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Hardware Implementation and Performance Analysis of Improved Sphere Decoder in Spatially Correlated Massive MIMO Channels

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Detection for high-dimensional multiple-input multiple-output (MIMO) and massive MIMO (MMIMO) systems is an active field of research in wireless communications. While most works consider spatially uncorrelated channels, practical MMIMO channels are correlated. This paper investigates the impact of correlation on Sphere Decoder (SD), for both single-user (SU) and multi-user (MU) scenarios. The complexity of SD is mainly determined by the initial radius (IR) method and the number of visited nodes during detection. This paper employs an efficient IR and proposes a new metric constraint in the tree searching algorithm, that significantly decrease the number of visited nodes and render SD feasible for large-scale systems. In addition, an introduced hardware implementation featured with a one-node-per-cycle architecture, minimizes the latency of the detection process. Trade-offs between bit error rate (BER) performance and computational complexity are presented. The trade-offs are achieved by either modifying the backtracking mechanism or limiting the number of radius updates. Simulation results prove that the proposed optimizations are effective for both correlated and uncorrelated channels, regardless of the level of noise. The decoding gain of SD compared to the low-complexity linear detectors (LD) is higher in the presence of correlation than in the uncorrelated case. However, as expected, spatial correlation adversely affects the performance and the complexity of SD. Simulation results reported here also confirm that correlation at the side equipped with more antennas is less detrimental. Hardware implementation aspects are examined for both a Virtex-7 FPGA device and a 28-nm ASIC technology.

Index Terms—correlation, initial radius, massive MIMO, maximum likelihood decoding, MIMO detection, multiple-input multiple-output, sphere decoding, hardware architecture.

I. INTRODUCTION

The increasing demand for higher data rates and for more connected devices has led to MMIMO technology. The use of a high number of antennas is no longer just a theoretical concept but a principal way of increasing the capacity of cellular communication systems [1]. The term “MMIMO” is mainly used to impose a MU system where the receiving antennas (N_R) outnumber the transmit antennas (N_T) [2]. In this work both SU and MU systems are examined. A MU system is referred to as MMIMO, while the SU scenario is referred to as high-dimensional MIMO. Both MIMO schemes are quoted as MMIMO channels.

A Maximum Likelihood (ML) detector yields the optimal solution in the MIMO detection problem but its complexity rises exponentially with the order of the modulation and with the number of transmit and receive antennas. On the contrary, Hassibi and Vikalo show that the expected complexity of SD is polynomial under certain circumstances [3], while Jaldén and Ottersten report that SD can exceed polynomial time in several use cases [4]. The reduced complexity is justified because SD detects the transmitted signal vector by searching only among candidate vectors which lie inside a hypersphere of radius R_0 around the received signal, in contrast to the ML detector where all vectors are examined [4], [5]. IR selection is critical for the complexity of the SD, as the size of hypersphere should be minimal while providing ML performance [5]–[7]. A large initial value of sphere radius can lead to an exhaustive search provoking exponential complexity; in contrast a very small radius may contain no lattice point inside the hypersphere and the search will have to be restarted with a new estimation of IR discarding the computations that had preceded up to that point. However in large-scale systems an effective IR is not always enough for complexity reduction and node pruning techniques are also required.

Several authors have studied IR selection and a wide range of variations of the SD algorithm have been proposed, targeting to decreasing the number of nodes visited during the search. In systems with limited N_T and low modulation order, a method for IR selection is optional, since the tree searching involves a small number of nodes; however in large-scale systems the absence of an efficient IR estimate leads to an exhaustive search, not feasible for hardware implementations. Defining a constant or the infinity value as IR [8]–[10] is applicable for small-scale systems only. Contrariwise, choosing the IR based on the noise distribution is efficient regardless of the tree length and has a beneficial effect on the computational complexity of SD [3], [11], [12]. A pre-processing block or an other low-complexity detector is also an effective IR technique [8], [9], diminishing the number of lattice points inside the hypersphere, but requires additional hardware resources. Recent works [13], [14] exploit deep learning in order to compute the IR in systems, where $N_T = N_R$. The results are encouraging in terms of searching time but the additional complexity of a neural network (NN) is not negligible especially for large-scale systems. In [13] the NN estimates a number of IR values, based on the noise statistics and on the structure of the channel matrix $\mathbf{H}$, implementing the increasing radius search algorithm [3], [15], while in [14] the NN predicts the minimum path metrics of sub-trees in order to optimize the search order and the IR estimate. Apart from IR selection, searching strategies are also crucial. In [16] statistical pruning is proposed, while in [17], [18] both algorithmic and architectural optimizations on sphere decoding are described.

This contribution is an extension of an earlier conference paper [19], where an implementation of Improved Sphere Decoder (ISD) is presented. The following points render the implementation of ISD feasible for large-scale systems:

- 1) a simple method for the estimation of an efficient IR;
- 2) an additional metric constraint at tree searching compared to the conventional SD algorithm; and
- 3) a one-node-per-cycle hardware architecture.

The employed IR technique is based on the noise variance and is just as efficient as the one proposed in [19], but less demanding in computational complexity. Simulation results from [19] show that the utilized IR combined with the extra metric constraint reduce significantly the number of nodes and consequently the total number of arithmetic operations and the latency of the detection process. SD is known for a non-deterministic behavior [5], in terms of the number of visited nodes, therefore a decrease in the required clock cycles is crucial. Apart from the reduction of the number of visited nodes, a one-node-per-cycle architecture also targets this problem by minimizing the latency. Assuming an uncorrelated channel, ISD achieves 0.5–1.25 dB decoding gain compared to LD and its implementation is attainable, both for an FPGA and an ASIC technology, for $E_b/N_0 \geq 4$ dB [19].

In this extended work, the effect of spatial correlation on ISD is investigated in terms of BER performance, number of visited nodes, computational complexity and detection throughput rate. Both SU and MU scenarios are examined. Simulation results are promising, proving that optimizations introduced in ISD are effective also for correlated channels. Correlation at receiver, given that $N_R \gg N_T$, is less harmful as expected [20], [21]. The decoding gain against LD depends on the level of correlation, but is found to be always higher compared to the uncorrelated case. Nevertheless, in the presence of spatial correlation, a higher Signal-to-Noise Ratio (SNR) is required to render ISD feasible for a hardware implementation. In addition trade-offs between complexity and performance become possible by exploiting an introduced constraint, which defines the maximum number of possible radius updates during tree searching. Modifying the backtracking mechanism of SD algorithm also leads to similar trade-offs. Besides, the problem of long required wordlength for data representation presented in [19] is solved in this paper by a suitable scaling of all tree distances. The achieved decrease in required word length reduces the area requirements for a one-node-per-cycle component, in comparison with [19]. The results of [19] deal with the calculations that are responsible for the assessment of the next level and the next node. In this work a complete version of the one-node-per-cycle component is described, including introduced functionality to control either a possible radius update or a possible termination of the search. This increases the critical path delay of the design, however maintaining the one-node-per-cycle architecture and high detection throughput rates.

LD have gained the interest of researchers in MMIMO and high-dimensional MIMO systems, where $N_R \gg N_T$. LD feature low complexity and their performance is quite satisfactory when the receiving antennas outnumber the transmitting antennas; but when $N_R = N_T$, their performance lags behind [22]. Common choices for LD in MMIMO channels are Zero-Forcing (ZF) and Minimum Mean-Square Error (MMSE). The Matched Filter (MF) is the simplest detector in terms of complexity; however it is efficient only for BPSK modulation

when $N_R = N_T$ [23], [24]. Under these circumstances, MF is able to replace ZF estimate in the SD algorithm. This simplification would further reduce the computational complexity of ISD. However for large-scale systems or for more dense modulation schemes, which relate to this contribution, MF is not applicable [23]–[25]. Furthermore results from [22] show that ZF or MMSE can achieve the performance of MF with a significantly reduced number of antennas. This paper focuses on the BER performance of ISD in comparison to ZF and MMSE detectors due to the inefficiency of MF.

A typical assumption in the literature is that the considered channels are spatially uncorrelated. Real testbeds prove the presence of correlation in MMIMO channels [26]–[28]. Antenna spacing and no rich scattering conditions are some of the main reasons due to which MMIMO channels suffer from correlated fading [22], [2], [29], [30]. In [27], the effect of antenna spacing is investigated. In large-scale systems, where $N_R \gg N_T$, the correlation normally exists at a lower level provided a rich enough angular spread [22], [26], [27]. Results of [26] demonstrate that the correlation, which always depends on the respective scattering environment, decreases as the quotient $\frac{N_R}{N_T}$ increases. The problem of correlation is more severe in SU than MU schemes [2]. This is reasonable since in MU systems a user is not necessarily close to an other and the possibility of having independent propagation paths is larger. Furthermore, as mentioned before, correlation on the side with fewer antennas is more detrimental [20], [21]. In the cases examined in this paper, this side refers to the transmitter since $N_R \gg N_T$.

Wubben *et al.* [31] study BER degradation and more generally the impact of spatially correlated channels on LD. Regarding SD, the effect of correlation is investigated for small-scale MIMO systems in [6], [7] and for spatial multiplexing MIMO of higher dimensions in [13], [32]. In these contributions, it is assumed that $N_R = N_T$ and all conclude that as correlation increases the complexity of SD rises and its performance deteriorates. Results in [6] show that one-side correlation imposes the same BER degradation irrespectively of whether it incurs at the transmitter or at the receiver. This is explained as the same number of antennas is used at both sides. The level of correlation determines the extent of the effect, in terms of the BER performance and the number of visited nodes. A common method to estimate the level of correlation is to derive the distribution of the channel eigenvalues; i.e., the eigenvalues of matrix $\mathbf{H}^H \mathbf{H}$. The statistical behavior of channel eigenvalues depends on the correlation [29] and determines the capacity of the system [33]. In a strongly correlated channel a few eigenvalues dominate [27], [30], [34], in contrast with weakly correlated scenarios where the eigenvalues are almost identical. Another technique, also based on the channel eigenvalues, is the computation of the condition number, defined in [26]. In addition, a feature of uncorrelated channels is the diagonal dominance [35], which does not apply in the presence of spatial correlation.

The remainder of this paper is structured as follows: Section II reviews the system model and the detection problem of SD. In addition, channel models for both SU and MU scenarios are derived. Section III describes the SD algorithm

and presents the proposed optimizations for a more efficient node pruning during tree searching. Section IV presents the employed IR method. Section V details the one-node-per-cycle architecture. Section VI shows the effect of spatial correlation and presents simulation results about BER performance, visited nodes, computational complexity and detection throughput rate. Hardware implementation aspects for both FPGA and ASIC implementation are revealed. Finally, Section VII discusses conclusions.

Notation: We use bold lower case for column vectors, $\mathbf{x}$, and upper case for matrices, $\mathbf{X}$. The transpose, the conjugate transpose (Hermitian) and the conjugate of a matrix or vector are denoted as $(\cdot)^T$, $(\cdot)^H$ and $(\cdot)^*$ respectively. The inverse and the square root of matrix $\mathbf{X}$ are denoted by $\mathbf{X}^{-1}$ and $\mathbf{X}^{\frac{1}{2}}$, respectively.

II. SYSTEM AND CHANNEL MODELS

This section describes the structure of the simulation model. Channel models for both SU and MU scenarios under correlation are also presented. Several spatially correlated channel models are employed in the literature, each adopting specific assumptions. This paper focuses on the analysis of the effects of correlated channels on sphere decoding methods, and not on identifying channels that best model the real propagation conditions.

A. System Model

A high-dimensional MIMO or a MMIMO system consists of N_T transmit and N_R receive antennas ($N_R \gg N_T$). In this work uplink communication is investigated, which means that the base station (BS) receives data from the N_T single-antenna users in the case of MU scenario or from a transmitter equipped with N_T antennas in the case of point-to-point spatial multiplexing MIMO. When correlation is not considered, a fully spatial multiplexing MIMO and a MU scheme are equivalent in terms of simulation. Results of [19] refer to both system models, since they assume an uncorrelated channel.

The transmitted data are mapped to constellation symbols by using Quadrature Amplitude Modulation (QAM) of M points. Let $\mathcal{O}^M$ denote the set of all possible lattice points. A flat-fading channel, instead of a frequency-selective fading channel, is assumed and is transformed into N flat-fading subchannels using Orthogonal Frequency Division Multiplexing (OFDM) transmission. Let N denote the number of subcarriers of an OFDM block. The $N_R \times N_T$ spatially correlated channel matrix $\mathbf{H}$ illustrates the complex transfer function from transmitter j to receiver i . In this work we assume that subcarriers which belong to the same OFDM block are subjected to the same fading, therefore $\mathbf{H}$ is common for these N subcarriers. In the case of an uncorrelated channel, all the h_{ij} are considered as Rayleigh fading channel coefficients. Additive white Gaussian noise (AWGN) is assumed. The received signal $\mathbf{Y}$ is defined as $\mathbf{Y} = \mathbf{H}\mathbf{S} + \mathbf{N}$, where the $N_T \times N$ matrix $\mathbf{S}$ denotes the N_T transmitted OFDM symbols, the $N_R \times N$ matrix $\mathbf{Y}$ denotes the N_R received OFDM symbols and $\mathbf{N}$ is the independent and identically distributed

complex AWGN with zero mean and variance N_0 . Provided that $\mathbf{H}$ is known at the receiver, the detection problem of SD [5] for one subcarrier is written as

$$\hat{\mathbf{s}}_{\text{SD}} = \arg \left\{ \min_{\mathbf{s} \in \mathcal{O}^M} \|\mathbf{y} - \mathbf{H}\mathbf{s}\|^2 \leq R_0^2 \right\}. \quad (1)$$

B. Channel Models

1) Single-user (SU) correlated channel

For point-to-point high-dimensional MIMO systems, the Kronecker model is employed. Kronecker model is mathematically manageable and allows transmitter side and receiver side correlation to be considered separately [6], [34]. The channel matrix is

$$\mathbf{H} = (\mathbf{R}_R)^{\frac{1}{2}} \mathbf{H}_u (\mathbf{R}_T)^{\frac{1}{2}}, \quad (2)$$

where $\mathbf{H}_u$ is the $N_R \times N_T$ Rayleigh fading channel matrix used for the uncorrelated scenario, $\mathbf{R}_R$ is the $N_R \times N_R$ correlation matrix of the receiver and $\mathbf{R}_T$ is the $N_T \times N_T$ transmit side correlation matrix. Both spatial correlation matrices are positive semi-definite generated by the exponential correlation model [36] as follows

$$r_{R_{ij}} = \begin{cases} \rho_\rho^{j-i}, & \text{if } i \leq j \\ r_{R_{ji}}^*, & \text{if } i > j \end{cases} \quad (3)$$

$$r_{T_{ij}} = \begin{cases} \rho_\tau^{j-i}, & \text{if } i \leq j \\ r_{T_{ji}}^*, & \text{if } i > j, \end{cases}$$

where ρ_ρ , ρ_τ are correlation coefficients with magnitude ≤ 1 and $r_{R_{ji}}^*$, $r_{T_{ji}}^*$ are the conjugates of $r_{R_{ij}}$ and $r_{T_{ij}}$ respectively. The exponential model is convenient since the level of correlation is determined by modifying one parameter, but it is not especially physically accurate [6]. In [33], it is reported that the exponential correlation model is reasonable in the case of a uniform linear array (ULA) in a richly scattering environment. The Kronecker model is assumed in [6], where the impact of spatial correlation on SD in small-scale point-to-point MIMO is examined. In [7], where the complexity of SD is investigated in the presence of correlation for a 4×4 MIMO system, the Kronecker model combined with the exponential model for the computation of $\mathbf{R}_R$, $\mathbf{R}_T$ is considered. Contributions in [33], [37] also use this model. Real measurements in [38], [39] prove the efficiency of the Kronecker model in MIMO systems. This paper focuses on the impact of correlation on SD and not on the channel model.

2) Multi-user (MU) correlated channel

For MMIMO schemes, the Gaussian local scattering model [40, Ch. 2.6] is illustrated. Recent works [2], [41] also assume this channel model. The local scattering model considers a BS equipped with a ULA of N_R antennas featured with half-wavelength spacing, which is elevated without scatterers in its near-field and is located at the center of a cell sector (cs). In this paper, both 60° and 120° cell sectors are examined. On the contrary, scattering is localized around the N_T single-antenna users, whose nominal angle ϕ_u is crucial for the generation of the $N_R \times N_R$ spatial correlation matrix $\mathbf{R}_u$. Apart from random ϕ_u , the assumed standard deviation $\sigma_\phi \geq 0$ is also important for modeling matrix $\mathbf{R}_u$. As σ_ϕ decreases, the correlation increases [40]. For the

simulation $\sigma_\phi = 10^\circ$ is selected, which represents strong spatially correlated channels and is considered as a plausible value in an urban environment [42]. The structure of the NLOS propagation model is depicted in [40] and in [41, Fig. 3].

The spatial correlation matrix $\mathbf{R}_u$, individually for each user, is derived from [40, Eq. (2.23)]. For a given user, the channel vector $\mathbf{h}_u$ is computed as

$$\mathbf{h}_u = \mathbf{R}_u^{\frac{1}{2}} \hat{\mathbf{e}} = \mathbf{V}_u \mathbf{E}_u^{\frac{1}{2}} \mathbf{V}_u^H \hat{\mathbf{e}}, \quad (4)$$

where $\mathbf{V}_u \mathbf{E}_u^{\frac{1}{2}} \mathbf{V}_u^H$ is the eigenvalue decomposition of $\mathbf{R}_u$ and $\hat{\mathbf{e}}$ is sampled from $\sim \mathcal{CN}(0, \mathbf{I}_{N_R})$. Let γ denote the rank of $\mathbf{R}_u$. The $\gamma \times \gamma$ $\mathbf{E}_u$ matrix is a diagonal matrix containing the γ positive non-zero eigenvalues of $\mathbf{R}_u$ and the $N_R \times \gamma$ $\mathbf{V}_u$ matrix consists of the corresponding eigenvectors, such that $\mathbf{V}_u^H \mathbf{V}_u = \mathbf{I}_\gamma$. Given that $\mathbf{R}_u$ is symmetric, the eigenvalue decomposition and the singular value decomposition are equivalent.

III. SPHERE DECODER (SD)

A. Sphere Decoder Algorithm

Sphere decoding algorithm of Fincke and Pohst is reviewed in this subsection [43]. Sphere constraint of (1) can be rewritten, according to [5], as

$$\hat{\mathbf{s}}_{\text{SD}} = \arg \left\{ \min_{\mathbf{s} \in \mathcal{O}^M} \|\mathbf{U}(\mathbf{s} - \hat{\mathbf{s}})\|^2 \leq R_0^2 \right\}, \quad (5)$$

where $\mathbf{U}$ is obtained by the Cholesky decomposition of matrix $\mathbf{H}^H \mathbf{H}$ and $\hat{\mathbf{s}}$ is the Least Squares estimate of $\mathbf{s}$,

$$\hat{\mathbf{s}} = (\mathbf{H}^H \mathbf{H})^{-1} \mathbf{H}^H \mathbf{y}. \quad (6)$$

SD performs a tree search through N_T levels, where every node has M branches. A limited number of possible candidate vectors is examined by SD depending on IR. The detection of the ML solution vector requires a metric constraint equal to R_0^2 . When a solution path is found, the sphere constraint updates its value with the new path metric. Let D_i denote the accumulated Euclidean metric down to antenna level i ,

$$D_i = \underbrace{u_{ii}^2 |s_i - z_i|^2}_{d_i} + \underbrace{\sum_{j=i+1}^{N_T} u_{jj}^2 |s_j - z_j|^2}_{D_{i+1}} \leq R_0^2, \quad (7)$$

where d_i is the partial Euclidean metric from antenna level i , D_{i+1} is the accumulated Euclidean metric down to antenna level $i+1$, s_i is the constellation symbol corresponding to the branch of level i and z_i is derived from

$$z_i = \hat{s}_i - \sum_{j=i+1}^{N_T} \frac{u_{ij}}{u_{ii}} (s_j - \hat{s}_j). \quad (8)$$

B. Improved Sphere Decoder (ISD)

In [44], depth-first tree traversal with IR selection combined with pruning techniques is presented as the most favorable strategy for SD implementations. ISD meets these specifications and as mentioned is characterized by an efficient IR, an additional constraint at each node, and can provide a one-cycle-per-node architecture. The conventional SD checks at each node only if the accumulated distance D_i is smaller

than the square of the sphere radius. On the contrary, ISD examines initially if the partial distance d_i exceeds the $\frac{1}{8}$ of IR. This value is constant during tree searching, in contrast with the sphere constraint in (7), and is hardware-friendly since it can be easily computed with shift operations. If this restriction is not met, ISD does not examine the sphere constraint of D_i and all the paths originating from that node are discarded. A similar concept, in which each node has its constraint, is mentioned in [5]. Additional restrictions are also reported in [45]–[47], where K -best SD implementations are examined. The reason which leads to the introduction of new constraints is the inefficient pruning of the conventional SD algorithm especially in upper tree levels, where D_i is a sum of a few partial distances. Effective node pruning in high levels achieves important savings, particularly in high-order modulation schemes.

Concerning ISD implementation, scaling of R_0 and of all d_i metrics is employed in order to prevent increase in word length. The employed IR and the one-node-per-cycle architecture are analyzed in next sections. ISD is very effective at node pruning having as lower bound of $N_T \times M$ visited nodes per tree searching. The lower bound on the number of visited nodes achieved by ISD equals to the total number of nodes required for K -best SD algorithm when $K = 1$. The main advantage of the K -best approach is the constant throughput and complexity [44], [46]. However, to prevent BER performance losses a large value of K is required [46]. Selecting a high value of K leads to a large number of nodes though. In [45], in order to achieve a detection performance close to ML, $K = 10$ is assumed for a 4×4 MIMO with 64-QAM modulation. Regarding the impact of the new metric on BER performance, simulations results in Section VI show that ISD achieves ML performance assuming uncorrelated channel and has a low possibility of an extremely small loss in the presence of correlation.

C. ISD with limited radius updates (LRU-ISD)

A variation of ISD, which leads to trade-offs between computational complexity and BER performance, is proposed here, namely LRU-ISD. Imposing a limit of L radius updates terminates the algorithm prematurely, decreasing the number of visited nodes, unless L is larger than the total number of radius updates that take place at ISD algorithm. When L exceeds the number of required radius updates, LRU-ISD and ISD are equivalent. It is reasonable that as the value of L falls, the number of visited nodes is reduced but its BER performance deteriorates. Especially for the case of $L = 1$, the solution path is the first path that emerges, satisfying the sphere constraint of (7) and the partial distance metric constraint of ISD, without the need for backtracking. The lower bound for this specific case regarding the visited nodes is $\frac{N_T \times M}{2}$, while for $L > 1$ is $N_T \times M$ as ISD. An early termination scheme is also presented in [14]. Simulations results of LRU-ISD are analyzed in Section VI.

D. ISD with backtracking constraint (BC-ISD)

Another variation of ISD for additional node pruning, but at the cost of losing BER performance, is introduced here

TABLE I
EFFECTIVENESS OF IR AND ISD IN TERMS OF NODE PRUNING

Feature	128×16	128×16	128×16	128×32
	$M = 16$	$M = 32$	$M = 64$	$M = 16$
Total nodes	1.97e+19	1.25e+24	8.05e+28	3.63e+38
$V_n^* (R_0 = \infty)$	5.68e+05	3.64e+06	6.45e+07	7.60e+08
$V_n^* (\text{IR})$	3229	20498	52930	72124
$V_n^* (\text{ISD})$	256	568	5261	1927

*assuming uncorrelated channel and $E_b/N_0 = 5$ dB

and modifies the backtracking mechanism. BC-ISD, in contrast with ISD, does not apply backtracking for the $\frac{N_T}{2}$ low levels of the tree when a path solution is found. Selecting the number of non-backtracking levels is a trade-off, since the more levels are omitted the more nodes are reduced. On the other hand, removing nodes arbitrarily from the search increases the likelihood that the optimal path will not be found, resulting in a drop in BER performance. The visited nodes of BC-ISD are always less than ISD and always more than LRU-ISD with $L = 1$. This is explained due to the fact that ISD backtracks without neglecting tree levels, while LRU-ISD with $L = 1$ terminates as soon as it finds the first solution path. Due to non-deterministic behavior of sphere decoding methods [5], in terms of the number of visited nodes, a combination of LRU-ISD and BC-ISD for further reduction of visited nodes is meaningless. If the ML solution path is close to the area of the first solution, then LRU-ISD is more efficient. However, if numerous radius updates are required through detection until the ML solution is found, BC-ISD is a more suitable approach. Further discussion about BC-ISD in Section VI.

IV. INITIAL RADIUS (IR)

IR computation is considered as a complex process in both high-dimensional MIMO and MMIMO schemes, therefore an effective and low-complexity IR method is required.

A simple IR technique, in terms of computational complexity, is to rely on the noise distribution. As already mentioned, this approach significantly reduces the number of lattice points inside the hypersphere and therefore the expected complexity of the SD [3], [11], [12]. In [3], [8], [9], [48], the IR is defined as a scaled variance of noise

$$R_0^2 = \alpha N_R \sigma^2, \quad (9)$$

in such a way that with a high probability, a lattice point lies inside the sphere

$$\int_0^{0.5\alpha N_R} \frac{\lambda^{0.5N_R-1}}{\Gamma(0.5N_R)} e^{-\lambda} d\lambda = 0.99. \quad (10)$$

Noting that $\|\mathbf{y} - \mathbf{H}\mathbf{s}\|^2 = \|\mathbf{n}\|^2$, where $\mathbf{n}$ is a column vector of N , is a Chi-square ($\tilde{\chi}^2$) random variable with N_R degrees of freedom and the integrand in (10) is the probability density function of this variable. The mean value is derived from (9), when $\alpha = 1$. This IR method, apart from being efficient in node pruning, is easily implemented on hardware and is the one which is employed for the simulations of this contribution.

Table I shows the effectiveness of the employed IR and compares ISD with the conventional SD algorithm in terms of

TABLE II
COMPUTATIONAL COMPLEXITY OF BASIC COMPONENTS

Component	Complex Multipliers		Multipliers	Adders	
	$N_T = 16$	$N_T = 32$		$N_T = 16$	$N_T = 32$
ZF detector	6.15e+05	1.51e+06	-	6.09e+05	1.50e+06
one-node-per-cycle	16	32	1	22	38

Fig. 1. Top-level block diagram of one-node-per-cycle component.

the number of visited nodes per detection. Let V_n denote the mean number of visited nodes per tree searching. Assuming an uncorrelated channel and $E_b/N_0 = 5$ dB, a significant decrease caused by the employed IR, in conventional SD algorithm, is observed for each examined system compared to the total number of visited nodes when no IR ($R_0 = \infty$) exists. Table I displays the substantial reduction of visited nodes due to ISD, achieved by employing the IR for both its sphere constraint and the introduced additional restriction on partial distance of each node. It is observed that in all cases we have over 90% savings attained by the introduced constraint of ISD.

V. A ONE-NODE-PER-CYCLE ARCHITECTURE

The number of visited nodes is a crucial factor for sphere decoding methods. It determines to a great extent not only the overall complexity but also the latency of the detection process. However, in a hardware implementation, apart from the number of visited nodes, the number of clock cycles required for processing at each node is also very important. In this contribution one clock cycle per node is required, which is optimal in terms of latency. It must be noted that in order to process more than one node in a clock cycle, there must be no data dependency, which does not apply in the proposed depth-first tree traversal. In [49], four nodes are investigated in parallel in a clock cycle, which is explained because it describes a breadth-first searching approach without data dependency between the nodes of the same level.

During tree traversal, ISD requires a series of tasks to be executed at each node. The top-level block diagram of the proposed one-node-per-cycle architecture is depicted in Fig. 1. Let en denote the enable signal of the component and sel denote the selection signal of the depicted multiplexers, which

Fig. 2. Component responsible for z_i computation. Fully loop unrolling is implemented.

are both driven by the Finite-State-Machine (FSM) of the design. When $en = 1$, tree searching takes place and the tree parameters are initialized before visiting the first node. The level i and the node enumeration nd , which declares which of the M branches is examined, are defined by the initial values i_0 and nd_0 respectively. As R_0 we assign the sphere radius defined in (9). Excluding the first node IR_n , i_n and nd_n , produced by this component, are selected as tree parameter inputs. Let s_c denote all the possible lattice points. In order to achieve the one-node-per-cycle processing in this work, all memory resources are fully partitioned. Multiplexing is utilized to deliver all the necessary data to the other subcomponents. In addition, the quotients $\frac{u_{ij}}{u_{ii}}$ in (8) and the value of u_{ii}^2 in (7) are computed before the tree exploration, since Cholesky decomposition has preceded. The total number of the required quotients is $\frac{N_T \times (N_T - 1)}{2}$, while the square of real-valued u_{ii} has to be calculated for N_T times. The calculation of z_i takes place initially and then it is the turn of the computation of d_i and the check of the partial distance constraint. Depending on that result, it is determined whether the calculation of D_i and the sphere constraint affect the value of pr . The 1-bit signal pr determines about pruning, while D_1 denotes the accumulated distance from the root up to the leaf node. Both signals contribute to the calculation of outputs. In addition, control mechanisms in order to decide whether the sphere constraint needs to be updated and whether the algorithm has to terminate having found the ML solution are applied and determine the values of the $rupd$ and $stop$ 1-bit signals respectively. The $stop$ signal is input to the FSM, while depending on the $rupd$ signal it is decided if the temporary path $path_{sl}$ has to be stored. When $rupd = 1$ the stored path vector is renewed, therefore the last update is the solution path of tree searching.

Apart from the memory partition and the precomputation of some data entries, another reason which makes possible the one-node-per-cycle architecture is the implementation of the circuit that computes z_i . Specifically, the implemented loop of (8) has to be fully unrolled. This implementation is presented in Fig. 2. The most demanding processing performed by the one-node-per-cycle component concerns the z_i computation, which is directly related to the number of

transmit antennas since the unroll factor equals to N_T . Loop unrolling contributes to the minimization of latency, but has an impact on area utilization. A rise in transmit antennas provokes increase in the required arithmetic units and in the critical path delay of the design, in which extra adders are introduced. Regarding the selection signals of N_T multiplexers, a $\log_2 N_T$ -to- N_T decoder, having as input the level i , is demanded. When $i = N_T$ all selection signals equal to 0, while for $i \neq N_T$ the sel_{N_T} is always equal to 1. The remainder of selection signals are determined according to the value of i .

In [44], a one-node-per-cycle architecture with variations in the tree searching procedure is also achieved. In this work, one unit specifies the next node regardless of whether the sphere constraint or the additional partial distance restriction of ISD are met. Contrariwise in [44] two units operate in parallel, where the one is utilized for forward node selection and the second one estimates the next node in case of pruning or when a temporary path solution is found. Besides, in [44] the SD algorithm is demonstrated for a limited number of levels assuming as IR the infinity value, so an initial sphere constraint does not exist in order to contribute to node pruning. However in this work IR is a key factor, where apart from the conventional sphere confinement, it also determines the partial distance restriction required by ISD during the entire search.

Tree searching occurs separately for each subcarrier of the OFDM block. For that reason the area required by a one-node-per-cycle component is critical, because increasing the number of instances of this unit leads to higher detection-throughput-rate implementations. The reduction in the utilized word length, due to the scaling of all the tree distances, reduces the required area of one-node-per-cycle component thus allowing more parallelism, compared to [19]. The computational complexity of the one-node-per-cycle component is presented in Table II. Timing and area reports and also trade-offs between area and detection throughput rate are analyzed at Section VI for both FPGA and ASIC implementations.

VI. EXPERIMENTAL RESULTS AND DISCUSSION

In this section, the proposed ISD and the introduced variations are evaluated, analyzing the effect of spatial correlation. As usual in systems where $N_R \gg N_T$, the results are subject to comparison with near-optimal LD. The linear detector chosen for comparisons is ZF, because for the systems studied here, it has a performance similar with MMSE and also the low-complexity MF can not be applied.

A. BER performance and visited nodes

The advantage of SD is that it yields the ML solution and thus surpasses the LD in terms of BER performance. The mean number of visited nodes during the detection process determines whether the decoder is attainable and provided that, what SNR value is required. Spatially correlated channels adversely affect both BER performance and visited nodes. Apart from this paper, in [6] it is also observed that the square values of the diagonal elements of the matrix resulting from the Cholesky decomposition decrease as the correlation level increases. The value of u_{ii}^2 is a crucial factor for a sphere

Fig. 3. BER performance of ISD existing correlation both at the transmitter and the receiver for 128×16 SU-MIMO, $M = 16$

decoding method, since it determines the number of candidate symbols per tree level [6]. Therefore in highly correlated systems, the total paths to be examined and consequently visited nodes increase. However the additional constraint of ISD imposed on the partial distance of each node substantially reduces this problem. Furthermore, correlation causes BER degradation at LD [31]. As a result ZF estimate, which is utilized by ISD during detection process, is less efficient compared to the uncorrelated scenario. The simulation results refer to the system presented in Section II, where, depending on whether it concerns the SU or the MU scenario, the corresponding correlated channel model is employed. For the uncorrelated case, a Rayleigh fading channel is assumed. The number of subcarriers equals to $N = 256$ and the total number of processed M -QAM symbols equals to $N_T \times N$.

1) SU-MIMO scenario

Fig. 3 depicts the BER performance of ISD, compared to ZF, under correlation on both sides for 128×16 SU-MIMO and 16-QAM modulation scheme. It is observed that raising the level of correlation both at transmitter and receiver side, the performance of ISD worsens taking into account the uncorrelated scenario but the decoding gain against the linear ZF detector increases. In [6], [13] BER degradation at SD due to spatial correlation is also noticed. For the same SU-MIMO system, Fig. 4 shows the mean number of visited nodes of ISD throughout the detection for various correlation values. As correlation increases, ISD examines more nodes in order to find the ML solution vector. However, regardless of the correlation level, ISD achieves a lower bound, which equals to $N_T \times M$ visited nodes. In highly correlated conditions, a

Fig. 4. Visited nodes of ISD existing correlation both at the transmitter and the receiver for 128×16 SU-MIMO, $M = 16$

larger E_b/N_0 value is demanded though. In [4], it is mentioned that the required operations and in general the complexity of SD decreases provided a high SNR value. This is also evident in Fig. 4, where fewer visited nodes in high SNR regions are translated into fewer arithmetic operations. In Figs. 5 and 6, the effect of one-side correlation is examined. It is inferred from both figures that transmit correlation affects the performance of ISD more than in the case when only correlation at the receiver is assumed. This outcome is expected, since as mentioned in [20], [21] the correlation on the side equipped with fewer antennas, which in this work refers to the transmitter, is more devastating. It goes without saying that the correlation on both sides is worse than one-side correlation.

A summary of the measurements for all the examined high-dimensional MIMO schemes of this contribution is presented in Tables III to VI. Modifying the coefficients ρ_τ and ρ_ρ , not only one-side but also correlation at both sides are investigated. The decoding gain compared to ZF detector and the required E_b/N_0 value in order ISD to be feasible for implementation for each test condition are displayed. Furthermore, Tables III to VI reveal the number of integral and fractional bits (int.frac) needed to represent data throughout the detection process. In this paper, the criterion which determines when the ISD is suitable for implementation is the upper bound of 1024 visited nodes. Computational complexity and latency are the reasons for establishing this threshold. Therefore, for the depicted E_b/N_0 value it is implied that ISD examines a maximum of 1024 nodes. It must be noted that the lower bound of ISD for $N_T = 16$ and $M = 64$ equals to the threshold.

According to Tables III to VI, the decoding gain of ISD against the linear detector ZF, which as already mentioned, grows as the correlation level increases, can reach up to 13 dB in extremely highly correlated scenarios. Correlation causes BER degradation at both ZF and ISD but the impact is greater on LD. Assuming an uncorrelated channel, the decoding gain ranges between 0.5 and 1.25 dB therefore the benefit

Fig. 5. BER performance of ISD existing correlation for 128×16 SU-MIMO, $M = 16$

Fig. 6. Visited nodes of ISD existing correlation for 128×16 SU-MIMO, $M = 16$

of ISD in spatially correlated channels is evident. However in a highly correlated environment, a larger SNR value is necessary, hence the benefit from BER performance may be expensive. The decoding gain of ISD when no correlation exists is in agreement with [50], which analyzes the BER performance of ML detector as opposed to ZF. On the contrary, simulation results reveal that there is a low probability of error in ISD when correlation is added to the channel, especially for denser constellations, but the decoding gain remains high. This is due to the additional partial distance constraint of

TABLE III
MEASUREMENTS FOR 128×16 SU-MIMO, $M = 16$.

Correlation		Decoding gain (dB)	$E_b/N_0 \geq$	Word format [†]
ρ_τ	ρ_ρ			
0	0	0.50	1 dB	10.4
0.1	0.1	2.75	3 dB	10.5
0.3	0.3	5.50	8 dB	11.6
0.5	0.5	7.70	15 dB	12.8
0.7	0.7	13.13	26 dB	14.10
0.1	0	2.26	2 dB	10.5
0.3	0	3.78	7 dB	11.5
0.5	0	6.10	12 dB	11.6
0.7	0	7.18	19 dB	12.7
0	0.1	0.55	2 dB	10.4
0	0.3	1.30	2 dB	10.5
0	0.5	3.38	5 dB	11.5
0	0.7	5.62	8 dB	11.6

[†]Word format $i.f$ denotes i integral and f fractional bits.

TABLE IV
MEASUREMENTS FOR 128×16 SU-MIMO, $M = 32$.

Correlation		Decoding gain (dB)	$E_b/N_0 \geq$	Word format [†]
ρ_τ	ρ_ρ			
0	0	0.60	5 dB	10.5
0.1	0.1	2.80	6 dB	10.5
0.3	0.3	5.59	11 dB	11.6
0.5	0.5	7.83	18 dB	13.8
0.7	0.7	12.46	29 dB	14.10
0.1	0	2.06	6 dB	10.5
0.3	0	4.00	10 dB	11.6
0.5	0	5.75	16 dB	11.8
0.7	0	7.00	23 dB	13.8
0	0.1	0.64	5 dB	10.5
0	0.3	1.32	6 dB	10.6
0	0.5	3.22	7 dB	11.6
0	0.7	6.06	12 dB	12.7

[†]Word format $i.f$ denotes i integral and f fractional bits.

ISD, which by dropping a node does not ensure that the ML solution path is not in the discarded ones and originated from that node. Furthermore, the results in Tables III to VI show that the transmit-side correlation leads to higher decoding gain and SNR requirements in comparison with the case assuming correlation at the receiver only, confirming what was mentioned before; i.e., correlation is more unfavorable on the side of the few antennas.

Besides, Tables III to VI show the word length employed for implementation. ISD requires as many bits as ZF for the tree searching process. The number of fractional bits is determined by the BER performance, while the integral bits are selected so as to avoid overflow. Further reduction of fractional bits causes BER degradation and increases the number of visited nodes. Scaling at all the tree distances prevents an increase in the required word length in contrast with [19]. In addition, it is observed that a rise of correlation increases the required word length.

In Figs. 7 and 8, the proposed variations of ISD algorithm are evaluated in terms of the BER performance and visited nodes. The conventional Fincke and Pohst SD and K -best implementations are also depicted. The results relate to a

TABLE V
MEASUREMENTS FOR 128×16 SU-MIMO, $M = 64$.

Correlation		Decoding gain (dB)	$E_b/N_0 \geq$	Word format [†]
ρ_τ	ρ_ρ			
0	0	0.70	9 dB	10.6
0.1	0.1	2.87	12 dB	10.7
0.3	0.3	5.50	18 dB	11.8
0.5	0.5	8.18	26 dB	13.9
0.7	0.7	10.91	38 dB	14.11
0.1	0	2.08	11 dB	10.7
0.3	0	4.19	16 dB	11.7
0.5	0	5.40	23 dB	12.8
0.7	0	6.51	29 dB	13.9
0	0.1	0.90	10 dB	10.6
0	0.3	2.00	12 dB	11.6
0	0.5	3.30	13 dB	11.7
0	0.7	5.97	18 dB	13.8

[†]Word format $i.f$ denotes i integral and f fractional bits.

TABLE VI
MEASUREMENTS FOR 128×32 SU-MIMO, $M = 16$.

Correlation		Decoding gain (dB)	$E_b/N_0 \geq$	Word format [†]
ρ_τ	ρ_ρ			
0	0	1.25	7 dB	10.6
0.1	0.1	4.20	9 dB	10.7
0.3	0.3	8.33	17 dB	12.8
0.5	0.5	11.40	26 dB	13.10
0.7	0.7	13.00	41 dB	15.11
0.1	0	3.10	8 dB	10.7
0.3	0	4.60	12 dB	11.7
0.5	0	6.20	20 dB	12.8
0.7	0	7.50	26 dB	13.10
0	0.1	2.00	8 dB	10.6
0	0.3	4.22	10 dB	10.7
0	0.5	5.50	14 dB	11.7
0	0.7	6.90	20 dB	13.8

[†]Word format $i.f$ denotes i integral and f fractional bits.

128×16 SU-MIMO system featured with 16-QAM modulation scheme, assuming $\rho_\tau = \rho_\rho = 0.3$. In Fig. 7, it is observed that the proposed ISD has a similar BER performance with the conventional sphere decoding algorithm. On the contrary, the contribution of ISD and its variations to the node pruning is displayed in Fig. 8, where the reduction in the number of visited nodes is remarkable. Regarding K -best implementation, Fig. 7 shows that even a large value of K ($K = 100$) is not enough to achieve BER performance of SD and ISD. The number of visited nodes during detection process, in case of K -best, is constant and equal to $M + (K \times M)(N_T - 1)$. If $K = 1$, this number is equivalent with the achieved lower bound of ISD.

LRU-ISD is evaluated for different values of permitted radius updates. A loss in BER performance is expected, therefore both LRU-ISD and BC-ISD are applicable when the decoding gain of ISD compared to ZF is adequate. The concept is to achieve a compromise between visited nodes and BER performance. Figs. 7 and 8 manifest in which E_b/N_0 value the proposed trade-offs are effective. The visited nodes should not exceed the threshold set in this work and a lower BER than ZF must be achieved simultaneously. In high noise conditions, it is observed that LRU-ISD does not perform as expected. This is explained by the fact that ISD needs to update its sphere constraint several times in these SNR regions. However, the performance of LRU-ISD changes at higher E_b/N_0 , where radius updates are limited due to the small variance of the noise which in turn leads to a more appropriate IR selection. It must be noticed that as L increases, the performance of LRU-ISD improves but the mean number of visited nodes increases. Regarding BC-ISD, the finite number of radius updates in high SNR regions rises the likelihood of not including the ML solution path. It is inferred from Fig. 7 that BC-ISD is more efficient than LRU-ISD at high noise levels. In addition, Fig. 8 shows that each variation achieves a lower bound for visited nodes. It equals to $\frac{N_T \times M}{2}$ for LRU-ISD featured with $L = 1$, while for $L > 1$ is $N_T \times M$ as ISD. As expected, BC-ISD always fluctuates between ISD and LRU-ISD with $L = 1$ in terms of visited nodes.

Fig. 7. BER performance of LRU-ISD and BC-ISD for 128×16 SU-MIMO, $M = 16$, $\rho_\tau = \rho_\rho = 0.3$

2) MU-MIMO scenario

As already mentioned, it is generally observed due to the random distribution of users that MU systems are less prone to spatial correlation than SU systems. Single-antenna users are assumed in this paper. The nominal angles ϕ_u are generated randomly for all the users and the obtained results are the average of five cases. A similar method is utilized in [41], where the final results are obtained as the mean value of 10 user drops. The standard deviation of the multipath angle distribution σ_ϕ is considered equal to 10° , as recommended in [42]. The same assumption is made in [41]. Figs. 9 and 10

Fig. 8. Visited nodes of LRU-ISD and BC-ISD for 128×16 SU-MIMO, $M = 16$, $\rho_\tau = \rho_\rho = 0.3$

concern a 128×16 MU-MIMO system featured with $M = 16$ and reveal the BER performance and the visited nodes of ISD under correlation. Both 60° and 120° cell sectors are examined. As expected and shown in the results, a smaller angle range leads to increased correlation among propagation paths, thus causing a BER degradation and an increase in the mean number of visited nodes in the detection process. Table VII depicts, for the examined systems and for both cell sectors, the decoding gain of ISD compared to ZF. In addition, the minimum E_b/N_0 value needed in order ISD to be feasible for implementation and the required number of bits are presented. For each MMIMO scheme, the E_b/N_0 value and the word length are identified for both cell sectors. The threshold of 1024 visited nodes applies as in SU system and $N_R = 128$ for all the examined cases. As opposed to SU-MIMO, a smaller decoding gain is observed and there are lower requirements in terms of word length and SNR. In the case of an 120° cell sector the results are similar with the uncorrelated scenario, while for 60° the decoding gain notes a slight increase. The assumptions of the employed channel model play a key role in this, since scatterers are located around the N_T users. Therefore transmit correlation, which is detrimental when $N_R \gg N_T$, is not assumed in this concept. According to Table VII the decoding gain compared to ZF in MU-MIMO reaches up to 2.15 dB. Tang *et al.* [35] examine expectation-propagation detection in real propagation conditions for NLOS MU-MIMO system achieving 4.3 dB. In comparison with SD, expectation-propagation detection lags behind in the matter

Fig. 9. BER performance of ISD existing correlation for 128×16 MU-MIMO, $M = 16$

Fig. 10. Visited nodes of ISD existing correlation for 128×16 MU-MIMO, $M = 16$

of BER performance [51], therefore ISD could demonstrate a higher decoding gain in real measurements.

B. Computational complexity

LD are widely used in both high-dimensional MIMO and MMIMO systems, due to their quite sufficient performance and low complexity. ISD is compared with ZF in this work, whose estimate is utilized for the tree exploration. Apart from ZF estimate, the Cholesky decomposition of matrix $\mathbf{H}^H \mathbf{H}$ and the IR computation are required before the execution of tree searching. The IR in (9) is estimated once for the

TABLE VII
MEASUREMENTS FOR MU-MIMO SYSTEM

System Model N_T	M	Decoding gain (dB)		$E_b/N_0 \geq$	Word format [†]
		120°cs	60°cs		
16	16	0.57	0.88	5 dB	10.4
16	32	0.75	1.00	5 dB	10.5
16	64	0.78	1.05	9 dB	10.6
32	16	1.33	2.15	7 dB	10.6

[†]Word format $i.f$ denotes i integral and f fractional bits.

TABLE VIII
OVERALL COMPUTATIONAL COMPLEXITY OF ISD COMPARED TO ZF
DETECTOR

V_n	128×16	128×16	128×16	128×32
	$M = 16$	$M = 32$	$M = 64$	$M = 16$
1024	$\times 7.82$	$\times 7.82$	$\times 7.82$	$\times 6.55$
$N_T \times M$	$\times 2.71$	$\times 4.41$	$\times 7.82$	$\times 3.78$
$(N_T \times M)/2$	$\times 1.85$	$\times 2.71$	$\times 4.41$	$\times 2.39$

$N_T \times N$ M -QAM symbols and is easily computed with one shift operation provided that $\alpha = 1$. If N_R is not a power number of 2 as in this work, then one multiplier is required. The complexity of ZF is in Table II, where for the required matrix inversion work from [52] is considered. Table II also presents the arithmetic operations, performed at each node. These are repeated for $N \times V_n$ times. Therefore node pruning is critical not only for the latency of the detection process but also for the computational complexity.

Table VIII demonstrates the computational complexity of the proposed ISD for the examined schemes, contingent upon the mean number of visited nodes per tree search. More specifically, it shows how many times its complexity is greater than that of ZF for the detection of an OFDM block. For the results, $N = 256$ is assumed. The ISD complexity shown in Table VIII refers to either SU-MIMO or MU-MIMO system. Depending on the correlation level, a suitable E_b/N_0 value is required to achieve the displayed bound of V_n . Having in mind the overall complexity and the latency of the implementation, the E_b/N_0 values depicted in Tables III to VII have been determined for each case. Operation at a lower E_b/N_0 than the one proposed, leads to more visited nodes during the detection and therefore to greater complexity. The computational complexity of the one-node-per-cycle component shown in Table II, does not differ significantly for LRU-ISD and BC-ISD; hence their complexity is modified only with respect to V_n . Table VIII shows the overall complexity considering the defined threshold of 1024 nodes, the lower bound of ISD as well as the bound of LRU-ISD featured with $L = 1$, which is the minimum number of visited nodes achieved in this paper. BC-ISD examines a number of candidate symbols between those of ISD and LRU-ISD with $L = 1$, therefore its complexity is intermediate. It must be supplemented, that the total arithmetic operations to be executed are not related to the number of instances of one-node-per-cycle components. The number of instances affects the area and the detection throughput rate, which are discussed below. From Table VIII it is inferred that the higher complexity is observed for $M = 64$, despite the fact that the examined scheme featured with $N_T = 32$ consists of a

tree representation with more nodes as described in Table I. Therefore it is concluded that a high number of tree levels is preferable for ISD than extremely dense constellations.

C. Detection throughput rate

Detection throughput rate is an important performance metric for the hardware implementation. Having reduced the mean number of visited nodes and achieved the minimum processing time per node, the throughput rate of tree searching process for both FPGA and ASIC implementations is presented here for ISD and its variations. FPGA implementations have been mapped to a Virtex-7 VT690T device. The ASIC results are obtained by synthesis using Mentor Graphics' Catapult as a front-end high-level synthesis tool and Cadence Genus and Innovus as back-end tools, targeting a 28-nm standard-cell library.

Tables IX and X show the maximum clock frequency ($f_{\max}$) and the area requirements of the one-node-per-cycle component for the FPGA and the ASIC implementation respectively. Trade-offs between area and throughput rate arise by replicating one-node-per-cycle components. Aiming at parallel detection FPGA resources usage reported in Table IX determines the maximum number of components. Regarding the ASIC implementation, the cost imposes area limits. Let α denote the number of components. The detection throughput rate is displayed in Table XI for both FPGA and ASIC implementations and is computed in terms of parameters α and V_n . Replacing the value of V_n according to the respective E_b/N_0 , the detection throughput rate can be computed for both ISD and its variations, since the one-node-per-cycle component for all variations has almost the same computational complexity; therefore the same upper bound on α is imposed. The detection rate is maximized for ISD and its variations when α receives its highest value and V_n equals to the lower bound of each proposed decoder. The decoded bits per tree searching for each examined scheme equals to $N_T \times \log_2 M$.

Tables IX to XI refer to all systems studied in this paper for both SU and MU scenarios. Since uncorrelated and MU correlated cases require exactly the same word length for detection, complexities derived for an uncorrelated channel are relevant for both cases. For the SU systems, except for the uncorrelated case, spatial channel correlation with $\rho_\tau = \rho_\rho = 0.3$ is examined. Table IX shows that as M rises, the one-node-per-cycle component requires more memory resources. The same holds for increasing N_T . In this case, a rise in DSP48 units is also observed, which are critical for the determination of the upper bound of α considering the resources of the selected board. Similarly Table X shows that the required area increases for the ASIC case. In addition, in both implementations the scheme equipped with more transmit antennas causes the greatest decrease in the $f_{\max}$ of the design. The results for the case with $N_T = 32$ are reasonable considering Fig. 2, where full loop unrolling is implemented for the computation of (8) with an unroll factor equal to N_T . This justifies the sharp increase both in the DSP48 units of the FPGA implementation and the critical path, since more arithmetic operations take place in one clock cycle. Apart from M and N_T , the word

TABLE IX
HARDWARE IMPLEMENTATION RESULTS OF ONE-NODE-PER-CYCLE COMPONENT - FPGA DEVICE

System model	$f_{\max}$ (MHz)		DSP48		FF		LUT	
$128 \times 16, M = 16$	39.43	38.56**	46 (1%)	46 (1%)**	352 (0%)	410 (0%)**	3729 (0%)	4555 (1%)**
$128 \times 16, M = 32$	38.63	38.02**	46 (1%)	46 (1%)**	352 (0%)	410 (0%)**	4039 (0%)	4565 (1%)**
$128 \times 16, M = 64$	38.12	33.50**	46 (1%)	92 (2%)**	370 (0%)	496 (0%)**	4312 (0%)	5149 (1%)**
$128 \times 32, M = 16$	34.74	30.83**	94 (2%)	188 (5%)**	658 (0%)	822 (0%)**	11805 (2%)	14689 (3%)**

*The percentages denote the utilization score according to the selected board ** $\rho_{\tau} = \rho_{\rho} = 0.3$

TABLE X
HARDWARE IMPLEMENTATION RESULTS OF ONE-NODE-PER-CYCLE COMPONENT - ASIC DEVICE FOR 28-nm TECHNOLOGY

System model	$f_{\max}$ (MHz)		Area*	
$128 \times 16, M = 16$	363.50	278.01**	33.68 k	44.36 k**
$128 \times 16, M = 32$	289.35	276.32**	43.41 k	52.83 k**
$128 \times 16, M = 64$	280.35	262.61**	63.45 k	79.10 k**
$128 \times 32, M = 16$	179.44	170.44**	77.30 k	105.61 k**

*referring to instances of cells of a 28-nm standard-cell library ** $\rho_{\tau} = \rho_{\rho} = 0.3$

TABLE XI
DETECTION THROUGHPUT RATE

System model	FPGA implementation (MHz)		ASIC implementation (MHz)	
$128 \times 16, M = 16$	$(2523.52 * \alpha) / V_n$	$(2467.84 * \alpha) / V_n^*$	$(23264.27 * \alpha) / V_n$	$(17792.60 * \alpha) / V_n^*$
$128 \times 16, M = 32$	$(3090.40 * \alpha) / V_n$	$(3041.60 * \alpha) / V_n^*$	$(23148.15 * \alpha) / V_n$	$(22105.55 * \alpha) / V_n^*$
$128 \times 16, M = 64$	$(3659.52 * \alpha) / V_n$	$(3216.00 * \alpha) / V_n^*$	$(26913.37 * \alpha) / V_n$	$(25210.08 * \alpha) / V_n^*$
$128 \times 32, M = 16$	$(4446.72 * \alpha) / V_n$	$(3946.24 * \alpha) / V_n^*$	$(22967.88 * \alpha) / V_n$	$(21816.94 * \alpha) / V_n^*$

* $\rho_{\tau} = \rho_{\rho} = 0.3$

length at each examined scheme affects the $f_{\max}$ and the area utilization.

Although the increase in transmit antennas is more detrimental concerning the critical path and the required area of the design, the lowest detection throughput rate is observed for the case of $M = 64$. According to the FPGA implementation results the maximum throughput rate for $N_T = 16$ and $M = 64$ is 278.75 Mbps compared to 338.71 Mbps for $N_T = 32$ and $M = 16$ assuming uncorrelated or correlated MU channel. Similar conclusions emerge also for the ASIC implementations. For the correlated SU scenario when $\rho_{\tau} = \rho_{\rho} = 0.3$ is 122.48 Mbps and 146.44 Mbps respectively. It follows due to fewer visited nodes in the case of $N_T = 32$, despite the fact that the system with $M = 64$ has the capability of using a larger number of parallel instances. The throughput rate of ISD and its variations is maximized for $N_T = M = 16$, where ISD reaches up to 768.89 Mbps and LRU-ISD featured with $L = 1$ up to 1537.77 Mbps regarding the FPGA implementation. These results are obtained for the maximum allowable α and the minimum V_n for each case. Furthermore, 7088.33 Mbps and 14176.66 Mbps are the maximum values respectively for the detection rate of the ASIC implementation, assuming the same number of parallel instances as in the FPGA implementation. As expected, ASIC implementations may support high throughput rates. The effect of correlation leads to a small reduction of the maximum throughput rate due to increase in wordlength. Furthermore a higher E_b/N_0 value compared to the uncorrelated case is required for both SU and MU systems in order to achieve the maximum detection rate.

Compared to [19], the architecture proposed here reduces the required area due to the decrease in the word length

achieved by scaling at all the tree distances, as described in Section V. However, the critical path increases because apart from the operations of [19] additional functionality for searching termination and IR updating are introduced in this work. Furthermore, the achieved detection throughput rate is higher compared to [19], despite the reduced $f_{\max}$. This is justified as the decrease in area utilization allows more one-node-per-cycle components to be implemented.

VII. CONCLUSION

In large-scale systems, a sphere decoding method is considered as prohibitive due to high computational complexity and latency, but the results of this paper disrupt conventional thinking and show that there may be a future in certain high-dimensional MIMO or MMIMO schemes. The proposed optimizations render ISD feasible for implementation, even in the presence of correlation, for both SU and MU scenarios. As the level of correlation rises the decoding gain compared to linear detectors increases, reaching up to 13 dB in highly correlated conditions, requiring though a larger value of E_b/N_0 . Variations of ISD algorithm are also presented leading to trade-offs between BER performance and computational complexity. The minimum complexity is achieved by LRU-ISD featured with $L = 1$ and equals to 1.85 times the complexity of ZF. In addition, one-side correlation is investigated for SU-MIMO schemes, confirming that the correlation on the side equipped with less antennas is more detrimental. Furthermore, the large gains observed in SU compared to MU systems are due to the greater effect of correlation, which is partly justified in the assumptions of the employed channel for the MU case.

Trade-offs between area and detection throughput rate are also presented for both FPGA and ASIC implementations reaching up to 1537.77 Mbps and 14176.66 Mbps respectively.

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