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Application of cylindrical IE-GSTC to physical metasurfaces

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Abstract—This work validates cylindrical IE-GSTC by applying it to physical metasurfaces, i.e. metasurfaces defined by material properties and dimensions rather than by susceptibility tensor components. Previously reported IE-GSTC which was formulated for zero thickness GSTC discontinuity is extended to handle finite thickness of physical metasurfaces. A simple analytical approach is used to extract the bianisotropic susceptibility tensor of concentric, multilayered, magneto-dielectric shell. Plane wave scattering by a physical metasurface constructed of four segments of multilayered, magneto-dielectric metasurface scatterers is used as an example problem to validate cylindrical IE-GSTC. A second example considers an opening on the cylindrical metasurface, confirming IE-GSTC can handle metasurfaces with openings. Good agreement is obtained between IE-GSTC results and full wave simulation results for both cases.

Index Terms—GSTC, Integral equation, MoM, Cylindrical, Metasurface, Boundary condition, Susceptibility, Electromagnetic discontinuity, COMSOL

I. INTRODUCTION

METASURFACES are deeply subwavelength surfaces which can manipulate electromagnetic waves in a desired manner [1]. Essentially, these are field transformers which are constructed by arrangement of subwavelength scatterers in a host medium. Metasurfaces have practical advantages over bulk metamaterials including easier fabrication, lower loss and less weight. Even though they have similarities with frequency selective surfaces, the design possibilities offered by metasurfaces are much broader. A review of metasurface applications is in [2]. Metasurfaces achieve its functionality by creating a spatial electromagnetic discontinuity. Mathematically, the discontinuity can be expressed by Generalized Sheet Transition Conditions (GSTC) which relates the electric and magnetic field discontinuities to the electric and magnetic surface polarization densities [3], [4]. It should be noted that even though physical metasurfaces have a finite subwavelength thickness, GSTCs which assume zero metasurface thickness are a very good approximation to such physical metasurfaces, i.e. scattered field due to the physical metasurface approximates the field scattered by GSTC interface conditions.

The vast majority of metasurfaces reported to date are planar. The modeling of planar GSTCs was previously reported using the Finite Difference Frequency Domain (FDFD) method [5] and Finite Element Method (FEM) [6]. Other

canonical shapes such as cylindrical metasurfaces [7]–[9] and spherical metasurfaces [10], [11] are now being studied. Integral Equation - Method of Moment (IE-MoM) was applied to circular cylindrical metasurfaces in [7]. This was extended to cylindrical metasurfaces of arbitrary cross-section in [8]. In [12], inverse source technique was used to synthesize surface susceptibility tensor components of metasurfaces of arbitrary shape. The work addressed metasurfaces capable of generating radiation patterns with specified half-power beamwidth, nulls, polarization etc. Even though mathematically rigorous, none of these works [7]–[10], [12], [13] have used physical metasurfaces. In this work, validity of the cylindrical GSTC [7] is confirmed by comparing the scattering due to a cylindrical, physical metasurface to scattering by a corresponding GSTC contour. Scattering by cylindrical GSTC contour was obtained by a slightly modified version of IE-GSTC formulation in [7]. The modification incorporates the finite thickness of physical metasurfaces.

The organization of the paper is as follows. Section II extends the IE-MoM in [7] to handle non-zero thickness of a physical metasurface. The contents in this section closely follows [7]. Only the significant differences are explained. This is followed by susceptibility tensor extraction of cylindrical, concentric, magneto-dielectric shell using an analytical approach in section III. Section IV provides two examples. The metasurfaces in the two examples are constructed using metasurface particles which are part of the shell structure in section III. The second example has an opening which can be taken care of by IE-GSTC formulation. IE-GSTC along with susceptibility tensors extracted using the method in section III are used to simulate the scattering problem, the results of which is compared with full wave simulation of the physical metasurface.

II. IE-MoM FOR FINITE THICKNESS CYLINDRICAL METASURFACE

The IE-MoM described in [7] was for a cylindrical GSTC contour of zero thickness whose surface polarization densities are related to electric and magnetic fields on either side of the metasurface by the bianisotropic susceptibility tensor model. That formulation need to be slightly extended to include the non-zero thickness of physical metasurfaces. Instead of using a single contour for calculation of MoM coefficients and integration of surface fields, two contours one external and other internal to the metasurface are used. This is shown in Fig. 1. Through out this paper, there is no variation of material properties or excitation along the z axis. The inner contour

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is Γ_2 and the outer contour is Γ_1 , separated by metasurface thickness δ . In this section, the extension of the formulation

Fig. 1. Finite thickness cylindrical metasurface

in [7] is outlined. A clear understanding of [7] is required to understand this section. The variables and notations follows [7]. The 2D IEs for TM polarization given in equation (6) of [7] are modified to

$$E_{z1}^{\text{inc}}(\bar{\rho}) + \oint_{\Gamma_1} \left[E_{z1}(\bar{\rho}_1') \frac{\partial G_0(\bar{\rho}; \bar{\rho}_1')}{\partial n'} - j\omega\mu_0 H_{\phi 1}(\bar{\rho}_1') G_0(\bar{\rho}; \bar{\rho}_1') \right] d\Gamma_1' =$$

$$= \begin{cases} E_{z1}(\bar{\rho}) & ; \bar{\rho} \in \Omega_1 \\ 0.5E_{z1}(\bar{\rho}) & ; \bar{\rho} \in \Gamma_1 \\ 0 & ; \bar{\rho} \in \Omega_2 \end{cases} \quad (1a)$$

$$E_{z2}^{\text{inc}}(\bar{\rho}) - \oint_{\Gamma_2} \left[E_{z2}(\bar{\rho}_2') \frac{\partial G_0(\bar{\rho}; \bar{\rho}_2')}{\partial n'} - j\omega\mu_0 H_{\phi 2}(\bar{\rho}_2') G_0(\bar{\rho}; \bar{\rho}_2') \right] d\Gamma_2' =$$

$$= \begin{cases} 0 & ; \bar{\rho} \in \Omega_1 \\ 0.5E_{z2}(\bar{\rho}) & ; \bar{\rho} \in \Gamma_2 \\ E_{z2}(\bar{\rho}) & ; \bar{\rho} \in \Omega_2 \end{cases} \quad (1b)$$

where $\bar{\rho}_1'$ and $\bar{\rho}_2'$ denote contour integration position variables along Γ_1 and Γ_2 respectively. The subscripts 1, 2 on the field variables denote the fields on domain Ω_1, Ω_2 respectively. Such a distinction was not made in [7]. $G_0(\rho; \rho_1') = -0.25jH_0^2(k_0|\bar{\rho} - \bar{\rho}_1'|)$ is the 2D Green's function. Derivative with respect to n denotes directional derivative along the radial direction (pointing outwards). The 2D IEs for TE polarization given in equation (7) of [7] are modified to

$$H_{z1}^{\text{inc}}(\bar{\rho}) + \oint_{\Gamma_1} \left[H_{z1}(\bar{\rho}_1') \frac{\partial G_0(\bar{\rho}; \bar{\rho}_1')}{\partial n'} + j\omega\epsilon_0 E_{\phi 1}(\bar{\rho}_1') G_0(\bar{\rho}; \bar{\rho}_1') \right] d\Gamma_1' =$$

$$= \begin{cases} H_{z1}(\bar{\rho}) & ; \bar{\rho} \in \Omega_1 \\ 0.5H_{z1}(\bar{\rho}) & ; \bar{\rho} \in \Gamma_1 \\ 0 & ; \bar{\rho} \in \Omega_2 \end{cases} \quad (2a)$$

$$H_{z2}^{\text{inc}}(\bar{\rho}) - \oint_{\Gamma_2} \left[H_{z2}(\bar{\rho}_2') \frac{\partial G_0(\bar{\rho}; \bar{\rho}_2')}{\partial n'} + j\omega\epsilon_0 E_{\phi 2}(\bar{\rho}_2') G_0(\bar{\rho}; \bar{\rho}_2') \right] d\Gamma_2' =$$

$$= \begin{cases} 0 & ; \bar{\rho} \in \Omega_1 \\ 0.5H_{z2}(\bar{\rho}) & ; \bar{\rho} \in \Gamma_2 \\ H_{z2}(\bar{\rho}) & ; \bar{\rho} \in \Omega_2 \end{cases} \quad (2b)$$

Following the same procedure as in [7], four integral equations are obtained using (1), (2) and circular metasurface synthesis equations given in equation (4) of [7]. These four equations are used to solve the four tangential fields $E_{z1}(\bar{\rho}_1')$, $H_{\phi 1}(\bar{\rho}_1')$, $E_{\phi 1}(\bar{\rho}_1')$ and $H_{z1}(\bar{\rho}_1')$ on the exterior contour Γ_1 . The equations are given as

$$\frac{E_{z1}(\bar{\rho}_1)}{2} - \oint_{\Gamma_1} \left[E_{z1}(\bar{\rho}_1') \frac{\partial G_0(\bar{\rho}_1; \bar{\rho}_1')}{\partial n'} - j\omega\mu_0 \right. \quad (3a)$$

$$\left. H_{\phi 1}(\bar{\rho}_1') G_0(\bar{\rho}_1; \bar{\rho}_1') \right] d\Gamma_1' = E_{z1}^{\text{inc}}(\bar{\rho}_1) ; \bar{\rho}_1 \in \Gamma_1$$

$$\oint_{\Gamma_2} \left[A_{11}(\bar{\rho}_c') E_{z1}(\bar{\rho}_1') + A_{12}(\bar{\rho}_c') H_{\phi 1}(\bar{\rho}_1') + A_{13}(\bar{\rho}_c') E_{\phi 1}(\bar{\rho}_1') \right. \\ \left. + A_{14}(\bar{\rho}_c') H_{z1}(\bar{\rho}_1') \right] \frac{\partial G_0(\bar{\rho}_2; \bar{\rho}_2')}{\partial n'} - j\omega\mu_0 \left[A_{21}(\bar{\rho}_c') E_{z1}(\bar{\rho}_1') + \right. \\ \left. A_{22}(\bar{\rho}_c') H_{\phi 1}(\bar{\rho}_1') + A_{23}(\bar{\rho}_c') E_{\phi 1}(\bar{\rho}_1') + A_{24}(\bar{\rho}_c') H_{z1}(\bar{\rho}_1') \right] \\ G_0(\bar{\rho}_2; \bar{\rho}_2') d\Gamma_2' + 0.5 \left[A_{11}(\bar{\rho}_c) E_{z1}(\bar{\rho}_1) + A_{12}(\bar{\rho}_c) H_{\phi 1}(\bar{\rho}_1) + \right. \\ \left. A_{13}(\bar{\rho}_c) E_{\phi 1}(\bar{\rho}_1) + A_{14}(\bar{\rho}_c) H_{z1}(\bar{\rho}_1) \right] = E_{z2}^{\text{inc}}(\bar{\rho}_2) ; \bar{\rho}_2 \in \Gamma_2 \quad (3b)$$

$$\frac{H_{z1}(\bar{\rho}_1)}{2} - \oint_{\Gamma_1} \left[H_{z1}(\bar{\rho}_1') \frac{\partial G_0(\bar{\rho}_1; \bar{\rho}_1')}{\partial n'} + j\omega\epsilon_0 \right. \quad (3c)$$

$$\left. E_{\phi 1}(\bar{\rho}_1') G_0(\bar{\rho}_1; \bar{\rho}_1') \right] d\Gamma_1' = H_{z1}^{\text{inc}}(\bar{\rho}_1) ; \bar{\rho}_1 \in \Gamma_1$$

$$\oint_{\Gamma_2} \left[A_{41}(\bar{\rho}_c') E_{z1}(\bar{\rho}_1') + A_{42}(\bar{\rho}_c') H_{\phi 1}(\bar{\rho}_1') + A_{43}(\bar{\rho}_c') E_{\phi 1}(\bar{\rho}_1') \right. \\ \left. + A_{44}(\bar{\rho}_c') H_{z1}(\bar{\rho}_1') \right] \frac{\partial G_0(\bar{\rho}_2; \bar{\rho}_2')}{\partial n'} + j\omega\epsilon_0 \left[A_{31}(\bar{\rho}_c') E_{z1}(\bar{\rho}_1') + \right. \\ \left. A_{32}(\bar{\rho}_c') H_{\phi 1}(\bar{\rho}_1') + A_{33}(\bar{\rho}_c') E_{\phi 1}(\bar{\rho}_1') + A_{34}(\bar{\rho}_c') H_{z1}(\bar{\rho}_1') \right] \\ G_0(\bar{\rho}_2; \bar{\rho}_2') d\Gamma_2' + 0.5 \left[A_{41}(\bar{\rho}_c) E_{z1}(\bar{\rho}_1) + A_{42}(\bar{\rho}_c) H_{\phi 1}(\bar{\rho}_1) + \right. \\ \left. A_{43}(\bar{\rho}_c) E_{\phi 1}(\bar{\rho}_1) + A_{44}(\bar{\rho}_c) H_{z1}(\bar{\rho}_1) \right] = H_{z2}^{\text{inc}}(\bar{\rho}_2) ; \bar{\rho}_2 \in \Gamma_2 \quad (3d)$$

where the $A_{ij}(\bar{\rho}_c')$ are elements of 4×4 matrix $\mathbf{A}(\bar{\rho}_c')$. This matrix which is a function of bianisotropic susceptibility tensor components (which vary azimuthally) relates the tangential electric and magnetic field components on the inner contour and the outer contour. $\bar{\rho}_c'$ denotes the center of the metasurface, i.e. $\bar{\rho}_c' = \bar{\rho}_1' - 0.5\delta\hat{\rho}$. Since the material properties of the metasurface varies azimuthally, the bianisotropic susceptibility tensor components and matrix $\mathbf{A}(\bar{\rho}_c')$ also varies with position. To be precise the matrix $\mathbf{A}(\bar{\rho}_c')$ is defined as

$$\mathbf{F}_2(\bar{\rho}_2') = \mathbf{A}(\bar{\rho}_c') \mathbf{F}_1(\bar{\rho}_1') \quad (4a)$$

$$\mathbf{A}(\bar{\rho}_c') = \mathbf{A}_2^{-1}(\bar{\rho}_c') \mathbf{A}_1(\bar{\rho}_c') \quad (4b)$$

where

$$\mathbf{F}_2(\bar{\rho}_2') = \begin{bmatrix} E_{z2}(\bar{\rho}_2') & H_{\phi 2}(\bar{\rho}_2') & E_{\phi 2}(\bar{\rho}_2') & H_{z2}(\bar{\rho}_2') \end{bmatrix}^T \quad (5a)$$

$$\mathbf{F}_1(\bar{\rho}_1') = \begin{bmatrix} E_{z1}(\bar{\rho}_1') & H_{\phi 1}(\bar{\rho}_1') & E_{\phi 1}(\bar{\rho}_1') & H_{z1}(\bar{\rho}_1') \end{bmatrix}^T \quad (5b)$$

The matrix $\mathbf{A}_1(\bar{\rho}'_c)$ and $\mathbf{A}_2(\bar{\rho}'_c)$ which are defined in equations (22), (23) in [7] are repeated in equations (6) and (7) for completeness.

The integration contours in (3a) - (3d) should be carefully noted. It is different from the corresponding equations in [7]. Similar to [7], the IEs (3a) - (3d) are converted to MoM system of linear equations using pulse basis functions and point matching. The MoM system of linear equations read

$$\begin{bmatrix} \mathbf{Z}_{11} & \mathbf{Z}_{12} & \mathbf{Z}_{13} & \mathbf{Z}_{14} \\ \mathbf{Z}_{21} & \mathbf{Z}_{22} & \mathbf{Z}_{23} & \mathbf{Z}_{24} \\ \mathbf{Z}_{31} & \mathbf{Z}_{32} & \mathbf{Z}_{33} & \mathbf{Z}_{34} \\ \mathbf{Z}_{41} & \mathbf{Z}_{42} & \mathbf{Z}_{43} & \mathbf{Z}_{44} \end{bmatrix} \begin{bmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \\ \mathbf{x}_3 \\ \mathbf{x}_4 \end{bmatrix} = \begin{bmatrix} \mathbf{b}_1 \\ \mathbf{b}_2 \\ \mathbf{b}_3 \\ \mathbf{b}_4 \end{bmatrix} \quad (8)$$

where the unknown vectors are

$$\mathbf{x}_1 = [E_{z1,1}, \dots, E_{z1,N}]^T \quad (9a)$$

$$\mathbf{x}_2 = [H_{\phi1,1}, \dots, H_{\phi1,N}]^T \quad (9b)$$

$$\mathbf{x}_3 = [E_{\phi1,1}, \dots, E_{\phi1,N}]^T \quad (9c)$$

$$\mathbf{x}_4 = [H_{z1,1}, \dots, H_{z1,N}]^T \quad (9d)$$

N is the total number of basis functions. The $\mathbf{Z}$ matrix coefficients and the excitation vector are as follows

$$Z_{11,mn} = p_{mn} \quad (10a)$$

$$Z_{12,mn} = r_{mn}^1 \quad (10b)$$

$$Z_{13,mn} = 0 \quad (10c)$$

$$Z_{14,mn} = 0 \quad (10d)$$

$$Z_{21,mn} = A_{11,n}q_{mn} - A_{21,n}r_{mn}^2 \quad (11a)$$

$$Z_{22,mn} = A_{12,n}q_{mn} - A_{22,n}r_{mn}^2 \quad (11b)$$

$$Z_{23,mn} = A_{13,n}q_{mn} - A_{23,n}r_{mn}^2 \quad (11c)$$

$$Z_{24,mn} = A_{14,n}q_{mn} - A_{24,n}r_{mn}^2 \quad (11d)$$

$$Z_{31,mn} = 0 \quad (12a)$$

$$Z_{32,mn} = 0 \quad (12b)$$

$$Z_{33,mn} = -s_{mn}^1 \quad (12c)$$

$$Z_{34,mn} = p_{mn} \quad (12d)$$

$$Z_{41,mn} = A_{41,n}q_{mn} + A_{31,n}s_{mn}^2 \quad (13a)$$

$$Z_{42,mn} = A_{42,n}q_{mn} + A_{32,n}s_{mn}^2 \quad (13b)$$

$$Z_{43,mn} = A_{43,n}q_{mn} + A_{33,n}s_{mn}^2 \quad (13c)$$

$$Z_{44,mn} = A_{44,n}q_{mn} + A_{34,n}s_{mn}^2 \quad (13d)$$

where $A_{ij,n}$ is the value of matrix elements A_{ij} at the n^{th} discretization segment. In fact, it is these matrix elements which connects IE-GSTC to susceptibility tensor components.

The coefficients $p_{mn}, q_{mn}, r_{mn}^1, r_{mn}^2, s_{mn}^1, s_{mn}^2$ are calculated as

$$p_{mn} = \frac{1}{2}\delta_{mn} - \int_{s_{n1}} \frac{\partial G_0(\bar{\rho}_{m1}; \bar{\rho}'_1)}{\partial n'} d\Gamma'_1 \quad (14a)$$

$$q_{mn} = \frac{1}{2}\delta_{mn} + \int_{s_{n2}} \frac{\partial G_0(\bar{\rho}_{m2}; \bar{\rho}'_2)}{\partial n'} d\Gamma'_2 \quad (14b)$$

$$r_{mn}^1 = j\omega\mu_0 \int_{s_{n1}} G_0(\bar{\rho}_{m1}; \bar{\rho}'_1) d\Gamma'_1 \quad (14c)$$

$$r_{mn}^2 = j\omega\mu_0 \int_{s_{n2}} G_0(\bar{\rho}_{m2}; \bar{\rho}'_2) d\Gamma'_2 \quad (14d)$$

$$s_{mn}^1 = j\omega\epsilon_0 \int_{s_{n1}} G_0(\bar{\rho}_{m1}; \bar{\rho}'_1) d\Gamma'_1 \quad (14e)$$

$$s_{mn}^2 = j\omega\epsilon_0 \int_{s_{n2}} G_0(\bar{\rho}_{m2}; \bar{\rho}'_1) d\Gamma'_2 \quad (14f)$$

s_{n1}, s_{n2} denote the n^{th} discretization segments on contours Γ_1, Γ_2 respectively. $\bar{\rho}_{m1}, \bar{\rho}_{m2}$ are the middle points of the m^{th} segments on contours Γ_1, Γ_2 respectively. δ_{mn} is the Kronecker delta function. The excitation vector components are given by

$$b_{1,m} = E_{z1}^{\text{inc}}(\bar{\rho}_{1,m}), \quad b_{2,m} = E_{z2}^{\text{inc}}(\bar{\rho}_{2,m}) \quad (15a)$$

$$b_{3,m} = H_{z1}^{\text{inc}}(\bar{\rho}_{1,m}), \quad b_{4,m} = H_{z2}^{\text{inc}}(\bar{\rho}_{2,m}) \quad (15b)$$

These four excitation vectors provide four possibilities: $b_{1,m}$ is for TM excitation external to the metasurface, $b_{2,m}$ is for TM excitation internal to the metasurface, $b_{3,m}$ is for TE excitation external to the metasurface and $b_{4,m}$ is for TE excitation internal to the metasurface.

III. SUSCEPTIBILITY EXTRACTION OF CYLINDRICAL, MULTILAYERED MAGNETO-DIELECTRIC SHELL

In this section, the bianisotropic susceptibility tensor components of a cylindrical, multilayered, magneto-dielectric shell is extracted. The geometry is shown in Fig. 2. Even though a three layered geometry is shown, the method can be used for any number of layers. The medium inside and outside the shell is vacuum. TM_z polarization is assumed. Following equa-

Fig. 2. Three layered cylindrical, magneto-dielectric shell

tion (4) in [7], the difference between the field components across the cylindrical metasurface $\Delta E_z, \Delta H_\phi, \Delta H_z, \Delta E_\phi$ is related to the average of fields across the metasurface

$$\mathbf{A}_1(\vec{\rho}'_c) = \begin{bmatrix} \frac{j\omega\epsilon_0\chi_{ee}^{\phi z}(\vec{\rho}'_c)}{2} & \frac{j\omega c_0^{-1}\chi_{em}^{\phi\phi}(\vec{\rho}'_c)}{2} & \frac{j\omega\epsilon_0\chi_{ee}^{\phi\phi}(\vec{\rho}'_c)}{2} & \frac{2+j\omega c_0^{-1}\chi_{em}^{\phi z}(\vec{\rho}'_c)}{2} \\ \frac{j\omega\epsilon_0\chi_{ee}^{zz}(\vec{\rho}'_c)}{2} & \frac{-2+j\omega c_0^{-1}\chi_{em}^{z\phi}(\vec{\rho}'_c)}{2} & \frac{j\omega\epsilon_0\chi_{ee}^{z\phi}(\vec{\rho}'_c)}{2} & \frac{j\omega c_0^{-1}\chi_{em}^{zz}(\vec{\rho}'_c)}{2} \\ \frac{-2+j\omega c_0^{-1}\chi_{me}^{\phi z}(\vec{\rho}'_c)}{2} & \frac{j\omega\mu_0\chi_{mm}^{\phi\phi}(\vec{\rho}'_c)}{2} & \frac{j\omega c_0^{-1}\chi_{me}^{\phi\phi}(\vec{\rho}'_c)}{2} & \frac{j\omega\mu_0\chi_{mm}^{\phi z}(\vec{\rho}'_c)}{2} \\ \frac{j\omega c_0^{-1}\chi_{me}^{zz}(\vec{\rho}'_c)}{2} & \frac{j\omega\mu_0\chi_{mm}^{z\phi}(\vec{\rho}'_c)}{2} & \frac{2+j\omega c_0^{-1}\chi_{me}^{z\phi}(\vec{\rho}'_c)}{2} & \frac{j\omega\mu_0\chi_{mm}^{zz}(\vec{\rho}'_c)}{2} \end{bmatrix} \quad (6)$$

$$\mathbf{A}_2(\vec{\rho}'_c) = \begin{bmatrix} \frac{j\omega\epsilon_0\chi_{ee}^{\phi z}(\vec{\rho}'_c)}{-2} & \frac{j\omega c_0^{-1}\chi_{em}^{\phi\phi}(\vec{\rho}'_c)}{-2} & \frac{j\omega\epsilon_0\chi_{ee}^{\phi\phi}(\vec{\rho}'_c)}{-2} & \frac{2-j\omega c_0^{-1}\chi_{em}^{\phi z}(\vec{\rho}'_c)}{2} \\ \frac{j\omega\epsilon_0\chi_{ee}^{zz}(\vec{\rho}'_c)}{-2} & \frac{-2-j\omega c_0^{-1}\chi_{em}^{z\phi}(\vec{\rho}'_c)}{2} & \frac{j\omega\epsilon_0\chi_{ee}^{z\phi}(\vec{\rho}'_c)}{-2} & \frac{j\omega c_0^{-1}\chi_{em}^{zz}(\vec{\rho}'_c)}{-2} \\ \frac{-2-j\omega c_0^{-1}\chi_{me}^{\phi z}(\vec{\rho}'_c)}{2} & \frac{j\omega\mu_0\chi_{mm}^{\phi\phi}(\vec{\rho}'_c)}{-2} & \frac{j\omega c_0^{-1}\chi_{me}^{\phi\phi}(\vec{\rho}'_c)}{-2} & \frac{j\omega\mu_0\chi_{mm}^{\phi z}(\vec{\rho}'_c)}{-2} \\ \frac{j\omega c_0^{-1}\chi_{me}^{zz}(\vec{\rho}'_c)}{-2} & \frac{j\omega\mu_0\chi_{mm}^{z\phi}(\vec{\rho}'_c)}{-2} & \frac{2-j\omega c_0^{-1}\chi_{me}^{z\phi}(\vec{\rho}'_c)}{2} & \frac{j\omega\mu_0\chi_{mm}^{zz}(\vec{\rho}'_c)}{-2} \end{bmatrix} \quad (7)$$

$E_{z,av}, H_{\phi,av}, E_{\phi,av}, H_{z,av}$, through the following metasurface synthesis expressions.

$$\begin{bmatrix} -\Delta H_z \\ \Delta H_\phi \end{bmatrix} = j\omega\epsilon_0 \begin{bmatrix} \chi_{ee}^{\phi\phi} & \chi_{ee}^{\phi z} \\ \chi_{ee}^{z\phi} & \chi_{ee}^{zz} \end{bmatrix} \begin{bmatrix} E_{\phi,av} \\ E_{z,av} \end{bmatrix} + j\omega\sqrt{\mu_0\epsilon_0} \begin{bmatrix} \chi_{em}^{\phi\phi} & \chi_{em}^{\phi z} \\ \chi_{em}^{z\phi} & \chi_{em}^{zz} \end{bmatrix} \begin{bmatrix} H_{\phi,av} \\ H_{z,av} \end{bmatrix} \quad (16a)$$

$$\begin{bmatrix} \Delta E_z \\ -\Delta E_\phi \end{bmatrix} = j\omega\sqrt{\mu_0\epsilon_0} \begin{bmatrix} \chi_{me}^{\phi\phi} & \chi_{me}^{\phi z} \\ \chi_{me}^{z\phi} & \chi_{me}^{zz} \end{bmatrix} \begin{bmatrix} E_{\phi,av} \\ E_{z,av} \end{bmatrix} + j\omega\mu_0 \begin{bmatrix} \chi_{mm}^{\phi\phi} & \chi_{mm}^{\phi z} \\ \chi_{mm}^{z\phi} & \chi_{mm}^{zz} \end{bmatrix} \begin{bmatrix} H_{\phi,av} \\ H_{z,av} \end{bmatrix} \quad (16b)$$

For the case of TM polarization, they reduce to

$$\Delta E_z = j\omega\sqrt{\mu_0\epsilon_0}\chi_{me}^{\phi z}E_{z,av} + j\omega\mu_0\chi_{mm}^{\phi\phi}H_{\phi,av} \quad (17a)$$

$$\Delta H_\phi = j\omega\epsilon_0\chi_{ee}^{zz}E_{z,av} + j\omega\sqrt{\mu_0\epsilon_0}\chi_{em}^{z\phi}H_{\phi,av} \quad (17b)$$

Since there are four unknown susceptibility components in the above equation, two different excitation problems are solved to obtain the four susceptibility components. The two excitations used are shown in Fig. 3. The first problem is that of a infinite, electric line source at origin and the second excitation problem is that of a cylindrical surface electric current density of radius a_s . a_s can be of arbitrary value which satisfies the condition, $a_s > a_4$. Both of these problems are simple, boundary value problems, which can be either solved analytically [14] or simulated using COMSOL. In the case of analytical solution,

Fig. 3. Bianisotropic susceptibility tensor extraction using two sources: (1) Infinite electric line source at origin, $\vec{J} = \delta(x)\delta(y)\hat{z}\text{Am}^{-2}$ shown as a blue dot (2) Cylindrical electric surface current density of radius a_s , $\vec{J}_s = \delta(\rho - a_s)\hat{z}\text{Am}^{-1}$ shown as a dashed, blue circle.

the vector magnetic potential, electric field and magnetic field in each region can be expressed in terms of Bessel functions.

$$A_{z,i} = C_i J_0(k_i \rho) + D_i Y_0(k_i \rho) \quad (18a)$$

$$E_{z,i} = -j\omega \left[C_i J_0(k_i \rho) + D_i Y_0(k_i \rho) \right] \quad (18b)$$

$$H_{\phi,i} = \frac{k_i}{\mu_i} \left[C_i J_1(k_i \rho) + D_i Y_1(k_i \rho) \right] \quad (18c)$$

where $J_0(\cdot), Y_0(\cdot)$ are Bessel function of first and second kind respectively. k_i, μ_i are the wavenumber and permeability of region i . For the case of cylindrical surface current density excitation of three layered, magneto-dielectric cylinder, the total number of regions is 6 (the surface current density discontinuity splits the external medium into two separate regions). Applying continuity of electric and magnetic fields across the region boundaries and magnetic field discontinuity caused by surface current density, the unknowns C_i, D_i can be obtained.

Once the two excitation problems are solved, the susceptibility tensor components are obtained as follows

$$j\frac{\omega}{c_0} \begin{bmatrix} E_{z,av}^{(1)} & \eta_0 H_{\phi,av}^{(1)} & 0 & 0 \\ 0 & 0 & \frac{E_{z,av}^{(1)}}{\eta_0} & H_{\phi,av}^{(1)} \\ E_{z,av}^{(2)} & \eta_0 H_{\phi,av}^{(2)} & 0 & 0 \\ 0 & 0 & \frac{E_{z,av}^{(2)}}{\eta_0} & H_{\phi,av}^{(2)} \end{bmatrix} \begin{bmatrix} \chi_{me}^{\phi z} \\ \chi_{mm}^{\phi\phi} \\ \chi_{ee}^{zz} \\ \chi_{em}^{z\phi} \end{bmatrix} = \begin{bmatrix} \Delta E_z^{(1)} \\ \Delta H_\phi^{(1)} \\ \Delta E_z^{(2)} \\ \Delta H_\phi^{(2)} \end{bmatrix} \quad (19)$$

where the superscript (1) denotes line source excitation fields and (2) denotes the cylindrical surface current density excitation.

IV. NUMERICAL VALIDATION

This section contains two examples of cylindrical metasurfaces which use IE-GSTC and the susceptibility extraction method described in the the previous sections. The number of MoM basis functions (or discretizations) was $N = 2000$. COMSOL software is used for full wave simulations.

A. Four segment cylindrical metasurface excited by plane wave

The cylindrical metasurface shown in Fig. 4 consists of four segments (quadrants). Each segment is a multilayered,

Quadrant	Material properties	Susceptibility tensor
Quadrant 1	$\epsilon_{r1} = 1, \mu_{r1} = 1$	$\chi_{me}^{\phi z} = j2.55236 \times 10^{-3}$
	$\epsilon_{r2} = 9, \mu_{r2} = 2$	$\chi_{mm}^{\phi\phi} = 6.08697 \times 10^{-2}$
	$\epsilon_{r3} = 4, \mu_{r3} = 1$	$\chi_{ee}^{zz} = 1.97682 \times 10^{-1}$
	$\epsilon_{r4} = 9, \mu_{r4} = 2$	$\chi_{em}^{z\phi} = j1.42107 \times 10^{-2}$
	$\epsilon_{r5} = 1, \mu_{r5} = 1$	
Quadrant 2	$\epsilon_{r1} = 1, \mu_{r1} = 1$	$\chi_{me}^{\phi z} = -j1.79047 \times 10^{-5}$
	$\epsilon_{r2} = 4, \mu_{r2} = 1$	$\chi_{mm}^{\phi\phi} = 5.27727 \times 10^{-3}$
	$\epsilon_{r3} = 3, \mu_{r3} = 1$	$\chi_{ee}^{zz} = 1.86202 \times 10^{-2}$
	$\epsilon_{r4} = 4, \mu_{r4} = 1$	$\chi_{em}^{z\phi} = j2.78948 \times 10^{-4}$
	$\epsilon_{r5} = 1, \mu_{r5} = 1$	
Quadrant 3	$\epsilon_{r1} = 1, \mu_{r1} = 1$	$\chi_{me}^{\phi z} = j1.45598 \times 10^{-3}$
	$\epsilon_{r2} = 4, \mu_{r2} = 1$	$\chi_{mm}^{\phi\phi} = 9.25367 \times 10^{-3}$
	$\epsilon_{r3} = 3, \mu_{r3} = 1$	$\chi_{ee}^{zz} = 2.80732 \times 10^{-2}$
	$\epsilon_{r4} = 6, \mu_{r4} = 2$	$\chi_{em}^{z\phi} = -j9.74236 \times 10^{-4}$
	$\epsilon_{r5} = 1, \mu_{r5} = 1$	
Quadrant 4	$\epsilon_{r1} = 1, \mu_{r1} = 1$	$\chi_{me}^{\phi z} = j2.83566 \times 10^{-4}$
	$\epsilon_{r2} = 1, \mu_{r2} = 1$	$\chi_{mm}^{\phi\phi} = 6.01626 \times 10^{-3}$
	$\epsilon_{r3} = 6, \mu_{r3} = 2$	$\chi_{ee}^{zz} = 1.44824 \times 10^{-2}$
	$\epsilon_{r4} = 1, \mu_{r4} = 1$	$\chi_{em}^{z\phi} = -j3.78786 \times 10^{-5}$
	$\epsilon_{r5} = 1, \mu_{r5} = 1$	

TABLE I

Material properties and susceptibility tensor of metasurface segments shown in Fig. 4. Permittivity and permeability have subscripts which represent corresponding regions.

concentric, magneto-dielectric shell. The susceptibility tensor components of each segment can be extracted using the method in the previous section. Referring to Fig. 4, different annular regions are identified as follows: region 1: $0 \leq \rho < a_1$, region 2: $a_1 \leq \rho < a_2$, region 3: $a_2 \leq \rho < a_3$, region 4: $a_3 \leq \rho < a_4$ and region 5: $a_4 \leq \rho < \infty$. The metasurface is surrounded by vacuum, i.e. region 1 and region 5 are vacuum. The material properties of each region (represented by subscript number) in the four quadrants are given in table I. The susceptibility tensors of the quadrants are also given. The dimensions are: $a_1 = 5\lambda$, $a_2 = 5\lambda + \frac{\lambda}{24}$, $a_3 = a_2 + \frac{\lambda}{24}$, $a_4 = a_3 + \frac{\lambda}{20}$.

Fig. 4. Four segment cylindrical metasurface.

The structure is excited by a plane wave $\vec{E}^{\text{inc}}(x) = e^{-jk_0 x} \hat{z}$. The total field along a line from $(0, 0)$ to $(10\lambda, 0)$ is plotted in Fig. 5 and Fig. 6. Good agreement between IE-

GSTC and full wave results obtained using COMSOL software can be seen. For the IE-GSTC result a small gap can be around the metasurface radius of $x = 5\lambda$. This is because IE results is valid only for regions interior and exterior to the metasurface. Minor discrepancies between the two results can be seen around the metasurface. This is probably due to MoM being inefficient in the near field of the scatterer as well as GSTC being an approximate interface condition.

Fig. 5. IE-GSTC vs. full wave simulation for example A. $|E_z(x)|$ vs. x from $(0, 0)$ to $(10\lambda, 0)$ Fig. 6. IE-GSTC vs. full wave simulation for example A. $\angle E_z(x)$ vs. x from $(0, 0)$ to $(10\lambda, 0)$

B. Cylindrical metasurface with opening excited by a line source

This example as shown in Fig. 7 is a four segment metasurface but with an opening. The opening subtends an angle of $\pi/2$ at the center and is equally divided between first and fourth quadrants. The material properties of the quadrants are same as that of previous example. The excitation is an infinite,

electric, line source of $\bar{J}(\bar{r}) = \delta(x)\delta(y)\hat{z} \text{ Am}^{-2}$. In IE-GSTC an opening is handled by setting $\mathbf{A}(\bar{\rho}_c) = \text{diag}(1, 1, 1, 1)$ [7]. This ensures that tangential fields on either side of the opening are equal. This is only an approximation because the opening has a width of fraction of a wavelength. In COMSOL full wave simulation, line source is modelled using the external current density option on a circle of radius much smaller than wavelength ($\sim \lambda/100$). IE-GSTC results and COMSOL

Fig. 7. Four segment cylindrical metasurface with opening

results on a circle of radius 8λ are plotted in Fig. 8 and Fig. 9. There is good agreement between the results. The discrepancies for $\phi < 1$ radian and $\phi > 5.5$ radian could be caused by edge diffraction which the GSTC does not take into account. Another reason could be the IE-GSTC approximation for opening, which ignores the finite width of the metasurface.

Fig. 8. IE-GSTC vs. full wave simulation for example B. $|E_z(\phi)|$ vs. ϕ on a circle of radius 8λ .

Fig. 9. IE-GSTC vs. full wave simulation for example B. $\angle E_z(\phi)$ vs. ϕ on a circle of radius 8λ .

V. CONCLUSION

In this work, IE-GSTC for circular cylindrical metasurface is validated for the case of physical metasurfaces. The essence of this work is that of circular metasurface scattering particles can be abstracted as few numbers, i.e. susceptibility tensor components. As far as to the author's knowledge, this work is the first time in which GSTC and bianisotropic susceptibility tensor model is validated for a physical, conformal metasurface. Even though the examples and susceptibility extraction was for TM polarization, it is easily extendable to TE polarization. The combination can be used to analyze and design circular metasurfaces for the case of circularly polarized incident wave. This work is important in the sense that it pushes the previous works on conformal GSTC [7]–[10] to physical implementation and optimization.

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