Comprehensive Analysis On A New Type VR-Resolver with Toroidal Windings Under Healthy and Eccentric Cases

Peyman Naderi , Reza Ghandehari

Abstract—A novel analysis for a new type Variable Reluctance Resolver (VR-Resolver) with toroidal windings is presented in this work. The resolver with various windings configurations as well as different rotor structures is considered for analysis. Position error of the resolvers in both healthy and eccentric cases are studied, where the effect of the eccentricity fault is analyzed for the resolvers with various windings configuration and rotor shapes. Hence, the analysis of the manufacturing tolerance due to the eccentric rotor is obtained by several analyses. Noticing the Magnetic Equivalent Circuit (MEC) method's flexibility as well as in regards to the analysis of the several structures, a well-known flexible MEC-based method with adjustable accuracy is considered for modeling. Hence shorter processing time and more flexibility compared to Finite-Element-Method (FEM) are needed. Finally, the effectiveness of the presented works is proved by the performed validation via both FEM and experimental results.

Index Terms—Eccentricity fault, Magnetic Equivalent Circuit, toroidal windings, variable reluctance resolver.

I. INTRODUCTION

VARIABLE reluctance resolvers (VR-Resolver) is an accurate position sensor that is necessary in many industries such as inverter-driven electric machines, robotic, and vehicular technologies [1]. Although optical encoders are well-known position sensors, they have important disadvantages such as working in environments with high temperature, smoke, and dust. Moreover, the encoders have speed limitation, where big position error is obtained in high-speed applications [2]. Resolvers are generally categorized into two types. The first type is Wound Rotor Resolver (WR-Resolver) [3]–[7] that have a connected voltage source by slip rings, and a rotary transformer is necessary [6]. In [6], its accuracy improvement by the arrangement of the windings is addressed, and in [7] the accuracy is analyzed under different installation

configurations. Although in [5] eliminating the core of the rotary transformer a new type VR-Resolver is presented, that is more costly [8] compared to the second type which is Variable Reluctance Resolver (VR-Resolver). In this type, the rotor is a solid magnetic material with m number of rotor saliency (called $m - X$ VR-Resolver), where the windings are located on the stationary part. The rotor shape of the VR-Resolvers is considered based on the sinusoidal permeance [9], [10]. Hence various rotor shapes [9], [11] and its optimization [2], [10], [11] are investigated in some researches. Noticing the VR-Resolvers benefit, there are some new researches which have focused on its development. Hall Effect Sensor (HS) usage in the VR-Resolvers is one of the new investigated techniques which is presented in [12]–[14]. In [12], [13] both excitation winding and PMs are used, where Back-Emf enhancement is performed by PMs and various VR-Resolvers are presented. In [14] Permanent Magnets (PMs) are used as the excitation and the position calculation is based on the measured fluxes by the HSs. Although the absence of the high-frequency excitation signal is known as an advantage, the HSs' sensitivity to the external magnetic fields and environment temperature are its disadvantages. The axial flux VR-Resolvers are another new type of resolvers that are investigated by researchers. In [15] the axial flux VR-Resolver is addressed, where a fast computational method is presented for its modeling. A novel VR-Resolver with toroidal windings is analyzed in [16], where two windings configurations are analyzed and the manufacturing tolerance of the resolver with both configurations is studied. Moreover, windings arrangement simplification is presented in [17] and position error analysis is done under the simplified case. Noticing the performed researches, there are three considered methods for the resolvers modeling. The Finite Element Method (FEM) is a powerful method that has been used many times but has some disadvantages such as long processing time and intensive processing. In [18], challenges of the FEM for resolvers modeling is illustrated and necessary considerations are analyzed. Magnetic Equivalent Circuit (MEC) is another choice that is more flexible compared to FEM, where less computational time can be obtained by that. This method is used recently for resolvers modeling thanks to its capability, where the WR and VR types are analyzed by that in [5] and [15], [19]–[21], respectively. Moreover, PM-Resolver model by MEC is addressed in [14], and the VR-Resolver with toroidal windings is modeled by

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that in [16]. In the presented works, the linear core is considered and the model accuracy is not adjustable. Although the resolvers operating point is low in the normal case, that may be near to the knee-point of the B-H curve in intensive eccentricity fault or with a low-frequency excitation in low-speed applications. Hence, the saturation effect is suitable for an integrated model of the resolvers.

As is illustrated previously, the VR-Resolver with toroidal windings is a new type of resolver which is presented firstly in [16]. In the presented work, two types of windings configurations are analyzed and the manufacturing tolerance is investigated for the resolver with 5 rotor saliencies (5-X VR-Resolver) by a simple MEC technique. This paper is an extended version of the mentioned work, where its novelties are summarized as follows:

- A flexible MEC-based method with adjustable accuracy is used for modeling.
- In addition to the presented windings configurations in [16], an additional configuration is analyzed.

- The position error under four different healthy rotor types are obtained.
- The position error under healthy and eccentric rotors are analyzed, where different windings configurations, as well as different rotor shapes, are considered.

Notably, since the static eccentricity has more impact on the resolvers' performance compared to dynamic type [20], [21], the static eccentricity is considered for the analysis.

II. ANALYZED RESOLVERS

The investigated resolvers are presented in Fig. 1, where the considered windings configurations and the rotor shapes are shown. The windings direction and its number of turners per slot are illustrated in Table I, where W_{sin} , W_{cos} , and W_{ext} denotes sine, cosine, and excitation windings, as well as N_s and N_e , are the maximum number of turn per toroidal coil of the signals and excitation windings, respectively.

TABLE I
CONSIDERED WINDINGS CONFIGURATION

WT1	S1	S2	S3	S4	S5	S6	S7	S8	S9	S10	S11	S12
W_{sin}	0	$N_s \uparrow$	0	0	0	$N_s \downarrow$	0	$N_s \downarrow$	0	0	0	$N_s \uparrow$
W_{cos}	0	0	$N_s \uparrow$	0	$N_s \uparrow$	0	0	0	$N_s \downarrow$	0	$N_s \downarrow$	0
W_{ext}	$N_e \uparrow$	0	0	$N_e \downarrow$	0	0	$N_e \uparrow$	0	0	$N_e \downarrow$	0	0
WT2	S1	S2	S3	S4	S5	S6	S7	S8	S9	S10	S11	S12
W_{sin}	$N_s \uparrow$	$N_s \downarrow$	0	0	$N_s \downarrow$	$N_s \uparrow$	$N_s \downarrow$	$N_s \uparrow$	0	0	$N_s \uparrow$	$N_s \downarrow$
W_{cos}	0	$N_s \downarrow$	$N_s \uparrow$	$N_s \downarrow$	$N_s \uparrow$	0	0	$N_s \uparrow$	$N_s \downarrow$	$N_s \uparrow$	$N_s \downarrow$	0
W_{ext}	$N_e \uparrow$	0	$N_e \downarrow$	$N_e \uparrow$	0	$N_e \downarrow$	$N_e \uparrow$	0	$N_e \downarrow$	$N_e \uparrow$	0	$N_e \downarrow$
WT3	S1	S2	S3	S4	S5	S6	S7	S8	S9	S10	S11	S12
W_{sin}	0	$0.5N_s \uparrow$	$0.87N_s \uparrow$	$N_s \uparrow$	$0.87N_s \uparrow$	$0.5N_s \uparrow$	0	$0.5N_s \downarrow$	$0.87N_s \downarrow$	$N_s \downarrow$	$0.87N_s \downarrow$	$0.5N_s \downarrow$
W_{cos}	$N_s \uparrow$	$0.87N_s \uparrow$	$0.5N_s \uparrow$	0	$0.5N_s \downarrow$	$0.87N_s \downarrow$	$N_s \downarrow$	$0.87N_s \downarrow$	$0.5N_s \downarrow$	0	$0.5N_s \uparrow$	$0.87N_s \uparrow$
W_{ext}	$N_e \uparrow$	$N_e \downarrow$	$N_e \uparrow$	$N_e \downarrow$	$N_e \uparrow$	$N_e \downarrow$	$N_e \uparrow$	$N_e \downarrow$	$N_e \uparrow$	$N_e \downarrow$	$N_e \uparrow$	$N_e \downarrow$

Fig. 1. Investigated resolvers with dimensions, various windings configurations, and rotor types, where S_i denotes i^{th} slot

 Fig. 2. Considered MEC model with a sample accuracy ($n_{f1} = 3$, $n_{f2} = 2$, $n_{f3} = 12$)

 TABLE II
NAMING OF THE ANALYZED RESOLVERS WITH TOROIDAL WINDINGS

		Windings Configuration (Stator)		
		WT1	WT2	WT3
Rotor	1-X	W1R1	W2R1	W2R1
	3-X	W1R3	W2R3	W3R3
	5-X	W1R5	W2R5	W2R5
	7-X	W1R7	W2R7	W3R7

It is notable that just two first windings configurations (WT1 and WT2) with a 5-X rotor are analyzed in [16] for healthy case, where all the structures are analyzed in this paper under both healthy and eccentric cases in this paper. In regards to Fig. 1, there are 12 possible structures for the analyzed VR-Resolvers, where their naming is introduced in Table II.

III. MEC-BASED MODEL OF THE RESOLVER

Since there are 12 cases for analysis, an 11-zone MEC-based technique is considered for simulations thanks to its flexibility and shorter processing time compared to FEM.

A. Considered MEC

The considered MEC-based model is flexible to the model of all rotor/stator structures with adjustable accuracy, where the number of flux tubes (FLs) in the sensitive zones can be adjusted by the user. The resolver structure and its accuracy can be adjusted by the number of FLs as written in Table III, where a sample accuracy ($n_{f1} = 3$, $n_{f2} = 2$, and $n_{f3} = 12$) is shown in Fig. 2. The R_{ij} and ϕ_{ij} denotes reluctance and flux of j^{th} FL in i^{th} zone, respectively. Each FL is modeled by Eq. (1), where li_j^c and li_j^g denotes length of the core and air parts of the j^{th} FL in i^{th} zone, respectively.

 TABLE III
SELECTIVE PARAMETERS OF MEC MODEL FOR VR-RESOLVER

Properties of m-X VR-Resolver	
Symbol	Definition
n	Number of stator teeth
m	Number of rotor poles (called $m - X$ resolver)
Number of flux tubes per each part	
Symbol	Definition
n_{f1}	Number of flux tubes per mover tooth
n_{f2}	Number of flux tubes per mover slot
n_{f3}	Number of flux tubes per rotor pole
Total number of flux tubes per each zone	
Zone number	Number of flux tubes in the zone
1,2,3,4,6	$2n$
5,7	$s = n \times (n_{f1} + n_{f2})$
8,9,10,11	$r = m \times n_{f3}$

$$R_{ij} = \frac{li_j^g}{\mu_0 Ai_j} + \frac{li_j^c}{\mu_0 \mu_r (Bi_j) Ai_j} \quad (1)$$

In Eq. (1), Bi_j and Ai_j are flux density and effective area of the FL, respectively. Moreover, $\mu_r(Bi_j)$ is defined as a functional relative permeability of the FL to model of the B-H curve nonlinearity. It is a keynote that normally the resolvers operating points are low thanks to high-frequency excitation, but that may be investigatable in some cases such as intensive eccentricity. Additionally, for low-speed applications, low-frequency excitation is possible. Hence, the operation point may be considerable in low-frequency excitation which is possible for low-speed applications. Therefore, modeling the saturation effect is necessary for an integrated model.

$$\mathbf{A}(\mathbf{X}(t))\mathbf{X}(t) = \mathbf{B}(V_e(t), V_e(t - \Delta t), \mathbf{X}(t - \Delta t)) \rightarrow \mathbf{X}(t) = \mathbf{A}(\mathbf{X}(t))^{-1} \mathbf{B}(V_e(t), V_e(t - \Delta t), \mathbf{X}(t - \Delta t)) \quad (2a)$$

$$\begin{bmatrix} \mathbf{MA}s & 0 & 0 & 0 & 0 & 0 & 0 & \Delta t/2\mathbf{R}s & 0 & 0 \\ \mathbf{M}1 & -\mathbf{R}3 & 0 & 0 & 0 & 0 & 0 & 0.5\mathbf{W} & 0 & 0 \\ \mathbf{M}2 & \mathbf{M}3 & 0 & -\mathbf{R}6' & 0 & 0 & 0 & -0.5\mathbf{W} & 0 & 0 \\ 0 & 0 & \mathbf{R}5 & \mathbf{R}6 & -\mathbf{R}7 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\mathbf{R}7 & 0 & 0 & 0 & \mathbf{A}s' & 0 \\ 0 & 0 & 0 & 0 & 0 & \mathbf{M}4 & \mathbf{M}5 & 0 & 0 & -\mathbf{A}r \\ 0 & 0 & 0 & 0 & 0 & -\mathbf{R}9 & \mathbf{M}6 & 0 & 0 & 0 \\ \mathbf{A}s & \mathbf{A}s & -\mathbf{A}45 & -\mathbf{A}46 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\mathbf{I} & -\mathbf{A}s' & 0 & 0 & 0 & 0 & \mathbf{A}s_s & \mathbf{A}s_r \\ 0 & 0 & 0 & 0 & 0 & 0 & -\mathbf{A}r & -\mathbf{A}r & \mathbf{A}r_s & \mathbf{A}r_r \end{bmatrix} \begin{bmatrix} \Phi 1 \\ \Phi 3 \\ \Phi 5 \\ \Phi 6 \\ \Phi 7 \\ \Phi 9 \\ \Phi 11 \\ \mathbf{I}s \\ \mathbf{U}1 \\ \mathbf{U}2 \end{bmatrix} = \begin{bmatrix} \mathbf{C}(t, \Delta t) \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad (2b)$$

There are some well-known equations which are used recently for relative permeability function. The below equation is used in this paper for that which allows having saturation and core nonlinearity effects [20], [21].

$$\mu_r(Bi_j) = \frac{a}{bBi_j^4 - cBi_j^2 + d} \quad (3)$$

B. MEC-Based Equations

Considering $V_e(t)$ as the excitation windings voltage, the $\mathbf{W}$ and $\mathbf{M}$ should be obtained based on the windings configuration. Moreover, by using Δt as the time-step, the whole equations can be written as Eq. (2). Details of the equations and solving procedure are completely illustrated in [20], [21]. Noticing Fig. 2 the $\mathbf{W}$ and $\mathbf{M}$ ($2n \times 3$ and $3 \times 2n$ matrices) in Eq. (2) should be computed as below, where N_{ai} , N_{bi} , and N_{ci} are the number of turns in the Si^{th} slot for $\mathbf{W}_{\sin}$, $\mathbf{W}_{\cos}$, and $\mathbf{W}_{\text{ext}}$, windings, respectively.

$$\mathbf{W} = \begin{bmatrix} N_{a2} & N_{b2} & N_{c2} \\ N_{a2} & N_{b2} & N_{c2} \\ N_{a3} & N_{b3} & N_{c3} \\ N_{a3} & N_{b3} & N_{c3} \\ \vdots & \vdots & \vdots \\ N_{an} & N_{bn} & N_{cn} \\ N_{an} & N_{bn} & N_{cn} \\ N_{a1} & N_{b1} & N_{c1} \\ N_{a1} & N_{b1} & N_{c1} \end{bmatrix}, \mathbf{M}^T = \begin{bmatrix} N_{a1} & N_{b1} & N_{c1} \\ N_{a2} & N_{b2} & N_{c2} \\ N_{a2} & N_{b2} & N_{c2} \\ N_{a3} & N_{b3} & N_{c3} \\ N_{a3} & N_{b3} & N_{c3} \\ \vdots & \vdots & \vdots \\ N_{an} & N_{bn} & N_{cn} \\ N_{an} & N_{bn} & N_{cn} \\ N_{a1} & N_{b1} & N_{c1} \end{bmatrix} \quad (4)$$

It is a keynote that the healthy and eccentric resolvers can be modeled by the considered MEC in Fig. 2. In [20], [21], the modeling technique for healthy and eccentric rotors are presented, where the calculation of the defined air-gap permeance in Eq. (5) based on the air-gap function $\text{gap}(\theta_{\text{act}}, i, \sigma_d, \sigma_s)$ are addressed in details. In Eq. (5), $0 \leq \sigma_d \leq 1$ and $0 \leq \sigma_s \leq 1$ denotes dynamic and static eccentricity indexes.

$$G_m(\theta_{\text{act}}, i, \sigma_s, \sigma_d) = \mu_0 \times l \times \frac{\min(\gamma_s R_{si}, \gamma_r R_{ro})}{\text{gap}(\theta_{\text{act}}, i, \sigma_s, \sigma_d)} \quad (5)$$

Moreover, as illustrated in [20], [21], the rotor shapes are considered by Eq. (6) to produce sinusoidal air-gap permeance.

$$l8_j^g + l10_j^g = \frac{1}{\alpha_1 - \alpha_2 \cos(m(j-1)\gamma_r)} - \delta = fg(j) \quad (6)$$

In Eq. (6) α_1 , α_2 and δ are defined for minimum and maximum air-gap lengths, and γ_s , and γ_r are FLs pitch as shown in Eq. (7).

$$\begin{cases} \frac{1}{\alpha_1 + \alpha_2} = \delta, \frac{1}{\alpha_1 - \alpha_2} = g_{\max} \\ \gamma_s = \frac{2\pi}{s}, \gamma_r = \frac{2\pi}{r} \end{cases} \quad (7)$$

IV. SIMULATION AND ANALYSIS

Simulations are performed for the resolver with considered properties in Table IV, where the resolver dimensions are shown in Fig. 1. In all simulations, 1000 rpm is considered as the rotor speed as well as a 4.2 V_{rms}, 5 kHz voltage is applied to the excitation winding. Simulations are performed for all defined resolvers in Table II, where the bellow scenarios are considered:

A)-Healthy resolvers.

B)-Static eccentric resolvers under $\sigma_s = 0.5$.

TABLE IV
PARAMETERS OF SIMULATED m-X TOROIDAL VR-RESOLVERS

Electrical and Structural Properties					
Parameter	Symb.	Value			
		R1	R3	R5	R7
Number of stator teeth/slots	n	12			
Number of rotor poles	m	1	3	5	7
Number of turns in signal winding	N_s	20			
Number of turns in excitation winding	N_e	20			
Resistance of excitation winding (Ω)	R_e	2			
Dimensions					
Arc of stator teeth (Deg.)	τ_{t1}	18			
Arc of stator slots (Deg.)	τ_{t2}	20			
Height of stator teeth tips (mm)	$h1, h5$	1			
Height of stator slots (mm)	$h2, h4$	2.5			
Height of stator yoke (mm)	$h3$	1			
Minimum length of air-gap (mm)	δ	0.5			
Maximum length of air-gap (mm)	g_{max}	5	2	2	2
stator outer radius (mm)	R_{so}	20			
Stator inner radius (mm)	R_{si}	12			
Rotor outer radius (mm)	R_{ro}	11.5			
Considered Accuracy in the MEC Model					
Number of FLs per stator tooth	n_{f1}	6			
Number of FLs per stator slot	n_{f2}	4			
Number of FLs per rotor poles	n_{f3}	180	60	32	22

The bellow considerations are used to obtain the results:

✧ Since the Resolver to Digital (R/D) converter is not considered in this paper, in all simulations, the Hilbert transformation is used to obtain the envelope signals, where a written m-file in the Matlab software is used for positive/negative half-cycles [21]. In each case, the envelope functions (V_{sine} and V_{cosine}) are obtained from the induced voltages in the signal windings ($V_{W\text{sin}}$ and $V_{W\text{cos}}$). Then, the calculated position and the position error are obtained by Eq. (8) for a m -X resolver.

$$\begin{cases} \theta_{\text{calculate}}(t) = \frac{1}{m} \tan^{-1} \left(\frac{V_{\text{sine}}(t)}{V_{\text{cosine}}(t)} \right) \\ \theta_{\text{error}}(t) = \theta_{\text{actual}}(t) - \theta_{\text{calculate}}(t) \end{cases} \quad (8)$$

✧ Considering 1000 rpm as the rotational speed, 60 ms duration time is necessary for a 360° rotation. In order to avoid the transient performance, 65 ms simulation is performed, where the results for $2.5^{\text{ms}} \leq t \leq 62.5^{\text{ms}}$ are analyzed. The actual initial position is tuned manually as $\theta_{\text{actual}}(0) = -15^\circ$. Hence $\theta_{\text{actual}}(2.5^{\text{ms}}) = 0^\circ$ is obtained for absolute position error analysis.

✧ Considering 5 kHz excitation frequency and 1000 rpm rotational speed, $\Delta t = 1.25 \mu\text{s}$ is used as the time step in the MEC method (see Eq. (2b)) to leads to trustable results.

✧ Noticing Eq. (8), the zero position is calculated by $V_{\text{sine}}(t) = 0$ which should be obtained at $\theta_{\text{actual}}(2k\pi)$ for $k = 0, 1, \dots$. Clearly, the mentioned values can be obtained by an ideal sensor. Hence, the calculated value by a resolver is considered as the absolute position error of the resolver.

A. Healthy Resolvers

Performance of the healthy resolvers is evaluated in this part. Since there are 24 analyzed cases, details of some cases are shown in Figs. 3-5. The induced voltages, envelope functions, and position errors are shown in the figures for W1R1, W1R5, and W3R5 cases. The maximum and absolute position errors, as well as peak of the induced voltages of the healthy resolvers, are listed in Table V under 1000 rpm, where 60 ms duration time denotes a 360° rotation.

Fig. 3. Induced voltages and envelopes functions for healthy W1R1

Fig. 4. Induced voltages and envelopes functions for healthy W1R5

Fig. 5. Induced voltages and envelopes functions for healthy W3R5

TABLE V
RESULTS OF THE ANALYSIS ON THE HEALTHY RESOLVERS

Maximum position error (Deg.)			
W1R1	W1R3	W1R5	W1R7
0.7	0.04	0.04	0.20
W2R1	W2R3	W2R5	W2R7
0.47	Not applicable	0.084	0.037
W3R1	W3R3	W3R5	W3R7
Not applicable	Not applicable	0.03	0.025
Absolute position error (Deg.)			
W1R1	W1R3	W1R5	W1R7
0.0057	0.009	0.0098	0.00095
W2R1	W2R3	W2R5	W2R7
-0.114	–	0.009	0.0002
W3R1	W3R3	W3R5	W3R7
–	–	0.0099	0.0075
Peak of the induced voltages (V)			
W1R1	W1R3	W1R5	W1R7
1.75	1.8	0.27	0.195
W2R1	W2R3	W2R5	W2R7
0.67	–	0.47	0.32
W3R1	W3R3	W3R5	W3R7
–	–	1.75	1.33

Noticing the obtained values, there are three non-applicable cases (W2R3, W3R1, and W3R3) in which non-sinusoidal induced voltages are produced. Moreover, the peak of the induced in the W1R5, W1R7, and W2R7 is low compared to other cases. Hence their signal-to-noise ratio is more considerable for them compared to other cases.

B. Eccentric Resolver

Eccentric resolvers performances are evaluated in this part, where the detail of some cases are shown in Figs. 6-8 as three samples. Noticing the figures, the asymmetry induced voltages (V_{Wsin} and V_{Wcos}) is produced due to eccentricity which caused to asymmetry envelope functions (V_{sine} and V_{cosine}). In regards to details of the sample results, the W1R5 and W1R7 resolvers have a very high-level sensitivity, where its position error reaches to infinite value in the eccentric case, while the W3R5 resolver has a low-level sensitivity. Now, the resolvers' sensitivity to the eccentric rotor (also manufacturing tolerance) can be analyzed. Results of the maximum and absolute position errors are tabulated in Table VI.

Fig. 6. Induced voltages and envelopes functions for eccentric W1R1

Fig. 7. Induced voltages and envelopes functions for eccentric W3R5

Fig. 8. Induced voltages and envelopes functions for eccentric W1R5

TABLE VI
MAXIMUM AND ABSOLUTE ERRORS FOR THE ECCENTRIC RESOLVERS

Maximum position error (Deg.)			
W1R1	W1R3	W1R5	W1R7
15.2	7.80	∞	∞
W2R1	W2R3	W2R5	W2R7
26	—	9.62	36.32
W3R1	W3R3	W3R5	W3R7
—	—	0.50	0.46
Absolute position error (Deg.)			
W1R1	W1R3	W1R5	W1R7
8.32	7.58	∞	∞
W2R1	W2R3	W2R5	W2R7
12.6	—	-10.21	-9.92
W3R1	W3R3	W3R5	W3R7
—	—	-0.16	-0.43

C. Discussion and Resolvers Evaluation

Regarding the performed analysis, the resolvers' capability and their applications can be discussed. Clearly, manufacturing tolerance is an existing natural phenomenon that should be considered in practical applications. Hence, a low-level eccentricity fault is a considerable fault that may naturally exist in a resolver. In addition, high voltage peak and low position error are valuable factors which should be compared for the analyzed resolvers. Result of the comparison is tabulated in Table VII. As is clear the W5R5 and W5R7 are trustable cases that can be considered as two practical resolvers.

D. Comparison with 2D-FEM

In order to validate and compare, the W2R5 resolver in both healthy and faulty cases is simulated by Maxwell software. The resolver geometry, flux density, and flux lines are shown in Fig. 9. Moreover, the induced voltages in both healthy and eccentric cases ($\sigma_s = 0.5$) are demonstrated in Fig. 10 in Maxwell environment. In addition, the obtained position error by both FEM and MEC are compared in Fig. 11 for the eccentric resolver. Noticing the obtained results, there is very good agreement between the results. Hence, the used flexible MEC method capability is confirmed.

TABLE VII
RESOLVERS EVALUATION

	WT1				WT2				WT3			
	W1R1	W1R3	W1R5	W1R7	W2R1	W2R3	W2R5	W2R7	W3R1	W3R3	W3R5	W3R7
Acceptable position error	No	Yes	Yes	No	No	—	Yes	Yes	—	—	Yes	Yes
Acceptable voltage peak	Yes	Yes	No	No	Yes	—	Yes	Yes	—	—	Yes	Yes
Acceptable sensitivity to SE	No	No	No	No	No	No	No	No	—	—	Yes	Yes
Applicable totally	No	No	No	No	No	No	No	No	No	No	Yes	Yes

Fig. 9. Flux lines and flux density at a sample moment for healthy W2R1

Fig. 10. Induced voltages for healthy and eccentric W2R1 under 1000 rpm obtained in Maxwell software

Fig. 11. Obtained position error for eccentric W2R1 by FEM and MEC

V. EXPERIMENTAL RESULTS

The analyzed resolver with two rotor shapes is manufactured for validation by the experimental setup. The 1-X and 5-X rotors in both healthy and eccentric cases are manufactured, where 0.25 mm eccentricity ($\sigma_s = 0.5$) is produced by the rotor center displacement in the manufacturing procedure as

shown in Fig. 12. The resolvers tests are performed under 1000 rpm rotor speed, where a 5 kHz 4.2 V_{rms} voltage is used by a signal generator as excitation. By using a 50 Hz universal motor, a 16000-pulse optical encoder (0.0225° resolution) is used as the position reference. The induced voltages sampling is performed by a 200 kS/s Advantech Multifunction USB Module to data import, where the Hilbert transformation and a written m-file are used for the envelopes computation.

Fig. 12. Manufactured resolvers (a)-shaft, cases, ballbearings, and placed stator (b)- stator and rotors

Fig. 13. Obtained V_{Wsin} and V_{Wcos} by experimental test with healthy and eccentric rotors (a)-for faulty W1R1 (b)-for healthy W3R5

Fig. 14. Difference between the obtained position by the manufactured resolvers and optical encoder in healthy and eccentric cases

The induced voltages for the eccentric W1R1 and healthy W3R5 resolvers are shown in Figs. 13a and 13b. Moreover, the obtained position errors (difference between the obtained values by resolvers and encoder) are presented in Fig. 14. As can be seen in the figures, there is very good agreement between the obtained experimental and simulation results (see Figs. 3-7).

VI. CONCLUSION

Variable reluctance resolver with toroidal windings is analyzed in this paper. Three windings types and four rotor shapes are considered for evaluation, where position error is analyzed in both healthy and faulty cases with static eccentricity fault. Regarding the analysis of the various structures, a well-known MEC method with adjustable accuracy is used for analysis thanks to its flexibility. Noticing the presented results, there are just two trustable structures that have high voltage peaks and low sensitivity to the eccentricity fault. Hence, a trustable resolver as an accurate position sensor can be obtained by sinusoidal windings configuration with 5-X and 7-X structures, where its sensitivity to the manufacturing tolerance is low. It is clarified that although the other configurations may have low position error in the healthy case, they are not practical due to high sensitivity to the eccentricity fault.

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