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BER-Aware Reflecting Elements Allocation for RIS-aided NOMA Networks over Ricean Fading

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Abstract—Reconfigurable intelligent surface (RIS) as a supportive technology for aiding downlink non-orthogonal multiple access (NOMA) can enhance the bit error rate (BER) performance. In this paper, a novel BER-aware reflecting elements allocation (REA) on an RIS is proposed to maintain the BER order among paired RIS-NOMA users. The RIS REA is useful for minimizing the average user BER, ompared with a system that allocates the same number of elements to all users. Additionally, the Ricean fading is considered instead of Rayleigh fading as it is more practical and general. Furthermore,an REA optimization objective function for equalizing the user BER is proposed. In order to solve the problem, a modified exhaustive search is proposed to reduce complexity. The distribution of the objective function is observed first; subsequently, the exhaustive search range is determined. Both the analytical and simulation results show that the proposed algorithm can minimize the average user BER.

Index Terms—Bit error rate (BER), non-orthogonal multiple access (NOMA), reconfigurable intelligent surface (RIS), Ricean fading, reflecting element allocation (REA)

I. INTRODUCTION

RECONFIGURABLE intelligent surfaces (RISs) is recently gaining a popularity as a technology to aid wireless technology for 5G and beyond, because of the capability to manipulate signal propagation for both downlink and uplink transmission. An RIS is defined as a planar surface comprising low-cost reflective elements that can work independently [1]. In [2], it is reported that each element can modify channel phase with a several tuning methods. Moreover, in [3] it is reported that an RIS performs better than relaying in term of spectral efficiency and deployment cost. Therefore, due to its numerous advantages, RIS has been integrated with many existing wireless technology as a primary transmitter, an access point (AP) or extending a network coverage, a dual hop communication (DH).

A diverse integration of RIS for aiding wireless telecommunications was performed, in particular, RIS-aided non-orthogonal multiple access. RIS-NOMA is recently gaining focus due to higher channel capacity and low error rates [4], [5]. NOMA is considered as a promising technologies for 5G and beyond that can fulfill the demands of high data rates, spectral efficiency and massive users connectivity [6]. This is due to the principal work of NOMA which allows user to

share a same time or frequency resources [7]. The principal work of power domain NOMA (PD-NOMA) is allocating the higher power allocation (PA) for adverse channel condition of cell edge users (CEU), while a cell center user (CCU) perform a successive interference cancellation (SIC) technique to retrieve information [8]. Compare to its predecessor, orthogonal multiple access (OMA), NOMA gives a better performance in a point of view of the ergodic capacity and outage probability for uplink [9] or downlink communication [10], [11].

A. Related Works

In NOMA, a tradeoff exists between high data rate and spectral efficiency, which results in a relatively high BER. This is because the CCU must perform SIC using the incoming superposition-coded (SC) signal. Therefore, the RIS facilitates the NOMA network because it improves the signal-to-noise ratio (SNR) with signal diversity. Recently, the integration RIS for assisting NOMA has become a topic of considerable research interest, in particular for assisting PD-NOMA. In [5], RIS single-input single-output (SISO) NOMA was proposed, as well as RIS passive beamforming for improving spectral efficiency. In [12], the joint optimization of active beamforming at the base station (BS) and passive beamforming at the RIS was proposed to maximize the signal-to-interference-plus-noise ratio (SINR) for all users. Therefore, the desired rate performance and user fairness can be achieved. In [13], a downlink PD-NOMA with RIS was proposed to enhance energy efficiency. It can be achieved through passive beamforming design and joint user clustering. In [14], a simple RIS-NOMA downlink scenario for multiple-input single-output (MISO) BS. It was presumed that the CCU had a line of sight (LoS) to the BS and multiplexed with spatial division multiple access (SDMA). At the same time, CEU had a no LoS to the BS and supported with RISs. In [15], a deployment and passive beamforming design for RIS aided NOMA with MISO BS. The main objective was to maximize energy efficiency and subjects to user data rate requirements. The joint optimization for the active beamforming of MISO-NOMA system was extended in [16], where the main objective was to reduce the total transmit power for multi-cluster user. In [17], proposed a passive beamforming desgin for RIS aided MISO-NOMA system. The idea was to optimize a beamforming weights in order to minimize intercluster interference. In [18], a combination of joint transmission coordinated multipoint (JT-CoMP) on an RIS was proposed to provide more CEU ergodic capacity without sacrificing CEU performane. In [19], it was examined the RIS phase shifter design, between the coherent discrete

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and the random discrete comprehensively. In [20], a capacity fairness performance of RIS SISO NOMA and RIS MIMO NOMA over Ricean fading channel was investigated. In [21], the symmetrical splitting of RIS elements for downlink CCUs and CEUs was proposed, and it was shown that RIS improved the BER performances of both CCUs and CEUs. In [22] proposed a joint optimization solution for resource allocation problem in RIS-NOMA system. Here, in order to maximize the system throughput, they optimize several aspects, including the channel assignments, power allocation, decoding order, and RIS reflection coefficient. Finally, in [23] considered a RIS-NOMA downlink scenario with element partitioning, aiming for user fairness. The proposed partitioning could enhance the outage probability and ergodic capacity.

B. Motivation and Contributions

The physical resource allocation scheme used in this paper is based on a previous works. It can be summarized as follows: The BS transmits a single SC signal to users via an RIS reflection (dual-hop communication), assuming all user positions are within the RIS coverage. The RIS phase and reflection coefficient are adjusted to jointly serve all users. In the proposed scheme, not all users will have the same distance to the BS and RIS. Therefore, physical resource allocation must not be equal among users, because every user has its own demands.

Inspired by [24], we propose a novel reflecting element allocation (REA) for RIS assisted NOMA systems and an efficient physical resource allocation. The main purpose of REA is to determine a suitable ratio of RIS to obtain equivalent BER of NOMA users without adjusting the PA. The RIS serves all NOMA users with different number ratio of elements for each users. The REA is beneficial to the RIS-NOMA systems because it provides a lower average users BER than fairly allocate an RIS elements for all users. The main contributions of this paper can be written as follows:

- The formulation of searching the suitable ratio of REA is proposed in present work, in addition to the result of the suitable ratio.
- In contrast to the conventional exhaustive search, we first examine and predict the distribution of the objective function based on a specified parameter. Furthermore, the range of exhaustive search can be narrowed to decrease system complexity.
- An exact analytical BER RIS-NOMA over DH Ricean fading channel is derived in this works, and the simulation and analytical results presented.

The rest of the paper is organized as follows: In Section II, the proposed system model is described in addition to the relevant mathematical expressions. The analytical or theoretical of BER RIS-NOMA over DH Ricean fading is derived in Section III-A. The problem formulation, solution, and the modified exhaustive search are described comprehensively in Section III-B and Section III-C. Results of computer simulations are provided in Section IV, followed by the conclusions in Section V.

Figure 1. The overall system model of BER-aware REA RIS aided NOMA system

The notation of this paper is written as follows: ${}_pF_q(\cdot; \cdot; \cdot)$ denotes a generalized hypergeometric function. ${}_1F_1(\cdot; \cdot; \cdot)$ denotes the confluent hypergeometric function of a first kind. $\mathcal{L}_n(\cdot)$ denotes a Laguerre Polynomial with n parameter. The $|\cdot|^2$ operator produces a scalar value and denotes the magnitude function of the complex vector. $\lceil \cdot \rceil$ and $\lfloor \cdot \rfloor$ represent ceiling and floor function, respectively. $\mathbb{C}^{a \times b}$ denotes complex-value array with length a and width b . $\text{diag}\{\cdot\}^d$ denotes a diagonal matrix with length and width d . $\mathcal{N}(A, \sigma^2)$ denotes a variable distribution with mean A and variance σ^2 . $I_0(\cdot)$ is the zero-order modified Bessel Function of the first kind. $Q_N(\cdot, \cdot)$ is the generalized marcum function with N parameter

II. SYSTEM MODEL

Based on [23] where N RIS elements split among NOMA user and the incoming NOMA signal is reflected to corresponding user. In this paper, we denotes reflecting element allocation L_k and prior to [20], a PD-NOMA downlink scheme is considered with single antenna equipped at base station. The initialization step from Fig.1 is that all the user symbols are modulated and mapped with an M-ary constellation which is denoted by S_k . Afterward, all of the symbols were multiplied with each user PA coefficient. Additionally, it is assumed that the value of PA is in equivalent to $0 < \alpha_k < 1$. Therefore, the superposition coding for this scenario can be expressed as $\chi = \sum_{k=1}^K \sqrt{\alpha_k P} S_k$, where P is the sum of transmitted power of the BS and l indicates an index of the k^{th} user from the total of K user. The rule of PA for K user is denoted by $\alpha_1 < \dots < \alpha_k < \dots < \alpha_K$ where user with $k = 1$ always had a better channel condition than the others.

In this paper, we considered a DH ricean fading scenario for two user NOMA, namely cell edge user (CEU) and cell center user (CCU). The single-hop ricean fading with the presence of line of sight (LOS) and non line of sight (NLOS) path is represented as [25], [26] :

$$h(t) = \underbrace{\sqrt{\frac{\nu\rho}{\nu+1}}}_{A:=} e^{j2\pi f_D \cos(\theta)t + \phi} + \underbrace{\sqrt{\frac{\rho}{\nu+1}}}_{S:=} g(t), \quad (1)$$

where θ is angle of arrival (AoA) at the reciver, ϕ the channel phase, $g(t)$ is a narrowband complex Gaussian process, and

f_D is a maximum Doppler Frequency. $\nu = \frac{A^2}{S^2}$ is the Ricean factor, which consists of the power ratio between LOS component (A^2) and scattered component (S^2). Here, ρ denotes the sum of total received signal power from both LOS and NLOS power components, which is expressed as $\rho = A^2 + S^2$. Based on [27], we define the channel fading matrix contains Gaussian random variables (RV) with non-zero mean A and standard deviation S as follows:

$$A = \sqrt{\frac{\nu}{2(\nu+1)}}, \quad S = \sqrt{\frac{\nu}{2(\nu+1)}}. \quad (2)$$

Based on Eq. (2), we can infer that $\nu = 0$ is the Rayleigh fading channel (pure scattered components) and $\nu = \infty$ is the LOS fading channel. The received signal of the k^{th} user of the proposed system is written as:

$$y_k = H G_k \Phi \chi + n_k, \quad (3)$$

where $[H \in \mathbb{C}^{L_k \times 1}, G_k \in \mathbb{C}^{L_k \times 1}]$ is a Ricean distributed fading channel $\mathcal{N}(A_H, S_H^2)$ within BS to RIS element and $\mathcal{N}(A_G, S_G^2)$ between RIS elements to k^{th} NOMA users, respectively. In addition, the ν factor for H and G are denoted with ν_H and ν_G , respectively. The Φ^{L_k} denotes a phase shift response [14] with ON/OFF switch and $\Phi^{L_k} \in \text{diag}^{L_k} \{e^{j\phi_1}, \dots, e^{j\phi_i}, \dots, e^{j\phi_{L_k}}\}$. Here, $e^{j\phi_i}$ denotes the response phase of i^{th} elements, which minimizes the channel phases, and n_k is an Additive White Gaussian Noise (AWGN) of user k with $\mathcal{N}(0, 1)$.

The capability of each i^{th} RIS element that can be easily turned ON and OFF, leads to the asymmetric element allocation for k^{th} NOMA users. Thus, the number of serials i elements that have to be activated to assist downlink communications denoted by L_k , and the elegant equation of the total of RIS elements is $N = \sum_{k=1}^K L_k$ where $L_1 < \dots < L_K$ is due to the lower value of ν that K^{th} user had. All users had the channel statistic information (CSI) on the receiver side, and $K - 1$ user were engaging perfect SIC to decode its data.

III. PROPOSED BER-AWARE REA

Throughout this section, the derivation for calculating the theoretical average bit error rate (BER) of user 1 and 2 RIS NOMA by exploiting the moment generating function (MGF) equation [28], [29]. On top of that, the problem formulation and solution approach of RIS EA in order to equalize the BER among NOMA users or minimizing BER for $k > 1$ NOMA users. Additionally, contrary to the previous work [21], here the QPSK modulation ($M = 4$) was employed for CCU and CEU for the sake of fair problem formulation.

A. RIS NOMA exact Error Probability

Suppose the channel magnitudes and phases are approximately denoted by $\alpha_i e^{j\psi_i}$ and $\beta_{i,l} e^{j\theta_{i,k}}$ for the channel h_i , $g_{i,k}$ respectively. Here, the effect of $e^{j\phi_i}$ is to manipulating the EM wave and maximize SNR by eliminating all the channel phases, hence the value of RIS phase response written as

$\phi_i = \psi_i + \theta_{i,k}$. Then, the instantaneous SINR in the OMA receiver is formulated as:

$$\gamma = \frac{P \left| \sum_{i=1}^N \alpha_i \beta_{i,k} e^{j(\phi_i - \theta_i - \psi_{i,k})} \right|^2}{N_0} = \frac{P \left| \sum_{i=1}^N \alpha_i \beta_{i,k} \right|^2}{N_0} \quad (4)$$

where P is a transmit power and N_0 is the noise power within the AWGN distribution. On the other hand, contrary to the OMA system, the SNR for the proposed RIS-NOMA system for two users is affected by inter-user interference. Thus it can be written as follows:

$$\begin{aligned} \gamma_1 &= \frac{\sqrt{\alpha_1 P} \left| \sum_{i=1}^{L_1} \alpha_i \beta_{i,1} \right|^2}{N_0}, \\ \gamma_2 &= \frac{\sqrt{(1 - \alpha_1) P} \left| \sum_{i=L_1+1}^{L_2} \alpha_i \beta_{i,2} \right|^2}{N_0 + \sqrt{\alpha_1 P} \left| \sum_{i=L_1+1}^{L_2} \alpha_i \beta_{i,2} \right|^2 + 1}, \end{aligned} \quad (5)$$

where $1 - \alpha_1$ is the PA of user 2. Considering α_i and $\beta_{i,l}$ are the independent ricean random variables. Mean and variance of proposed system was derived and proved by Theorem and Lemma below.

Theorem 1 (Sum of Ricean Variables theorem). *Let $R_i \sim \mathcal{N}(A_i, \sigma^2)$, $i = 1, \dots, N$ be a set of Ricean random variables. The summation of Ricean Random Variables is denoted as [30] [31]:*

$$SR = \sum_{i=1}^N R_i^2(A_i, \sigma^2) \sim \text{nc} - \chi_{2N}^2 \left(\frac{x}{\sigma^2}, \frac{A_N^2}{\sigma^2} \right) \quad (6)$$

with χ_{2N}^2 a $\text{nc} - \chi^2$ with $2N$ degrees of freedom and $A_N^2 = \sum_i |A_i|^2$.

Refer to the theorem 1 is the statement in the next lemma.

Lemma 1. *Define two independent ricean random variables α_i and $\beta_{i,k}$ respectively. The mean and variance for N element RIS are denoted as follows:*

$$E[\alpha_i \beta_{i,l}] = \mu_1 = \mu = N \sqrt{\frac{\pi}{2}} S \mathcal{L}_{1/2}(-\nu) \quad (7)$$

$$VAR[\alpha_{i,l} \beta_i] = \sigma^2 = N \left[A^2 + S^2 - \frac{\pi S^2}{2} \mathcal{L}_{1/2}^2(-\nu) \right]. \quad (8)$$

μ is the first raw moments of $\mu'_k = \sigma^k 2^{k/2} \Gamma(1 + k/2) \mathcal{L}_{k/2}(-\nu)$ and σ^2 is the second central moments of μ'_k [32].

Proof. To prove it, we refer to the closed form approximations to Ricean sum distribution proposed by J.Hu at all [30]. First, define probability density function (PDF) for Ricean fading with parameter A (non centrality parameter) and S (Gaussian scatter components) from 2:

$$f_{R_i}(x_i) = \frac{x_i}{S_i^2} \exp\left(-\frac{x_i^2 + A_i^2}{2S_i^2}\right) I_0\left(\frac{x_i A_i}{S_i^2}\right), \quad x_i \geq 0, \quad (9)$$

where x_i is the amplitude of N RIS elements independent RV's. Then, we integrate ν factor, the PDF becomes:

$$f_{R_i}(x_i) = \frac{2x_i(\nu_i + 1)}{\rho_i} \exp\left(-\nu_i - \frac{(\nu_i + 1)x_i^2}{\rho_i}\right) \times I_0\left(2x_i\sqrt{\frac{\nu_i(\nu_i + 1)}{\rho_i}}\right). \quad (10)$$

The envelope of the signal component at combiner output of an equal gain combining (EGC) [33] is defined as $Z = \sum_{i=1}^N R_i$. The RV S in Eq. (6), has a noncentral chi-square distribution with $2N$ degrees of freedom. Therefore, with the expressed PDF condition in Eq. (9) and [28], the PDP of SR denotes by:

$$f_{SR}(x) = \frac{1}{2S^2} \left(\frac{x}{A^2}\right)^{\frac{N-1}{2}} \exp\left(-\frac{x + A^2}{2S^2}\right) \times I_{N-1}\left(\frac{\sqrt{x}A}{S^2}\right), \quad x \geq 0, \quad (11)$$

where $A^2 = \sum_{i=1}^N A_i^2$ and $I_{N-1}(\cdot)$ is the $(N-1)^{th}$ order modified Bessel Function of the first kind. Obtain a CDF by integrating Eq. (11) and resulting in:

$$F_{SR}(x) = 1 - Q_N\left(\frac{A}{S}, \frac{\sqrt{x}}{S}\right), \quad (12)$$

where RV SR represents a envelope of the signal component at the combiner output of a MRC system in Ricean fading [34]. Thus the approximation for the CDF of RV SR , in respect to N elements is defined as:

$$F_N = 1 - Q_N\left(\frac{b}{c_1}, \frac{t}{c_2}\right), \quad (13a)$$

$$b = \sqrt{\frac{N\rho\nu}{\nu + 1}}, \quad (13b)$$

where c_1 and c_2 are the constants, optimally obtained from [35] and t contains a normalized arguments, defined as $\frac{x}{\sqrt{N}}$. Finally, the PDF of SR was obtained by first derivative order of CDF Eq. (13) and it is expressed as follows:

$$f_N \triangleq \frac{dF_N(t)}{dt} = \frac{t^N}{c_2^2} \left(\frac{c_1}{c_2 b}\right)^{N-1} \exp\left(-\frac{t^2}{2c_2^2} - \frac{b^2}{2c_1^2}\right) \times I_{N-1}\left(\frac{tb}{c_1 c_2}\right). \quad (14)$$

Based on proposed CDF and PDF as denoted in Eq. (13) and Eq. (14) respectively, we can conclude that the number of N is impacting the skewness of both density function. However, to make the analytical approach and simulation result is match, we follow a central limit theorem where the number N is far greater than 1 ($N \gg 1$). Additionally, with exact PDF approximation based on CDF, we can prove that number of elements are affecting mean and variance as mentioned in Eq. (7) and Eq. (8), respectively. ■

Lemma 2. The total of channel power ρ for dual hop Ricean fading channel is represented as:

$$\rho = A^2 + A^2 + 2S^2 \quad (15)$$

Proof. Fading channel H and G are consist of complex numbers, and both of it are Independent and identically distributed (IID) RV with mean $[A_H, A_G]$ respectively, and standard deviation S . Here, to prove it, we utilize the ricean RV identity as follows :

$$V \triangleq \sqrt{H^2 + G^2} \quad (16a)$$

$$E\{H^2\} = S^2 + A_H^2 \quad (16b)$$

$$E\{G^2\} = S^2 + A_G^2 \quad (16c)$$

$$E\{(H - A_H)^2\} = E\{(G - A_G)^2\} = S^2 \quad (16d)$$

$$E\{V^2\} = E\{H^2\} + E\{G^2\} \quad (16e)$$

$$E\{V^2\} = 2S^2 + A_H^2 + A_G^2. \quad (16f)$$

■

Here, it was predefined that $[\alpha_i, \beta_{i,l}]$ were non-central chi square RV with a single degree of freedom (DoF) [28], therefore, $MGF(\omega) = \left(\frac{1}{\sqrt{1-2\sigma^2\omega}}\right) \exp\left(\frac{\mu^2\omega}{1-2\sigma^2\omega}\right)$ whereas ω denoting the ω^{th} moment of MGF. With the derived $[\mu, \sigma^2]$, thus the formulation of MGF with a certain P/N_0 is written as:

$$MGF(\omega) = \sqrt{\frac{1}{1 - \frac{\omega N [A^2 + S^2 - \frac{\pi S^2}{2} \mathcal{L}_{1/2}^2(-\nu)] \rho P}{N_0}}} \exp\left(\frac{\frac{\omega N^2 \pi S^2 \mathcal{L}_{1/2}^2(-\nu) \rho P}{2} \frac{\rho P}{N_0}}{1 - \frac{\omega N [A^2 + S^2 - \frac{\pi S^2}{2} \mathcal{L}_{1/2}^2(-\nu)] \rho P}{N_0}}\right). \quad (17)$$

Since the proposed system is utilizing M -ary Modulation and the emitted signal transmitted through the Rayleigh fading channel, suppose M -PSK modulation. The MGF for analytical symbol error rate (SER) becomes [36]:

$$Pe_{\gamma_s} = \frac{1}{\pi} \int_0^{\frac{(M-1)\pi}{M}} MGF_{\gamma_s}\left(-\frac{\sin(\pi/M)}{\sin^2 \phi}\right) d\phi. \quad (18)$$

Therefore, by operating substitution (17) and (18), the SER for RIS aided OMA system is expressed as:

$$\overline{P_e} = \frac{1}{\pi} \int_0^{\frac{(M-1)\pi}{M}} \left(\frac{1}{1 + \frac{N [A^2 + S^2 - \frac{\pi S^2}{2} \mathcal{L}_{1/2}^2(-\nu)]^2 \sin(\pi/M) \rho P}{\sin^2 \xi N_0}} \right)^{\frac{1}{2}} \exp\left(\frac{-\frac{N^2 \pi S^2 \mathcal{L}_{1/2}^2(-\nu) \sin(\pi/M) \rho P}{2 \sin^2 \xi} \frac{\rho P}{N_0}}{1 + \frac{N [A^2 + S^2 - \frac{\pi S^2}{2} \mathcal{L}_{1/2}^2(-\nu)]^2 \sin(\pi/M) \rho P}{\sin^2 \xi N_0}}\right) d\xi. \quad (19)$$

Further, for deriving analytical SER of RIS-NOMA user 2 where the user is treating y_1 as noise (5) and implementing ML detector to detect its symbol is initialized by determining the decision boundary [29]. According to ML decision boundary

with QPSK modulation, user 2 decode symbol in signal constellation (SC) IQ plane within boundaries $\Re\{y_2\} < 0$ or $\Re\{y_2\} \geq 0$ and $\Im\{y_2\} < 0$ or $\Im\{y_2\} \geq 0$, where $\Re\{\cdot\}$ $\Im\{\cdot\}$ denoting in-phase and quadrature component of SC received signal y_2 respectively. Therefore, the bit error probability (BEP) for the in-phase component of SC in Q-function form is denoted by:

$$P_{e2,\Re} = \frac{1}{2} \left[Q(\sqrt{\Upsilon_a^2 \zeta_2}) + Q(\sqrt{\Upsilon_b^2 \zeta_2}) \right], \quad (20)$$

with $\Upsilon_a = \sqrt{\varepsilon_2} + \sqrt{\varepsilon_1}$; $\Upsilon_b = \sqrt{\varepsilon_2} - \sqrt{\varepsilon_1}$.

Further, Υ is denoting the baseband SC symbols for users, ε_2 is indicating user 2 PA $\sqrt{P(1-\alpha_1)}$ while ε_1 denoting user 1 PA $\sqrt{P\alpha_1}$. $Q(\cdot)$ is representing the Gaussian error performance $Q(x) = (1/\sqrt{2\pi}) \int_x^\infty \exp(-u^2/2) du$. Additionally, ζ_2 represents the user 2 channel gain and noise power $\frac{P_2}{N_0}$ since the received signal y_2 . Similarly, with the in-phase component of SC, the BEP for SC of quadrature component had the same formulation with (20). Therefore $P_{e2} = (P_{e2,\Re} + P_{e2,\Im})/2$. At this point, it is clear that the BEP of user 2 is multiplied by 0.5 due to the symmetrical QPSK SC symbols. Finally, the exact SER for user 2 based on BEP (20) is formulated as in (21), where $A(L_2) = \sigma^2 \sin(\pi/M)$, $B(L_2) = -\mu^2 \sin(\pi/M)$ and $A(L_i)$ and $B(L_i)$ denoting the multiplication between ω^{th} moment of MGF with μ and σ^2 in Eq. (7) and Eq. (8), respectively.

However, the BEP of user 1 where the user is performing SIC was defined as $P_{e1} = P_{e1}(e | \text{correct } b_2) + P_{e1}(e | \text{error } b_2)$. In other words, the P_{e1} is obtained by adding the BEP under condition b_2 (bit user 2) detected correctly and erroneously. Thus, in the form of Q-function, the BEP of the bit that detected correctly at the in-phase component SC is:

$$P_{e1,\Re}(e | \text{correct } b_2) = \frac{1}{2} [2Q(\sqrt{\varepsilon_1 \zeta_1}) - Q(\sqrt{\Upsilon_a^2 \zeta_1})], \quad (23)$$

where ζ_1 is the notation of the user channel 1 $\frac{P_1}{N_0}$. In a different manner with the BEP under condition first bit detected correctly, then for the condition when b_2 detected erroneously is expressed as:

$$P_{e1,\Re}(e | \text{error } b_2) = \frac{1}{2} [Q(\sqrt{\Upsilon_b^2 \zeta_1}) - Q(\sqrt{\Upsilon_c^2 \zeta_1}) + Q(\sqrt{\Upsilon_d^2 \zeta_1})], \quad (24)$$

with $\Upsilon_c = 2\sqrt{\varepsilon_2} - \sqrt{\varepsilon_1}$; $\Upsilon_d = 2\sqrt{\varepsilon_2} + \sqrt{\varepsilon_1}$.

Similarly, for calculating BEP at SC $\Im$ component by following (23) and (24) thoroughly. Additionally, it is identical with the calculation of P_{e2} , 0.5 is attached to the $P_{e1,\Re}$ to calculate the average BEP of both in-phase and quadrature SC, hence SER calculation for user 1 is formulated in (22).

B. Problem Formulation

The principal work of asymmetrical REA for RIS NOMA users is aiming at the same order of SER among downlink users. In simple words, the proposed system attempts to choose the ideal amount of elements allocation for each users until

Figure 2. (a) The result of $\tau(r)$ distribution for different transmit power and element ratio. (b) The simulation and analytical result of equilibrium point (pointed by arrow) with $d = 0.1$

it had an approximately similar SER $P_{e2} \approx P_{e1}$. Thus, in the case of two users SER value is modelled as $|P_{e2} - P_{e1}| < 0.1$. Then, the optimization problem of REA RIS-NOMA with finite N is defined as:

$$\begin{aligned} & \text{Minimize}_{\{\alpha_1, N, \Omega_i\}} \quad \tau = |\overline{P_{e2}}(L_2) - \overline{P_{e1}}(L_1)| \\ & \text{Subject to} \quad \tau < 0.1, \quad \tau = (10^{-2}, 10^{-3}), \\ & \quad e^{j\phi_i} = (0, 2\pi], \\ & \quad L_1 \neq L_2, \quad L_1 < L_2, \quad N = L_1 + L_2, \\ & \quad 0 \leq \alpha_1 \leq 0.5, \quad \alpha_2 = 1 - \alpha_1. \end{aligned} \quad (25)$$

where τ is the difference between SER user 1 and 2, moreover, the proposed system is aiming the high similarity for both user ($\tau < 0.1$) therefore the τ value is in between $10^{-2} \leq \tau \leq 10^{-3}$. The other important thing to notice that these problem formulation constraints is the fixed PA for both users (α_1). $e^{j\phi_i}$ denoting the discrete RIS response phase of i^{th} element [37].

C. Problem solution

In order to solve Eq. (25), we define τ as an objective function, $\tau(r)$ as a scalar value of objective function with parameter r , and S as a feasible value of interest of r . However, solving the problem with the conventional exhaustive search Optimization of number element has previously proposed in [38] where the monotonically increasing number of elements resulting in unimodality of energy consumption. Therefore, we inspect first the distribution of objective function $\tau(r)$ with ratio test $r \in [0.1, 0.1 + 1d, 0.1 + 2d, \dots, 1]$, where $d = 0.01$ as a interval. Fig. 2(a) shows that the distribution of $\tau(r)$ is decreasing first then increasing again after one point reached. The same behaviour was shown for different P where $P \in [-24, -26, 28, -30]$. Fig. 2(b) is pointing out that by using the same P and N for both user will be resulting in equilibrium point for a certain value of r (pointed by arrow). Therefore, based on the figure, we define that $\tau(r)$ is unimodal and the definition is written as follows:

Definition 1 (Unimodal Function). A function $\tau(r)$ is unimodal on the interval $r - d \leq r \leq r + d$ if and only if it is

$$\overline{P_{e2}} = \frac{1}{2\pi} \int_0^{\frac{3\pi}{4}} \left(\frac{1}{1 + \frac{A(L_2)\Upsilon_a^2\zeta_2}{\sin^2 \xi}} \right)^{\frac{1}{2}} \exp \left(\frac{B(L_2)\Upsilon_b^2\zeta_2}{1 + \frac{A(L_2)\Upsilon_a^2\zeta_2}{\sin^2 \xi}} \right) + \left(\frac{1}{1 + \frac{A(L_2)\Upsilon_b^2\zeta_2}{\sin^2 \xi}} \right)^{\frac{1}{2}} \exp \left(\frac{B(L_2)\Upsilon_a^2\zeta_2}{1 + \frac{A(L_2)\Upsilon_b^2\zeta_2}{\sin^2 \xi}} \right) d\xi. \quad (21)$$

$$\begin{aligned} \overline{P_{e1}} = \frac{1}{2\pi} \int_0^{\frac{3\pi}{4}} & 2 \left(\frac{1}{1 + \frac{A(L_1)\varepsilon_1\zeta_1}{\sin^2 \xi}} \right)^{\frac{1}{2}} \exp \left(\frac{B(L_1)\varepsilon_1\zeta_1}{1 + \frac{A(L_1)\varepsilon_1\zeta_1}{\sin^2 \xi}} \right) - \left(\frac{1}{1 + \frac{A(L_1)\Upsilon_a^2\zeta_1}{\sin^2 \xi}} \right)^{\frac{1}{2}} \exp \left(\frac{B(L_1)\Upsilon_a^2\zeta_1}{1 + \frac{A(L_1)\Upsilon_a^2\zeta_1}{\sin^2 \xi}} \right) + \\ & \left(\frac{1}{1 + \frac{A(L_1)\Upsilon_b^2\zeta_1}{\sin^2 \xi}} \right)^{\frac{1}{2}} \exp \left(\frac{B(L_1)\Upsilon_b^2\zeta_1}{1 + \frac{A(L_1)\Upsilon_b^2\zeta_1}{\sin^2 \xi}} \right) - \left(\frac{1}{1 + \frac{A(L_1)\Upsilon_c^2\zeta_1}{\sin^2 \xi}} \right)^{\frac{1}{2}} \exp \left(\frac{B(L_1)\Upsilon_c^2\zeta_1}{1 + \frac{A(L_1)\Upsilon_c^2\zeta_1}{\sin^2 \xi}} \right) + \\ & \left(\frac{1}{1 + \frac{A(L_1)\Upsilon_d^2\zeta_1}{\sin^2 \xi}} \right)^{\frac{1}{2}} \exp \left(\frac{B(L_1)\Upsilon_d^2\zeta_1}{1 + \frac{A(L_1)\Upsilon_d^2\zeta_1}{\sin^2 \xi}} \right) d\xi. \end{aligned} \quad (22)$$

monotonic on either side of the single optimal point r^* in the interval. In other words, if r^* is the single minimum point of $\tau(r)$ in the range $r - d \leq r \leq r + d$, then $\tau(r)$ is unimodal on the interval if and only if for any two points r_1 and r_2

$$\begin{aligned} r^* \leq r_1 \leq r_2 \quad & \text{implies that} \quad \tau(r^*) \leq \tau(r_1) \leq \tau(r_2) \\ r^* \geq r_1 \geq r_2 \quad & \text{implies that} \quad \tau(r^*) \leq \tau(r_1) \leq \tau(r_2) \end{aligned}$$

Definition 2 (Global Minimum). A function $\tau(r)$ defined on a set S attains its global minimum at a point $r^{**} \in S$ if and only if

$$\tau(r^{**}) \leq \tau(r) \quad \forall r \in S$$

Definition 3 (Local Minimum). A function $\tau(r)$ defined on S has a local minimum (relative minimum) at a point $r^* \in S$ if and only if there exist an $\varepsilon > 0$ such that

$$\tau(r^*) \leq \tau(r)$$

for all $r \in S$ satisfying $|r - r^*| < \varepsilon$

Remark. Based on the definition, Algorithm 1 was constructed with remarks as follows:

- 1) The objective function is unimodal, thus the local minimum automatically becomes the global minimum.
- 2) Special case with $d \ll 0.01$, more than one local minimum are possible and the global minimum can only found by locating all local optima and selecting the minimum one, written as:

$$r^{**} = \min\{[r_1^*, r_2^*, \dots, r_s^*] \in S\}, \quad (26)$$

where S is a set of local minimum.

- 3) The unimodality of $\tau(r)$ can decrease the exhaustive search iteration by 50%.

In Algorithm 1 the input is total elements N , r , d , P , ν_k factor for each user and α . On this algorithm we following the PD-NOMA rules where user with lower PA (CCU) needs more N than CEU. Therefore, exhaustive search starts from $r = 0.5$ and keeps increasing until $r = 1$. The initialization part started by calculating REA L_1 and L_2 , respectively. Then calculate analytical $\tau(r)$ with SER analytical approach from Eq. (21)

and Eq. (22), respectively. Afterward, calculate $\tau(r + d)$ and compare it with $\tau(r)$. If the condition is $\tau(r + d) > \tau(r)$ then update $S = \tau(r)$. However, if the condition is $\tau(r + d) < \tau(r)$ then update $S = \tau(r + d)$. Afterward, the proposed system do an iteration with parameter r until $r = 1$. Finally, to calculate r^{**} utilize the equation Eq. (26).

Algorithm 1: REA Calculation

Input: N ; d ; P ; ν_k ; $r = r_{CCU} = 0.5$; $r_{CEU} = 1 - r_{CCU}$; $[\alpha_1, \alpha_2]$; ρ

Output: Suitable r^{**}

Initialization:

Calculate $L_1 = \lfloor N \times r_{CCU} \rfloor$ and $L_2 = \lceil N \times r_{CEU} \rceil$.

Calculate analytical $\tau(r)$

Determine r^{**} :

while $r < 1$ **do**

$r_{CCU} = r + d$

$r_{CEU} = 1 - r_{CCU}$

Calculate L_1 and L_2

Calculate Analytical SER $[P_{e1}(L_1), P_{e2}(L_2)]$

Calculate $\tau(r + d)$

if $\tau(r + d) > \tau(r)$ **then**

Set $\tau(r^*) = \tau(r)$

else

Set $\tau(r^*) = \tau(r + d)$

end if

Update S element with $\tau(r^*)$

end while

$r^{**} = \min\{S\}$

IV. NUMERICAL RESULTS

The other parameters which were considered for Algorithm 1 and for the purpose to obtain r^{**} are $\alpha_1 = 0.3$; $\alpha_2 = 0.7$, $P \in [-60 \leq \dots \leq -14] \text{ dB}$, $N_0 = 1$, $d = 0.001$, ρ from Eq. (2), 10^5 monte carlo simulation, $N = 128, 256, 512$. Additionally, α value was determined based on optimal PA for NOMA 2 users [39]. In order to calculate BER, the conversion

Table I
THE RESULT OF ALGORITHM 1 FOR DIFFERENT P (dB), N , AND
 $\nu_k = [\nu_{CCU}; \nu_{CEU}]$

P (dB)	[$\nu = 8$ dB; 5 dB]			[$\nu = 6$ dB; 4 dB]		
	$N = 128$	256	512	128	256	512
-60	0.69	0.69	0.74	0.69	0.7	0.73
-56	0.69	0.72	0.79	0.69	0.72	0.79
-52	0.71	0.76	0.8	0.71	0.76	0.8
-48	0.74	0.8	0.74	0.74	0.81	0.75
-44	0.79	0.77	0.68	0.79	0.78	0.69
-40	0.8	0.72	0.61	0.8	0.72	0.62

Figure 3. Simulation and Analytical BER result for CCU and CEU user over different value of ν factor.

Figure 4. Average user BER result comparison between BER Aware system and fair allocation over different value of ν factor

is $\approx \overline{P_e}/\log_2(M)$ [40] and $M = 4$ QPSK modulation. The ν factor for dual hop channel are $\nu_H = \nu_G = \nu_k$.

The result of Algorithm 1 is presented in Table I with determined transmit SNR P . Here, we can see that the different number of ν factor of users had, is not giving a significant impact to the result of r^{**} . Therefore, it is feasible to find the suitable ratio with various number of ν factor. Based on this table, it can be inferred that the r^{**} is increased to 0.8 to a certain P then decreasing again for higher P value.

Fig. 3 and Fig. 4 show the result of BER with various ν_{test} factor where $\nu_{test} = \nu \in [1, 2, \dots, 10]$. On this figure, we set parameter $\nu_{test} = \nu_{CEU} = \nu_{CCU}$ for fair comparison. Fig. 3 shows that REA obtained from Algorithm 1 (trained with $\nu_k, \nu_{CCU} = 6$ dB, $\nu_{CEU} = 4$ dB) is feasible throughout

every value of ν . It can be inferred that BER for both user is similar for higher value of ν factor as well as the analytical and simulation. In this figure, the impact of higher N is resulting in better BER with the same transmit power level ($P = 28$ dB). We also investigate the benefit of BER-aware in Fig. 4 compare to fair elements allocation. Define that $\nu = 3$ and BER is average users BER $mean\{P_{eCCU}, P_{eCEU}\}$. With transmit power ($P = -24$ dB) for proposed system is 6.74×10^{-3} whilst the fair allocation is 4.22×10^{-3} , respectively. $P = -26$ dB is 7.87×10^{-4} and 4.23×10^{-4} . $P = -28$ dB is 3.25×10^{-5} and 1.1×10^{-5} .

Fig. 5 shows a result of BER over different transmit SNR P with r^{**} obtained from Table I. Fig. 5(a) ($\nu = 6, 4$) shows the suitable ratio r for transmitting SNR. Targeting the BER $< 10^{-3}$ for $N = 128$ is approximately acquired at $P = -20$ dB while $N = 256$ and $N = 512$ were $P \approx -24$ dB and $P \approx -28$ dB respectively. Thus, it can be concluded that increasing the N elements two times higher than before is lowering the minimum requirement of P by ≈ -4 dB in order to achieve the targeted BER. Moreover, in comparison with the proposed system, Fig. 5(b) shows the BER for both users with $r = 0.5$ for all P . On this figure, with the total available elements $N = 128$, to achieve targeted BER for CCU is improving ≈ 2 dB while the trade-off for CEU is the degradation value of P as far as ≈ 3 . Here, it can be seen that the more PA that CEU had, means more degradation of CCU BER. Hence increasing number of RIS elements is required to mitigate the worst channel condition. Fig. 5(c) shows that the system with proposed system BER aware, the average user BER is improving. With the different number of elements N the improvement remains the same. This figure shows that a same number of N and P giving a better performance. The low value of P is not giving a significant improvement for proposed system BER-aware. However, for higher P is giving a better performance. Therefore, define $N = 512$ with $P = -30$ dB, the error performance for fair r is on order 10^{-4} while BER-Aware system is 10^{-5} . This situation also applied for $N = 128$ with $P = -18$ dB and $N = 256$ with $P = -24$ dB, respectively.

V. CONCLUSIONS

In this letter, a novel BER-aware reflecting element allocation (REA) in RIS was proposed to maintain the same order of BER among paired RIS-NOMA users. Both analytic and simulation results showed that the proposed algorithm could achieve the same order of BER among multiple RIS-NOMA users. Moreover, it was shown that the proposed algorithm could be applicable to all ν factor. The proposed algorithm could enhance the average BER among multiple RIS aided NOMA users. Optimal element allocation in uplink RIS aided NOMA network with specific target BER could be one of future works.

Figure 5. The simulation result of suitable ratio r^{**} . (a) The result of r^{**} over different value of P and N . (b) Comparison between proposed system and fair REA for different number of N . (c) The comparison of average users BER between BER aware and fair REA.

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