cos \psi - l \ddot{\psi} \sin \theta \sin \psi \right. \\ \left. - 2l \dot{\theta} \dot{\psi} \cos \theta \sin \psi - l (\dot{\theta}^2 + \dot{\psi}^2) \sin \theta \cos \psi \right) \\ + 2m_w \ddot{x}_C = \lambda_1 \cos \psi + \lambda_2 \cos \psi + 2\lambda_3 \sin \psi. \end{aligned}$$

Step 2: y_C

$$\begin{aligned} m_B \left(\ddot{y}_C + l \ddot{\theta} \cos \theta \sin \psi + l \ddot{\psi} \sin \theta \cos \psi \right. \\ \left. + 2l \dot{\theta} \dot{\psi} \cos \theta \cos \psi - l (\dot{\theta}^2 + \dot{\psi}^2) \sin \theta \sin \psi \right) \\ + 2m_w \ddot{y}_C = \lambda_1 \sin \psi + \lambda_2 \sin \psi - 2\lambda_3 \cos \psi. \end{aligned}$$

Step 3: θ

$$\begin{aligned} m_B (x_C \cos \psi + y_C \sin \psi) l \cos \theta + (I_2 + m_B l^2) \ddot{\theta} \\ - (I_1 - I_3 + m_B l^2) \dot{\psi}^2 \sin \theta \cos \theta - m_B g l \sin \theta \\ = - (T_L + T_R) + c_\alpha (\dot{\gamma}_L - \dot{\theta}) + c_\alpha (\dot{\gamma}_R - \dot{\theta}). \end{aligned}$$

Step 4: ψ

$$\begin{aligned} \{ I_3 + m_W d^2 / 2 + 2K + (I_1 - I_3) \sin^2 \theta \} \ddot{\psi} \\ + m_B \{ -\ddot{x}_C \sin \psi + \ddot{y}_C \cos \psi - (\dot{x}_C \cos \psi + \dot{y}_C \sin \psi) \dot{\psi} \} \\ l \sin \theta + m_B (\dot{x}_C \cos \psi + \dot{y}_C \sin \psi) \dot{\psi} l \sin \theta \\ + 2(I_1 - I_3) \dot{\theta} \dot{\psi} \sin \theta \cos \theta \\ = - \frac{d\lambda_1}{2} + \frac{d\lambda_2}{2}. \end{aligned}$$

Step 5: γ_L

$$J\ddot{\gamma}_L = -r\lambda_1 + T_L - c_\alpha(\dot{\gamma}_L - \dot{\theta}).$$

Step 6: γ_R

$$J\ddot{\gamma}_R = -r\lambda_1 + T_R - c_\alpha(\dot{\gamma}_R - \dot{\theta}). \quad (36)$$

Then we have

$$T_L = K_m i_L, T_R = K_m i_R. \quad (37)$$

From (22), (23), (34) and (35), one has

$$\left\{ m_B + 2m_W + \frac{2J}{r^2} \right\} \ddot{x} - m_B l (\dot{\theta}^2 + \dot{\psi}^2) \sin \theta + (m_B l \cos \theta) \ddot{\theta} + \frac{2}{r} c_\alpha \left(\frac{\dot{x}}{r} - \dot{\theta} \right) = \frac{K_m(i_L + i_R)}{r}, \quad (38)$$

$$(I_2 + m_B l^2) \ddot{\theta} + (I_3 - I_1 - m_B l^2) \dot{\psi}^2 \sin \theta \cos \theta + (m_B l \cos \theta) \ddot{x} - m_B l \sin \theta - 2c_\alpha \left(\frac{\dot{x}}{r} - \dot{\theta} \right) = -K_m(i_L + i_R), \quad (39)$$

$$\left\{ I_3 + 2K + (m_W + \frac{J}{r^2}) \frac{d^2}{2} - (I_3 - I_1 - m_B l^2) \dot{\psi}^2 \sin^2 \theta \right\} \ddot{\psi} + \left\{ m_B l \dot{x} - 2(I_3 - I_1 - m_B l^2) \dot{\theta} \cos \theta \right\} \dot{\psi} \sin \theta + c_\alpha \dot{\psi} \frac{d^2}{2r^2} = (i_R - i_L) K_m \frac{d}{2r}. \quad (40)$$

Now, we can obtain the kinetic equation for two-wheel robots as follows,

$$\dot{x} = f(x) + g(x)u = F(x, u), \quad (41)$$

where

$$\begin{aligned} \dot{x}_1 &= x_4, \dot{x}_2 = x_5, \dot{x}_3 = x_6, \dot{x}_4 = F_4(x, u) = \frac{\Omega_1 + \Omega_2}{\Omega} \\ \dot{x}_5 &= F_5(x, u) = \frac{\Omega_3 + \Omega_4}{\Omega}, \dot{x}_6 = F_6(x, u) = \frac{\Omega_5}{\Omega_6} \\ \Omega_1 &= r^2(m_B l^2 + I_2) \left\{ \begin{aligned} &K_m \frac{u_1 + u_2}{r} + \frac{2c_\alpha}{r} \left(x_5 - \frac{x_4}{r} \right) \\ &+ m_B l \sin(x_2) (x_5^2 + x_6^2) \end{aligned} \right\}, \\ \Omega_2 &= m_B l r^2 \cos(x_2) \left\{ \begin{aligned} &-\cos(x_2) \sin(x_2) \times \\ &(m_B l^2 + I_1 - I_3) x_6^2 \\ &+ K_m(u_1 + u_2) \\ &+ 2c_\alpha \left(x_5 - \frac{x_4}{r} \right) \\ &- m_B g l \sin(x_2) \end{aligned} \right\}, \\ \Omega_3 &= [2J + (m_B + 2m_W)r^2] \left\{ \begin{aligned} &\cos(x_2) \sin(x_2) \times \\ &(m_B l^2 + I_1 - I_3) x_6^2 \\ &- K_m(u_1 + u_2) \\ &- 2c_\alpha \left(x_5 - \frac{x_4}{r} \right) \\ &+ m_B g l \sin(x_2) \end{aligned} \right\}, \\ \Omega_4 &= -m_B l r^2 \cos(x_2) \left\{ \begin{aligned} &K_m \frac{u_1 + u_2}{r} \\ &+ \frac{2c_\alpha}{r} \left(x_5 - \frac{x_4}{r} \right) \\ &+ m_B g l \sin(x_2) (x_5^2 + x_6^2) \end{aligned} \right\}, \\ \Omega_5 &= 2r^2 \left\{ \begin{aligned} &-K_m d \frac{u_1 + u_2}{2r} - x_6 \sin(x_2) \times \\ &[m_B l x_4 + 2x_5 \cos(x_2)(m_B l^2 + I_1 - I_3)] \\ &- \frac{c_\alpha d^2 x_6}{2r^2} \end{aligned} \right\} \end{aligned}$$

$$\Omega_6 = (2I_3 + 4K + m_W d^2) r^2 + (2(m_B l^2 + I_1 - I_3) \sin^2(x_2)) r^2 + J d^2,$$

$$\Omega = (m_B l r)^2 [1 - \cos^2(x_2)] + 2I_2 J + 2J m_B l^2 + (I_2 m_B + 2I_2 m_W + 2m_B m_W l^2) r^2.$$

B. Linearization of Two-Wheel Robots

Assume that x_e is a balanced point of a two-wheel robot. That is, $x_e = [* \ 0 \ * \ 0 \ 0 \ 0]^T$ is the solution of $F(x_e, 0) = 0$, where $*$ is a random value of x_1 and x_3 . If $* = 0$, $x_e = 0$. Hence, linearizing the two-wheel robot model around x_e is given by

$$\dot{x} = Ax + Bu,$$

where

$$A = \frac{\partial F}{\partial x} \Big|_{x=x_e} = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & a_{42} & 0 & a_{44} & a_{45} & 0 \\ 0 & a_{52} & 0 & a_{54} & a_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & a_{66} \end{bmatrix}$$

$$B = \frac{\partial F}{\partial u} \Big|_{x=x_e} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ b_{41} & b_{42} \\ b_{51} & b_{52} \\ b_{61} & b_{62} \end{bmatrix}$$

$$\begin{aligned} a_{42} &= \frac{-(m_B l r)^2 g}{\Delta}, a_{44} = \frac{-2c_\alpha(m_B l^2 + I_2 + m_B l r)}{\Delta}, \\ a_{45} &= \frac{2c_\alpha r(m_B l^2 + I_2 + m_B l r)}{\Delta}, \\ a_{52} &= \frac{m_B g l (2J + m_B r^2 + 2m_W r^2)}{\Delta}, \\ a_{54} &= \frac{2c_\alpha m_B l + 2c_\alpha (2J + m_B r^2 + 2m_W r^2)/r}{\Delta}, \\ a_{55} &= \frac{-2c_\alpha m_B l r - 2c_\alpha (2J + m_B r^2 + 2m_W r^2)}{\Delta}, \\ a_{66} &= \frac{-c_\alpha d^2}{J d^2 + (2I_3 + 4K + 2d^2) r^2}, \\ b_{41} &= b_{42} = \frac{K_m r(m_B l^2 + I_2 + m_B l r)}{\Delta}, \\ b_{51} &= b_{52} = \frac{-K_m(2J + m_B r^2 + 2m_W r^2 + m_B l r)}{\Delta}, \\ b_{61} &= -b_{62} = \frac{-K_m d r}{J d^2 + (2I_3 + 4K + m_W d^2) r^2}, \\ \Delta &= 2I_2 J + 2J m_B l^2 + I_2 m_B r^2 + 2I_2 m r^2 + 2M l^2 m r^2. \end{aligned}$$

C. Simulation Results

To demonstrate effectiveness of the proposed approach, we implemented it in controlling a two-wheel robot in a synthetic environment, where the robot parameters were set as follows. $m_B = 0.5(kg)$, $m_W = 0.04(kg.m^2)$, $l = 0.08(m)$, $d = 0.16(m)$, $r = 0.033(m)$, $g = 9.81(m/s^2)$, $c_\alpha =$

Fig. 2: Two-wheel robot simulation results obtained by the proposed algorithm.

$5.10 - 4(Ns/m)$ and $K_m 0.412(Nm/A)$. The cost function of the optimal control problem is defined by

$$J(x, u) = \int_0^\infty (x^T Q x + u^T R u) dt,$$

where $Q = \text{diag}(1, 0.5, 2, 0.05, 0.05, 1)$ and $T = 0.01s$. The algorithm is executed every 2 s. The control objective is minimize both the inclined and directional angles while the robot is tracking the reference. For instance, in the x direction, in the first 10 s, the robot was expected to track at 0 m from its initial position of 0.5 m. And in the next 10 s, the robot was required to stay at 1 m. To this end, we initially set the state feedback controller as follows,

$$K_0 = \begin{bmatrix} -0.2 & -0.8 & -0.3 & -0.2 & -0.1 & -0.2 \\ -0.2 & -0.8 & 0.3 & -0.2 & -0.1 & 0.2 \end{bmatrix}.$$

And the controller was adaptively updated by the proposed algorithm after every iteration. For instance, at 3 running iterations, the controller was

$$K_3 = \begin{bmatrix} -0.189 & -0.617 & -0.489 & -0.178 & -0.078 & -0.231 \\ -0.282 & -0.666 & 0.693 & -0.226 & -0.083 & 0.233 \end{bmatrix}.$$

The simulation results of the two-wheel robot during first 20 s are demonstrated in Fig. 3, which promise practicality of the proposed approach.

V. CONCLUSIONS

An efficient method using reinforcement learning based adaptive dynamic programming to control a two-wheel robot has been presented in the paper. The proposed algorithm guarantees finding the optimal state controller where its convergence has theoretically been proved. The optimal control scheme was successfully implemented in a synthetic robot, where the obtained results promise its practicality.

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