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Evaluating the Performance of Various ACOPF Formulations Using Nonlinear Interior-Point Method

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Abstract—Alternating current optimal power flow (ACOPF) problem is a non-convex and a nonlinear optimization problem. Similar to most nonlinear optimization problems, ACOPF is an NP-hard problem. On the other hand, Utilities and independent service operators (ISO) require the problem to be solved in almost real-time. The real-world networks are often large in size and developing an efficient and tractable algorithm is critical to many decision-making processes in electricity markets. Interior-point methods (IPMs) for nonlinear programming are considered one of the most powerful algorithms for solving large-scale nonlinear optimization problems. However, the performance of these algorithms is significantly impacted by the optimization structure of the problem. Thus, the choice of the formulation is as important as choosing the algorithm for solving an ACOPF problem. Different ACOPF formulations are evaluated in this paper for computational viability and best performance using the interior-point line search (IPLS) algorithm. Different optimization structures are used in these formulations to model the ACOPF problem representing a range of varying sparsity. The numerical experiments suggest that the least sparse ACOPF formulation with polar voltages yields the best computational results. A wide range of test cases, ranging from 500-bus systems to 9591-bus systems, are used to verify the test results.

Index Terms—ACOPF, interior-point methods, optimal power flow, line search algorithm, sparsity, power system economics, ACOPF formulations.

I. INTRODUCTION

THE aim of power system operation is to deliver power to customers in a reliable and cost-effective manner. Optimal and reliable power dispatch is an optimization problem, which in its original form is referred to as the “alternating current optimal power flow” (ACOPF) problem [1], [2]. ACOPF is a nonlinear optimization problem with non-convex constraints, which makes it NP-hard [3].

The computational challenge is also significant because of the large size of real-world networks. The power grid in the United States is the largest machine ever built by humans, with hundreds of thousands of buses, transmission elements, and generators in three synchronous interconnections. According to the U.S. Energy Information Administration, the aggregate sum of annual retail transactions in the US power grid now exceeds USD 400 billion [4]. Clearly, a cost-effective operation is important due to the economic impacts of this crucial infrastructure.

Because of the difficulty of solving the ACOPF problem, the majority of the grid operators use some form of a direct current optimal power flow (DCOPF) formulation [5]. The

DCOPF linearization used in electricity market problems is based on a few assumptions that help convert the nonlinear ACOPF problem to a linear formulation [6], [7]. Although the implementations enable solving optimal power flow within the practical time constraints, rough approximations of voltage and reactive power flow are applied, requiring the application of conservative transmission limits. This can most likely result in an inefficient and sub-optimal operation of the grid.

According to a staff report by the U.S. Federal Energy Regulatory Commission (FERC), the differences between DCOPF and more efficient operations that may be achieved with ACOPF may incur 10% additional cost due to the sub-optimal operation of the grid [1]. In the United States, the economic costs represent tens of billions of dollars each year. Moreover, a good solution to ACOPF is critical to many complex decision-making processes, including consideration of trade-offs between active and reactive power flows [8], [9]. Thus, developing a method that efficiently solves the ACOPF problem can have a significant economic impact.

Over the past few decades, many methods have been proposed to solve the ACOPF problems. However, the majority of them overlook the impact of formulation on the performance of ACOPF solutions, especially when solving large-scale problems. Many methods have been proposed and tested for solving ACOPF, including a variety of convex relaxation techniques [10]–[13]. Another approach that has recently attracted a growing interest among researchers and power engineers is nonlinear programming (NLP) methods [14]–[22].

Finally, in [23] different optimal power flow formulations are presented and [24] numerically analyzed the performance of three different ACOPF formulations. However, the analysis are limited to branch flow models. These models do not encompass the least sparse formulations for an ACOPF problem.

The past work presents a great deal of progress in solving ACOPF problems by presenting different choices for the selection of an efficient algorithm. However, the performance of these algorithms can be significantly enhanced by the appropriate choice of the ACOPF formulation. None of the past work extensively explores the impact of formulation structure on the performance of their proposed algorithm, the impact of which could be significant, especially for large-scale problems.

This paper explicitly evaluates the performance of seven

different ACOPF formulations with different optimization structures. The study includes both branch flow and nodal injection models for an ACOPF problem. The performance of these formulations are evaluated using the Interior-Point Optimizer (IPOPT) implementation of interior-point nonlinear programming method. According to [25], IPLS is a powerful algorithm for solving large-scale nonlinear optimization problems. IPOPT efficiently implements the IPLS filter method, which is the state-of-the-art method, and it is typically used as a benchmark by most of the recent work on ACOPF algorithms [3], [26], [27]. The contribution of this paper can be summarized as follows:

- 1) Formulation of ACOPF problem with different optimization structures ranging from most sparse to least sparse formulations.
- 2) Evaluation of the performance of different ACOPF formulations and the complexity of their sub-problems.
- 3) Finally, identification of the computationally best performing formulations.

Similarly, different techniques, equipment, solvers, and algorithms can have different performance behavior corresponding to different ACOPF formulations and an overall computational impact on ACOPF solution. Some of the techniques include employing box-constraints, different linear solvers, and active-set. Furthermore, shunt FACTS devices can also have a computational impact on ACOPF by availing additional reactive power at different nodes. Finally, The convergence pattern might differ when used in other methods. Thus, as a future work it is worth investigating the performance of different ACOPF formulations using different techniques, equipment, solvers and algorithms such as sequential linear programming (SLP), sequential quadratic programming (SQP), and alternating direction method of multipliers (ADMM).

The rest of the paper is organized as follows. In Section II, optimal power flow and its different formulations are introduced. In section III, the results of numerical experiments are presented. Finally, in section IV the paper is concluded.

II. OPTIMAL POWER FLOW (OPF)

An optimal power flow constitutes the core of power system operations. It is an optimization extension of the power flow analysis [28], where in addition to feasibility, an optimal dispatch is also desired. The objective of power system operation is to serve the customers at the minimum cost, taking into consideration the physical constraints of the system. This can be expressed as an optimization problem, referred to as optimal power flow (OPF). In addition to power system operations, a solution to the optimal power flow is also essential for power system planning. In an ACOPF problem, the most important equations are about the representation of transmission branches. There are different approaches to model a transmission branch in a power system. The most commonly used representation of the transmission branch is called the π -model, which is applied in this paper.

Fig. 1: A general representation of a π -model of a transmission branch, where y_{ij} is the branch series admittance, y_{ij}^s is the branch shunt admittance, y_{ij}^M is the branch mutual admittance, and τ_{ij} is the branch complex tap ratio. If a branch is only a transmission line then $\tau_{ij}^i = 1$ and $y_{ij}^M = 0$. If a branch is only a transformer, then all the line parameters are zero. Thus, the figure represents a general model that can represent any type of transmission branch by simply adjusting the values of the parameters shown in the figure.

Let $y_{ij}^{net} = y_{ij}^M + (y_{ij} + y_{ij}^s) \cdot |\tau_{ij}^i|^2$, $y_{ji}^{net} = y_{ij} + y_{ij}^s$, and $2\mathcal{K} = \{i, j\} \in \mathcal{K} \cup \{j, i\} \in \mathcal{K}$. The admittance matrix can be written as shown in (1).

$$Y_{ii} = \sum_{k=1, k \neq i}^{\mathcal{K}_i^f} y_{ik}^{net} + \sum_{k=1, k \neq i}^{\mathcal{K}_i^t} y_{ki}^{net}, \quad (1a)$$

$$Y_{ij} = -y_{ij} \cdot \tau_{ij}^{i*} \quad (1b)$$

$$Y_{ji} = -y_{ij} \cdot \tau_{ij}^i \quad (1c)$$

An ACOPF problem can be formulated as an optimization problem as shown in (2).

$$\min \sum_{g \in \mathcal{G}} c_{2g} \cdot (\Re[s_g])^2 + c_{1g} \cdot \Re[s_g] + c_{0g} \quad (2a)$$

$$\text{s.t.} \quad s_{ij} = v_i \cdot (y_{ij}^{net} \cdot v_i + Y_{ij} \cdot v_j)^*, \quad \forall \{i, j\} \in 2\mathcal{K}. \quad (2b)$$

$$\sum_{g \in \mathcal{G}_n} s_g - \sum_{d \in \mathcal{D}_n} s_d = \sum_{k \in \mathcal{K}_n^f} s_{nk} + \sum_{k \in \mathcal{K}_n^t} s_{kn} + Y_n^L \cdot |v_n|^2, \quad \forall n \in \mathcal{N}. \quad (2c)$$

$$V_n^{Min} \leq |v_n| \leq V_n^{Max}, \quad \forall n \in \mathcal{N}. \quad (2d)$$

$$|s_{ij}| \leq I_{ij}^{Max} \cdot |v_n|, \quad \forall \{i, j\} \in 2\mathcal{K}. \quad (2e)$$

$$s_g^{Min} \leq s_g \leq s_g^{Max}, \quad \forall g \in \mathcal{G}. \quad (2f)$$

Equation (2) represents one of the general representations of an ACOPF problem in its complex form based on the grid model presented in [29]. c_{2g} , c_{1g} , and c_{0g} are the coefficients of quadratic cost function of generator g , $s_g = p_g + jq_g$ is the apparent power output of generator g , $v_n = v_n^r + jv_n^i = V_n e^{j\theta_n}$ is the voltage at bus n , $S_{ij} = P_{ij} + jQ_{ij}$ is the line apparent power flow from node i to j , $s_d = p_d + jq_d$ is the complex load, V_n^{Min} and V_n^{Max} are lower and upper voltage bounds at bus n , I_{ij}^{Max} is the current thermal limit of line $\{i, j\}$, $s_g^{Min} = p_g^{Min} + jq_g^{Min}$, $s_g^{Max} = p_g^{Max} + jq_g^{Max}$ are the complex generator g bounds, and $Y_n^L = G_n^L - jB_n^L$ is the complex Shunt element connected to bus n . $\mathcal{N}$ is the set of

buses, $\mathcal{G}$ is the set of generators, and $\mathcal{K}$ is the set of branches in the network.

Power generation output can be separated into real and reactive power components and therefore, the constraints in (2f) can be written as (3).

$$p_g^{Min} \leq p_g \leq p_g^{Max}, \forall g \in \mathcal{G}. \quad (3a)$$

$$q_g^{Min} \leq q_g \leq q_g^{Max}, \forall g \in \mathcal{G}. \quad (3b)$$

An ACOPF problem in its complex form can be expanded into different solvable optimization formulations as shown in fig. 2. While these formulations represent the same original ACOPF problem, they have different optimization structures that can have different convergence patterns. The ACOPF formulations can be broadly classified as either of the following.

- 1) Branch Flow Models
- 2) Nodal Injection Models

Fig. 2: Classification of different ACOPF formulations.

In addition, the flow can be either represented by power equations or current equations. Finally, voltages are either represented by polar form, rectangular form, or W -model in modeling different ACOPF formulations.

A. Branch Flow Model

In a branch flow model ACOPF problem, a power or current flow in each branch is explicitly represented as variables. To use the power equations, the branches can be modeled as (4).

$$s_{ij} = v_i \cdot i_{ij}^* = v_i \cdot (y_{ij}^{net} \cdot v_i + Y_{ij} \cdot v_j)^*, \forall \{i, j\} \in 2\mathcal{K} \quad (4)$$

To use current equations, the branches can be modeled as (5).

$$i_{ij} = y_{ij}^{net} \cdot v_i + Y_{ij} \cdot v_j, \forall \{i, j\} \in 2\mathcal{K} \quad (5)$$

These branch variables are then used in the node balance equations.

B. Nodal Injection Model

In this model, flows are not represented by explicit variables and instead are integrated in node power balance equations using admittance matrix and voltages as shown in (6).

$$\sum_{g \in \mathcal{G}_n} s_g - \sum_{d \in \mathcal{D}_n} s_d = v_n \cdot \sum_{k \in \mathcal{N}} Y_{nk}^* \cdot v_k^* + Y_n^L \cdot |v_n|^2, \forall n \in \mathcal{N}. \quad (6)$$

Three different approaches for modeling voltages in an ACOPF problem are proposed. of

C. Voltage Polar Form

In this category, the voltages are modeled in polar form. Using the polar form of the voltages in ACOPF problem, the branch power flow can be modeled as shown in (7).

$$P_{ij} = g_{ij}^{net} \cdot V_i^2 + (G_{ij} \cdot \cos(\theta_i - \theta_j) + B_{ij} \cdot \sin(\theta_i - \theta_j)) \cdot V_i \cdot V_j, \forall \{i, j\} \in 2\mathcal{K}. \quad (7a)$$

$$Q_{ij} = -b_{ij}^{net} \cdot V_i^2 + (G_{ij} \cdot \sin(\theta_i - \theta_j) - B_{ij} \cdot \cos(\theta_i - \theta_j)) \cdot V_i \cdot V_j, \forall \{i, j\} \in 2\mathcal{K}. \quad (7b)$$

Correspondingly, the node power balance can be written as shown in (8).

$$\sum_{g \in \mathcal{G}_n} p_g - \sum_{d \in \mathcal{D}_n} p_d = \sum_{k \in \mathcal{K}_n^f} P_{nk} + \sum_{k \in \mathcal{K}_n^t} P_{kn} + G_n^L \cdot V_n^2, \forall n \in \mathcal{N}. \quad (8a)$$

$$\sum_{g \in \mathcal{G}_n} q_g - \sum_{d \in \mathcal{D}_n} q_d = \sum_{k \in \mathcal{K}_n^f} Q_{nk} + \sum_{k \in \mathcal{K}_n^t} Q_{kn} - B_n^L \cdot V_n^2, \forall n \in \mathcal{N}. \quad (8b)$$

For branch flow, the power thermal limit of transmission branches can be enforced as shown in (9).

$$P_{ij}^2 + Q_{ij}^2 \leq (I_{ij}^{Max})^2 \cdot V_n^2, \forall \{i, j\} \in 2\mathcal{K}. \quad (9)$$

Similarly, for the nodal injection models with polar admittance, the nodal power balance can be written as shown in (10).

$$\sum_{g \in \mathcal{G}_n} p_g - \sum_{d \in \mathcal{D}_n} p_d = \sum_{k \in \mathcal{N}} |Y_{nk}| \cdot V_n \cdot V_k \cdot \cos(\theta_n - \theta_k - \theta_{Y_{nk}}^L) + G_n^L \cdot V_n^2, \forall n \in \mathcal{N}. \quad (10a)$$

$$\sum_{g \in \mathcal{G}_n} q_g - \sum_{d \in \mathcal{D}_n} q_d = \sum_{k \in \mathcal{N}} |Y_{nk}| \cdot V_n \cdot V_k \cdot \sin(\theta_n - \theta_k - \theta_{Y_{nk}}^L) - B_n^L \cdot V_n^2, \forall n \in \mathcal{N}. \quad (10b)$$

Similarly, for the nodal injection model with rectangular admittance, the nodal power balance can be written as shown in (11).

$$\begin{aligned} \sum_{g \in \mathcal{G}_n} p_g - \sum_{d \in \mathcal{D}_n} p_d &= \sum_{k \in \mathcal{N}} V_n \cdot V_k \cdot (G_{nk} \cdot \cos(\theta_n - \theta_k) \\ &\quad + B_{nk} \cdot \sin(\theta_n - \theta_k)) \\ &\quad + G_n^L \cdot V_n^2, \forall n \in \mathcal{N}. \end{aligned} \quad (11a)$$

$$\begin{aligned} \sum_{g \in \mathcal{G}_n} q_g - \sum_{d \in \mathcal{D}_n} q_d &= \sum_{k \in \mathcal{N}} V_n \cdot V_k \cdot (G_{nk} \cdot \sin(\theta_n - \theta_k) \\ &\quad - B_{nk} \cdot \cos(\theta_n - \theta_k)) \\ &\quad - B_n^L \cdot V_n^2, \forall n \in \mathcal{N}. \end{aligned} \quad (11b)$$

For nodal injection models, the current thermal limit of transmission branches can be enforced as shown in (12).

$$\begin{aligned} 2 \cdot |y_{ij}^{net}| \cdot |Y_{ij}| \cdot V_i \cdot V_j \cdot [\\ \cos(\theta_n - \theta_k + \theta_{nk}^Y)] \\ + |y_{ij}^{net}|^2 \cdot V_i^2 + |Y_{ij}|^2 \cdot V_j^2 \leq (I_{ij}^{Max})^2, \forall \{i, j\} \in 2\mathcal{K}. \end{aligned} \quad (12a)$$

The voltage magnitude limits can be enforced using (13).

$$V_n^{Min} \leq V_n \leq V_n^{Max}, \forall n \in \mathcal{N}. \quad (13)$$

D. Voltage Rectangular Form

In this category, voltages are modeled in rectangular form. Using the rectangular form of the voltage in the ACOPF problem, the branch power flow can be modeled as shown in (14).

$$\begin{aligned} P_{ij} &= g_{ij}^{net} \cdot [(v_i^r)^2 + (v_i^i)^2] + G_{ij} \cdot v_i^r \cdot v_j^r + B_{ij} \cdot v_i^i \cdot v_j^r \\ &\quad + G_{ij} \cdot v_i^i \cdot v_j^i - B_{ij} \cdot v_i^r \cdot v_j^i, \forall \{i, j\} \in 2\mathcal{K}. \end{aligned} \quad (14a)$$

$$\begin{aligned} Q_{ij} &= -b_{ij}^{net} \cdot [(v_i^r)^2 + (v_i^i)^2] + G_{ij} \cdot v_i^i \cdot v_j^r - B_{ij} \cdot v_i^r \cdot v_j^r \\ &\quad - G_{ij} \cdot v_i^r \cdot v_j^i - B_{ij} \cdot v_i^i \cdot v_j^i, \forall \{i, j\} \in 2\mathcal{K}. \end{aligned} \quad (14b)$$

Correspondingly, the node power balance can be written as shown in (15).

$$\begin{aligned} \sum_{g \in \mathcal{G}_n} p_g - \sum_{d \in \mathcal{D}_n} p_d &= \sum_{k \in \mathcal{K}_n^f} P_{nk} + \sum_{k \in \mathcal{K}_n^t} P_{kn} \\ &\quad + G_n^L \cdot [(v_n^r)^2 + (v_n^i)^2], \forall n \in \mathcal{N}. \end{aligned} \quad (15a)$$

$$\begin{aligned} \sum_{g \in \mathcal{G}_n} q_g - \sum_{d \in \mathcal{D}_n} q_d &= \sum_{k \in \mathcal{K}_n^f} Q_{nk} + \sum_{k \in \mathcal{K}_n^t} Q_{kn} \\ &\quad - B_n^L \cdot [(v_n^r)^2 + (v_n^i)^2], \forall n \in \mathcal{N}. \end{aligned} \quad (15b)$$

For branch power flow, the thermal limit of transmission branches can be enforced as shown in (16).

$$P_{ij}^2 + Q_{ij}^2 \leq (I_{ij}^{Max})^2 \cdot [(v_n^r)^2 + (v_n^i)^2], \forall \{i, j\} \in 2\mathcal{K}. \quad (16)$$

Using the rectangular form of the voltage in the ACOPF problem, the branch current flow can be modeled as shown in (17).

$$\begin{aligned} i_{ij}^r &= g_{ij}^{net} \cdot v_i^r - b_{ij}^{net} \cdot v_i^i + G_{ij} \cdot v_j^r \\ &\quad - B_{ij} \cdot v_j^i, \forall \{i, j\} \in 2\mathcal{K}. \end{aligned} \quad (17a)$$

$$\begin{aligned} i_{ij}^i &= g_{ij}^{net} \cdot v_i^i + b_{ij}^{net} \cdot v_i^r + G_{ij} \cdot v_j^i \\ &\quad + B_{ij} \cdot v_j^r, \forall \{i, j\} \in 2\mathcal{K}. \end{aligned} \quad (17b)$$

Correspondingly, the node power balance can be written as shown in (18).

$$\begin{aligned} \sum_{g \in \mathcal{G}_n} p_g - \sum_{d \in \mathcal{D}_n} p_d &= \sum_{\{n,k\} \in \mathcal{K}_n^f \cup \mathcal{K}_n^t} (v_n^r \cdot i_{nk}^r + v_n^i \cdot i_{nk}^i) \\ &\quad + G_n^L \cdot [(v_n^r)^2 + (v_n^i)^2], \forall n \in \mathcal{N}. \end{aligned} \quad (18a)$$

$$\begin{aligned} \sum_{g \in \mathcal{G}_n} q_g - \sum_{d \in \mathcal{D}_n} q_d &= \sum_{\{n,k\} \in \mathcal{K}_n^f \cup \mathcal{K}_n^t} (v_n^i \cdot i_{nk}^r + v_n^r \cdot i_{nk}^i) \\ &\quad - B_n^L \cdot [(v_n^r)^2 + (v_n^i)^2], \forall n \in \mathcal{N}. \end{aligned} \quad (18b)$$

where $i_{ij} = i_{ij}^r + j i_{ij}^i$ is the complex line current flow from node i to j .

For branch current flow, the thermal limit of transmission branches can be enforced as shown in (19).

$$(i_{ij}^r)^2 + (i_{ij}^i)^2 \leq (I_{ij}^{Max})^2, \forall \{i, j\} \in 2\mathcal{K}. \quad (19)$$

Similarly, for the nodal injection model with rectangular admittance, the nodal power balance can be written as shown in (20).

$$\begin{aligned} \sum_{g \in \mathcal{G}_n} p_g - \sum_{d \in \mathcal{D}_n} p_d &= \sum_{k \in \mathcal{N}} (G_{nk} \cdot v_n^r \cdot v_k^r + B_{nk} \cdot v_n^i \cdot v_k^r \\ &\quad + G_{nk} \cdot v_n^i \cdot v_k^i - B_{nk} \cdot v_n^r \cdot v_k^i) \\ &\quad + G_n^L \cdot [(v_n^r)^2 + (v_n^i)^2], \forall n \in \mathcal{N}. \end{aligned} \quad (20a)$$

$$\begin{aligned} \sum_{g \in \mathcal{G}_n} q_g - \sum_{d \in \mathcal{D}_n} q_d &= \sum_{k \in \mathcal{N}} (G_{nk} \cdot v_n^i \cdot v_k^r - B_{nk} \cdot v_n^r \cdot v_k^r \\ &\quad - G_{nk} \cdot v_n^r \cdot v_k^i - B_{nk} \cdot v_n^i \cdot v_k^i) \\ &\quad - B_n^L \cdot [(v_n^r)^2 + (v_n^i)^2], \forall n \in \mathcal{N}. \end{aligned} \quad (20b)$$

For nodal injection models, the thermal limit of transmission branches can be enforced as shown in (21).

$$\begin{aligned} (g_{ij}^{net} \cdot v_i^r - b_{ij}^{net} \cdot v_i^i \\ + G_{ij} \cdot v_j^r - B_{ij} \cdot v_j^i)^2 \\ + (g_{ij}^{net} \cdot v_i^i + b_{ij}^{net} \cdot v_i^r \\ + G_{ij} \cdot v_j^i + B_{ij} \cdot v_j^r)^2 \leq (I_{ij}^{Max})^2, \forall \{i, j\} \in 2\mathcal{K}. \end{aligned} \quad (21a)$$

The voltage magnitude limits can be enforced using (22).

$$(V_n^{Min})^2 \leq (v_n^r)^2 + (v_n^i)^2 \leq (V_n^{Max})^2, \forall n \in \mathcal{N}. \quad (22)$$

E. Voltage W-Model

In this category, the square of voltage magnitudes are replaced by a variable notated as W^d . This will make the optimization structure of the problem further sparse. In this model, V_n^2 variables or its equivalent expressions are substituted with W_n^d variables.

The node power balance for the branch current flow can be modeled as shown in (23).

TABLE I: The table analyzes optimization structure of different ACOPF formulations with objective function (2a). Typically, for a transmission system $\mathcal{K} > \mathcal{N} > \mathcal{G}$.

Sno.	ACOPF Formulations	Constraints	Num. of total constraints	Num. of NL constraints
1	Branch Power Flow Polar Voltage (BFPV)	(3),(7),(8),(9),(13)	$4\mathcal{G} + 4\mathcal{N} + 6\mathcal{K}$	$2\mathcal{N} + 6\mathcal{K}$
2	Branch Power Flow Rectangular Voltage (BPRV)	(3),(14),(15),(16),(22)	$4\mathcal{G} + 4\mathcal{N} + 6\mathcal{K}$	$4\mathcal{N} + 6\mathcal{K}$
3	Branch Current Flow Rectangular Voltage (BCFRV)	(3),(17),(18),(19),(22)	$4\mathcal{G} + 4\mathcal{N} + 6\mathcal{K}$	$4\mathcal{N} + 2\mathcal{K}$
4	Branch Current Flow W-Model (BCFW)	(3),(17),(23),(19),(24a),(24b)	$4\mathcal{G} + 5\mathcal{N} + 10\mathcal{K}$	$3\mathcal{N} + 6\mathcal{K}$
5	Nodal Injection Polar Admittance Polar Voltage (NIPAPV)	(3),(10),(12),(13)	$4\mathcal{G} + 4\mathcal{N} + 2\mathcal{K}$	$2\mathcal{N} + 2\mathcal{K}$
6	Nodal Injection Rectangular Admittance Polar Voltage (NIRAPV)	(3),(11),(12),(13)	$4\mathcal{G} + 4\mathcal{N} + 2\mathcal{K}$	$2\mathcal{N} + 2\mathcal{K}$
7	Nodal Injection Rectangular Admittance Rectangular Voltage (NIRARV)	(3),(20),(21),(22)	$4\mathcal{G} + 4\mathcal{N} + 2\mathcal{K}$	$4\mathcal{N} + 2\mathcal{K}$

$$\sum_{g \in \mathcal{G}_n} p_g - \sum_{d \in \mathcal{D}_n} p_d = \sum_{\{n,k\} \in \mathcal{K}_n^f \cup \mathcal{K}_n^t} (v_n^r \cdot i_{nk}^r + v_n^i \cdot i_{nk}^i) + G_n^L \cdot W_n^d, \forall n \in \mathcal{N}. \quad (23a)$$

$$\sum_{g \in \mathcal{G}_n} q_g - \sum_{d \in \mathcal{D}_n} q_d = \sum_{\{n,k\} \in \mathcal{K}_n^f \cup \mathcal{K}_n^t} (v_n^i \cdot i_{nk}^r + v_n^r \cdot i_{nk}^i) - B_n^L \cdot W_n^d, \forall n \in \mathcal{N}. \quad (23b)$$

The W-Model and the limits can be formed using the voltage relationship shown in (24).

$$(V_n^{Min})^2 \leq W_n^d \leq (V_n^{Max})^2, \forall n \in \mathcal{N}. \quad (24a)$$

$$W_n^d = (v_n^r)^2 + (v_n^i)^2, \forall n \in \mathcal{N}. \quad (24b)$$

Table I tabulates all seven ACOPF formulations using different combination of the equations derived above. All seven formulations use (2a) as their objective function.

III. NUMERICAL EXPERIMENTS

Various numerical experiments were performed on an 8-core Intel(R) i9-9980 HK CPU @2.40 GHz with 32 GB RAM system to test and validate the performance of different ACOPF formulations. The programming for numerical experiments are implemented in Julia v1.5.3 using IPOPT v3.13.2 and JuMP v0.21.5, in a Linux Ubuntu operating system.

The numerical experiments were performed on a set of synthetic networks varying from 500-bus systems to 9591-bus systems. Table II presents the description of different test cases used in the numerical experiments. The test results are tabulated in Table III and IV. Table III shows the computation time for each test case using different ACOPF formulations. Similarly, table IV shows the number of iterations needed to solve each test case using different ACOPF formulations.

Performance Profile

To assess the performance of different ACOPF formulations solved with IPOPT solver, we plot the performance profile [30] using different scenarios of a wide range of test cases shown in Table II.

Let $\mathcal{F}$ be the set of ACOPF formulations. For each problem $p \in \mathcal{P}$ and formulation $f \in \mathcal{F}$, let t_{pf} be the solution time for solving problem p using formulation f . We define the performance ratio

$$r_{pf} := \frac{t_{pf}}{\min_{f \in \mathcal{F}} t_{pf}} \quad (25)$$

that represents the performance of formulation f as compared with the best performance by any formulation on problem p . Now we define the performance of formulation f on any given problem as the probability for formulation f that a performance ratio r_{pf} is within a factor τ of the best possible ratio; that is,

$$\rho_f(\tau) := \frac{1}{|\mathcal{P}|} |\{p \in \mathcal{P} : r_{pf} \leq \tau\}| \quad (26)$$

Using (26) we plot the performance profile of different ACOPF formulations as shown in fig. 3.

The performance profile shows that IPLS algorithm, in general, converges faster with dense (least sparse) ACOPF formulations. Similarly, it also converges faster when parameters, particularly, voltages are modeled using polar form. Therefore, the best performing formulations are the least sparse formulations with polar voltages. Modeling current flow in branches instead of power also helps branch flow ACOPF formulations with rectangular voltages. However, using voltages in polar form always outperforms other branch flow ACOPF formulations presented in this paper.

TABLE II: The table describes the size of different test networks used in the numerical experiments.

Networks	Buses	Gens	Branches
Network_01R-040	500	90	594
Network_03R-050	793	210	912
Network_05R-018	2000	384	3639
Network_07R-122	2312	617	3013
Network_81R-050	3288	379	4871
Network_84R-100	4601	410	7304
Network_09R-070	4918	1340	6727
Network_12R-030	9591	365	15915

Finally, for better visualization and breakdown of performance for each ACOPF formulation, a bar chart is plotted in fig. 4 to show the computation time, and in fig. 5 to show the number of iterations needed to solve different ACOPF formulations for *Network_12R-030*, which is a large-scale network. As can be seen in fig. 4 and 5, different ACOPF formulations can have different complexity of sub-problems in IPLS Algorithm for the same ACOPF problem. For instance, in *Network_12R-030* the BFPV formulation has the highest computation time while solving the ACOPF problem in fewer iterations than BCFRV and NIPARV. This shows that the

Fig. 3: Performance profile of different ACOPF formulations using IPOPT solver for different scenarios of a wide range of test cases shown in Table II.

TABLE III: The table presents the computation time of one scenario of various test networks using different ACOPF formulations.

Network	BFPFV	BPFVRV	BCFRV	BCFW	NIPAPV	NIRAPV	NIPARV
Network_01R-040	3.73068	4.982912	4.076387	2.738256	1.91067	1.755969	3.081178
Network_03R-050	3.732486	3.773625	3.080637	3.440697	1.510793	1.443192	3.046712
Network_05R-018	16.363282	16.898092	412.73109	68.677693	14.070075	15.309972	28.015885
Network_07R-122	48.769313	59.856143	147.61875	67.495748	17.206433	15.67697	51.931805
Network_81R-050	96.409814	82.167673	66.547177	81.969888	36.648551	33.459432	57.04276
Network_84R-100	210.508382	175.200104	163.442526	152.807441	40.084922	43.241946	73.861782
Network_09R-070	44.880277	46.045293	39.612629	47.888543	22.53276	19.634313	38.124695
Network_12R-030	561.562603	368.828423	457.325533	392.646498	122.726361	133.988779	484.546118

TABLE IV: The table presents the number of barrier iterations needed to solve one scenario of various test networks using different ACOPF formulations.

Network	BFPFV	BPFVRV	BCFRV	BCFW	NIPAPV	NIRAPV	NIPARV
Network_01R-040	45	49	62	50	46	41	46
Network_03R-050	22	23	28	34	21	19	28
Network_05R-018	31	36	709	120	40	41	56
Network_07R-122	58	86	253	112	47	43	101
Network_09R-070	32	38	41	46	30	28	43
Network_12R-030	132	104	147	131	68	71	198
Network_81R-050	68	59	82	76	60	56	73
Network_84R-100	142	130	152	133	62	62	77

subproblem in BFPFV formulation has higher complexity than the subproblem in BCFRV and NIPARV formulations.

IV. CONCLUSION

The paper presents seven different ACOPF formulations, each representing a unique optimization structure and sparsity. The work includes an explicit investigation of the impact of these ACOPF formulations on the performance of ACOPF algorithms. The paper employs the Interior-Point Line Search Algorithm for Nonlinear Programming implemented by IPOPT solver to evaluate these formulations. Results show that the choice of formulation, which is often overlooked, can significantly impact the computation time of an algorithm,

especially for large-scale networks. Numerical results in the paper suggest a consistently superior performance by Nodal Injection Polar Admittance Polar Voltage (NIPAPV) and Nodal Injection Rectangular Admittance Polar Voltage (NIRAPV) formulations among different ACOPF formulations studied in this paper and across all ranges of test networks. The main contributing factors in the superior performance of NIPAPV and NIRAPV are the least sparsity of their optimization structures and modeling voltages in polar form. The complexity of the sub-problem of these two formulations compared to the other formulations has also consistently remained the least.

Fig. 4: The bar chart shows the computation time of IPLS algorithm to solve each formulation using the same scenario of *Network_{12R} - 030*.

Fig. 5: The bar chart shows the number of iterations needed by IPLS Algorithm to solve each formulation for the same scenario of *Network_{12R} - 030*.

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