Average Contiguous Duration – A Novel Metric for Characterizing Wireless Fading Channels

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Abstract— We define the average contiguous duration (ACD) of the received fading signal as the average time duration during which the signal envelope contiguously remains within a bounded amplitude interval. Also, the multi-interval ACD or K -ACD function is defined as the set of ACD values for K non-overlapping amplitude-intervals whose union spans the received envelope's amplitude range. We derive closed-form expressions of the ACD for Rayleigh, Rice, and Nakagami- m fading signals, which are widely analyzed in the literature. The derived expressions hold practical significance for several signal processing applications such as non-uniform quantization of channel samples for secret key generation (SKG) in physical layer security (PLS) techniques. The well-known channel fading metric of average fade duration (AFD) is shown as a special case of the proposed ACD metric and the proposed theoretical analysis is validated by simulations.

Index Terms— Average fade duration, characterization, contiguous duration, fading channel

I. INTRODUCTION

Various metrics have been discussed in the literature to quantify the behaviour of fading signals. The probability distribution function (PDF) of the received signal's envelope can be used to predict the likelihood of the envelope residing in a certain interval. The standard deviation (SD), variance, and root mean square (RMS) metrics quantify the signal dispersion in relation to its mean. To quantify the envelope fluctuation rate, the level-crossing rate (LCR) and average-fade duration (AFD) metrics are widely used [1]. The LCR is the average rate at which the signal envelope crosses a given threshold-level with a positive or a negative slope. The AFD is the average duration for which the signal envelope remains below some given threshold-level. The AFD is suitable for applications where signal fluctuations relative to a single threshold are of interest, e.g., to compare signal strength with the minimum threshold for reliable communication. However, in other signal processing applications such as quantization and encoding, signal fluctuations may need to be quantified in relation to multiple thresholds simultaneously. In this context, this letter proposes a novel metric termed *average contiguous duration (ACD)*, which is the average time duration for which a given signal's envelope contiguously remains within a bounded amplitude interval. Moreover, the multi-interval ACD or K -ACD function is defined as the set of ACD values for K non-overlapping amplitude-intervals whose union spans the

received envelope's amplitude range. The ACD metric has uses in several signal processing applications such as in determining appropriate non-uniform quantization (NUQ) intervals for the channel samples in secret key generation (SKG) for physical layer security (PLS). The ACD-based NUQ intervals can offer promising trade-offs between the key generation rate (KGR), key agreement probability (KAP), and key randomness property (KRP). The rest of this letter is organized as follows: In Sec. II, the proposed ACD metric is defined and closed-form expressions of ACD for the Rayleigh, Rice, and Nakagami- m distributions are derived. In Sec. III, the multilevel K -ACD metric is defined. In Sec. IV, applications of the proposed metrics are highlighted with emphasis on SKG for PLS. Finally, Sec. V concludes this letter.

II. AVERAGE CONTIGUOUS DURATION (ACD)

We define ACD as the average time duration for which the received signal envelope $|r(t)|$ remains within an amplitude range of interest (ARoI) that is bounded by two threshold levels ρ_1 and ρ_2 . Fig. 1 shows M contiguous duration (CD) instances, ξ_m $m = 1, 2, \dots, M$. For this case the ACD is obtained by averaging as $\Xi_{\rho_1}^{\rho_2} = \frac{1}{M} \sum_{m=1}^M \xi_m$. The ACD can be defined more rigorously as

$$\Xi_{\rho_1}^{\rho_2} = \frac{1}{\mathcal{M}} \text{Prob}(\rho_1 < |r| \leq \rho_2), \quad (1)$$

where $\mathcal{M}$ represents the rate (CD instances per second) at which the signal envelope enters the ARoI with either a positive or a negative slop. In Fig. 1 the start and end points of each of the $M = 7$ CD instances is marked with $\circ$ and $\bullet$, respectively. A CD may start with the signal envelope entering the ARoI with a negative slope from an amplitude greater than ρ_2 or with a positive slope from an amplitude smaller than ρ_1 . The *level-crossing rate* (LCR) of the signal envelope with reference to some threshold $\rho_{(\cdot)}$ is the average number of times per second that the envelope crosses $\rho_{(\cdot)}$ with a positive or a negative slope ($N_r^+(\rho_{(\cdot)}) = N_r^-(\rho_{(\cdot)})$) [2]. The LCR can be expressed as $N_r(\rho) = \int_0^\infty \dot{r} f(\rho, \dot{r}) d\dot{r}$, where $\dot{r}$ represents the time derivative of the signal envelope r , ρ is the predefined threshold, and $f(\cdot, \cdot)$ represents the joint density function. The parameter $\mathcal{M}$ can be interpreted as a sum of the LCR for ρ_2 (with negative slope) and the LCR for ρ_1 (with positive slope), i.e., $\mathcal{M} = N_r^-(\rho_1) + N_r^+(\rho_2)$. Then the ACD from (1) can be expressed as

$$\Xi_{\rho_1}^{\rho_2} = \frac{1}{N_r(\rho_1) + N_r(\rho_2)} \int_{\rho_1}^{\rho_2} f(r) dr = \frac{F_r(\rho_2) - F_r(\rho_1)}{N_r(\rho_2) + N_r(\rho_1)}. \quad (2)$$

where $f(r)$, $F_r(\rho_{(\cdot)})$, $N_r(\rho_{(\cdot)})$ represent the PDF, CDF, and LCR of signal envelope, respectively. The *Average fade*

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Fig. 1: Demonstration of ACD and AFD calculations for a sample function of Rayleigh fading signal.

duration (AFD), $\tau_r(\rho)$, represents the average duration for which the signal envelope stays below a given threshold level ρ , i.e., it is the ratio between the total time the envelope remains below the threshold and the total number of times the envelope crosses the threshold with positive or a negative slope. The AFD can be obtained as a special case of the ACD by substituting $\rho_2 = \rho$ and $\rho_1 = 0$. The AFD can be expressed in terms of the ACD as

$$\tau_r(\rho) = \Xi_0^\rho = \frac{\text{Prob}(0 < r \leq \rho)}{N_r(\rho) + N_r(0)} = \frac{1}{N_r(\rho)} \int_0^\rho f(r) dr = \frac{F_r(\rho)}{N_r(\rho)}. \quad (3)$$

The ACD metric can help determine the appropriate sampling rate to ensure that a certain number of contiguous signal samples fall within the ARoI. For example, by setting the sampling interval as $T_s = \Xi_{\rho_1}^{\rho_2}/N$, on average N contiguous samples will lie within the ARoI $\rho_2 - \rho_1$. Alternatively, an ACD value can be converted to an integer sample-count as $\Xi_{\rho_1}^{\rho_2} = \text{floor}(\Xi_{\rho_1}^{\rho_2}/T_s)$.

For reference, Table I [2] lists the mathematical expressions of the PDF, CDF, LCR, and AFD for Rayleigh, Rice, and Nakagami- m distributed signal envelopes and we provide their closed-form ACD expressions below.

a) *Rayleigh*: By substituting the expressions of Rayleigh CDF and LCR from Table I into (2) and after simplification the ACD for Rayleigh signal envelope can be expressed as

$$\Xi_{\rho_1}^{\rho_2} = \frac{\sqrt{\Omega} \left(\exp\left(\frac{\rho_2^2}{\Omega}\right) - \exp\left(\frac{\rho_1^2}{\Omega}\right) \right)}{f_m \sqrt{2\pi} \left(\rho_1 \exp\left(\frac{\rho_2^2}{\Omega}\right) + \rho_2 \exp\left(\frac{\rho_1^2}{\Omega}\right) \right)}, \quad (4)$$

where f_m is the maximum Doppler shift, $\Omega = E[r^2] = 2\sigma_r^2$ represents the average signal power, σ_r is the standard deviation of signal amplitude, and $E[\cdot]$ denotes statistical expectation.

b) *Rice*: By substituting Rician CDF and LCR expressions from Table I into (2) and after simplifying the ACD for Rician case can be expressed as

$$\Xi_{\rho_1}^{\rho_2} = \frac{\Omega \exp(k)}{f_m \sqrt{2\pi(k+1)}} \left\{ Q_1 \left(\sqrt{2k}, \sqrt{\frac{2\rho_1^2(k+1)}{\Omega}} \right) - Q_1 \left(\sqrt{2k}, \sqrt{\frac{2\rho_2^2(k+1)}{\Omega}} \right) \right\} \left\{ I_0 \left(2\rho_1 \sqrt{\frac{k(k+1)}{\Omega}} \right) \times \rho_1 \exp\left(-\frac{\rho_1^2(k+1)}{\Omega}\right) + I_0 \left(2\rho_2 \sqrt{\frac{k(k+1)}{\Omega}} \right) \rho_2 \exp\left(-\frac{\rho_2^2(k+1)}{\Omega}\right) \right\}^{-1}, \quad (5)$$

where $k \geq 0$ is the Rice parameter, $Q(\cdot, \cdot)$ is the Marcum-Q function, and $I_0(\cdot)$ is the 0th-order modified Bessel function of first-kind.

c) *Nakagami- m* : By substituting the Nakagami- m CDF and LCR expressions from Table I into (2) the Nakagami- m ACD can be expressed as

$$\Xi_{\rho_1}^{\rho_2} = \frac{\rho_1 \rho_2 \exp\left(\frac{m}{\Omega} (\rho_1^2 + \rho_2^2)\right) m^{\frac{1}{2}-m} \left(\Gamma\left(m, \frac{m\rho_2^2}{\Omega}\right) - \Gamma\left(m, \frac{m\rho_1^2}{\Omega}\right) \right)}{f_m \sqrt{2\pi\Omega} \left(\rho_2 \rho_1^{2m} \exp\left(\frac{m}{\Omega} \rho_2^2\right) + \rho_1 \rho_2^{2m} \exp\left(\frac{m}{\Omega} \rho_1^2\right) \right)}, \quad (6)$$

where m is the Nakagami- m fading severity parameter and $\Gamma(\cdot, \cdot)$ is the incomplete Gamma function.

III. MULTI-INTERVAL ACD (K -ACD) FUNCTION

For the K non-overlapping intervals spanning the envelope range $0 - \rho_{\max}$ of the received fading envelope, the multi-interval ACD or K -ACD function can be expressed as

$$\Theta_0^{\rho_{\max}}(K) = \left[\Xi_{\rho_0=0}^{\rho_1}, \Xi_{\rho_1}^{\rho_2}, \dots, \Xi_{\rho_{K-1}}^{\rho_K=\rho_{\max}} \right]. \quad (7)$$

By setting the non-uniform widths z_k of the K ARoI intervals appropriately, the probability of attaining an identical ACD for the K intervals can be maximized. For this task, after setting $\rho_0 = 0$ and $\rho_K = \rho_{\max}$, the remaining $K - 1$ separating thresholds $(\rho_1, \dots, \rho_{K-1})$ can be determined by manipulating the set of expressions

$$\Xi_{\rho_0=0}^{\rho_1} = \Xi_{\rho_1}^{\rho_2} = \dots = \Xi_{\rho_{K-1}}^{\rho_K=\rho_{\max}} = \Psi, \quad (8)$$

where Ψ is the constant ACD value across the K intervals. For SKG quantization purposes, a suitable K can be set to achieve $\Psi/T_s \geq 1$, i.e., one or more envelope sample falls

TABLE I: PDF, CDF, LCR, and AFD for Rayleigh, Rice, and Nakagami- m distributions [2]

Distribution	PDF, $f(r)$	CDF, $F_r(\rho)$	LCR, $N_r(\rho)$	AFD, $\tau_r(\rho)$
Rayleigh	$\frac{2r}{\Omega} \exp\left(-\frac{r^2}{\Omega}\right)$	$1 - \exp\left(-\frac{\rho^2}{\Omega}\right)$	$\frac{\sqrt{2\pi} f_m \rho}{\sqrt{\Omega}} \exp\left(-\frac{\rho^2}{\Omega}\right)$	$\frac{(\exp(\rho^2/\Omega) - 1) \sqrt{\Omega}}{\sqrt{2\pi} f_m \rho}$
Rician	$\frac{2(k+1)r}{\Omega} \exp\left(-k - \frac{(k+1)r^2}{\Omega}\right) \times I_0\left(2r\sqrt{\frac{k(k+1)}{\Omega}}\right)$	$1 - Q_1\left(\sqrt{2k}, \sqrt{\frac{2(k+1)\rho^2}{\Omega}}\right)$	$\sqrt{2\pi(k+1)} f_m \frac{\rho}{\Omega} \exp\left(-k - \frac{(k+1)\rho^2}{\Omega}\right) \times I_0\left(2\rho\sqrt{\frac{k(k+1)}{\Omega}}\right)$	$\frac{1 - Q_1\left(\sqrt{2k}, \sqrt{\frac{2(k+1)\rho^2}{\Omega}}\right)}{\sqrt{2\pi(k+1)} f_m \rho I_0\left(2\rho\sqrt{\frac{k(k+1)}{\Omega}}\right)} \times \Omega \exp\left(k + \frac{(k+1)\rho^2}{\Omega}\right)$
Nakagami- m	$\frac{2m^m r^{2m-1}}{\Omega^m \Gamma(m)} \exp\left(-\frac{mr^2}{\Omega}\right)$	$\frac{\Gamma\left(m, \frac{m\rho^2}{\Omega}\right)}{\Gamma(m)}$	$\frac{\sqrt{2\pi} f_m}{\Omega \Gamma(m)} m^{m-1/2} \rho^{2m-1} \exp\left(-\frac{m\rho^2}{\Omega}\right)$	$\frac{\Gamma\left(m, \frac{m\rho^2}{\Omega}\right) \Omega \exp\left(\frac{m\rho^2}{\Omega}\right)}{\sqrt{2\pi} f_m m^{m-1/2} \rho^{2m-1}}$

in each quantization interval, on average. For given channel conditions Ψ decreases with increasing K .

2-ACD Function Example: For $K = 2$, the separating threshold ρ_1 that divides the received envelope range into two intervals of identical ACD, i.e., $\Xi_0^{\rho_1} = \Xi_{\rho_1}^{\rho_{\max}}$ can be determined as

$$\frac{F_r(\rho_1)}{N_r(\rho_1)} = \frac{1 - F_r(\rho_1)}{N_r(\rho_1)}. \quad (9)$$

For a Rayleigh-distributed envelope, the separating threshold ρ_1 for the 2-ACD function can be expressed in closed-form as

$$\rho_1 = \sqrt{\Omega \ln 2}. \quad (10)$$

For a Rice-distributed envelope, the separating threshold ρ_1 for the 2-ACD function can be obtained by manipulating the expression

$$1 - 2 Q_1\left(\sqrt{2k}, \sqrt{\frac{2\rho_1^2(k+1)}{\Omega}}\right) = 0, \quad (11)$$

where the value of ρ_1 can be obtained by numerical inversion of the Marcum Q-function [3]. Alternatively, a closed-form approximation to ρ_1 can be obtained by using the first order Marcum Q-function approximation [4].

For the Nakagami- m distributed envelope, the separating threshold ρ_1 for the 2-ACD function can be obtained by manipulating the expression

$$\Gamma(m) - 2 \Gamma\left(m, \frac{m\rho_1^2}{\Omega}\right) = 0. \quad (12)$$

Similarly, the separating thresholds for $K > 2$ case can also be obtained.

IV. APPLICATION EXAMPLES AND NUMERICAL EVALUATION

This section discusses some applications of the proposed ACD metric and K -ACD function. Most notably, numerical evaluation of the proposed ACD metric-based non-uniform quantization (NUQ) is conducted in the context of SKG for PLS.

a) SKG for PLS: PLS is an emerging paradigm of augmenting secure communications at the physical layer of wireless communication systems. PLS techniques can be key-based or key-less. The generation of symmetric encryption keys from independent observations of the common wireless channel between the legitimate nodes circumvents the need to

distribute secret keys between the legitimate nodes, which in turn enhances the system secrecy performance. The behaviour of the common reciprocal channel between the legitimate nodes (say Alice and Bob) is a function of their spatial positions and it cannot be measured by the eavesdropper/Eve for sufficiently rich scattering. Fig. 2 illustrates the system model and channel profiles observed independently at Alice and Bob within a channel coherence time T_c . The $K = 4$ level quantization and corresponding 4-ACD function are also shown in the figure. Practically, differences between hardware and noise conditions of Alice and Bob lead to differences between their measurements of the ideally reciprocal channel. For example, the dissimilar channel envelope measurements can be modeled by a Gauss Markov model as [5] $h_{ab} = \hat{\rho} h_{ba} + \sqrt{1 - \hat{\rho}^2} \varepsilon$, where $\hat{\rho} \in [0, 1]$ is the correlation coefficient between envelope measured at Bob h_{ab} and envelope measured at Alice h_{ba} . Furthermore, $\varepsilon \sim \mathcal{N}(0, 1)$ represents the Gaussian-distributed error in envelope measurement at Bob. The symmetric SKG algorithms aim to generate identical secret keys independently at Alice and Bob, despite their dissimilar channel measurements. The KGR, KAP, and KRP are some of the conventional metrics [5] employed to measure the effectiveness of SKG algorithms and they will also be used in the numerical results provided here. Among the several SKG algorithms proposed in the literature, a notable algorithm proposed by Ye et al. [6] uses for SKG only those contiguous envelope samples, whose sequence length exceeds some predefined threshold (Ξ_{th}). This algorithm increases the KAP at the cost of reduced KGR. However, a favorable trade-off between KGR and KAP can be achieved by appropriate selection of Ξ_{th} . For this task we propose to set the multi-level common ACD value as the acceptance/rejection threshold for sample excursions, i.e., $\Xi_{th} = \Psi$. This ACD-based quantization also leads to improved KRP as shown later by the presented numerical results.

The application of proposed ACD metric in 4-level NUQ for SKG algorithm of [6] is depicted in Fig. 3. The legitimate channels h_{ab} and h_{ba} are modeled as Rayleigh fading and 10^5 channel samples are generated for obtaining the presented results. We assume that the legitimate nodes have prior knowledge of $\hat{\rho}$, which is set to $\hat{\rho} = 0.84$ for the displayed results. As evident in the figure, the ACD-based quantization for SKG ensures performance improvement in terms of KGR and KRP, while maintaining the same KAP as that of the conventional uniform quantization (UQ) scheme. KRP performance offered by NUQ and ACD-based

Fig. 2: K -ACD for SKG at Alice and Bob – PLS Example

NUQ is comparable. The NUQ scheme without ACD can provide promising KRP compared to UQ by maintaining same KGR and KAP [5] but the proposed ACD-based NUQ simultaneously improves both the KGR and KRP for the same KAP. This is due to the increased likelihood of identical length of contiguously located samples in each quantization interval in the ACD-based NUQ scheme. Fig. 3(b) shows the KRP evaluated by using the NIST test suite. The P value is plotted for several NIST tests including the frequency, block frequency, run, discrete Fourier transform, Maurer, binary matrix rank, cumulative sum forward, and cumulative sum reversed tests for the proposed ACD-based NUQ and the conventional UQ schemes. As shown by Fig. 3(a), the performance improvement in KGR is more evident when the acceptance/rejection threshold Ξ_{th} for sample excursions is set in accordance with the ACD metric, where $\Psi = \text{round}(\Psi/T_s)$. This indicates the scope for improving the SKG algorithms, e.g., [6] by utilizing the ACD information to further increase the KGR. A potential extension in the SKG algorithm can be the characterization of CD instances that qualify the minimum acceptance criteria (i.e., Ξ_{th}) into multiple CD length bins with codes of different sizes associated to each bin. This will result in the generation of more key-bits for longer CD instances and eventually result in further increased KGR.

b) Other Applications: The proposed ACD metric can be exploited to ensure high compression rate. For example, the number of K -ACD function-based NUQ intervals and sampling rate can be jointly optimized such that a minimal number of consecutive samples (ideally a single sample) fall in each quantization interval (i.e., each adjacent sample falls in different quantization intervals). Moreover, instead of encoding all the samples contiguously falling in a quantization interval, only the quantization interval index and ACD information can be used to encode the samples. A high compression rate can be achieved by interpreting the ACD behaviour of the signal under consideration to jointly devise appropriate values of the sampling rate, number and bounding thresholds of NUQ intervals. Furthermore, the code size associated with each quantization interval can also be manipulated by using the ACD information, e.g., ACD-based revisited Huffman coding.

Fig. 3: KGR, KAP, and KRP performance of 4-ACD quantization based SKG

Also, the proposed K -ACD function can be used for quantifying signal randomness. For example, a random number generator algorithm can be set to ensure a low ACD value for different amplitude intervals, for a given probability density function.

Angular spread quantification is critical in characterizing wireless channels and in the design of multi-antenna transceivers. Several angular spread quantifiers such as the true standard deviation are proposed in the literature [7]. The proposed ACD metric can be applied with modifications to the angular-domain for angular spread quantification.

V. CONCLUSIONS

In this letter, a novel ACD metric for the characterization of wireless fading channels has been proposed. Mathematical expressions of ACD metric for Rayleigh, Rice, and Nakagami- m distributions have been derived. A multi-level ACD (i.e., K -ACD) function has also been proposed. Various applications of the proposed ACD metric have been highlighted, where quantization for SKG has been numerically investigated.

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