Communication

Design of a Highly Miniaturized, Inherently Matched, Spherical Folded Dipole Antenna and Evaluation of its Quality Factor

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Abstract—A highly miniaturized three-dimensional spherical folded dipole antenna has been reported, for which inherent impedance matching is achieved with respect to a practical source impedance by employing a simple series- LC loading combination, thereby engineering its input impedance. In order to maximize its bandwidth, the miniaturized antenna employs a bifilar spherical helix that maximizes use of the volume of the corresponding Chu sphere. Full-wave simulations show that the resulting antenna demonstrates excellent impedance matching and over 95% radiation efficiency even when miniaturized by 85% in comparison to a resonant dipole operating at the same frequency. Furthermore, various quality-factor definitions for the miniaturized spherical folded dipole antenna were estimated. It is observed that, owing to the use of proximate but non-overlapping resonances, two of these definitions fall below the Chu lower bound around the frequency of operation, resulting in a fractional bandwidth larger than that predicted by the Chu limit.

Index Terms—Electrically small antenna, folded dipole antenna, miniaturization, reactive loading, spherical helix, quality factor.

I. INTRODUCTION

Although the interest in electrically small antennas (ESAs) in the antenna community has grown significantly in recent years, the use of ESAs goes back as far as the late 1800s when Edison used vertical top-loaded grounded antennas for communicating between two distant points on land and/or sea, or Hertz performed his groundbreaking spark-gap experiments using a top-loaded dipole antenna; and in the early 1900s, when Marconi achieved trans-Atlantic communication using a fan-shaped monopole [1]. Today, ESAs are used extensively in a range of applications from broadcasting to mobile communications technologies like NFC, WiFi, WiMAX, MIMO etc., and sensing and identification applications like biomedical/environmental monitoring, machine-to-machine communications, body-centric applications, and RFID [2] – [8]. For most of these applications, dipole and monopole antennas along with their folded counterparts are often preferred, particularly due to their simple design and analysis, low cost, compact size, and well-defined radiation properties.

Despite being a popular choice for modern wireless systems, there exist well-known fundamental limits that put certain restrictions on the performance of an ESA in terms of its bandwidth and radiation efficiency [9]. The performance metrics of an ESA are usually linked to the electrical size ka

of the antenna (where, $k = 2\pi/\lambda$ is the wavenumber, a is the radius of the smallest sphere circumscribing the antenna (also known as the Chu sphere), and λ is the corresponding wavelength in free space) through the antenna's quality factor, Q , which is physically expected to have an inverse relationship with the antenna bandwidth [10]. As suggested by Chu and others [11] – [13], there exists a lower bound on the Q factor, which in turn determines the maximum attainable bandwidth for an antenna of a given electrical size. With increasing miniaturization levels (i.e., decreasing electrical size), the Q factor of the antenna increases, decreasing the maximum attainable bandwidth.

In order to achieve the maximum possible bandwidth, an ESA must occupy all the volume inside the corresponding Chu sphere. This is a particular challenge for wire-type antennas such as dipoles. However, by realizing the dipole arm in the form of a spherical helix, one can maximize its bandwidth [14] – [16]. On the other hand, the process of miniaturization also results in a small input resistance and a large input reactance for the antenna, making it very difficult to match to a practical source impedance. This can typically be resolved by employing external matching networks, however at the expense of added loss and increased antenna footprint. In [14] – [16], the matching problem was resolved by using a multi-arm folded dipole antenna whose arms were realized in the shape of a spherical helix, and the miniaturized antennas were operated at their strongly reduced half-wavelength resonance frequency.

In [17] – [18], an alternative mechanism was proposed for folded dipole antennas in order to achieve good impedance matching at high miniaturization levels, without the need for an external matching network; that is, the matching is inherent to the design of the antenna element. Unlike the technique described in [14] – [16], this new technique makes use of the intrinsic first antiresonance of the folded dipole antenna that occurs naturally at a lower frequency than the half-wavelength resonance, and engineers the input impedance around it by applying simple series inductive and series capacitive loading to facilitate matching. It was also shown that this technique could more generally be used to match complex source impedances, and thus is suitable for such narrowband applications as RFID [19]. However, since the miniaturized folded dipole antennas reported in [17] – [18] were not spherical in shape, there remains an opportunity to enhance their bandwidths using such an approach. Thus, this work develops a 3-D version of the folded dipole antenna presented in [18], where the folded arm has been realized in the shape of a spherical helix in order to maximize the bandwidth of the antenna, while the driven

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Fig. 1. (a) The 75% miniaturized folded dipole antenna reported in [17], and (b) its input impedance response.

arm has been realized using a simple cylindrical shape loaded with two parallel plate capacitors in order to maintain inherent matching with a prescribed source impedance. The resulting antenna is 85% miniaturized relative to a half-wavelength dipole operating at the same frequency, and it exhibits a wider bandwidth than a similarly miniaturized spherical folded dipole antenna [16]. Furthermore, a discussion is provided on how to estimate the Q factor for the proposed antenna, and how it can be related to the corresponding antenna bandwidth.

This article is organized as follows. Section II describes the design principle and the structure of an 85% miniaturized folded dipole antenna, along with the simulated results. Section III discusses considerations for the evaluation of Q for the designed antenna and its comparison with respect to the corresponding lower bound.

II. ANTENNA DESIGN AND SIMULATION

A. Design Principle

As described in [17], a folded dipole antenna is more amenable to inherent impedance matching at a high miniaturization level than a conventional dipole antenna, while exhibiting radiation performance comparable to its conventional counterpart miniaturized to the same level; this is because of the presence of an additional antiresonance before the conventionally used half-wavelength resonance. Although the antiresonance is not typically used due to very high input resistance, it was shown that the input impedance of the folded dipole antenna can be easily engineered around the antiresonance to yield an impedance equal to or approximately

equal to the source impedance (with respect to which matching is to be done), just by applying a simple asymmetric loading combination of series inductive (L) and series capacitive (C) elements. The 75% miniaturized folded dipole antenna described in [17] is shown in Fig. 1 along with its input impedance response. Whereas the series inductive loading was responsible for providing the necessary miniaturization level by shifting the inherent resonances of the folded dipole towards lower frequencies, the series capacitive loading introduced a new resonance $f_{R,N}$ before the antiresonance $f_{AR,1}$ of the folded dipole antenna, which is also the first inherent resonance of the antenna. Thereafter, by varying the inductive and capacitive loading values, the resonances $f_{R,N}$ and $f_{AR,1}$ could be brought closer to each other, and their mutual positions could be adjusted in order to achieve the desired $\Re\{Z_{in}(f_{R,N})\}$. This results in impedance matching at $f_{R,N}$, which is accomplished inherently by the abovementioned design of the antenna and without an external matching network.

It was also shown in [17] that the currents on the folded dipole arm are directed opposite to those on the driven arm, unlike the examples given in [16] where these currents are co-directed and radiate in phase. However, due to the fact that the magnitude of currents on the folded arm was much higher than that on the driven arm, there occurred a net current imbalance, which afforded high radiation efficiency. Clearly, the radiation efficiencies for the antenna structures reported in [16] were much higher ($>90\%$) compared to the radiation efficiency reported in [17] (72.3%). However, it must be noted that achieving inherent impedance matching for arbitrary miniaturization levels and source impedances is more flexible in the approach of [17] than that of [16]. This can be explained by studying the different current components of a folded dipole antenna. It is well known that the overall current on the folded dipole arms can be decomposed into two components: (1) a loop-like or contra-directed current component (also known as the differential mode, odd mode, or transmission-line mode (TLM)), and (2) a co-directed current component (also known as the common mode, even mode, or antenna mode (AM)) that accounts for most of the radiation. While the TLM current does not contribute strongly to radiation, both TLM and AM currents do influence the input impedance of the antenna, and thus both are required to efficiently match the antenna.

The highly miniaturized and inherently matched spherical folded dipole antenna reported in this article is designed following the same working principle described in [17], and as mentioned earlier, is an evolution of the fully printed, planar, and highly miniaturized inherently matched folded dipole antenna reported in [18]. It is designed to exhibit a wider bandwidth at a lower miniaturization level and possesses a higher co-to-cross-polarization separation owing to its unique geometry; these attributes will be demonstrated in the next subsection.

B. Antenna Structure

The different stages of the antenna construction are displayed in Fig. 2. In the first stage, the driven arm is constructed

Fig. 2. Step-by-step construction of the miniaturized spherical helix loaded folded dipole antenna: (a) cylindrical driven arm, (b) application of capacitive loading, and (c) complete antenna structure with both inductive and capacitive loading (the left-hand-side image shows the antenna in profile, whereas the right-hand-side image shows the antenna's E-plane cross-section).

by realizing an electrically small cylindrical dipole antenna of length l and radius of cross-section r . The cylindrical driven arm is shown in Fig. 2(a). In the next phase, parallel-plate capacitors are introduced on the cylindrical driven arm at a distance $l/4$ from the antenna ends, as shown in Fig. 2(b), following the loading-capacitance locations used in [17] – [19]. The individual parallel plate is circular in shape, with a radius r_c and a thickness t_c . The gap between the parallel plate w_c together with the plate radius r_c determine the amount of series capacitive loading. In the final phase, a spherical helical folded arm is added between the ends of the antenna in order to produce the required distributed inductive loading for miniaturization. While the radius of the spherical helix ($l/2$) is limited by the overall dimension of the antenna, the number of turns can be optimized in order to realize the required loading inductance. Furthermore, it was shown in [16] that the bandwidth of a spherical folded helix antenna can be improved by increasing the number of folded helical arms. For the folded dipole antenna shown in Fig. 2, although it is possible to employ an arbitrary number of folded arms, additional arms may cause difficulties in feeding the antenna, as the feeding circuit needs to extend through the spherical

TABLE I
OPTIMIZED DIMENSIONS FOR THE MINIATURIZED SPHERICAL HELIX
LOADED FOLDED DIPOLE ANTENNA

Dimension	Value
l	44 mm
r	1 mm
f_w	1 mm
r_c	3.86 mm
t_c	1 mm
w_c	1.5 mm
r_{fd}	0.5 mm
Number of helix turns	3.8

helix into the centre of the antenna. As a result, only two folded arms were used in this design with a 180° separation between them. Finally, as shown in Fig. 2(c), the sense of rotation of the helical folded arm changes halfway between the antenna ends in order to reduce cross-polarized radiation in the broadside direction ($\theta = 90^\circ$). The overall length of the antenna l was chosen to be 44 mm, and the other dimensions were optimized for operation at an 85% miniaturization level (i.e., at 515 MHz where $l \approx 0.075\lambda$) compared to an unloaded slender half-wavelength dipole antenna operating at the same frequency. The optimized antenna dimensions are given in Table I.

C. Simulated Results

While a miniaturized folded dipole antenna can be designed for matching with respect to practically any source impedance less than $\Re\{Z_{in}(f_{AR,1})\}$ using the technique described in [17], the source impedance for the antenna reported in this article was selected to be 100Ω . Simulations were performed in Ansys HFSS, where bulk copper was used as the conducting material for the antenna, and no additional losses (e.g., surface roughness) were introduced. For the dimensions listed in Table I, the spherical folded dipole antenna produced very good return loss of 39.7 dB at 515 MHz, providing $ka = 0.237$. The input impedance and the S_{11} of the optimized antenna are shown in Figs. 3(a) and 3(b), respectively, and the 10-dB return-loss bandwidth of the antenna was found to be 8 MHz ($\sim 1.55\%$). Its corresponding 3-dB return-loss bandwidth is 4.47%, which is better than the 2.37% 3-dB bandwidth reported for the 83% miniaturized ($ka = 0.265$) four-arm folded dipole antenna in [16].

The E-plane and the H-plane radiation patterns are shown in Figs. 4(a) and 4(b), respectively. It can be seen from Fig. 4 that the broadside gain of the antenna at 515 MHz is -4.4 dBi, and the co-to-cross-polarization separation is 71.93 dB at broadside. The radiation efficiency of the antenna at 515 MHz was found to be 95.6%. Finally, the current distribution of the antenna is shown in Fig. 5. As can be seen, most of the radiation at the operating frequency is owed to the helical folded arm, similar to the case described in [17].

III. ESTIMATION OF QUALITY FACTOR

The Q factor of an antenna is a valuable metric that connects the electrical size of the antenna, ka , to its bandwidth and

Fig. 3. (a) Input impedance and (b) S_{11} of the spherical helix loaded folded dipole antenna.

radiation efficiency, and is particularly useful for describing the performance of an ESA. For a given electrical size, expressions can be found for the lower bound of the Q , which in turn determines the maximum attainable bandwidth for the antenna. For a generally lossy antenna with a radiation efficiency of η_{rad} , the Chu lower bound of Q (denoted hereafter as Q_{lb}) can be given as [15]:

$$Q_{lb} = \eta_{rad} \left[\frac{1}{ka} + \frac{1}{(ka)^3} \right]. \quad (1)$$

Numerous approaches exist in the literature for estimating the Q of an ESA; however, arguably the most widely used expressions for determining Q are given by Yaghjian and Best [10]. Three definitions can be found in [10] for the quality factor of an antenna, the first of which is the Q determined from antenna fields:

$$Q_F(\omega_0) = \left| \frac{\omega_0}{2R_{in}(\omega_0)} X'_{in}(\omega_0) - \frac{2\omega_0}{|I_0|^2 R_{in}(\omega_0)} [W_{\mathcal{L}}(\omega_0) + W_{\mathcal{R}}(\omega_0)] \right| \quad (2)$$

where ω_0 is the operating frequency, $R_{in}(\omega_0)$ and $X_{in}(\omega_0)$ are the real and imaginary parts of the input impedance Z_{in}

Fig. 4. Radiation patterns at 515 MHz: (a) E-plane and (b) H-plane (Note: the range shown spans up to -80 dB).

Fig. 5. Current distribution on the spherical helix loaded folded dipole antenna at 515 MHz (arrows indicate the direction of the current on the biconical driven arm and one of the helical folded arms).

at the operating frequency, respectively, and $X'_{in}(\omega_0)$ is the frequency derivative of $X_{in}(\omega_0)$. $W_{\mathcal{L}}$ and $W_{\mathcal{R}}$ are dispersion energies that are directly related to the lost and radiated powers, respectively, and are calculated using the fields and

currents of the antenna. The second definition of quality factor is given entirely in terms of the input impedance of the antenna:

$$Q_Z(\omega_0) = \frac{\omega_0}{2R_{in}(\omega_0)} |Z'_{in}(\omega_0)|$$

$$= \frac{\omega_0}{2R_{in}(\omega_0)} \sqrt{[R'_{in}(\omega_0)]^2 + \left[X'_{in}(\omega_0) + \frac{X_{in}(\omega_0)}{\omega_0}\right]^2} \quad (3)$$

where once again primes indicate frequency derivatives. The final definition of quality factor relates directly to the fractional bandwidth FBW of the antenna for a given VSWR level s , and is given as

$$Q_{BW}(\omega_0) = \frac{2\sqrt{\beta}}{FBW(\omega_0)}, \quad \sqrt{\beta} = \frac{s-1}{2\sqrt{s}} \quad (4)$$

All the three quality factors are independent of whether they are estimated at the resonance or the antiresonance of the antenna, and are approximately equal for an antenna tuned at its resonance/antiresonance that does not have another resonance/antiresonance within its defined VSWR bandwidth. Furthermore, the quality factors exhibit an inverse relationship with the bandwidth, which is arguably the most important property of the quality factor of a system. However, the equality of the different Q factors and their inverse proportionality with bandwidth have often been observed to fall apart for the case of a multi-resonant antenna, where multiple resonances are employed in order to enhance the antenna bandwidth or achieving better impedance matching. In [20], it was shown that for such a multi-resonant antenna, the three quality factors are no longer comparable near the closely-spaced resonances. Moreover, although Q_F does not violate the lower bound, it is no longer inversely related to the bandwidth, which raises a question about the validity of Q_F for the multi-resonant cases. On the other hand, despite Q_Z violating the lower bound, it continues to demonstrate an inverse relationship with bandwidth in such cases. As such, it was mentioned in [20] that Q_Z might still be a meaningful quantity for multi-resonant antennas. Furthermore, it remains comparable with Q_{BW} for a small enough VSWR level, such that the defined bandwidth does not contain more than one resonance/antiresonance [20]–[22]. It should also be noted at this point that Q_{lb} is estimated for an antenna radiating a single TE/TM mode in its operating bandwidth, which raises questions about its applicability for the case of a multi-resonant antenna. Nevertheless, it was also shown in [20] that when the matched resonance is sufficiently isolated from its neighbouring resonances, the parameter Q_Z no longer violates Q_{lb} , and is almost equal to Q_F .

A graph was produced to compare Q_Z and Q_{BW} with Q_{lb} over the whole frequency range. While Q_{lb} and Q_Z may be easily estimated over a range of frequencies using (1) and (3), respectively, one must match the antenna at each of the frequency points in order to obtain the correct Q_{BW} and estimate the corresponding bandwidths from the S_{11} response at the chosen VSWR levels. In this work, matching was obtained at select frequencies over the band using a unique (and fully reactive) L-section matching network designed for each frequency in order to ensure that the source impedance

Fig. 6. Comparison of quality factors for the miniaturized spherical helix loaded folded dipole antenna. The vertical dashed lines mark the positions of the matched resonance $f_{R,N}$ and the neighbouring antiresonance $f_{AR,1}$.

TABLE II
CALCULATION OF Q FACTORS FOR THE MINIATURIZED SPHERICAL HELIX LOADED DIPOLE ANTENNA

Antenna	ka	η_r	Q_{lb} (including losses)	Q_Z
Spherical folded helix from [16]	0.265	97.4%	55.6	84.64
This work	0.237	95.6%	75.58	45.63

could be maintained at 100Ω (of course, no such matching network is needed at the antenna operating frequency of $f_{R,N} = 515$ MHz). A comparison of the different quality factors for the designed antenna is shown in Fig. 6. As can be seen, despite the fact that the antiresonance $f_{AR,1}$ is sufficiently isolated from the matched resonance $f_{R,N}$ (as can be seen in Fig. 6), the parameter Q_Z still violates the lower bound Q_{lb} . However, Q_Z remains approximately equal to Q_{BW} for small VSWR levels, e.g., $s = 1.9$ (10-dB return loss) and $s = 1.2$ (20-dB return loss), as expected. In Table II, the quality factors for the proposed miniaturized antenna are compared with those of the spherical folded helix antenna described in [16] at their respective operating points, since these antennas have comparable ka values. $Q_{BW} < Q_{lb}$ is consistent with the observation in Sec. IIC, i.e., the FBW of the proposed antenna is greater than that of a similarly miniaturized spherical dipole antenna reported in [16].

It may be observed that the use of multiple, proximate resonances in this work, with the result of enhanced FBW and Q_{BW} violating the Chu lower bound, is reminiscent of the approach of Bode-Fano matching, where an external lossless matching network is typically employed in order to achieve a broad bandwidth with respect to a complex load (e.g., an electrically small antenna in this context) [23] – [25]. The introduction of this matching network helps engineer the overall input impedance by manipulating the resonances and antiresonances of the complex load, and thus may result in a bandwidth greater than the bandwidth calculated from Q_{lb} of

the miniaturized antenna. However, it should be emphasized that the suggested impedance-response engineering approach is more intuitive and inherent in the antenna structure (thus obviating the need for an external matching network at the antenna operating frequency).

IV. CONCLUSION

In this work, an alternative technique for achieving inherent impedance matching has been demonstrated for a three-dimensional highly miniaturized folded dipole antenna, while simultaneously maximizing the bandwidth by employing an overall spherical shape. The technique enables matching with respect to a wide range of source impedances and results in a larger fractional bandwidth compared to other techniques reported in the literature for antennas of comparable electrical size. The designed antenna, which is 85% miniaturized compared to an unloaded resonant dipole operating at the same frequency, exhibits a bandwidth wider than that of an equivalently miniaturized spherical folded dipole antenna reported in the literature, while exhibiting over 95% radiation efficiency. The challenging fabrication and measurement of the proposed antenna is underway, and is expected to be reported in a future communication.

Two quality factors, Q_Z and Q_{BW} , were computed and compared with the corresponding lower bound Q_{lb} . As expected, Q_Z was found to be approximately equal to Q_{BW} for sufficiently low VSWR levels. Moreover, it was observed that both Q_Z and Q_{BW} violate the lower bound Q_{lb} around the frequency of operation, despite there being only one resonance within the defined VSWR bandwidth of the antenna and neighbouring resonances being sufficiently displaced. It was suggested that this violation of the Chu lower bound may be akin to that observed using similar impedance-engineering techniques employed in Bode-Fano matching, but that the proposed antenna achieves this result at its operating frequency without the use of an external matching network.

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