Formulation of Stray Loss in Medium Power Inverter-fed Induction Motors

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Abstract—The work presents a fast and accurate empirical stray loss model for inverter fed three-phase induction motors (IMs). The model is intended to quantify the SL both at the design stage and, normal operation of the IM. The development of the model considers the variations of frequency, switching frequency and motor's loading. Test data of 128 operating points of two training IMs are used to develop the model. The accuracy and prediction capability of the proposed model is presented with the measured SL values for 128 operating points of another set of two IMs of the same specifications and different geometries.

Index Terms—Induction motor, core loss, stray loss, motor efficiency, inverter, pulse-width modulation (PWM) supply.

I. INTRODUCTION

Growing concern of energy saving has made motor efficiency a prime necessity among design engineers and motor-driven systems manufacturers. An accurate measure of motor efficiency is always required to operate an existing motor in the high-efficiency region and to design an energy-efficient motor [1]. The efficiency estimation uses two terms evaluated precisely viz. motor's output power and the sum of all the losses incurred. The motor's output power can be measured directly with high precision [2], whereas the losses in the motor, especially the magnetic losses, have always been challenging. These magnetic losses include core losses (CL) and stray load losses (SL). Both these losses depend upon the time and space distribution of flux density in a motor. If the motor is supplied power with a PWM inverter, these magnetic losses become more prominent and complex to evaluate due to higher-order harmonics. The evaluation of CL for an inverter fed motor is widely researched and, there exist several models of CL (from modified Steinmetz equation [3] to two or three terms CL model with variable loss coefficients and, hybrid CL models [4]. On the other hand, the SL still lacks a straightforward and precise model representation for inverter fed motors.

SL in an induction motor (IM) is the additional CL, attributed to the various space and time harmonics of magnetic flux density [5], [6]. The space-harmonics results from the placement of coils in the slotted stator and variation of flux-path permeance [7]. In contrast, the time-harmonics are linked to the non-linearity (in MMF-flux linkage) introduced by the magnetic saturation and the inverter voltage harmonics. The impact of time harmonics depends upon the ratio of fundamental to the switching frequency [8]. This ratio varies

from 20 (medium power application) to 200 (small power applications) in the majority of inverter fed IM-drives [9], [10].

When addressing the SL, literature is broadly restricted to the space harmonics and sinusoidal excitation [7], [11], [12]. [7] and [11], present theoretical expressions to estimate the SL in a grid-connected IM. These expressions are rather complex, require the motor's complete geometry details, and need modifications as the slots' geometry changes. In a lumped element-based approach, Alger et al. present an extended equivalent circuit of IM to include the impact of harmonics [11]. Direct and exact evaluation of these coupled lumped parameters is strenuous.

Evaluation of SL in the post-process of FEA results is another approach used in recent works [13], [14]. However, unlike CL, which has three constituents, SL consists of multiple sources, making the post-process estimation complex in all the aspects-time, data storage, and computation [15].

In the attempt to redress these evaluation intricacies, [12], [16], [17] present different SL empirical models. The models presented in these works are developed with a logical hypothesis supported by the statistical analysis on a large set of measured SL data. Such models accelerate the iterative motor design process and require less response time when incorporated in motor control than purely physics-based methods. However, all the available SL models [12], [16], [17] are exclusively developed for a sinusoidal power supply. The SL models for PWM excited IMs are not addressed extensively and, literature still lacks a simple and generalised formulation of SL in inverter fed IM.

The work presented in this paper aimed to develop a simple and accurate SL model for an inverter fed IM, which:

- can be used in the design phase to minimise the SL.
- applies to the motors already in operation, whose detailed design and material data are not available except for the name-plate data.
- can distinguish the IMs of the same power rating *w.r.t.* their SL values. For example, two motors of the same power ratings but different efficiency classes will have different SL values, and the model should predict it.
- be applied to determine SL for different loading of the motors.

The model presented in this work uses IM's equivalent circuit parameters, output power, current, current's total harmonic distortion as input variables. All these input variables are available at both the design and testing stages, making the model capable of fulfilling the four criteria mentioned above.

The organisation of the paper is as follows: Section II reports the theoretical framework of the SL in an IF-IM. Section III presents SL measurements and loss segregation. Proposed SL formulation and its estimation accuracy is detailed in Section IV. Section V concludes the paper.

II. BACKGROUND: STRAY LOSSES IN INVERTER FED IM

Primary sources of SL in an IM are the various leakage fluxes, space and, time harmonics of the magnetic flux density ($B(t)$). These space and time harmonics of $B(t)$ are traced to the variation of magnetomotive force MMF. This section describes the space and harmonics in an inverter fed IM and their correlation with SL.

A. Space harmonics with sinusoidal excitation

Fourier decomposition of the magnetomotive force (MMF) generated by a distributed winding of one-phase (in a three-phase machine) is given as [11]:

$$F_A(\theta, t) = \frac{2}{\pi} N_S i(t) \sum_{v=1}^n \frac{K_W(v)}{v} \sin(vp\theta) \quad (1)$$

where,

$$i(t) = I_m \sin(\omega t) \quad (2)$$

, $K_W(v)$ is the winding factor of the v -th space harmonic. N_S and, p are, respectively, per phase number of turns in stator winding and, pole-pairs. For a three-phase machine drawing a balanced set of currents, the net MMF is given as the sum of MMFs of all the three phases, which are 120 shifted in space and it is:

$$F_S(\theta, t) = F_A(\theta, t) + F_B(\theta, t) + F_C(\theta, t) \quad (3)$$

Using (1) and (2), the net MMF is given as:

$$F_S(\theta, t) = \frac{3}{2} \left[A_1 \cos(\omega t - \theta) - \frac{A_5}{5} \cos(\omega t + 5\theta) + \frac{A_7}{7} \cos(\omega t - 7\theta) \dots \right] \quad (4)$$

where,

$$A_v = \frac{2}{\pi} N_S I_m K_W(v) \quad (5)$$

Expression (3) represents multiple travelling waves rotating in different directions. All these MMF harmonic waves travel with a speed less than the synchronous speed i.e., the speed of the fundamental MMF-wave. The MMF waves corresponding to space harmonic number $v = 6k - 1$ rotate in the reverse direction of the MMF waves of fundamental and, $6k + 1$ harmonic numbers. The space harmonics of order higher than 7 are negligible [6] and hence are neglected.

B. Space harmonics with inverter fed excitation

For an inverter-fed IM, the motor currents are expressed as:

$$i_a(t) = \sum_{n=1}^h I_m(n) \sin(n\omega t + \phi(n)) \quad (6)$$

$$i_b(t) = \sum_{n=1}^h I_m(n) \sin(n\omega t + \phi(n) - \frac{2\pi}{3}) \quad (7)$$

$$i_c(t) = \sum_{n=1}^h I_m(n) \sin(n\omega t + \phi(n) + \frac{2\pi}{3}) \quad (8)$$

Evaluation of (1)-(3) for the MMF in relation to the current expressions (5)-(6) gives:

$$F_S(\theta, t) = F_1(\theta, t) + F_5(\theta, t) + F_7(\theta, t) + \dots \quad (9)$$

where,

$$F_1(\theta, t) = \sum_{v=1}^n A_v \cos(v\omega t - p\theta) \quad (10)$$

$$F_5(\theta, t) = \sum_{v=1}^n A_v \cos(v\omega t + 5p\theta) \quad (11)$$

$$F_7(\theta, t) = \sum_{v=1}^n A_v \cos(v\omega t - 7p\theta) \quad (12)$$

From expressions (10)-(12), it is seen that the harmonics in current superimpose various high-frequency waves to the MMF waves corresponding to the fundamental frequency. All these rotating MMF waves produce additional SL.

C. Correlation of SL with the inverter power supply

The major components of SL viz. surface core loss, tooth flux pulsation loss and, phase belt pulsation loss are the additional eddy current losses in reaction to the space harmonics of the $B(t)$. The expressions of these losses derived using the diffusion equation and Faraday's law show that these losses vary directly as the square of MMF [6], [7]. Hence, SL in a cumulative effect of all the MMF harmonics for an inverter fed motor can be expressed as:

$$SL = \sum_{v=1}^n SL(i) = \sum_{v=1}^n K_i F_i^2 \quad (13)$$

$$\Rightarrow SL = K_1 F_1^2 \left(1 + k_2 \left(\frac{F_2}{F_1} \right)^2 + k_3 \left(\frac{F_3}{F_1} \right)^2 + \dots \right) \quad (14)$$

where, $SL(i)$ is the net SL with i -th time-harmonic and all the space harmonics. F_i is the magnitude of the MMF corresponding to the i -th current harmonic (as shown by (7)-(9)). K_i is the proportional coefficient of the SL-MMF relation. With a linear relation of MMF with current, the series sum used in (11) is directly proportional to the total harmonic distortion (THD) in the motor-current (a directly measurable quantity), and hence the SL in an inverter fed motor can be written as

$$SL = K_1 F_1^2 f(THD) \quad (15)$$

where, $f(THD)$ is a single valued monotonic function of motor's current THD. The determination of this function is explained in section-IV, which uses the experimental results of SL. The test procedure and the experimental results are detailed in the next section, i.e. section-III.

III. EXPERIMENTAL ESTIMATION OF SL

Total four squirrel cage induction motors of the same power rating but different slot geometries are selected to record the

variation of SL with inverter power supply. Nameplate details of these motors are given in Table-I and, Table-III shows their slot geometries. Two prominent sources of the SL *i.e.*, tooth-saturation and permeance variation are very specific to the slot shape. The major limitations of existing analytical SL methods [7], [11], [14] is that, they are developed for the basic geometries of the slots and, hence it is not possible with these expressions to differentiate two IMs of same number of slots but different slot geometries (except the slot-opening) for SL. The purpose of selection of these motors is, hence, to examine the SL variation among the IMs of the same power rating and, pole pairs but different slot geometries.

TABLE I
SPECIFICATIONS OF TESTED IMs

Line voltage/Frequency: 200V/50Hz		Pole-pairs(p): 2	$P_{Out}(kW)$: 2.5	
Stator slots: 36		Rotor slots: 28		
Motor	Design	Rated Speed (rpm)	Cage bar material	
M-1	SD-1	RD-1	1482	Copper
M-2	SD-2	RD-1	1482	Copper
M-3	SD-2	RD-2	1482	Copper
M-4	SD-1	RD-2	1482	Copper

A. Test operating points

The ratio of switching frequency to the fundamental frequency varies highly among the various applications (from 10 to 200). Therefore, the IMs selected in this work (as shown Table I) are tested for a wide range of this ratio. Another important parameter affecting the SL is the fundamental frequency and loading of the motor. The SL measurements are performed to analyse all these parameters' impact on all the operating points as given in Table II.

TABLE II
IMs OPERATION DOMAIN

	f_0 (Hz)	Torque (Nm)	f_0/f_{sw} Ratio
Minimum	30	0	10
Maximum	60	14	200
Steps	4	5	5

B. Measurement procedure

SL for these four IMs is evaluated using the input-output test. The test is reported to be the most accurate among all the SL evaluation methods [18]. Since the power ratings of the IMs are not very high and they can be loaded upto their rated power, the input-output test is preferred. If the IM is inverter fed, all the CL and SL related tests require two sets of measurements- a)- load and no-load test with PWM power supply b)-load and no-load tests with sinusoidal excitation. These tests are performed as follows:

1) *Rated load test*: The test IM is coupled with magnetic power brake as load and loaded for different torque values as shown in Table-II. Voltages, currents and shaft torque are recorded at the thermal equilibrium.

TABLE III
SLOT SHAPES OF THE TWO DESIGN PROTOTYPES

Stator slots: Initial design	SD1	Stator slots: Modified design	SD-2
	Dim. (mm) $B_{s0}=2.4$ $B_{s1}=6.0$ $B_{s2}=8.5$ $R_s=2.75$ $H_{s0}=1.0$ $H_{s1}=0.67$ $H_{s2}=18.33$		Dim. (mm) $B_{s0}=1.5$ $B_{s1}=6.0$ $B_{s2}=9.5$ $R_s=2.5$ $H_{s0}=0$ $H_{s1}=1.67$ $H_{s2}=18.0$
Rotor slots: Initial design	RD1	Rotor slots: Modified design	RD-2
	Dim. (mm) $B_{r0}=0$ $B_{r1}=5.5$ $B_{r2}=6.32$ $B_{r3}=3.17$ $H_{r0}=0.3$ $H_{r1}=1.0$ $H_{r2}=2.5$ $H_{r3}=14.0$		Dim. (mm) $B_{r0}=0$ $B_{r1}=5.0$ $B_{r2}=6.32$ $B_{r3}=1.50$ $H_{r0}=0.3$ $H_{r1}=2.25$ $H_{r2}=1.25$ $H_{r3}=1.80$ $H_{r4}=14.0$ $R_1=2.4$ $R_2=3.12$

2) *No-load test*: The no-load (NL) test is performed with the hot motor after the load test. The friction and windage losses and core loss are obtained from the no-load test at different supply voltages with sinusoidal power supply. Fig. 1 (a)-(b) show the NL-test results. Detailed step-wise procedure for these test in line with IEC-60034-3-2 is discussed in [19],[20]. Given the fact that, SL-estimation is highly sensitive to the torque and current measurements [29], the equipments used are selected in compliance with IEEE 112-B (*i.e.*, 0.2% accuracy for voltage and current measurements and, 0.5% accuracy for torque and speed measurements).

Fig. 1. No-load test results for at(a) M1, (b) M2.

The measurement set-up for these test (as shown in Fig.2 comprises: a).torque sensor (Kistler 4503B) and CoMo Torque Type 4700 data acquisition system. b) 2-level, H-bridge three-phase inverter c). 1.5 kW IM motor prototype d). Digital oscilloscope: YOKOGAWA DLM2054 with voltage and current probes e). Voltage and current transducers: LEM sensors LV25-P and LA 15P f). dSpace DS1103 for data acquisition g).Photo-tachometer.

SL from these test data is obtained as difference of CL between the loaded and no-load conditions of the IM [21],[14] *i.e.*,

$$SL = P_{Core_L} - P_{Core_NL} \quad (16)$$

where, P_{Core_L} and, P_{Core_NL} are, respectively, the CL at loaded and no-load conditions for the same voltage, f and f_{sw} . The CL, here, is calculated using the cycle-averaged value of

Fig. 2. Experimental setup: Schematic and test-bench

Fig. 3. Measurement results for IM-M1 (a) Variation of SL with T_L^2 for high and small values of f_{sw}/f at, (b) Correlation of SL and THD variations with f_{sw}/f , (c) Variation of SL with f_{sw}/f for different T_L , (d) Variation of THD with f_{sw}/f for different T_L , (e) Impact of loading on THD at high and small f_{sw}/f , (f) SL variation with f at constant f_{sw}/f ($=10$) and, T_L ($=10\text{Nm}$). f , in all these plots, $=30\text{Hz}$.

input power (P_{In}) and then, applying the input-output power balance as:

$$P_{Core} = P_{In} - P_{CuS} - P_{CuR} - P_{Out} \quad (17)$$

where, P_{CuS} and, P_{CuR} are the stator and, rotor copper losses, respectively, determined using the frequency and temperature dependant values of stator and rotor resistances (i.e., $R_S(T, f_i)$,

Fig. 4. Measurement results for IM-M1 (a) Variation of SL with T_L^2 for high and small values of f_{sw}/f , (b) Correlation of SL and THD variations with f_{sw}/f , (c) Variation of SL with f_{sw}/f for different T_L , (d) Variation of THD with f_{sw}/f for different T_L , (e) Impact of loading on THD at high and small f_{sw}/f , (f) SL variation with f at constant f_{sw}/f ($=10$) and, T_L ($=10\text{Nm}$). f , in all these plots, $=30\text{Hz}$.

$R_R(T, f_i))$ and stator-current harmonics I_i as

$$P_{CuS} = 3 \sum_{i=1}^{100} I_i^2 R_S(T, f_i) \quad (18)$$

,

$$P_{CuR} = 3R_R(T, f)I_{R1}^2 + 3 \sum_{i=2}^{100} R_R(T, f) \left(I_i \frac{X_{mi}}{X_{lRi} + X_{mi}} \right)^2 \quad (19)$$

here, X_{mi} and, X_{lRi} are the magnetising and rotor-leakage reactances w.r.t. i -th harmonic,

$$P_{Out} = P_{Shaft} + P_{Windage} + P_{Bearing} \quad (20)$$

and,

$$P_{Shaft} = T_{Shaft} \omega_r \quad (21)$$

T_{Shaft} and ω_r are the measured values of shaft torque and rotor speed in rad/s respectively. To reduce the measurement discrepancy, the obtained SL test data is smoothened (for correlation coefficient, $R^2 > 0.95$) by applying linear regression analysis with squared value of the torque. The process needs repeated measurements. The SL for the rated operating point is then calculated by multiplying the slope of the plot with

Fig. 5. Equivalent circuit of sinusoidally excited 3-phase IM.

Fig.2 (a)-(e) show the SL values of M1 and M2 plotted against the fundamental and switching frequency for different torque values (operating points of Table II). Similarly, Fig. 6 show the measured SL values for M3 and, M4. Examination and comparison of these results for different IMs draw following observations:

- As it can be seen, the SL of the M1 is of highest value always and, lowest for M3 in all the experiments. SL values for M2 and M3 always lie between the same of M1 and M3. For example, at the rated torque at 30Hz frequency with 300Hz switchings, they amount to 62.18W, 57.83 W, 48.30 W, and 52.25W for M1, M2, M3 and M4, respectively.
- SL notably increases with frequency. As Fig.2 (f) shows, the order of variation of SL with frequency is greater than one.
- The impact of carrier frequency on SL is quite prominent as expected. Low switching frequency (f_{sw}) operation makes the motor current highly distorted and consequently, raise the net SL. As shown in Fig.2 (a)-(d), the impact of f_{sw} shows a rapid decrement in the f_{sw}/f range of [10,50]. For very high f_{sw}/f (i.e. 100-200), SL with inverter fed IM is almost equal to that of a sinusoidally excited IM.

IV. PROPOSED SL FORMULATION

This section presents the development of the SL model for inverter fed IM using the measured values of SL (Section III). The exact analytical formulation of the SL is not possible [22] due to non-linear behaviour of magnetic core, dependence of iron-loss coefficients on magnetisation and frequency, impact of magnetic field distribution on current density distribution within the conductors (proximity and skin effect) and, the impact of slot-tapering on flux pulsation. Development of an empirical model is, hence, the most suitable option to account quantitatively the SL. [23] presents an empirical SL model for grid-connected IMs, which includes the impact of motor geometry on SL as well. The model in [23] uses motor inductances, the number of pole pairs and, output power as input variables. However, this model is developed neglecting the impact of power supply harmonics, which limits its application to the grid connected IMs. The model will need to incorporate the following additional variables when modified to use in inverter fed IM: THD, f_{sw} (or f_{sw}/f).

A. SL Model structure

In case of inverter fed IM, the variation in SL is associated with the changes in three power supply parameters: 1.) Output

power (P_{out}), 2.) Supply frequency (f) and, 3.) f/f_{sw} ratio (τ) or current THD in motor current.

Incremental change in SL, corresponding to the changes in above three parameters is:

$$dSL = \frac{\partial SL}{\partial P_{out}} dP_{out} + \frac{\partial SL}{\partial f} df + \frac{\partial SL}{\partial \tau} d\tau \quad (22)$$

Since, the three variables (P_{out}), f and, τ) are mutually exclusive, the variation (22) can be expressed as:

$$SL = f_1(P_{out})f_2(f)f_3(\tau) \quad (23)$$

Here, variation of SL with P_{out} (i.e. $f_1(P_{out})$ in (22)) is recognised as reported in [23] i.e.,

$$f_1(P_{out}) = K_{SL}P_{out} = \left(\frac{1}{p} \frac{L_{ls}}{L_m}\right)^m \left(\frac{1}{p} \frac{L_{lr}}{L_m}\right)^n P_{out} \quad (24)$$

where, p is number of pole-pairs and, L_m, L_{ls}, L_{lr} are the magnetising inductance, stator leakage inductance and rotor leakage inductance, respectively (as shown in Fig.5).

Formulation of $f_2(f)$ and, $f_1(\tau)$ uses following measurement observations (Section III):

1) *Impact of frequency (for $f_2(f)$):* As shown in Fig. 2 (f), SL increases with the fundamental frequency for constant f_{sw} and, load torque(T_L). With two trend related observations that, the rate of increment of SL with f is higher than one and, SL is zero at zero f , the simplest form of is given as:

$$f_2(f) = \alpha \left(\frac{f}{f_r}\right)^\beta \quad (25)$$

where, α and β are the constants and f_r is rated frequency i.e. supply frequency for rated speed, rated torque.

2) *Impact of THD in motor-current (for $f_3(\tau)$):* Fig.2(a)-(b) show the measured values of current THD and SL for different switching frequency-ratio. Here it is evident that SL decreases monotonously with increment in f_{sw}/f ratio following the grid SL value asymptotically. Also, the rate of change of SL with f_{sw}/f depends upon the f_{sw}/f . This rate decreases as the ratio f_{sw}/f increases i.e. the impact of switching frequency variation is high at low switching frequency. Hence, the fractional change in SL (due to change in τ only) shows a decreasing trend with τ i.e.,

$$\frac{df_3(\tau)}{f_3(\tau)} = \frac{d\tau}{(a+b\tau)} \quad (26)$$

Solving (26) provides the general solution as:

$$f_3(\tau) = \frac{k}{(a+b\tau)^r} + C \quad (27)$$

where, a, b, r, k and, C are the equation constants. The expression (27) relates SL (analogously the current THD) variation with f/f_{sw} for constant T_L . However, the current THD decreases as the T_L is increased (Fig. 2(e)). This is because only the fundamental current increases when T_L is increased. The rotor resistance to slip ratio for higher order harmonic currents observe negligible change due to higher value of their synchronous speed and thus, the change in harmonic current

with T_L (or slip) is very small. To incorporate this, $f_3(\tau)$ is further multiplied with a factor $(I/I_0)^2$. Here, I_0 is the no-load current at the same f_{sw} . The final expression of SL with the $f_2(f)$ and $f_3(\tau)$ is

$$SL = \underbrace{\left(\left(\frac{1}{p} \frac{L_{IS}}{L_m} \right)^m \left(\frac{1}{p} \frac{L_{IR}}{L_m} \right)^n P_{Out} \right)}_{f_1(P_{Out})} \underbrace{\left(\alpha \left(\frac{f}{f_r} \right)^\beta \right)}_{f_2(f)} \underbrace{\left(\left(\frac{I}{I_0} \right)^2 \left(C + \frac{k}{(a+b\tau)^r} \right) \right)}_{f_3(\tau)} \quad (28)$$

The expression (28) is summarised as

$$SL = K_{Grid} K_{Inv} P_{Out} \quad (29)$$

where,

$$K_{Grid} = \left(\left(\frac{1}{p} \frac{L_{IS}}{L_m} \right)^m \left(\frac{1}{p} \frac{L_{IR}}{L_m} \right)^n \right) \quad (30)$$

and,

$$K_{Inv} = \left(\alpha \left(\frac{f}{f_r} \right)^\beta \right) \left(\frac{I}{I_0} \right)^2 \left(C + \frac{k}{(a+b\tau)^r} \right) \quad (31)$$

B. Parameter identification

Extraction of parameters of the SL model (28) uses both the end-conditions ($f=f_r$ & $\tau=0$) and, non-linear regression. At first, α is determined as $f_2(f_r)=1 \Rightarrow \alpha=1$. Similarly, at $\tau=\infty$, $f_3(\tau)=1$ which implies, $C=1$. For identification of remaining constants, non-linear regression with the least squares technique is applied and, the best empirical fit for K_{Inv} is exhibited by the relation:

$$SL = \underbrace{\left(\left(\frac{1}{p} \frac{L_{IS}}{L_m} \right)^m \left(\frac{1}{p} \frac{L_{IR}}{L_m} \right)^n P_{Out} \right)}_{f_1(P_{Out})} \underbrace{\left(\left(\frac{f}{f_r} \right)^2 \right)}_{f_2(f)} \underbrace{\left(\left(\frac{I}{I_0} \right)^2 \left(1 + \frac{1}{(1+0.5\tau)} \right) \right)}_{f_3(\tau)} \quad (32)$$

The obtained value of the coefficient of correlation (R^2) with (31) is 0.98. The values of the exponents m and, n are 0.95 and 0.34, respectively.

C. Statistical evaluation

The errors spectrum with the proposed model for results is given in Table IV. There is no reference model in available in the literature to compare with (in terms of relative error), and, hence, only the absolute error with the measurements is compared. The model parameters are trained using the test data of M1 and M2 and, in further verification of these parameters, the SL for M3 and M4 is calculated. Comparison of the calculated and measured SL for M3 and M4 for the two extremes of f_{sw}/f and T_L is shown in Fig. (6).

Other important parameters of error-statistics wrt the proposed SL relation are given in Table V.

Noteworthy observations, derived from the Table IV and Fig. 6, regarding the prediction capability of the proposed SL model are mentioned below as:

- The proposed method ensures the accuracy of SL estimation with the estimation error less than 30% for 53.27%

Fig. 6. Measured (M) and calculated (C) SL values for (a) M-3, (b) M-4.

TABLE IV
ERROR SPECTRUM OF THE PROPOSED SL MODEL FOR FOUR IMS

Error(%)	No. of test-points	% of test-points
10-20	44	17.19
21-30	118	46.09
31-50	52	20.31
50-70	30	11.72
70-100	10	3.91
>100	0	0

operating points of the test sample. Test points with error higher than 50% are 16.41% of the test domain.

- Measured SL is maximum for M1 and minimum for M3, with respective SL values equal to 1.44% and 1.03% of the rated output power. Hence, the slot geometry plays a significant role in SL deviation.

V. CONCLUSION

In this work, a fast and easy-to-apply empirical model is presented to estimate the SL in inverter fed IM. Experimental data of a wide range of inverter power supply parameters (*i.e.*, f and f_{sw}/f) and motors' loading (T_L) of two IMs is analysed to develop this model. Total 128 IMs' operating points are tested in this dimensional space of f , f_{sw}/f , T_L . The model uses IMs equivalent circuit parameters, nameplate details, and directly measurable electrical quantities to estimate the SL, making the model applicable for both design and operation stages. To check the accuracy of the proposed model, two IMs (of similar rated characteristics but different geometries) are tested for SL for another 128 operating points. Comparing the calculated SL with the measured SL shows the 50% interval of the accuracy to have 83.89% of the total 256 operating points. Also, the R^2 with the proposed model is estimated to be 0.997.

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