Eddy Currents Based Stray Loss Model For Variable Load Conditions of Medium Power Induction Motors

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Abstract—This paper presents an eddy current based stray loss model for induction motors taking into account the impact of motor's loading. The model uses the various motor inductances and, other nameplate data of an induction motor as primary variables. The Magnetic field distribution in a motor is prominently affected by the stator and rotor slot geometries. Distortions in the magnetic field have a direct impact on SL as well as on various inductances of the motor. The development and validation of the model is accomplished by testing total 26 numbers of induction motors of different geometries and materials at different loading conditions. In a further step, the applicability of the model in designing an IM is presented by re-designing a 75kW induction motor and comparing the experimental test results with the initial IM.

Index Terms—Induction motors, stray losses, equivalent circuit parameters, core loss, modelling

I. INTRODUCTION

Design and operation of energy efficient motor-driven systems (MDS) require accurate estimates of all the losses incurred in the motor. Mechanical, stator copper, rotor copper, iron, and stray load losses (SL) are the five significant losses noted for the motors [1]. All these losses, except the SL, are thoroughly analyzed and reported in the literature. Several fairly accurate models exist for these losses, whereas the formulation of SL is still a challenge [2]. This is due to multiple factors influencing the SL. These factors include slot MMF and permeance harmonics due to stator and rotor slots, magnetic non-linearity of the ferromagnetic core, and various leakage fluxes [3], [4]. The typical value of the SL in a motor, at its rated operation, lies between 0.5% and 2.5% of the motor's power rating [5], [6], significant enough to affect the motor's efficiency estimation. Further, the SL becomes more critical for premium efficiency motors as all the other losses are small.

Literature broadly discusses SL's evaluation by four approaches: viz. testings [7], numerical [8], equivalent circuit representation [9], and empirical formulation [10], [11]. Among these methods, testing is not applicable in design phase. High implementation time, large data storage, and complex computation of numerical methods led to empirical representation of SL. Given the fact that, in industrial applications, 44% of machines in the industrial sector run with loads less than or equal to 40% of their rated value [12], the available literature, in our findings, still lacks an SL model, which can:

- 1) approximate the SL values for different load-conditions of an IM while designing it so that SL can be minimised in the design phase by revamping the IM's geometry for a wide range of its operation.
- 2) distinguish the IMs of the same power rating with respect to (*w.r.t.*) their SL values. For example, two motors of the same power ratings, number of slots, and material but different slot shapes would show different SL values. The SL model should apprehend this.
- 3) also be used for motors already in operation, whose detailed design and material data are not available except for the name-plate data;

This paper presents an ECP based SL model to cater to all the above requirements mentioned with the accomplishment of the following objectives:

- 1) to assess and formulate the SL variation with motors' loading.
- 2) to assist the motor designer to evaluate and minimise the SL while designing the motor using the formulated SL model. The model should demonstrate a good sensitivity with the motor's geometry to lead the designer to a minimal SL design.
- 3) propose an SL model applicable to a wide range of medium power motors *i.e.*, from 0.75kW to 375kW.

ECPs and nameplate details are chosen as the primary variables for the proposed SL model. The ECPs are the function of motor's geometry and, can be evaluated at both the stages *i.e.* during the design (from the geometry details of the motor [9], [13]) and, after the prototyping (by standard tests [5], [14]).

The paper's organisation is as follows: Section II describes SL variation with the motor's loading. Section III pertains to the model formulation based upon the experimental results of section II. Validation of the proposed formulation is presented in Section IV with using a set of eight IMs of different slot geometries and rotor materials and, by re-designing a 75kW IM. Finally, section V concludes the paper.

II. MEASUREMENT OF SL

To trace and formate the SL in an IM and its variation with loading, two sets (*i.e.*, set-1 and set-2) of measured SL values of a total of 26 IMs are used. Set-1 comprises rated SL values of 18 IMs of different geometries, power ratings and number of poles. Set-2, used to study the SL variation with

TABLE I
IMs' TEST SAMPLE DISTRIBUTION FOR RATED SL

Poles	Number of IMs	Minimum power	Maximum power
2	3	75 kW	160 kW
4	10	75 kW	375 kW
6	5	75 kW	200 kW

loading, belongs to the eight IMs of the same power ratings, the same number of poles, same outer dimensions but different slot shapes. Subsections A and B present the SL measurements for set-1 and set-2, respectively.

A. SL values at rated loadings

Variation of SL (at rated P_{Out}) among the IMs is figured out using the test data of set-1. Sample distribution of the set-1 is given in Table I. Nameplate details of these IMs are given in annexure-1. All these 18 IMs of set-1 are tested by TMEIC-Medium rotating machinery department. SL values for these IMs are measured from the load test with torque measurement as per the section 8.2.2.5.1 of IEC60034-2-1 and, they are shown in Fig. 1.

Fig. 1. Test bench with the 75kW IM

B. SL variation with loading

Eight cage-bar induction motors are prototyped to understand the trend of SL variation with the loading of IM. The set of these motors (*i.e.*, set-2) consists:

- eight motors of same power ratings but different slot geometries.
- four motors of same slot geometries (and power ratings) but different rotor material, *i.e.* two aluminium rotor motors and two copper rotor motors.

The purpose of prototyping eight such diverse IMs is to check the model's sensitivity to SL when motors are very similar except for the slot geometries. Nameplate data of these motors are given in annexure-2. All these 2.5kW rating motors have the same diameters, stack length, slot area and end-rings area. The only difference is in the slot geometries and rotor bar material. These motors (Fig. 3(a)) are tested for SL at different operating points (including the rated) as per the relevant and latest test standard-IEC60034-2-1. The test procedure, test results are detailed in the following sub-section.

C. Experiments and SL test results

The SLs, for the eight IMs (mentioned in subsection B) are measured using the Eh-star test, following the procedure given

Fig. 2. Test results: variation of SL with loading for the 2.5kW IMs: (a) Cu motors, (b) Al motors

Fig. 3. (a) 2.5kW, 4p IMs, (b) Test setup

in [5]. The test set-up is shown in Fig. 3(b). The analysis of the results shown in Fig.4(a)-4(b) show that:

- even though the motors M1 to M8 have the same rated output power, the SL is different.
- SL of a motor does not vary in the same proportion of output power (*i.e.* $\frac{\partial(SL)}{\partial SL} \neq \frac{\partial(P_{Out})}{P_{Out}}$), which implies that the relationship between SL and P_{Out} is not linear. The SL varies as an exponential function of output power, and hence a percentage-based formulation of SL *w.r.t.* P_{Out} is not justified for variable load scenarios.
- the motors with aluminium rotors show higher SL values, vis-a-vie copper rotors, for the same output power. This is because the aluminium rotor motors, as opposed to copper rotor motors, run at a relatively higher slip for the same P_{Out} . As a result of this, the stator current and the leakage fluxes increase, resulting in higher SL.

An empirical relation of SL is formulated based on the above observations and rated SL values of set-1. The next section explains the model's development in detail.

III. PROPOSED MODELING OF SL FOR VARIABLE LOADING CONDITIONS

This section presents the formulation of SL model in subsection-A and, required further modifications to take into account the impact of loading in subsection-B.

A. Modeling of SL

Various components of SL are, in fact, the additional eddy current losses (P_{Eddy}) in response to the space and time harmonics. The expression for P_{Eddy} for a sinusoidal excitation of flux-density magnitude B_m and, frequency f is given as:

$$P_{Eddy} = k_e \omega^2 B_m^2 \quad (1)$$

SL is the portion of the net P_{Eddy} in a machine, generated by the space and time harmonics. These harmonics are generated from air-gap flux variation due to the slots, magnetic

saturation and, leakage flux [8-11]. Net P_{Eddy} produced by all the harmonics is:

$$P_{Eddy} = k_e B_{m1}^2 \omega_1^2 + \sum_{v=2}^n k_e B_{mv}^2 \omega_v^2 \quad (2)$$

where, B_{mv} and ω_v are the flux density magnitude and frequency of the v -th space harmonic, respectively. Denoting $k_e B_{m1}^2 \omega_1^2$ as P_{Eddy1} i.e., eddy current loss corresponding to the fundamental air-gap flux density (B_1) and fundamental frequency (ω_1),

$$P_{Eddy} = P_{Eddy1} \left(1 + \sum_{v=2}^n \left(\frac{B_{mv}}{B_1} \right)^2 \left(\frac{\omega_{mv}}{\omega_1} \right)^2 \right) \quad (3)$$

The SL due to v -th harmonic is:

$$SL_v = P_{Eddy1} \left(\frac{B_{mv}}{B_1} \frac{\omega_{mv}}{\omega_1} \right)^2 \quad (4)$$

SL is hence,

$$SL = P_{Eddy1} \left(\sum_{v=2}^n \left(\frac{B_{mv}}{B_1} \right)^2 \left(\frac{\omega_{mv}}{\omega_1} \right)^2 \right) \quad (5)$$

Two of major SL components in a non-skewed IM are surface eddy current losses and, tooth flux pulsation losses [4], [8], [13].

1) *Surface eddy current losses:* Modulation of the air-gap flux density due to slots results higher order space harmonics which creates additional eddy current losses on the surface of stator and rotor laminations. The impact of stator slot openings is prominent and hence, the surface losses produced by the rotor slots is negligible [13]. Harmonic numbers of the travelling flux-density waves of these harmonics are $v = N_s \pm p$ [13]. Frequency of the additional eddy currents induced on the rotor surface (corresponding to the $v = N_s \pm p$) is hence,

$$f_v = s_v f_1 \Rightarrow f_v = \left(1 - \frac{N_s \pm p}{p} \right) f_1 = \frac{N_s}{p} f_1 \quad (6)$$

The magnitudes of these harmonics are determined from the magnitudes of the variations in MMF due to slots as [15]

$$\frac{B_{mv}}{B_1} = \frac{F_{mv} \lambda_v}{F_1 \lambda_1} = \frac{F_{mv} \lambda_{sv}}{F_1 \lambda_{1s}} = \frac{L_{msv}}{L_{m1}} \quad (7)$$

where, F_{mv}, F_1 are, respectively, the v -th harmonic and fundamental MMFs. λ_m, λ_1 are specific permeances of v -th harmonic and fundamental flux paths, respectively. L_{mv} is magnetising inductance corresponding the v -th harmonic expressed as [9]

$$L_{mv} = \frac{6\mu_o(N_s K_w)^2}{\pi v^2} \left(\frac{L D}{p^2 g K_C K_{Sv}} \right) \quad (8)$$

SL due to the eddy current losses corresponding to these harmonics (as per (5)) is:

$$SL_{surf} = P_{Eddy1} \left(\left(\frac{N_s}{p} \right)^2 \sum_{v=2}^n \left(\frac{B_{mv}}{B_1} \right)^2 \right) \quad (9)$$

from (7),(8)

$$SL_{surf} = P_{Eddy1} \left(\left(\frac{N_s}{p} \right)^2 \sum_{v=1}^n \left(\frac{L_{msv}}{L_{m1}} \right)^2 \right) \quad (10)$$

Using the expression for zig-zag leakage inductances (zig-zag leakage inductance is the sum of all the harmonic magnetising inductances other than fundamental [13]),

$$L_{lz} = \sum_{v=2}^n L_{mv} \quad (11)$$

From (10) and, (11),

$$SL_{surf} = P_{Eddy1} \left(\frac{N_s}{p} \right)^2 \left(\frac{L_{lsz}}{L_{m1}} \right)^2 \quad (12)$$

2) *Flux pulsation losses:* The variation in the main flux path permeance due to change in rotor position results in flux pulsation losses both in stator and rotor. The impact of rotor slot is very small in case closed or narrow-opening rotor slots. The frequency of eddy currents generated by the flux pulsation due to stator slot openings is $f_v = \frac{N_s}{p} f_1$ and, the same due to rotor slot openings is $f_v = \frac{N_r}{p} f_1$. The magnitudes of these harmonics are given as [4]

$$B_{mtv} = K_{tv} B_1 \left(\frac{t_s}{b_{s0}} \right) \quad (13)$$

where, neglecting the skewing,

$$K_{tv} = 1 - \frac{L_{mv} + L_{lsz}}{L_{mv} + L_{lsz} + L_{Slot}} = \frac{L_{Slot}}{L_{mv} + L_{lsz} + L_{Slot}} \quad (14)$$

here, L_{Slot} is the slot leakage inductance. With $L_{mv} \gg L_{Slot} + L_{lsz}$,

$$B_{mtv} = \left(\frac{L_{Slot}}{L_{mv}} \right) \left(\frac{t_s}{b_{s0}} \right) B_1 \quad (15)$$

where, t_s and, b_{s0} , respectively represent the slot pitch and, slot opening. SL corresponding to these harmonics in stator is calculated as

$$SL_{pulsS} = P_{Eddy1} \left(\left(\frac{N_s}{p} \right)^2 \sum_{v=2}^n \left(\frac{B_{mtv}}{B_1} \right)^2 \right) \quad (16)$$

With using (13)-(16),

$$SL_{pulsS} = P_{Eddy1} \left(\left(\frac{N_s}{p} \right)^2 \left(\frac{L_{SlotS}}{L_{m1}} \frac{t_{sS}}{b_{s0S}} \right)^2 \right) \quad (17)$$

similarly, for rotor

$$SL_{pulsR} = P_{Eddy1} \left(\left(\frac{N_r}{p} \right)^2 \left(\frac{L_{SlotR}}{L_{m1}} \frac{t_{sR}}{b_{s0R}} \right)^2 \right) \quad (18)$$

With (12) and, (17)

$$SL = SL_{surf} + SL_{pulsS} + SL_{pulsR} \quad (19)$$

$$\Rightarrow SL = P_{Eddy1} \left(\frac{N_s}{p} \right)^2 \left(\left(\frac{L_{lsz}}{L_{m1}} \right)^2 + \left(k_s \frac{L_{SlotS}}{L_{m1}} \right)^2 + \left(k_r \frac{L_{SlotR}}{L_{m1}} \right)^2 \right) \quad (20)$$

where, $k_s = \frac{t_{sS}}{b_{s0S}}$ and, $k_r = \frac{N_r t_{sR}}{N_s b_{s0R}}$. Equation (20) is written in form of IM's output power with using the relations shown in (21)-(24)

$$P_{Out} = 3 \left(\frac{\omega^2 \lambda_s^2}{Z_s} \right) \cos \phi \quad (21)$$

where, ω, λ_s, Z_s and, $\cos\phi$ denote supply frequency, stator flux linkage, input impedance and, power factor of a three phase IM. Rewriting (21) in terms of average air-gap flux density (B) results:

$$P_{Out} = 3 \left(\frac{\omega^2 (T_s^2 B^2 A^2)}{Z_s} \right) \frac{R_R}{Z_s} \Rightarrow P_{Out} = 3 \left(\frac{T_s^2 A^2 R_R}{Z_s^2} \right) \omega^2 B^2 \quad (22)$$

where, T_s, A, R_R are, respectively, the stator number of turns/phase, area under a pole, and rotor resistance. The expression of eddy current loss is:

$$P_{Eddy} = k_e \omega^2 B^2 \quad (23)$$

Comparing (22) and (23),

$$\Rightarrow P_{Eddy} = k^2 P_{Out} \quad (24)$$

From (20) and (24),

$$SL = k^2 \left(\frac{1}{p} \right)^2 \left(\left(\frac{L_{lsz}}{L_{m1}} \right)^2 + \left(k_s \frac{L_{SlotS}}{L_{m1}} \right)^2 + \left(k_r \frac{L_{SlotR}}{L_{m1}} \right)^2 \right) P_{Out} \quad (25)$$

Since, in an IM $L_{lSlotS} \gg L_{lsz}$, $L_{lS} \approx L_{lSlotS}$ and, $L_{lR} \approx L_{lSlotR}$, the expression (25), with denoting L_{m1} as L_m can be written as

$$SL = \left(\left(\frac{k k_s L_{lS}}{p^2 L_m} \right)^2 + \left(\frac{k k_r L_{lR}}{p^2 L_m} \right)^2 \right) P_{Out} \quad (26)$$

3) *Statistical evaluation*: SL for 18 IMs of set-1 is calculated as per (26) using the inductances L_m, L_{lS} and, L_{lR} . The values of these inductances are given in annexure-3. However, high difference between the measured and calculated values with (26) (of the range 200%-300%) suggests further improvement in (26). Hence, three another empirical representations of SL are explored using the same prime variables of (26) i.e., $P_{Out}, \frac{L_{lS}}{p L_m}, \frac{L_{lR}}{p L_m}$. The three empirical representations of SL are:

$$SL = k \left(\left(\frac{1}{p} \frac{L_{lS}}{L_m} \right)^m + \left(\frac{1}{p} \frac{L_{lR}}{L_m} \right)^n \right) P_{Out} \quad (26a)$$

$$SL = k \left(\frac{1}{p} \frac{L_{lS}}{L_m} + \frac{1}{p} \frac{L_{lR}}{L_m} \right)^m P_{Out} \quad (26b)$$

$$SL = k \left(\frac{1}{p} \frac{L_{lS}}{L_m} \right)^m \left(\frac{1}{p} \frac{L_{lR}}{L_m} \right)^n P_{Out} \quad (26c)$$

where, k is the proportional constant and, m and n are the exponents chosen to reflect the impact of rotor and stator slot geometries on the SL. These parameters are determined with the nonlinear least-squares regression. Values of the exponents and, calculated SL with these three models are given in annexure-3. The average errors with (26a), (26b) and (26c) are 42.42%, 25.89% and, 24.56%, respectively. Standard deviation of errors (26a), (26b) and (26c) are 27.82%, 18.88% and, 15.44%, respectively. The relative capability of these three models is further checked with relative-quality-estimates viz. Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC). The AIC and BIC weights are calculated for each model as per [16] using number of model parameters

(n) and, the residual sum of error-squares (σ_k). Values of these scores are given in Table II. The model with least AIC and BIC score (or with zero values of ΔAIC and ΔBIC) is considered the best estimator. The prediction capabilities (C) of the three models are described as

$$C_{26c} > C_{26b} > C_{26a}$$

TABLE II
QUALITY ESTIMATES OF SL MODELS

SL Model	n	σ_k	AIC	ΔAIC	BIC	ΔAIC
26a	4.00	5.88	-12.14	19.59	-8.58	18.70
26b	5.00	3.43	-19.83	11.91	-15.38	11.91
26c	5.00	1.77	-31.73	0	-27.28	0

B. Adapting SL model for loading condition

The model development discussed in subsection-A uses a single value of measured SL and, single set of model parameters (i.e., $P_{Out}, \frac{L_{lS}}{p L_m}, \frac{L_{lR}}{p L_m}$) for each of the 18 IMs of set-1. This gives a linear trend of SL with P_{Out} . However, the variation of SL w.r.t. output power is not linear. Hence, the expression (26c) needs further modifications to account for the nonlinearity associated with the SL- P_{Out} relation. The following modifications in are feasible:

- make L_{lS} and L_{lR} functions of stator and rotor currents, respectively.
- make L_m a function of stator current.
- make the exponents m and n function of slip.

The L_{lS} and L_{lR} do not vary much with loading due to a large air in the path of leakage flux lines in comparison to the magnetising flux lines (till $I_S/I_{Srated} < 2$) [13], [15]. Since the L_{lS} and L_{lR} do not show a significant variation w.r.t load current, they considered constant (as determined from no-load and block rotor test). On the other hand, the L_m changes with the loading because variation in load results in a change of the magnetising voltage due to the change in stator resistive drop as:

$$E_m = V_S - I R_S$$

The magnetising flux-linkage corresponding to the is:

$$\psi_m = \int E_m dt = -\frac{1}{\omega_e} j E_m$$

$$L_m(\psi_m) = \frac{\psi_m}{I_m(\psi_m)}$$

From the above expression, it is evident that L_m is a function of flux-linkage ψ_m , and magnetising current (I_m). Since both, ψ_m and I_m change due to loading, the L_m also changes with loading. The variation of L_m can be evaluated using the no-load test at different voltages, and Fig.4(b) shows a typical L_m variation. It is clear from Fig.4(b) that L_m rises with an increase in load current. However, the L_m variation (as shown in Fig.7(b)) does not take into account the impact of leakage fluxes as it uses no-load test data and the leakage flux is minimal at no-load conditions.

Fig. 4. Variations due to load-change: (a) K_{SL} with slip for M-1, (b) L_m with ψ_m .

The other factor that affects L_m is tooth-bridge saturation. As the leakage fluxes increase with the load current, the tooth-bridge begins to saturate. Thereby, the permeance of the slots reduces leading to decrease in L_m . Since, both the above phenomena tend to have an opposing influence on L_m , the L_m variation w.r.t loading is dependent on the motor's geometry. For example, a motor with a higher tooth width has a different L_m variation compared to a motor with a narrower tooth. To incorporate the effect of L_m variation on SL (i.e. flux dependent variation), the relation (26c) is modified for load-dependent L_m as:

$$K_{SL} = \frac{SL}{P_{Out}} = \left(\frac{1}{p} \frac{L_{ls}}{L_m(I_m)} \right)^m \left(\frac{1}{p} \frac{L_{lr}}{L_m(I_m)} \right)^n \quad (27)$$

The third option is to make the exponents m and n function of slip. Hence, based on the above discussion, the modified version of (27) is:

$$K_{SL} = \left(\frac{1}{p} \frac{L_{ls}}{L_m(I_m)} \right)^{m(s)} \left(\frac{1}{p} \frac{L_{lr}}{L_m(I_m)} \right)^{n(s)} \quad (28)$$

To keep the number of variables small, exponent $n(s)$ is taken as a constant multiplier of $m(s)$ with zero offset, i.e. $n(s) = k \cdot m(s)$. Using this, (28) becomes:

$$K_{SL} = \left(\frac{1}{p} \frac{L_{ls}}{L_m(I_m)} \right)^{m(s)} \left(\frac{1}{p} \frac{L_{lr}}{L_m(I_m)} \right)^{k \cdot m(s)} \quad (29)$$

Determination of the exponents $m(s)$ and $n(s)$ uses measured

Fig. 5. (a) SL variation: Measured and estimated with the three variants of SL model for M-1, (b) Variation of the exponent $m(s)$ corresponding to (30)

values of SL fractions in total output power i.e., SL/P_{Out} at different loadings of the motor. This fraction is the measure of K_{SL} by its definition. III-B shows the K_{SL} variation for the motor M-1. The relation between these K_{SL} values and the $m(s)$ is given as:

$$\frac{1}{m(s)} \log(K_{SL}) = \log(A) + k \log(B) \quad (30)$$

where,

$$A = \frac{1}{p} \left(\frac{L_{ls}}{L_m} \right) \text{ and } B = \frac{1}{p} \left(\frac{L_{lr}}{L_m} \right).$$

The right term in (30) is solely a geometry and material dependant term and can be evaluated during the design or the testing. However, neglecting the saturation makes this term a constant and as a result, the variation of $m(s)$ w.r.t. loading (or the P_{Out} or the measured K_{SL}) will always lead to a constant:

$$\frac{1}{m(s)} \log(K_{SL}) \quad (31)$$

The $m(s)$ variation meeting this constraint with correlation $R^2 = 0.99$ is deduced as:

$$m(s) = (1 - s)^{\sqrt{2}\pi} \quad (32)$$

$$k = \frac{1}{2\sqrt{2}} \quad (33)$$

Fig.5(b) shows the trend of $m(s)$ correspond to the constraint (30) for the motor M-1.

IV. VALIDATION AND DESIGN IMPLICATION OF THE MODIFIED SL MODEL

This section presents the validation of the proposed load dependant SL model wherein, the SL measurements of eight 2.5kW prototype IMs are used. In further step, a 75kW IM is re-designed in congruence with the proposed model's notion to verify the model's capability. Subsection-A and B detail the analysis for the eight IMs and, the 75kW IM, respectively. Stator tooth flux density variation is used to correlate the model's predictions.

A. Validation with Motors of Same Power

Three variants of (29), inferred from the discussion in subsection-B of the previous section, are used to validate the proposed method and, these are:

- model with variable L_m and constant exponents (m and n). (i.e., expression (29) with load-dependent L_m): F-6a;
- model with constant L_m . (i.e., expression (29) with rated L_m values): F-6b
- model with load-dependent L_m : F-6c.

SL estimates with the three variants of the model for the eight IMs are shown in Table III.

The careful investigation of these tables and, Fig.5, shows that:

- the model F-6a always underestimates the SL w.r.t the measurement. Also, the trend of SL variation with F-6a, as shown in Fig.5(a), is different from the measurement. The mismatch in the trend asserts the need for load-dependent exponents m and n .
- model F-6b and F-6c show SL trend similar to the measurement. However, the variant F-6b always overestimates SL.

TABLE III
MEASURED AND ESTIMATED SL VALUES WITH MODIFIED SL MODEL
VARIANTS

IM	P_{Out} (kW)	SL (Measured)	Estimated SL with:		
			F-6a	F-6b	F-6c
M-1	1.95	19.05	13.95	21.34	20.43
	2.14	22.86	14.87	25.37	23.72
	2.32	26.98	16.81	29.89	28.99
	2.5	31.32	17.79	35.80	34.22
M-2	1.95	15.26	12.05	20.73	17.61
	2.14	18.34	13.03	24.35	20.47
	2.32	21.62	14.72	29.07	25.42
	2.5	25.09	15.49	34.84	29.94
M-3	1.95	15.24	12.05	19.43	17.60
	2.14	18.29	13.03	23.14	20.93
	2.32	21.59	14.72	27.67	26.08
	2.5	25.05	15.49	33.21	29.83
M-4	1.95	18.31	13.14	20.14	19.08
	2.14	21.92	14.59	24.61	23.66
	2.32	25.83	16.60	29.37	29.45
	2.5	29.14	16.56	35.65	33.48
M-5	1.95	28.58	13.86	30.72	29.27
	2.14	34.29	15.56	39.28	38.24
	2.32	40.48	17.86	46.98	47.84
	2.5	46.98	17.41	64.34	60.70
M-6	1.95	25.27	11.92	29.00	24.71
	2.14	30.32	13.56	36.60	32.36
	2.32	35.79	15.57	45.65	42.36
	2.5	41.54	15.09	59.72	51.30
M-7	1.95	23.04	12.11	24.89	22.47
	2.14	27.65	13.92	35.44	33.49
	2.32	32.64	16.05	44.14	43.82
	2.5	37.88	15.04	56.65	50.57
M-8	1.95	26.46	13.00	26.78	25.23
	2.14	31.75	15.22	30.52	30.47
	2.32	37.48	17.59	45.07	47.28
	2.5	43.50	16.17	56.93	52.79

- the SL estimates with F-6c have 8.13% (22.57%-14.44%, as shown in Table IV) lesser average error in comparison to F-6b. This suggests that the variation of L_m if taken into account gives SL values closer to the measurement in variable slip models.
- the motors M-1 to M-8 have the same specifications (as shown in annexure-2) but different slot geometries. Due to the difference in slot geometry, they have different levels of distortions in stator-tooth flux densities (as shown in Fig. 6) measured SL values. The variants F-6b and F-6c also give different values of SL for the eight motors.
- the slip-dependent exponents make the model a function of the rotor-bar material. The motors M-1 and M-6 have same geometry with M-1 having a copper (Cu) rotor and M-6 an aluminium (Al) rotor. Since the Al rotor runs at higher slip than Cu rotor for the same P_{out} , its exponents in (3) will be lesser than Cu motor. Hence, the SL of Al motor will be higher than the Cu motor (as shown in column-6: row-17-20 and row-2-5).

From the above discussion, it is clear that both the modified variants F-6b and F-6c (the models with variable exponents) predict the SL trend with a small relative difference. The variant F-6c needs multiple tests to encase the L_m variation while F-6b uses only one L_m value. Hence, F-6b is a convenient

TABLE IV
PERCENTAGE ERRORS IN CALCULATED SL VALUES

IM	P_{Out} (kW)	SL (Measured)	Percentage error with:		
			F-6a	F-6b	F-6c
M-1	1.95	19.05	26.78	12.00	7.25
	2.14	22.86	34.95	10.99	3.78
	2.32	26.98	37.69	10.77	7.43
	2.5	31.32	43.19	14.30	9.26
M-2	1.95	15.26	21.02	35.87	15.40
	2.14	18.34	28.94	32.76	11.63
	2.32	21.62	31.91	34.46	17.56
	2.5	25.09	38.26	38.88	19.35
M-3	1.95	15.24	20.92	27.47	15.49
	2.14	18.29	28.75	26.50	14.44
	2.32	21.59	31.82	28.17	20.82
	2.5	25.05	38.17	32.56	19.10
M-4	1.95	18.31	28.23	10.00	4.19
	2.14	21.92	33.43	12.26	7.93
	2.32	25.83	35.75	13.72	14.01
	2.5	29.14	43.17	22.33	14.90
M-5	1.95	28.58	51.51	7.52	2.42
	2.14	34.29	54.63	14.56	11.54
	2.32	33.45	55.86	16.07	18.19
	2.5	38.82	62.95	36.96	29.22
M-6	1.95	25.27	52.83	14.77	2.21
	2.14	30.32	55.26	20.72	6.73
	2.32	35.79	56.49	27.54	18.34
	2.5	41.54	63.68	43.76	23.49
M-7	1.95	23.04	47.43	8.02	2.49
	2.14	27.65	49.64	28.16	21.11
	2.32	32.64	50.82	35.22	34.23
	2.5	37.88	60.30	49.53	33.49
M-8	1.95	26.46	50.88	1.21	4.66
	2.14	31.75	52.06	3.86	4.04
	2.32	37.48	53.05	20.27	26.15
	2.5	43.50	62.81	30.88	21.36
Average error:			43.85	22.57	14.44
Standard deviation:			12.76	12.37	9.01

choice.

B. Validation with Design of 75kW IM

This section explains the use of the proposed model in designing an IM for lower SL. For this, an existing 4-pole, 400V, 75kW IM's stator, and rotor slot are redesigned to achieve a lower SL. It is noteworthy to mention here that, the original motor already possesses un-skewed slots and 5/6 short-pitched stator windings- the design attributes of lower SL. The design method used here to lower the SL uses these two prime variables. Since the winding scheme and geometries other than slot geometries are unchanged during modifications, the objective to reduce L_{IS}/L_m and L_{IR}/L_m in (6) can be summarised as reduction of term:

$$F_{obj} = K_C K_S (\lambda_{lg} + \lambda_{ISS}) \quad (34)$$

Slot areas are kept same while applying the (34). The changes in the initial IM are:

- variation airgap conductance harmonics (a_n) with the slot opening is recorded and the B_{S0} of stator is set to the minimum value constrained by the winding placement. In response, the carter's coefficient and a_n are reduced and consequently, the L_m is improved. (Fig.7 shows the magnitudes of a_n up to 4th order for both the IMs.)

TABLE V
SLOT SHAPES OF THE INITIAL AND MODIFIED 75kW IM

Stator slots: Initial design		Stator slots: Modified design
Dim. (mm) $B_{s0}=3.0$ $B_{s1}=8.08$ $B_{s2}=10.78$ $R_s=5.00$ $H_{s0}=1.0$ $H_{s1}=1.67$		Dim. (mm) $B_{s0}=2.6$ $B_{s1}=6.9$ $B_{s2}=9.5$ $R_s=3.00$ $H_{s0}=0$ $H_{s1}=1.30$
Rotor slots: Initial design		Rotor slots: Modified design
Dim. (mm) $B_{r01}=0$ $B_{r0}=6.00$ $B_{r1}=8.51$ $B_{r2}=5.16$ $H_{r1}=1.50$ $H_{r2}=12.5$		Dim. (mm) $B_{r01}=0$ $B_{r0}=7.0$ $B_{r1}=8.80$ $B_{r2}=5.70$ $H_{r1}=3.50$ $H_{r2}=8.60$ $R_1=3.5$ $R_2=1.95$

(a) Tooth flux density (B_t) in M1 (b) Tooth flux density (B_t) in M2

(c) Tooth flux density (B_t) in M3 (d) Tooth flux density (B_t) in M4

Fig. 6. Stator tooth flux density: for four 2.5kW IMs of copper cage rotor

Fig. 7. Harmonic spectrum of airgap conductance for the initial and modified 75kW IM.

- B_{S1} is made zero and the tooth width is increased by 1mm to reduce the a_n .
- H_{S2} is increased slightly to compensate the decrease in λ_{ISS} due to lower B_{S0} and H_{S1} (to limit the blocked rotor current).
- in rotor slot, all the sharp edges are replaced with arcs to avoid the leakage flux concentration. B_{S0} and B_{S0} are increased and to keep the net rotor leakage inductance as it is, slot height is slightly increased.

Table V shows the initial and modified slot shapes of the 75kW IMs.

1) *Experimental determination of SL:* Both the modified and original 75kW motors are tested for variable SL by changing the load at 400V, 50Hz sinusoidal power supply. The SL measurement is done as per the procedure in IEC60034-2-1, section 8.2.2.5. Table VI and VII show the test results.

2) *Verification of the proposed modified model F-6b:* The calculated values of SL using the proposed modified SL model are shown in Table VIII and Table IX for the original and modified 75kW IM, respectively.

Careful examination of these values reveals that:

- the motor designed with modified geometry has 4.5% smaller SL in comparison to the original IM.
- despite both IMs having the same power ratings, the

Fig. 8. Test bench with the 75kW IM

number of poles, winding schemes and volumes, the proposed model is capable enough to distinguish them for SL.

- though the model captures the SL trend in line with measurement, the error in SL prediction is high at light load and overload conditions.
- the results with the modified model show correlation coefficients of 0.99 with the corresponding measured SL values for both the 75kW IMs.

V. CONCLUSION

This work presents a stray loss model for a grid-connected IM for variable loading. The development of the model uses measurement data of 8 different IMs. The model is developed

TABLE VI
TEST RESULTS FOR SL OF ORIGINAL IM

Sr. No.	Load (%)	Supply Voltage (V)	Current (A)	Slip (%)	Power factor	SL (W)
1	50	400	77.1	0.51	74.2	106
2	75	400	102.6	0.79	83.3	237
3	100	400	131	1.08	87.1	425
4	125	400	161.5	1.39	88.8	659
5	150	400	193.6	1.7	89.4	956

TABLE VII
TEST RESULTS FOR SL OF MODIFIED IM

Sr. No.	Load (%)	Supply Voltage (V)	Current (A)	Slip (%)	Power factor	SL (W)
1	50	400	76.8	0.55	74.1	104
2	75	400	102.7	0.84	82.9	229
3	100	400	131.6	1.17	86.6	406
4	125	400	162.5	1.48	88.2	641
5	150	400	195.6	1.83	88.7	915

TABLE VIII
CALCULATED AND MEASURED SL:ORIGINAL 75kW IM

Sr. No.	P_{Out} (kW)	Slip (pu)	$m(s)$	$n(s)$	SL (cal-W)	SL (meas-W)	% error
1	37.5	0.01	0.98	0.35	149.16	106	40.72
2	56.25	0.01	0.96	0.34	240.25	237	1.37
3	75	0.01	0.95	0.34	347.07	425	18.34
4	93.75	0.01	0.94	0.33	467.38	659	29.08
5	112.5	0.02	0.92	0.33	609.46	956	36.25
Average error:							25.75
Standard deviation of error:							15.75
Correlation coefficient:							0.99

TABLE IX
CALCULATED AND MEASURED SL:MODIFIED 75kW IM

Sr. No.	P_{Out} (kW)	Slip (pu)	$m(s)$	$n(s)$	SL (cal-W)	SL (meas-W)	% error
1	37.5	0.01	0.98	0.35	125.88	104	21.04
2	56.25	0.01	0.97	0.34	202.68	229	11.49
3	75	0.01	0.95	0.34	290.61	406	28.42
4	93.75	0.01	0.94	0.34	392.28	641	38.8
5	112.5	0.02	0.93	0.33	507.92	915	44.49
Average error:							28.85
Standard deviation of error:							13.28
Correlation coefficient:							0.99

by testing the eight numbers of different small power IMs of same power ratings at different loads. The design applicability of the model is verified successfully by re-designing a medium power 75kW IM for SL minimisation. The results comparing the measured and estimated SL with the proposed model lay the following conclusions:

- The model is capable of determining the SL for both the objectives: SL at design stage and SL for a motor in operation.
- SL is not the same, even when the motors have the same output specifications. The proposed model is capable of segregating the IMs of the same specifications and different geometries for their SL values, which helps in designing an IM of smaller SL.
- SL in an IM is load dependent loss; the proposed model expresses this variation in line with the measurements. This is experimentally verified for both the small and medium power IMs here.
- The modified model also incorporates the impact of rotor

bar material on SL in terms of change in the exponents formulated.

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