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Far-Field Radar Cross-Section Determination From Near-Field 3-D Synthetic Aperture Imaging With Arbitrary Antenna Scanning Surfaces

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Abstract—In this study, we propose a generalized algorithm for far-field radar cross-section determination from 3-D synthetic aperture imaging with arbitrary antenna scanning surfaces. This method belongs to the class of techniques called image-based near-field to far-field transformation. The previous image-based approaches were formulated based on a specific antenna scanning surface or trajectory such as a line, a plane, a circle, a cylinder, and a sphere—and the majority of them considered 2-D radar images to determine the azimuth radar cross-section. We generalize the conventional image-based technique to accommodate an arbitrary antenna scanning surface, and consider a 3-D radar image for radar cross-section prediction in both the azimuth and zenith directions. We demonstrate the proposed algorithm via numerical simulations and anechoic chamber measurements.

Index Terms—Near-field to far-field transformation, radar cross-section, synthetic aperture radar, image reconstruction

I. INTRODUCTION

RADAR cross-section (RCS) is an essential quantity in characterizing a target or an object to be measured using radar systems. Because RCS is defined in the far-field distance, which is proportional to the electrical size of the target under test, direct RCS measurements of a large target require a long distance between the target and the radar antenna. However, the spatial requirement or the cost of the measurement facility for the direct measurement of an electrically large target is often unacceptable—and one demands an alternative method to measure or predict the RCS in a shorter measurement distance.

One approach is the use of a compact range system in which a transmitted and reflected spherical wave is physically transformed to a plane wave by a large reflector. The advantage of this method is that, because a target can be irradiated by a physical plane wave, the measured RCS is free of conversion errors associated with signal processing. However, a larger target requires a sufficiently larger reflector compared to the target geometry, which is very expensive to build and maintain.

Other approaches are based on signal processing, which converts the scattered wave measured in the near-field into far-field data. This technique is known as the near-field to far-field transformation (NFFFT), and various NFFFT methods have

been developed for both RCS and antenna measurements. The well-known existing approaches are: the method based on the physical optics approximation [1], the plane-wave synthesis [2]–[4], and the image-based method [5]–[14] which is treated in this research.

The theoretical underpinning of image-based methods is the synthetic aperture radar (SAR), or inverse SAR (ISAR) imaging [15]–[18]. The majority of the methods in this class explicitly generate radar images to obtain the spatial distribution of the scattering centers, and then the obtained image is used to infer the far-field scattered data [5]–[12]. Other methods avoid explicit image generation to accelerate the computation [13], [14]. The process of the explicit image-based methods can be interpreted as follows: first, we solve the inverse scattering problem of the wave equation under the Born or single-scattering approximation [18] to reconstruct the secondary source distribution induced by the incident wave. Next, we solve the forward problem of the wave equation in which the reconstructed radar image is used as the source term in order to determine the field distribution at an arbitrary location produced by that source.

Radar images have useful applications, in spite of their computational cost, such as the determination of a strong source of scattering for the design and evaluation of low RCS products. Moreover, a spatial filter can be applied to extract the desired part of the target under testing, or to reject unwanted scattering sources [17], [19]. In this research, we focus on the explicit image-based method; unless otherwise specified, we use the term ‘image-based method’ to refer the explicit approach throughout this paper.

As pointed out in [8], [11], [12], one of the challenges in the image-based method is the measurement of a strongly asymmetric target under a shorter antenna separation from the target. Without taking any correction for this situation into account in the imaging process, the accuracy of the RCS prediction is degraded. In [8], the correction factor was introduced in the 2-D image reconstruction with a circular scanning trajectory around the target, such that the integral transformation in the radar imaging is self-consistent for every electrically small scatterer. In [11], [12], this correction factor was extended to 3-D cylindrical scanning geometries.

In this study, we propose a generalization of the above-mentioned image-based NFFFT algorithms with a correction factor. Most notably, we consider an arbitrary antenna scanning surface for the near-field measurement—including a canonical surface such as a plane, a cylinder, and a sphere—

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and any other numerically described surfaces for which an analytical expression may be unavailable. We derive a general expression for the correction factor that can be used for an arbitrary scanning surface, and the specific correction factors for the planar, cylindrical, and spherical scanning surfaces are derived as special cases. Furthermore, we provide a numerical computation scheme for the correction factor, which enables the use of a numerically described scanning surface. The derivation of this generalized correction factor is the major novelty and contribution of this study.

As we consider 2-D scanning surfaces in the 3-D spatial domain, the 3-D radar image is utilized rather than the 2-D image to derive the far-field RCS. Image-based methods using circular scanning assume that the height of the target is sufficiently small such that the far-field criterion is satisfied with respect to the height dimension. By generating a 3-D radar image with a 3-D scanning geometry, such as a cylindrical or spherical surface, this restriction can be removed [10]–[12]. Moreover, the NFFFT based on the 3-D image can produce the RCS pattern in both the azimuth and zenith (or elevation) directions—and the location of the scattering source can be determined in the 3-D spatial domain.

The remainder of this paper is organized as follows. In Section II, we present the formulation of the proposed image-based NFFFT algorithm. We provide the system and signal model of the 3-D synthetic aperture imaging with an arbitrary antenna scanning surface, and derive the generalized correction factor for the imaging to accurately predict the far-field RCS from the radar image. Section III discusses the numerical simulation of the proposed NFFFT algorithm using a simple model consisting of point scatterers. In addition to the standard cylindrical and spherical scanning surfaces, we provide a curved cylindrical surface in which the cylinder radius is a function of the height, as an example of an arbitrary antenna scanning surface. Section IV presents the experimental validation performed in an anechoic chamber. In this experiment, we employed a metallic aircraft model as the test target. Finally, Section V summarizes this study presented in the paper.

II. PROBLEM FORMULATION

In this section, we discuss the formulation of the proposed NFFFT algorithm. We consider a monostatic scattering experiment in which the transmitting and receiving antennas are scanned over an arbitrary surface around the target under test. For this measurement configuration, we derive a correction factor used in the image reconstruction procedure, which improves the accuracy of the following RCS computation.

A. System and Signal Model

Fig. 1 depicts the system model of the synthetic aperture imaging with an arbitrary antenna scanning surface. We consider a monostatic scattering measurement in the 3-D spatial domain (x, y, z) , and denote a general location in this space as $\mathbf{r}$. The transmitting and the receiving antennas are located on an arbitrary scanning surface $\mathbf{r}_0(u, v)$ parametrized by the coordinates on the surface (u, v) . For simplicity, we assume a simply connected closed surface for $\mathbf{r}_0(u, v)$ which completely

Fig. 1. System model of the synthetic aperture imaging with an arbitrary antenna scanning surface.

encloses the spatial domain to be imaged. However, we can employ an open surface and a discrete set of antenna locations for the measurement and the image reconstruction, as discussed later.

We define the vectors $\mathbf{r}$ and $\mathbf{r}_0(u, v)$ as

$$\mathbf{r}_0(u, v) = x_0(u, v)\hat{\mathbf{x}} + y_0(u, v)\hat{\mathbf{y}} + z_0(u, v)\hat{\mathbf{z}} \quad (1a)$$

$$\mathbf{r} = x\hat{\mathbf{x}} + y\hat{\mathbf{y}} + z\hat{\mathbf{z}} \quad (1b)$$

where $\hat{\mathbf{x}}$, $\hat{\mathbf{y}}$, and $\hat{\mathbf{z}}$ are the unit vectors along the x -, y -, and z -directions, respectively. For example, if we use a spherical scanning surface of radius r_0 , and denote the azimuth and zenith angles as $u = \phi_0$ and $v = \theta_0$, respectively, the surface can be expressed as

$$x_0(\phi_0, \theta_0) = r_0 \cos \phi_0 \sin \theta_0 \quad (2a)$$

$$y_0(\phi_0, \theta_0) = r_0 \sin \phi_0 \sin \theta_0 \quad (2b)$$

$$z_0(\phi_0, \theta_0) = r_0 \cos \theta_0. \quad (2c)$$

The vector $\mathbf{R}$ from point $\mathbf{r}$ to the antenna location $\mathbf{r}_0(u, v)$ on the scanning surface can be determined via

$$\mathbf{R}(u, v) = \mathbf{r}_0 - \mathbf{r} = R_x\hat{\mathbf{x}} + R_y\hat{\mathbf{y}} + R_z\hat{\mathbf{z}} \quad (3a)$$

$$R_x(u, v) = x_0(u, v) - x \quad (3b)$$

$$R_y(u, v) = y_0(u, v) - y \quad (3c)$$

$$R_z(u, v) = z_0(u, v) - z. \quad (3d)$$

Using vector $\mathbf{R}$ in Eq. (3), the distance between the arbitrary location $\mathbf{r}$ and the antenna location $\mathbf{r}_0(u, v)$ is expressed as follows:

$$R(u, v) = |\mathbf{R}(u, v)| = |\mathbf{r}_0 - \mathbf{r}| = \sqrt{R_x^2 + R_y^2 + R_z^2}. \quad (4)$$

The transmitting antenna at the location $\mathbf{r}_0(u, v)$ on the scanning surface emits an electromagnetic wave, and the receiving antenna at the same location collects the scattered wave. A similar measurement was repeated at all the desired antenna locations. We denote the angular frequency of the transmitted wave as ω , and the corresponding wavenumber as $k = \omega/c$, where c is the speed of wave propagation.

We denote the reflection coefficient of a point scatterer at location $\mathbf{r}$ as C . For this single target, the scattered wave $E^s(k, \mathbf{r}_0)$ recorded at the antenna location $\mathbf{r}_0$ is given by

$$E^s(k, \mathbf{r}_0) = P^2(k, \mathbf{r}_0, \mathbf{r}) \frac{k^2 C e^{-2jk|\mathbf{r}_0 - \mathbf{r}|}}{\sqrt{4\pi} |\mathbf{r}_0 - \mathbf{r}|^2} \quad (5)$$

where $P(k, \mathbf{r}_0, \mathbf{r})$ is the directive pattern of the antenna. If the reflection coefficient is continuously distributed over the spatial volume V , the received signal can be expressed as

$$E^s(k, \mathbf{r}_0) = \frac{k^2}{\sqrt{4\pi}} \int_V P^2(k, \mathbf{r}_0, \mathbf{r}) C(\mathbf{r}) \frac{e^{-2jk|\mathbf{r}_0 - \mathbf{r}|}}{|\mathbf{r}_0 - \mathbf{r}|^2} d\mathbf{r} \quad (6)$$

where we denote the reflection coefficient as a function of the spatial variable $\mathbf{r}$ as $C(\mathbf{r})$.

If the target is a cloud of discrete point scatterers located at $\mathbf{r}_i, i \in \{1, 2, \dots\}$, the reflection coefficient $C(\mathbf{r})$ can be defined as

$$C(\mathbf{r}) = \sum_i C_i \delta(|\mathbf{r}_0 - \mathbf{r}_i|) \quad (7)$$

where C_i represents the reflectivity of the i th point scatterer. For electrically small spheres of radii a_i ($ka_i < 0.4$), we can use the following expression for C_i [8], [20].

$$C_i = 3\sqrt{\pi}a_i^3. \quad (8)$$

Substituting Eq. (7) into Eq. (6) yields the following discrete scatterer model.

$$E^s(k, \mathbf{r}_0) = \frac{k^2}{\sqrt{4\pi}} \sum_i P^2(k, \mathbf{r}_0, \mathbf{r}_i) C_i \frac{e^{-2jk|\mathbf{r}_0 - \mathbf{r}_i|}}{|\mathbf{r}_0 - \mathbf{r}_i|^2}. \quad (9)$$

The discrete scatterer model in Eq. (9) is used to test the proposed algorithm in Section III.

B. Image Reconstruction

Next, we discuss the image reconstruction problem. We consider an ideal 3-D spatial image $\psi(x, y, z)$ of an electrically small scatterer (a point scatterer) at the location $\mathbf{r}_1 = (x_1, y_1, z_1)$, defined as

$$\psi(\mathbf{r}) = C_1 \delta(x - x_1) \delta(y - y_1) \delta(z - z_1) \quad (10)$$

where $\delta(\cdot)$ is a delta function. Our objective is to reconstruct the 3-D spatial image $\psi(\mathbf{r})$ of Eq. (10) from the measured scattered signal $E^s(k, \mathbf{r}_0)$ of Eq. (5). The general expression for the image reconstruction is given by the following integral transformation of the received signal $E^s(k, \mathbf{r}_0)$.

$$\begin{aligned} \psi(\mathbf{r}) &= \int_0^\infty \int_D E^s(k, \mathbf{r}_0) F(k, \mathbf{r}_0, \mathbf{r}) d\mathbf{r}_0 dk \\ &= \int_0^\infty \int_{D_v} \int_{D_u} E^s(k, \mathbf{r}_0) F(k, \mathbf{r}_0, \mathbf{r}) du dv dk. \end{aligned} \quad (11)$$

where D , D_v , and D_u are the domains of the integration of $\mathbf{r}_0$, u , and v , respectively. The weighting function $F(k, \mathbf{r}_0, \mathbf{r})$ is referred to as the focusing factor and is given by:

$$F(k, \mathbf{r}_0, \mathbf{r}) = g(k, \mathbf{r}_0, \mathbf{r}) \frac{|\mathbf{r}_0 - \mathbf{r}|^2}{P^2(k, \mathbf{r}_0, \mathbf{r})} e^{2jk|\mathbf{r}_0 - \mathbf{r}|}. \quad (12)$$

In Eq. (12), $g(k, \mathbf{r}_0, \mathbf{r})$ is the correction factor to be determined. Note that the standard SAR or ISAR formulation corresponds to $g(\mathbf{r}_0, \mathbf{r}) = 1$. If the antenna can be placed at a sufficiently large distance compared to the target dimension, the NFFT can be precisely performed without the correction factor.

Fig. 2. Local coordinate system centered at the scatter location $\mathbf{r}_1$.

The numerical implementation of Eq. (12) can be obtained by simply replacing the triple integral in Eq. (12) with the following triple summation.

$$\psi(\mathbf{r}) \simeq \sum_\ell \sum_m \sum_n E^s[k_\ell, \mathbf{r}_0(u_m, v_n)] F[k_\ell, \mathbf{r}_0(u_m, v_n), \mathbf{r}] \quad (13)$$

where k_ℓ is the discrete sample of the wavenumber k , u_m and v_n are the discrete coordinates on the scanning surface, and ℓ , m , and n are the indices that represent each sample value.

For an efficient implementation of (13), we can use a method utilizing the fast Fourier transformation (FFT) for the range profile generation, as discussed in [7], [17]. Moreover, a graphics processing unit can be used to further accelerate the calculation.

C. Correction Factor

In this section, we derive the correction factor $g(k, \mathbf{r}_0, \mathbf{r})$, which appears in Eq. (12). The derivation is similar to those discussed in [8], [11], [12], except that the problem is generalized to the 3-D case of an arbitrary scanning surface. The correction factor should be properly selected such that the integral transformation of Eq. (11) is self-consistent for every electrically small scatterers; that is, the Eq. (11) should be reduced to the 3-D spatial image defined as (10).

According to Eq. (5), we consider the scattered wave produced by a point scatterer located at $\mathbf{r}_1$ as

$$E_1^s(k, \mathbf{r}_0) = P^2(k, \mathbf{r}_0, \mathbf{r}_1) \frac{k^2 C_1}{\sqrt{4\pi}} \frac{e^{-2jk|\mathbf{r}_0 - \mathbf{r}_1|}}{|\mathbf{r}_0 - \mathbf{r}_1|^2}. \quad (14)$$

Substituting the received signal from Eq. (14) into Eq. (11) yields the following equation:

$$\begin{aligned} \psi(\mathbf{r}) &= \int_0^\infty \int_{D_v} \int_{D_u} g(k, \mathbf{r}_0, \mathbf{r}) \frac{P^2(k, \mathbf{r}_0, \mathbf{r}_1)}{P^2(k, \mathbf{r}_0, \mathbf{r})} \\ &\quad \cdot \frac{|\mathbf{r}_0 - \mathbf{r}|^2}{|\mathbf{r}_0 - \mathbf{r}_1|^2} e^{j2ks} k^2 du dv dk \end{aligned} \quad (15a)$$

$$s = |\mathbf{r}_0 - \mathbf{r}| - |\mathbf{r}_0 - \mathbf{r}_1|. \quad (15b)$$

Here, we consider the local coordinate system centered at the scatterer location $\mathbf{r}_1$ as shown in Fig. 2. In this local

coordinate system, the azimuth angle $\alpha(u, v)$ and zenith angle $\beta(u, v)$ are expressed as follows:

$$\alpha(u, v) = \tan^{-1} [R_y(u, v)/R_x(u, v)] \quad (16a)$$

$$\beta(u, v) = \tan^{-1} [\rho/R_z(u, v)] \quad (16b)$$

$$\rho(u, v) = \sqrt{R_x^2(u, v) + R_y^2(u, v)}. \quad (16c)$$

Throughout this study, the arctangent $\tan^{-1}(\cdot/\cdot)$ was assumed to be calculated within the range $(-\pi, \pi]$ depending on the signs of the numerator and denominator.

Using the angles α and β , we can approximate the quantity s in the vicinity of the scatterer at $\mathbf{r}_1$ as [8], [11], [12]

$$s \approx (x - x_1) \sin \beta \cos \alpha + (y - y_1) \sin \beta \sin \alpha + (z - z_1) \cos \beta. \quad (17)$$

Substituting Eq. (17) into Eq. (15) yields the following approximation of the spatial image.

$$\begin{aligned} \psi(\mathbf{r}) \approx & \int_0^\infty \int_{D_v} \int_{D_u} g(k, \mathbf{r}_0, \mathbf{r}) \\ & \cdot \exp[j2k(x - x_1) \sin \beta \cos \alpha] \\ & \cdot \exp[j2k(y - y_1) \sin \beta \sin \alpha] \\ & \cdot \exp[j2k(z - z_1) \cos \beta] k^2 du dv dk. \end{aligned} \quad (18)$$

In the above Eq. (18), we consider the variable transformations defined as follows:

$$k_x = 2k \sin \beta \cos \alpha, \quad (19a)$$

$$k_y = 2k \sin \beta \sin \alpha, \quad (19b)$$

$$k_z = 2k \cos \beta. \quad (19c)$$

Eq. (19) represents the spatial frequency of the local coordinate centered at the point $\mathbf{r}_1$. If the scanning surface is a closed surface that completely encloses the area to be imaged, the range of the local azimuth angle α and zenith angle β are $[0, 2\pi]$ and $[0, \pi]$, respectively. Therefore, the range of integration with respect to the spatial frequency variables k_x , k_y , and k_z is $[-\infty, \infty]$. A similar discussion can also be made for the case of an open surface—that is, one can virtually extend the surface so that it becomes a closed surface, and consider the received signal at the extended part of the surface to be zero.

Substituting Eq. (19) into Eq. (18) yields the following integration:

$$\begin{aligned} \psi(\mathbf{r}) = & \int_{-\infty}^\infty \int_{-\infty}^\infty \int_{-\infty}^\infty g(k, \mathbf{r}_0, \mathbf{r}) \\ & \cdot e^{jk_x(x-x_1)+jk_y(y-y_1)+jk_z(z-z_1)} \frac{k^2}{|J|} dk_x dk_y dk_z \end{aligned} \quad (20)$$

where J represents the Jacobian which is defined as

$$J = \frac{\partial(k_x, k_y, k_z)}{\partial(k, u, v)} = \begin{vmatrix} \frac{\partial k_x}{\partial k} & \frac{\partial k_x}{\partial u} & \frac{\partial k_x}{\partial v} \\ \frac{\partial k_y}{\partial k} & \frac{\partial k_y}{\partial u} & \frac{\partial k_y}{\partial v} \\ \frac{\partial k_z}{\partial k} & \frac{\partial k_z}{\partial u} & \frac{\partial k_z}{\partial v} \end{vmatrix}. \quad (21)$$

The elements of Eq. (21) are given as follows:

$$\frac{\partial k_x}{\partial k} = 2 \cos \alpha \sin \beta \quad (22a)$$

$$\frac{\partial k_x}{\partial u} = 2k (\beta_u \cos \alpha \cos \beta - \alpha_u \sin \alpha \sin \beta) \quad (22b)$$

$$\frac{\partial k_x}{\partial v} = 2k (\beta_v \cos \alpha \cos \beta - \alpha_v \sin \alpha \sin \beta) \quad (22c)$$

$$\frac{\partial k_y}{\partial k} = 2 \sin \alpha \sin \beta \quad (22d)$$

$$\frac{\partial k_y}{\partial u} = 2k (\beta_u \cos \alpha \cos \beta + \alpha_u \sin \alpha \sin \beta) \quad (22e)$$

$$\frac{\partial k_y}{\partial v} = 2k (\beta_v \cos \alpha \cos \beta + \alpha_v \sin \alpha \sin \beta) \quad (22f)$$

$$\frac{\partial k_z}{\partial k} = 2 \cos \beta \quad (22g)$$

$$\frac{\partial k_z}{\partial u} = -2k \beta_u \sin \beta \quad (22h)$$

$$\frac{\partial k_z}{\partial v} = -2k \beta_v \sin \beta. \quad (22i)$$

In Eq. (22), we have used the following definitions.

$$\alpha_u = \frac{\partial \alpha}{\partial u} = \frac{1}{\rho^2} (R_x y_{0u} - R_y x_{0u}) \quad (23a)$$

$$\alpha_v = \frac{\partial \alpha}{\partial v} = \frac{1}{\rho^2} (R_x y_{0v} - R_y x_{0v}) \quad (23b)$$

$$\beta_u = \frac{\partial \beta}{\partial u} = \frac{1}{R^2} (R_z \rho_u - \rho z_{0u}) \quad (23c)$$

$$\beta_v = \frac{\partial \beta}{\partial v} = \frac{1}{R^2} (R_z \rho_v - \rho z_{0v}) \quad (23d)$$

$$\rho_u = \frac{\partial \rho}{\partial u} = \frac{1}{\rho} (R_x x_{0u} + R_y y_{0u}) \quad (23e)$$

$$\rho_v = \frac{\partial \rho}{\partial v} = \frac{1}{\rho} (R_x x_{0v} + R_y y_{0v}) \quad (23f)$$

$$\chi_\xi = \frac{\partial \chi(u, v)}{\partial \xi}, \quad \chi \in \{x_0, y_0, z_0\}, \quad \xi \in \{u, v\}. \quad (23g)$$

For a parametric surface defined by an analytical expression such as a plane, cylinder, and sphere, the exact expression for the partial derivative $\chi_\xi(u, v)$ is available, and the exact correction factors for these surfaces can be derived—as discussed in Section II-D. However, for an arbitrary surface, the corresponding partial derivative $\chi_\xi(u, v)$ may be unavailable in an analytical form. In such a case, we must employ a numerical computation of the partial derivative; we describe this case in Section II-E. For now, we assume that the partial derivative $\chi_\xi(u, v)$ can be computed by considering the scanning surface.

From Eq. (22) and Eq. (23), the Jacobian J of Eq. (21) reduces to

$$J = -8k^2 \sin \beta (\alpha_u \beta_v - \alpha_v \beta_u). \quad (24)$$

We select the focusing factor $g(k, \mathbf{r}_0, \mathbf{r})$ as

$$g(k, \mathbf{r}_0, \mathbf{r}) = \frac{|J|}{2\pi^{\frac{5}{2}} k^2} = \frac{4}{\pi^{\frac{5}{2}}} |\sin \beta| |\alpha_u \beta_v - \alpha_v \beta_u| \quad (25)$$

and comparing the integral form of the 3-D delta function

$$\begin{aligned} \delta(x - x_1) \delta(y - y_1) \delta(z - z_1) = & \frac{1}{(2\pi)^3} \int_{-\infty}^\infty \int_{-\infty}^\infty \int_{-\infty}^\infty \\ & \cdot e^{jk_x(x-x_1)+jk_y(y-y_1)+jk_z(z-z_1)} dk_x dk_y dk_z \end{aligned} \quad (26)$$

Fig. 3. Canonical antenna scanning surfaces. (a) Planar surface. (b) Cylindrical surface (c) Spherical surface.

with Eq. (20), we can confirm that Eq. (20) reduces to Eq. (10), and that the desired image reconstruction is achieved.

Therefore, the focusing factor $F(k, \mathbf{r}_0, \mathbf{r})$ in Eq. (12) can be given as follows:

$$F(k, \mathbf{r}_0, \mathbf{r}) = g(\mathbf{r}_0, \mathbf{r}) \frac{|\mathbf{r}_0 - \mathbf{r}|^2}{P^2(k, \mathbf{r}_0, \mathbf{r})} \quad (27)$$

$$= \frac{4}{\pi^{5/2}} |\sin \beta| |\alpha_u \beta_v - \alpha_v \beta_u| \frac{|\mathbf{r}_0 - \mathbf{r}|^2}{P^2(k, \mathbf{r}_0, \mathbf{r})}.$$

The derived correction factor in Eq. (25) and the corresponding focusing factor of Eq. (27) are the general expressions for an arbitrary scanning surface, from which we can derive the analytical correction and focusing factors for a special surface given by a parametric equation, as discussed in Section II-D.

D. Exact Correction Factors for Special Surfaces

Here, we show how the derived general expression for the correction factor in Eq. (25) can be used to produce the specific correction factors for a planar, a cylindrical, and a spherical antenna scanning surface. Fig. 3 shows the special surfaces considered in this section.

1) *Planer Scanning*: Fig. 3(a) shows a planar antenna scanning surface. We assume that the scanning plane is parallel to the x - and z -axis, and fixed at $y = a_0$. The surface is defined as the following expression.

$$x_0(u, v) = u \quad (28a)$$

$$y_0(u, v) = a_0 \quad (28b)$$

$$z_0(u, v) = v. \quad (28c)$$

In this case, we can readily obtain the partial derivatives χ_ξ, α_ξ , and β_ξ ($\chi \in \{x_0, y_0, z_0\}$, $\xi \in \{u, v\}$) as

$$x_{0u} = 1, \quad x_{0v} = 0 \quad (29a)$$

$$y_{0u} = 0, \quad y_{0v} = 0 \quad (29b)$$

$$z_{0u} = 0, \quad z_{0v} = 1 \quad (29c)$$

and

$$\alpha_u = -R_y/\rho^2, \quad \alpha_v = 0 \quad (30a)$$

$$\beta_u = R_x R_y / (R^2 \rho), \quad \beta_v = -\rho / R^2. \quad (30b)$$

Therefore, the correction factor is given by

$$g(k, \mathbf{r}_0, \mathbf{r}) = \frac{4}{\rho R^2 \pi^{5/2}} |\sin \beta| |R_y|. \quad (31)$$

2) *Cylindrical Scanning*: Fig. 3(b) shows a cylindrical antenna scanning surface expressed as

$$x_0(u, v) = \rho_0 \cos u \quad (32a)$$

$$y_0(u, v) = \rho_0 \sin u \quad (32b)$$

$$z_0(u, v) = v \quad (32c)$$

where ρ_0 is the constant radius of the cylinder. In this case, the partial derivatives χ_ξ, α_ξ , and β_ξ are given by

$$x_{0u} = -\rho_0 \sin u, \quad x_{0v} = 0 \quad (33a)$$

$$y_{0u} = \rho_0 \cos u, \quad y_{0v} = 0 \quad (33b)$$

$$z_{0u} = 0, \quad z_{0v} = 1 \quad (33c)$$

and

$$\alpha_u = \frac{r_0}{\rho^2} (R_x \cos u + R_y \sin u), \quad \alpha_v = 0 \quad (34a)$$

$$\beta_u = \frac{R_z \rho_0}{R^2 \rho} (R_y \cos u - R_x \sin u), \quad \beta_v = -\frac{\rho}{R^2}. \quad (34b)$$

Therefore, the correction factor is given by

$$g(k, \mathbf{r}_0, \mathbf{r}) = \frac{4\rho_0}{\rho R^2 \pi^{5/2}} |\sin \beta| |R_x \cos u + R_y \sin u|. \quad (35)$$

Another expression for the cylindrical case has been derived in literature [11], [12].

3) *Spherical Scanning*: Fig. 3(c) shows a spherical antenna scanning surface expressed as

$$x_0(u, v) = r_0 \cos u \sin v \quad (36a)$$

$$y_0(u, v) = r_0 \sin u \sin v \quad (36b)$$

$$z_0(u, v) = r_0 \cos v \quad (36c)$$

where r_0 is the constant radius of the sphere. In this case, the partial derivatives χ_ξ, α_ξ , and β_ξ are given by

$$x_{0u} = -r_0 \sin u \sin v, \quad x_{0v} = r_0 \cos u \cos v \quad (37a)$$

$$y_{0u} = r_0 \cos u \sin v, \quad y_{0v} = r_0 \sin u \cos v \quad (37b)$$

$$z_{0u} = 0, \quad z_{0v} = -r_0 \sin v \quad (37c)$$

and

$$\alpha_u = \frac{r_0 \sin v}{\rho^2} (R_x \cos u + R_y \sin u) \quad (38a)$$

$$\alpha_v = \frac{r_0 \cos v}{\rho^2} (R_x \sin u - R_y \cos u) \quad (38b)$$

$$\beta_u = \frac{R_z r_0 \sin v}{\rho R^2} (R_y \cos u - R_x \sin u) \quad (38c)$$

$$\beta_v = \frac{r_0}{\rho R^2} [R_z \cos v (R_x \cos u + R_y \sin u) + \rho^2 \sin v]. \quad (38d)$$

The correction factor is then computed via Eq. (25).

E. Numerical Correction Factor

In this section, we propose a numerical computation of the correction factor, which can be applied for antenna scanning over an arbitrary surface.

We consider a discrete subpatch of the scanning surface, as shown in Fig. 4. A discrete point on the subpatch is denoted as (u_m, v_n) , where $u_m, m \in \{1, 2, \dots\}$ and

Fig. 4. Discrete subpatch of the scanning surface.

$v_n, n \in \{1, 2, \dots\}$ are the discrete spatial coordinates along the surface—corresponding to u and v , respectively. For notational simplicity, we define the indexes $m = 1$ and $n = 1$ to represent the smallest spatial variables within this subpatch, and $m = M$ and $n = N$ as the indices that represent the largest spatial variables within the subpatch. Moreover, the spatial variables $\chi(u_m, v_n), \chi \in \{x_0, y_0, z_0\}$ are denoted by $\chi_{m,n}$.

Using the aforementioned notation, we describe the following two types of numerical differentiation of the subpatch; namely, the finite difference approximation, and the polynomial approximation.

1) *Finite Difference Approximation*: A simple solution for computing the Jacobian in Eq. (12) is to approximate the partial derivative with respect to the spatial variable using the finite difference. By employing the central difference, we can numerically compute the partial differences as

$$\chi_u(u_m, v_n) \approx \frac{\chi_{m+1,n} - \chi_{m-1,n}}{u_{m+1} - u_{m-1}} \quad (39a)$$

$$\chi_v(u_m, v_n) \approx \frac{\chi_{m,n+1} - \chi_{m,n-1}}{v_{n+1} - v_{n-1}}. \quad (39b)$$

Using Eq. (39), Eq. (23) and the correction factor defined in Eq. (25) can be approximated and computed for an arbitrary scanning surface.

Note that if the forward sample (i.e., $m+1$ and/or $n+1$) or the backward sample (i.e., $m-1$ and/or $n-1$) is unavailable, one can use the backward or forward difference, respectively—instead of the central difference of Eq. (39).

2) *Polynomial Approximation*: The finite difference approximation can be expected to work well for stable antenna scanning, such as those using a turntable and a vertical scanner to create a cylindrical scanning surface. However, if the antenna scanning is carried out via a moving platform such as a vehicle or aircraft, the finite difference can be affected by the fluctuation of the scanning path or surface. In this case, polynomial-based numerical differentiation can be a useful alternative approach.

We consider the following polynomial expansion of $\chi_{m,n}$ at coordinates (u_m, v_n) .

$$\chi_{m,n} = \sum_{p=0}^K \sum_{q=0}^L a_{p,q}^{(\chi)} u_m^p v_n^q \quad (40)$$

where $a_{p,q}^{(\chi)}$ is an unknown expansion coefficient to be determined, $[\cdot]^{(\chi)}$ indicates that this coefficient is related to the

element $\chi \in \{x_0, y_0, z_0\}$ of the position vector $\mathbf{r}_0$, and K and L represent the order of the polynomial for u_m and v_n , respectively.

By constructing Eq. (40) for all the sample points within the subpatch, we obtain the simultaneous equations with the unknown coefficients $a_{p,q}^{(\chi)}$. Then, by solving this equation for $a_{p,q}^{(\chi)}$, the antenna scanning surface can be approximated by the polynomial equations within the subpatch—making it possible to readily evaluate the partial derivative $\chi_\xi, \xi \in \{u, v\}$. In the following discussion, we provide a least-squares solution to Eqs. (40).

Eq. (40) can be equivalently expressed by the following vector and matrix notation.

$$\chi_{m,n} = \mathbf{u}_m^T \mathbf{A}^{(\chi)} \mathbf{v}_n \quad (41a)$$

$$\mathbf{u}_m = [1, u_m, u_m^2, \dots, u_m^p, \dots, u_m^K]^T \quad (41b)$$

$$\mathbf{v}_n = [1, v_n, v_n^2, \dots, v_n^q, \dots, v_n^L]^T \quad (41c)$$

$$\mathbf{A}^{(\chi)} = \begin{bmatrix} a_{0,0}^{(\chi)} & a_{0,1}^{(\chi)} & \dots & a_{0,q}^{(\chi)} & \dots & a_{0,L}^{(\chi)} \\ a_{1,0}^{(\chi)} & a_{1,1}^{(\chi)} & \dots & a_{1,q}^{(\chi)} & \dots & a_{1,L}^{(\chi)} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ a_{p,0}^{(\chi)} & a_{p,1}^{(\chi)} & \dots & a_{p,q}^{(\chi)} & \dots & a_{p,L}^{(\chi)} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ a_{K,0}^{(\chi)} & a_{K,1}^{(\chi)} & \dots & a_{K,q}^{(\chi)} & \dots & a_{K,L}^{(\chi)} \end{bmatrix} \quad (41d)$$

$$\mathbf{a}_p^{(\chi)} = [a_{p,0}^{(\chi)}, a_{p,1}^{(\chi)}, \dots, a_{p,q}^{(\chi)}, \dots, a_{p,L}^{(\chi)}]^T \quad (41e)$$

where $[\cdot]^T$ represents the transpose operation. Eq. (41) can be further reformulated into the following form.

$$\chi_{m,n} = (\mathbf{u}_m \otimes \mathbf{v}_n)^T [\mathbf{a}_0^{(\chi)T}, \mathbf{a}_1^{(\chi)T}, \dots, \mathbf{a}_p^{(\chi)T}, \dots, \mathbf{a}_K^{(\chi)T}]^T \quad (42a)$$

$$= \mathbf{w}_{m,n}^T \mathbf{a}^{(\chi)} \quad (42b)$$

$$\mathbf{w}_{m,n} = (\mathbf{u}_m \otimes \mathbf{v}_n) \quad (42b)$$

$$\mathbf{a}^{(\chi)} = [\mathbf{a}_0^{(\chi)T}, \mathbf{a}_1^{(\chi)T}, \dots, \mathbf{a}_p^{(\chi)T}, \dots, \mathbf{a}_K^{(\chi)T}]^T \quad (42c)$$

where $\otimes$ represents the Kronecker product. Considering the above representation for all sample points within the subpatch, we arrive at the following expression:

$$\boldsymbol{\chi} = \mathbf{W} \mathbf{a}^{(\chi)} \quad (43a)$$

$$\boldsymbol{\chi} = [\chi_1^T, \chi_2^T, \dots, \chi_m^T, \dots, \chi_M^T]^T \quad (43b)$$

$$\mathbf{W}_m = [\chi_{m,1}, \chi_{m,2}, \dots, \chi_{m,n}, \dots, \chi_{m,N}]^T \quad (43c)$$

$$\mathbf{W} = [\mathbf{W}_1^T, \mathbf{W}_2^T, \dots, \mathbf{W}_m^T, \dots, \mathbf{W}_M^T]^T \quad (43d)$$

$$\mathbf{W}_m = [\mathbf{w}_{m,1}, \mathbf{w}_{m,2}, \dots, \mathbf{w}_{m,n}, \dots, \mathbf{w}_{m,N}]^T \quad (43e)$$

Then, the least-squares solution to the coefficient vector $\mathbf{a}^{(\chi)}$ can be constructed as follows:

$$\mathbf{a}^{(\chi)} = (\mathbf{W}^T \mathbf{W})^{-1} \mathbf{W}^T \boldsymbol{\chi}. \quad (44)$$

Using the coefficient vector $\mathbf{a}^{(x)}$, we can approximate subpatch as

$$\chi(u, v) \approx \mathbf{u}(u) \mathbf{A}^{(x)} \mathbf{v}^T(v) \quad (45a)$$

$$\mathbf{u}(u) = [1, u, u^2, \dots, u^m, \dots, u^K]^T \quad (45b)$$

$$\mathbf{v}(v) = [1, v, v^2, \dots, v^n, \dots, v^L]^T. \quad (45c)$$

Therefore, the partial difference $\chi_\xi(u, v)$ can be numerically evaluated via

$$\chi_u(u, v) = \frac{\partial \chi(u, v)}{\partial u} \approx \mathbf{u}'(u) \mathbf{A}^{(x)} \mathbf{v}^T(v) \quad (46a)$$

$$\chi_v(u, v) = \frac{\partial \chi(u, v)}{\partial v} \approx \mathbf{u}(u) \mathbf{A}^{(x)} \mathbf{v}'^T(v) \quad (46b)$$

$$\mathbf{u}'(u) = [0, 1, 2u, \dots, mu^{(m-1)}, \dots, Ku^{(K-1)}]^T \quad (46c)$$

$$\mathbf{v}'(v) = [0, 1, 2v, \dots, nv^{(n-1)}, \dots, Lv^{(L-1)}]^T. \quad (46d)$$

The abovementioned procedure is applied for x_0, y_0 , and z_0 to obtain the partial derivative $\chi_\xi(u, v)$, and the correction factor of Eq. (25) and the focusing factor of Eq. (12) are numerically computed. A similar procedure was repeated for all subpatches within the scanning surface.

In the following discussion, we use a polynomial approximation to compute the numerical correction factor.

F. Radar Cross-Section Computation

The final step of the image-based NFFFT is to compute the far-field RCS from the reconstructed radar image. Because the 3-D radar image $\psi(x, y, z)$ can be considered as a cluster of point scatterers, the scattered field at an arbitrary location $\mathbf{r}_0(\phi_0, \theta_0)$, where the spherical coordinate is given by Eq. (2), can be computed according to Eq. (6) as follows:

$$E^s(k, \phi_0, \theta_0) = \frac{k^2}{\sqrt{4\pi}} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \psi(x, y, z) \cdot \frac{e^{-2jk|\mathbf{r}_0 - \mathbf{r}|}}{|\mathbf{r}_0 - \mathbf{r}|^2} dx dy dz \quad (47)$$

Note that we have defined $P(k, \mathbf{r}_0, \mathbf{r}) = 1$.

If the observation point $\mathbf{r}_0(\phi_0, \theta_0)$ lies in the far-field region, that is, $|\mathbf{r}_0| \gg |\mathbf{r}|$, the distance between the point within the image $\mathbf{r}$, and the observation point $\mathbf{r}_0(\phi_0, \theta_0)$ can be expanded as [18]

$$\begin{aligned} |\mathbf{r}_0 - \mathbf{r}| &= \sqrt{(\mathbf{r}_0 - \mathbf{r}) \cdot (\mathbf{r}_0 - \mathbf{r})} \\ &= \sqrt{|\mathbf{r}_0|^2 - 2\mathbf{r}_0 \cdot \mathbf{r} + |\mathbf{r}|^2} \\ &= |\mathbf{r}_0| \left[1 - \frac{\hat{\mathbf{r}}_0 \cdot \mathbf{r}}{|\mathbf{r}_0|} + O\left(\frac{|\mathbf{r}|^2}{|\mathbf{r}_0|^2}\right) \right] \\ &= r_0 - \hat{\mathbf{r}}_0 \cdot \mathbf{r} + O(r^2/r_0) \end{aligned} \quad (48a)$$

$$\begin{aligned} \hat{\mathbf{r}}_0(\phi_0, \theta_0) &= \mathbf{r}_0(\phi_0, \theta_0)/r_0 \\ &= \sin \theta_0 \cos \phi_0 \hat{\mathbf{x}} + \sin \theta_0 \sin \phi_0 \hat{\mathbf{y}} + \cos \theta_0 \hat{\mathbf{z}} \end{aligned} \quad (48b)$$

where we have used the Taylor series expansion $\sqrt{1+a} = 1 + a/2 + O(a^2)$. Using Eq. (48), the following approximation can be made.

$$1/|\mathbf{r}_0 - \mathbf{r}|^2 \approx 1/r_0^2 \quad (49a)$$

$$e^{-2jk|\mathbf{r}_0 - \mathbf{r}|} \approx e^{-j2kr_0} e^{j2k\hat{\mathbf{r}}_0 \cdot \mathbf{r}} = e^{-j2kr_0} e^{j\mathbf{k}_r \cdot \mathbf{r}} \quad (49b)$$

$$\mathbf{k}_r(k, \phi_0, \theta_0) = 2k\hat{\mathbf{r}}_0(\phi_0, \theta_0). \quad (49c)$$

Substituting Eq. (49) into Eq. (47) yields the following scattered field representation in the far-field region.

$$\begin{aligned} E^s(k, \mathbf{r}_0) &\approx \frac{k^2 e^{-j2kr_0}}{r_0^2 \sqrt{4\pi}} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \psi(x, y, z) \\ &\quad \cdot \exp(j\mathbf{k}_r \cdot \mathbf{r}) dx dy dz \\ &= \frac{k^2 e^{-j2kr_0}}{r_0^2 \sqrt{4\pi}} \mathcal{F}_{(x,y,z)}^{-1} [\psi(x, y, z)] \end{aligned} \quad (50)$$

where $\mathcal{F}_{(x,y,z)}^{-1}[\cdot]$ represents the 3-D inverse Fourier transform with respect to the spatial variables (x, y, z) .

The far-field expression in Eq. (50) can be used to derive the RCS, which is defined as follows:

$$\sigma[\mathbf{k}_r(k, \phi_0, \theta_0)] = \lim_{r_0 \rightarrow \infty} 4\pi r_0^2 \left| \frac{E^s(k, \phi_0, \theta_0)}{E_0^i(k, \phi_0, \theta_0)} \right|^2 \quad (51)$$

where $E_0^i(k, \phi_0, \theta_0)$ represents the incident field given by

$$E_0^i(k, \phi_0, \theta_0) = \frac{e^{-jk|\mathbf{r}_0|}}{|\mathbf{r}_0|} \quad (52)$$

Substituting Eq. (50) and Eq. (52) into Eq. (51), we arrive at the following representation of the RCS in terms of the 3-D spatial image $\psi(x, y, z)$.

$$\sigma[\mathbf{k}_r(k, \phi_0, \theta_0)] = k^4 \left| \mathcal{F}_{(x,y,z)}^{-1} [\psi(x, y, z)] \right|^2 \quad (53)$$

Therefore, once the 3-D radar image is reconstructed, the far-field RCS is given by its inverse spatial Fourier transform. In the digital implementation, a discrete Fourier transform (DFT) can be used. Alternatively, we can implement Eq. (53) using the 3-D inverse fast Fourier transform (IFFT), followed by the interpolation of the (k_x, k_y, k_z) domain to obtain the (ϕ_0, θ_0) domain pattern at the desired frequency.

G. Summary of the Proposed Algorithm

The proposed NFFFT can be summarized as follows:

- 1) Select the image location (x, y, z) to be reconstructed.
- 2) For the selected image location (x, y, z) , compute the antenna location (R_x, R_y, R_z) in the local coordinate system centered at the image location (x, y, z) based on Eq. (3). Then compute $\rho(u, v)$ based on Eq. (16c).
- 3) Using the results of 2), compute the partial derivatives $\alpha_u, \alpha_v, \beta_u$, and β_v based on Eq. (23). If the analytical expressions of the antenna scanning surface are available, the exact correction factor can be derived—as explained in Sec II-D. Otherwise, we can use the numerical correction factor described in Sec. II-E.
- 4) Using the results of 3), compute the Jacobian J based on Eq. (24).

- 5) Using the obtained Jacobian J , compute the correction factor $g(\mathbf{r}_0, \mathbf{r})$ at the image location (x, y, z) based on Eq. (25), and the corresponding focusing factor $F(k, \mathbf{r}_0, \mathbf{r})$ based on Eq. (27).
- 6) Using the obtained focusing factor $F(k, \mathbf{r}_0, \mathbf{r})$, reconstruct the image at (x, y, z) based on Eq. (11) or the discretized Eq. (13).
- 7) Repeat the above procedures from 1) to 6) for all the desired image locations.
- 8) Compute the RCS $\sigma(\mathbf{k}_r)$ from the reconstructed 3-D spatial image $\psi(x, y, z)$ at the desired azimuth angle ϕ_0 and the zenith angle θ_0 based on the inverse spatial Fourier transform of Eq. (53).

III. NUMERICAL SIMULATION

In this section, we demonstrate the proposed image-based NFFFT using the discrete scatterer model defined in Eq. (9). In addition to spherical and cylindrical antenna scanning surfaces, we provide an example using curved cylindrical scanning for data acquisition—demonstrating the ability of the proposed method to handle a nonstandard scanning surface.

A. Overview of the Simulation

Fig. 5 shows the location of the point scatterers and the antenna scanning surfaces, and Table I lists the simulation parameters. The three spherical scatterers of the equivalent radius of 1 mm ($ka_i < 0.3$) were placed at the different 3-D spatial locations as shown in Fig. 5(a). We used Eq. (8) and Eq. (9), with $i \in \{1, 2, 3\}$ and $\mathbf{r}_i$ defined in Table I, to generate the received signal.

Fig. 5(b) and (c) represent the cylindrical and the spherical surfaces defined in Eq. (32) and Eq. (36), respectively. The curved cylinder shown in Fig. 5(d) is defined as

$$x_0(u, v) = [\rho_0 + \rho_\delta(v)] \cos u \quad (54a)$$

$$y_0(u, v) = [\rho_0 + \rho_\delta(v)] \sin u \quad (54b)$$

$$z_0(u, v) = v \quad (54c)$$

$$\rho_\delta(v) = A \sin(2\pi v/T) \quad (54d)$$

where $\rho_\delta(v)$ modulates the radius of the cylinder ρ_0 as a sine function of v , and we selected $A = 10$ cm and $T = 50$ cm.

In this simulation, all correction factors were computed based on the numerical method described in Sec. II-E. Note that we tested both the exact correction factor and numerical correction factor for the cylindrical and spherical scanning surfaces, and confirmed that similar results were obtained.

We compared the reconstructed RCS with the theoretical RCS of a cloud of point scatterers of reflectivity C_i , given as follows:

$$\sigma = k^4 \left| \sum_i C_i \exp(jk_x x_i + jk_y y_i + jk_z z_i) \right|^2 \quad (55)$$

where the spatial frequency (k_x, k_y, k_z) was determined using Eq. (19). It is assumed that the local coordinate (x', y', z') coincides with the global coordinate (x, y, z) .

Fig. 5. Scatterer location and scanning surfaces. (a) Scatterer location. (b) Cylinder. (c) Sphere. (d) Curved cylinder.

TABLE I
SIMULATION PARAMETERS.

Center frequency	10.2 GHz
Bandwidth	4 GHz
Frequency step	10 MHz
Antenna scanning surface	Sphere, Cylinder Curved cylinder
Angular interval in azimuth	$[-180^\circ, 180^\circ]$
Angular interval in zenith	$[0^\circ, 180^\circ]$
Angular step	0.8°
Height (z-direction) interval	$[-25, 25]$ cm
Height (z-direction) step	1 cm
Radius of the surfaces	$\rho_0 = r_0 = 1$ m
Number of points in a subpatch	$M = N = 11$
Order of the polynomial	$K = L = 2$
Sphere location	$\mathbf{r}_1 = (0, 10, -10)$ cm $\mathbf{r}_2 = (20, 20, 0)$ cm $\mathbf{r}_3 = (25, 20, 10)$ cm
Radius of the spherical scatterer	$a_1 = a_2 = a_3 = 1$ mm
Voxel spacing of the spatial image	4 mm

To quantify the prediction error of the NFFFT algorithm, we defined the average difference between the theoretical far-field RCS and the predicted RCS on a decibel scale, as follows:

$$\sigma_{e,\zeta}^{\text{dB}} = \sum_{n=1}^{N_\zeta} \frac{|\sigma_{\zeta[n]}^{\text{dB}} - \tilde{\sigma}_{\zeta[n]}^{\text{dB}}|}{N_\zeta} \quad (56)$$

where $\sigma_{\zeta[n]}^{\text{dB}}$ and $\tilde{\sigma}_{\zeta[n]}^{\text{dB}}$ ($\zeta \in \{\phi_0, \theta_0\}$) are the n th sample of the true and predicted RCS patterns, respectively, in the azimuth ($\zeta = \phi_0$) and zenith ($\zeta = \theta_0$) directions. N_ζ is the number of samples in the decibel scale in each direction. For the cylinder and the curved cylinder, we only considered the zenith angle within the interval $\theta_0 \in [80^\circ, 100^\circ] \cup [-100^\circ, -80^\circ]$ for the

computation of Eq. (56). This is because the data outside these intervals were unavailable owing to the finite cylinder length.

The center frequency of the transmitted wave was 10.2 GHz, and the bandwidth was 4 GHz. Within this bandwidth, the frequency was discretized into 10 MHz, resulting in 401 frequency samples at each antenna location. Note that these frequency parameters were selected to ensure consistency with the experimental validation given in Section IV.

The antenna pattern was assumed to be isotropic, such that $P(k, \mathbf{r}_0, \mathbf{r}) = 1$. The antenna scanning surfaces were cylindrical, spherical, and curved cylindrical surfaces, as shown in Fig. 3(b), (c), and (d), respectively. The radii of the cylinder, the curved cylinder, and the sphere were selected as $\rho_0 = r_0 = 60$ cm. The scanning ranges were $u \in D_u : [-180^\circ, 180^\circ]$ and $v \in D_v : [-25, 25]$ cm for the cylinder and curved cylinder, and $u \in D_u : [-180^\circ, 180^\circ]$ and $v \in D_v : [0^\circ, 180^\circ]$ for the sphere. The angular sample spacing was 0.8° for all surfaces, and the height sampling interval was 1 cm for the cylinder and curved cylinder.

B. Simulation Results and Discussion

Fig. 6 to Fig. 9 show the simulation results for the cylindrical and the spherical antenna scanning. In each of these figures, (a), (b), (c), and (d) show the reconstructed 3-D spatial image, the spatial frequency spectrum of the spatial image, computed via the 3-D IFFT, the azimuth RCS pattern at the zenith angle $\theta_0 = 90^\circ$, and the zenith RCS pattern at the azimuth angle $\phi_0 = 0^\circ$, respectively. The spatial image and the spatial frequency spectrum are normalized by the maximum value of each image. The spatial frequency spectra within the (k_x, k_y) -plane, ($\theta_0 = 90^\circ$) and the (k_y, k_z) -plane ($\phi_0 = 0^\circ$) are shown in these figures. All of these RCS patterns were computed for 10 GHz. The solid lines represent the far-field RCS reconstructed via the NFFFT algorithm, and the dashed lines represent the theoretical RCS determined from Eq. (55).

1) *Cylindrical Scanning*: Fig. 6 and Fig. 7 show a comparison of the simulation results for the cylindrical scanning without [$g(k, \mathbf{r}_0, \mathbf{r}) = 1$], and with the correction factor, respectively. Both methods produced similar 3-D spatial images; however, from Fig. 6(a) and Fig. 7(a), the point scatterer response seems more focused when the proposed correction factor is used for image reconstruction.

The spatial frequency spectra of 6(a) and Fig. 7(a) are almost uniformly distributed over all the azimuth angles in the (k_x, k_y) -plane, whereas the spectra are partially distributed around the zenith angle $\phi_0 = 90^\circ$ in the (k_y, k_z) -plane. This is because while the cylindrical scanning surface completely encloses the spatial domain to be imaged, the cylinder with the finite length means that the measurement can be performed within the partial interval of the zenith angle.

Comparing Fig. 6(c) and (d) with Fig. 7(c) and (d), we can note the clear improvement by introducing the proposed focusing factor into the image reconstruction. As explained above, the zenith RCS is partially available around the angle $\phi_0 = 90^\circ$, as shown in Fig. 6(d) and Fig. 7(d).

Fig. 6. NFFFT with a cylindrical scanning surface (conventional, simulation). (a) Spatial image. (b) Spatial frequency spectrum. (c) Azimuth RCS. (d) Zenith RCS.

Fig. 7. NFFFT with a cylindrical scanning surface (improved, simulation). (a) Spatial image. (b) Spatial frequency spectrum. (c) Azimuth RCS. (d) Zenith RCS.

2) *Spherical Scanning*: Fig. 8 and Fig. 9 represent the simulation results for the spherical scanning with and without the correction factor, respectively. As shown in Fig. 8(a), the undesired artifact appeared around the scatterers. The spatial image of a point target is expected to be symmetric around its location, however, the reconstructed image was blurred and asymmetric owing to the artifact. We can see from Fig. 8(b)

Fig. 8. NFFFT with a spherical scanning surface (conventional, simulation). (a) Spatial image. (b) Spatial frequency spectrum. (c) Azimuth RCS. (d) Zenith RCS.

Fig. 9. NFFFT with a spherical scanning surface (improved, simulation). (a) Spatial image. (b) Spatial frequency spectrum. (c) Azimuth RCS. (d) Zenith RCS.

that the spatial frequency spectrum is concentrated around the zenith angles $\theta_0 = 0^\circ$ and $\theta_0 = 90^\circ$ in the (k_y, k_z) -plane. This is due to the fact that, although angular sample spacing is uniform in both the azimuth and zenith directions, the spatial density of the sampling points on the spherical surface becomes higher around these zenith angles. Because of this effect, a significant error was observed in the reconstructed

Fig. 10. Comparison of the NFFFT with the cylindrical and the curved cylindrical scanning surfaces (improved, simulation). (a) Azimuth RCS. (b) Zenith RCS.

Fig. 11. Averaged RCS prediction error (simulation). (a) Azimuth RCS. (b) Zenith RCS.

zenith RCS as shown in Fig. 8(d).

By using the proposed correction factor, a clear improvement was obtained, as shown in Fig. 9(a); each point target showed the symmetric response with respect to the corresponding scatterer location. As a result, the spatial frequency spectrum shown in Fig. 9(b) is uniformly distributed over all the azimuth and zenith angles. The resultant RCS showed an excellent agreement with the theoretical value, as shown in Fig. 9(c) and (d).

3) *Curved Cylindrical Scanning*: For the curved cylindrical scanning, we confirmed that the spatial image and spatial frequency distribution were similar to those of regular cylindrical scanning, as shown in Fig. 6 and Fig. 7. Fig. 10(a) and (b) show the azimuth and the zenith RCS for the curved cylindrical scanning, respectively, plotted with the RCS obtained by the cylindrical scanning—showing the agreement between both scanning surfaces. From this result, we can confirm that the proposed correction factor works well for a nonstandard antenna scanning surface.

4) *RCS Prediction Error*: Fig. 11(a) and (b) show the averaged RCS prediction error evaluated using Eq. (56) for the azimuth and the zenith RCS, respectively, for each antenna scanning surface. As it can be expected from the abovementioned simulation results, the improved NFFFT shows a lower prediction error compared to the conventional method (without correction factor) for all the scanning surfaces. The spherical surface shows the lowest prediction error of 0.1 dB and 0.2 dB

Fig. 12. Aircraft model used in the experiment. (a) Top view. (b) Perspective view. (c) Side view. (d) Front view.

for the azimuth and the zenith RCS, respectively.

For the spherical surface, the conventional method shows a significant prediction error in the zenith direction as compared to the other scanning surfaces, as shown in Fig. 11(b). As discussed previously, this is because the spatial sampling density around the north and south poles ($\phi_0 = \pm 90^\circ$) on the spatial frequency distribution increases, and this effect is uncompensated in the conventional method. In contrast, the effect of the scanning geometry on the prediction error of the conventional method was insignificant in the azimuth direction.

The cylindrical and curved cylindrical geometries show similar RCS patterns, as shown in Fig. 10. The prediction error for these surfaces were found to be comparable.

IV. EXPERIMENTAL VALIDATION

In this section, we discuss the experimental validation of the proposed algorithm. The experiment was conducted in an anechoic chamber, and a metallic aircraft model was used for this experiment.

A. Overview of the Experiment

Fig. 12 illustrates the geometry of the aircraft model used in this experiment; where (a), (b), (c), and (d) show the top, the perspective, the front, and the right views, respectively. The total length, wingspan, and total height of the aircraft model were 52 cm, 33.2 cm, and 8.8 cm, respectively. The model is made of aluminum.

Fig. 13 shows the setup of the experiment, and Fig. 14 shows photographs of the experiment. The target was fixed on a styrene foam support, and the support was placed on a turntable for azimuth rotation. As shown in Fig. 13, the target was intentionally off-centered to demonstrate the effectiveness of the proposed correction factor; the center of the model was $(-15, 15, 0)$ cm, and the front of the aircraft was directed toward the x -axis. By this configuration, the target was strongly asymmetric about the origin of the coordinate system.

Four standard gain horn antennas (Narda Model 640), which cover the frequency band from 8.2 GHz to 12.4 GHz, were used as the transmitting and receiving antennas. The horizontal

Fig. 13. Geometry of the experiment. The target is intentionally off-centered to demonstrate the effectiveness of the proposed correction factor.

Fig. 14. Photographs of the experiment. The target was placed on a styrene-foam support. Four horn antennas were used for transmission and reception.

and vertical separations of these antennas were approximately 11 cm, as shown in Fig. 14. The two horn antennas were horizontally polarized, and the other antennas were vertically polarized. These antennas were mounted on a vertical scanner that was able to move the antennas in the vertical (height) direction. We carried out cylindrical antenna scanning, and the shape of the scanning surface was the same as that depicted in Fig. 5(b).

A vector network analyzer (VNA, Keysight N5242A) was used as the transmitter and receiver. Horizontally and vertically polarized horn antennas were connected to the VNA through coaxial cables. As shown in Fig. 13, the low noise amplifiers were inserted in the receiving paths. Under this experimental configuration, the two co-polarized channels, namely, the HH- (horizontally polarized transmission and reception) and VV- channel (vertically polarized transmission and reception), were measured. Note that this setup was strictly in a bistatic configuration; however, we assumed that the monostatic approximation was valid, where the monostatic antenna location was assumed to be located at the center of the antenna separation.

Table II lists the experimental parameters. Parameters similar to the previous numerical simulation were employed for this experiment. The center frequency of the transmitted wave was 10.2 GHz, and the bandwidth was 4 GHz. Within this bandwidth, the frequency was discretized into 10 MHz, re-

TABLE II
EXPERIMENTAL PARAMETERS.

Center frequency	10.2 GHz
Bandwidth	4 GHz
Frequency step	10 MHz
Polarization	HH, VV
Antenna scanning surface	Cylinder
Angular interval in azimuth	$[-180^\circ, 180^\circ]$
Angular step	0.8°
Height (z -direction) interval	$[-25, 25]$ cm
Height (z -direction) step	1 cm
Radius of the surfaces	$\rho_0 = 1$ m
Number of points in a subpatch	$M = N = 11$
Order of the polynomial	$K = L = 2$
Target	Aircraft model
Voxel spacing of the spatial image	4 mm

sulting in 401 frequency samples at each antenna location. To compensate for the effect of the antenna pattern, the theoretical pattern of the horn antenna was computed using an analytical expression given in [21]. The radius of the antenna scanning cylinder was selected as $\rho_0 = r_0 = 60$ cm. The scanning range was $u \in D_u : [-180^\circ, 180^\circ]$ and $v \in D_v : [-25, 25]$ cm. The angular sample spacing was 0.8° , and the height sampling interval was 1 cm.

The far-field RCS of the target was computed using a multilevel fast multipole method (MLFMM) solver. Because we were unable to measure the far-field RCS directly owing to the spatial limitation of the anechoic chamber used in this experiment, we assumed the MLFMM results to be the true far-field RCS of the experimental model to evaluate the prediction accuracy of the proposed NFFFT. To quantify the prediction error, we computed the average RCS prediction error—as defined in Eq. (56). As in the previous simulation, the angular range for averaging was $\phi_0 \in [-180^\circ, 180^\circ]$ for the azimuth RCS, and $\theta_0 \in [-100^\circ, -80^\circ] \cup [80^\circ, 100^\circ]$ for the zenith RCS.

B. Experimental Results and Discussion

Fig. 15(a) and (b) show the reconstructed 3-D spatial image and the spatial frequency spectrum, respectively—for HH-polarization. These images were generated using the proposed correction factor. The spatial image and the spatial frequency spectrum were normalized by the maximum value of each image. The spatial frequency spectra within the (k_x, k_y) -plane ($\theta_0 = 90^\circ$) and the (k_y, k_z) -plane ($\phi_0 = 0^\circ$) are shown in Fig. 15(b). In Fig. 15(a), backscattering from the aircraft model was clearly imaged. In the (k_x, k_y) -plane, we can see the bright response around $\phi_0 = \pm 90^\circ$ which corresponds to the backscattering from the body of the aircraft. In the (k_y, k_z) -plane, the strong response around $\theta_0 = -90^\circ$ (i.e., the negative side of the k_x domain) was owing to the backscattering from the tail of the aircraft model (see Fig. 14).

Fig. 16 to Fig. 19 compare the RCS predicted from the (a) conventional, and (b) improved NFFFT—where Fig. 16 and Fig. 17 show the azimuth RCS for HH and VV polarization, respectively. Fig. 18 and Fig. 19 show the zenith RCS for HH and VV polarization, respectively. All of these RCS patterns were computed for 10 GHz. The solid lines represent the

Fig. 15. Spatial image and spatial frequency spectrum (experiment, HH-polarization, improved). (a) Spatial image. (b) Spatial frequency spectrum

Fig. 16. Azimuth RCS of the aircraft model (experiment, HH-polarization). (a) Conventional (b) Improved.

Fig. 17. Azimuth RCS of the aircraft model (experiment, VV-polarization). (a) Conventional (b) Improved.

far-field RCS reconstructed via the NFFFT algorithm, and the dashed lines represent the far-field RCS computed by a MLFMM solver.

Comparing Fig. 16 and Fig. 17, we notice that the HH-polarized channel shows much weaker backscattering around the azimuth angle 0° ; in contrast to the strong backscattering around that angle in the VV-polarized channel (Fig. 17). This can be attributed to the geometry of the aircraft model, where a horizontal aperture (air intake) existed in the front of the model, as shown in Fig. 14.

Comparing (a) and (b) in Fig. 16 and Fig. 17 (azimuth RCS for HH and VV polarization), we can confirm the improvement of the prediction accuracy of the azimuth RCS by using the proposed correction factor for both polarization channels. In Fig. 18 and Fig. 19 (zenith RCS for HH and VV polarization),

Fig. 18. Zenith RCS of the aircraft model (experiment, HH-polarization). (a) Conventional (b) Improved.

Fig. 19. Zenith RCS of the aircraft model (experiment, VV-polarization). (a) Conventional (b) Improved.

Fig. 20. Averaged RCS prediction error (experiment). (a) Azimuth RCS. (b) Zenith RCS.

the improvement was relatively small compared to those in the azimuth RCS; and since the height of the scanning cylinder is so small, we could not evaluate the prediction accuracy for a wider angular range.

Fig. 20(a) and (b) show the averaged prediction error for the azimuth and the zenith RCS. As mentioned above, we used the MLFMM results as the true far-field RCS. From Fig. 20, both the azimuth and the zenith RCS show the improvement by using the proposed correction factor for both the HH and VV polarization channels.

From the abovementioned results, we experimentally confirmed the effectiveness of the proposed correction factor.

V. CONCLUSION

In this study, we propose a novel generalized image-based near-field to far-field transformation algorithm for radar cross-section determination, that can accommodate an arbitrary antenna-scanning surface. We determine the correction factor for the conventional focusing operator so that the reconstructed image is reduced to the point scatterer response (delta function). Based on the derived general expression for the correction factor for 3-D problems, we provide the analytical forms of the correction factor for special antenna scanning geometries—that is, for planar, cylindrical, and spherical surfaces. Furthermore, we propose a numerical computation scheme for the correction factor that can be used for an arbitrary antenna scanning surface. The numerical simulation and experimental results demonstrate the validity of the proposed algorithm. In future work, an extension of the proposed approach to the bistatic case, and an efficient numerical implementation of the imaging algorithm should be considered.

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