Information's Relativistic Convey with Matter Wave's Non-Dispersive Propagation

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Abstract

The Wave-Particle Duality is a basic property of microscopic particles. As a basic concept of quantum mechanics, the wave-particle duality theory from elementary particles to big molecules had been verified by lots of experiments. Different from electromagnetic wave, the matter wave's propagation is not only fast but also adjustable. According to the special relativity theory, the group velocity with which the overall envelope shape of the wave, namely the related particle's propagation and information convey speed is changeable with its energy and related wavelength, among which only the energy exceeds over the minimum value, the propagation can be starting and the velocity is not allowed to surpass the maximum value i.e. the light speed in vacuum. Take electron as an example, if the free electron beam gains energy higher than around $8.187 \times 10^{-14} \text{J}$ and the related wavelength is shorter than around $5.316 \times 10^{-3} \text{nm}$, the matter wave can start to propagate.

Wave-Particle Duality is a basic property of microscopic particles. Inspired by Albert Einstein's light wave-particle duality theory, Louis de Broglie in 1924 proposed that all particles exhibit wave-particle duality. This is referred as matter wave and also called as de Broglie wave.⁽¹⁾ This hypothesis was first confirmed by George Paget Thomson's cathode ray diffraction experiment in 1927 and then The Fullerene molecule in 1999,⁽²⁾ the Phthalocyanine molecule (consist of 58 atoms) in 2012⁽³⁾ and even the big molecule with 25,000 amu in 2019⁽⁴⁾ were discovered in succession with wave-like behaviors.

According to the wave-particle duality theory, de Broglie combined the Einstein mass- energy equation $E=mc^2$, one base of the relativity theory related to mass and energy with the Planck relation $E=h\nu$, one base of quantum mechanics theory related to wave frequency and energy then equalized both to: $mc^2 = h\nu$. However, De Broglie considered that not all matter move with the velocity of light speed in vacuum, so he changed the mass- energy equation from $E=mc^2$ to $E=mv^2$ by replacing c with the actual velocity v and equal both as $mv^2 = h\nu$. This important change means that the de Broglie hypothec did not take the relativity theory as its theoretical base and the propagation is non-relativistic⁽⁶⁾

This article deals with the matter wave's relativistic propagation on bases of the special relativity theory and the Planck relation.

In the matter wave's propagation, the group velocity of a wave is the velocity with which the overall envelope shape of the wave's amplitudes –known as the modulation or envelope of the wave propagates through space and is often thought of as the velocity at which energy or information is conveyed along a wave. The group velocity v_g is defined by the equation (1) :

$$v_g = \frac{\partial \omega}{\partial \kappa} = \frac{\partial E}{\partial p} \quad (1)$$

Where E , p , ω and κ denote total energy, momentum, angular frequency and angular wavenumber. Matter wave's non-dispersive propagation means its propagation in space without effects caused either by interaction with transmitting medium or by geometric boundary condition.

De Broglie concluded that the velocity of a particle should always be equal to the group velocity of the corresponding wave. This conclusion had been proved for free particle's non-relativistic propagation⁽⁶⁾ and also can be proved for free particle's relativistic propagation as below:

According to the special relativity theory, particle's total energy E is

$$E^2 = (m_0 c^2)^2 + (pc)^2 \quad (2)$$

Where m_0 denotes rest(intrinsic) mass, p is momentum, c is light speed in vacuum. If v denotes particle's propagate speed and γ represents Lorentz factor:

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \quad (3)$$

From formula (1)

$$v_g = \frac{\partial E}{\partial p} \quad (4)$$

$$= \frac{\partial}{\partial p} \sqrt{(m_0 c^2)^2 + (pc)^2}$$

$$= \frac{pc^2}{E} \quad (5)$$

$$= \frac{\gamma m_0 v c^2}{\gamma m_0 c^2}$$

$$= v \quad (6)$$

From (4) to (6) de Broglie's conclusion proved.

The matter wave's relativistic propagation can be classified into two types as below:

1) Rest mass $m=0$

$E=pc$ thus

$$V_g = \frac{\partial E}{\partial p} = c \quad (7)$$

This is the group velocity of light, namely the photon's propagation speed in vacuum.

2) Rest mass $m \neq 0$

$$E^2 = (m_0 c^2)^2 + (pc)^2$$

$$v_g = \frac{\partial E}{\partial p}$$

$$= \frac{\partial}{\partial p} \sqrt{(m_0 c^2)^2 + (pc)^2}$$

$$= \frac{pc^2}{E}$$

And

$$E = \gamma m_0 c^2 \quad (8)$$

$$p = \gamma m_0 V \quad (9)$$

$$\gamma^2 = \frac{1}{1 - \frac{V^2}{c^2}} = \frac{c^2}{c^2 - V^2} = \frac{E^2}{m_0^2 c^4} \quad (10)$$

Thus

$$V = \sqrt{c^2 - \frac{m_0^2 c^6}{E^2}} \quad (11)$$

$$\text{As } c^2 - \frac{m_0^2 c^6}{E^2} > 0 \quad \text{so}$$

$$E > m_0 c^2 \quad (12)$$

Let E_s to denote the minimum energy needed to start matter wave's propagation

$$E_s = m_0 c^2 \quad (13)$$

And the related wavelength λ , from de Broglie relation:

$$p = h / \lambda \quad (14)$$

$$\lambda = h / p = h / (\gamma m_0 V) \quad (15)$$

Formula (11) indicates the relation between the particle propagation's velocity and the related particle's energy in relativistic propagation.

To start the relativistic propagation, a minimum energy E_s is needed shown in formula (13).

Take electron as an example, the electron's rest mass, $m_e=9.109 \times 10^{-31} \text{kg}$, the light speed in vacuum, $c=299,792,458 \text{m.s}^{-1}$, the Planck constant, $h=6.62607015 \times 10^{-34} \text{J.s}$. so, the start energy

$$E_s = m_0 c^2$$

$$= 8.187 \times 10^{-14} \text{ J}$$

The related start velocity is

$$V_s = \sqrt{c^2 - \frac{m_0^2 c^6}{E_s^2}}$$

$$= 0$$

If the particle with energy, $E=9 \times 10^{-14} \text{ J}$, the propagation velocity will be

$$V = \sqrt{c^2 - \frac{m_0^2 c^6}{E^2}}$$

$$= 1.245 \times 10^8 \text{ m.s}^{-1}$$

$$= 0.415c$$

According to the de Broglie relation, the related wavelength should be

$$p = h/\lambda \quad (16)$$

$$\lambda = h/(\gamma m_0 V) \quad (17)$$

$$= 5.316 \times 10^{-12} \text{ m} = 5.316 \times 10^{-3} \text{ nm}$$

In (11), when the energy E tends to infinitely great,

$$E \rightarrow \infty$$

the velocity approaches to the light speed c .

$$V \rightarrow c$$

Facts and Data

The electron microscope uses a beam of accelerated electrons under a very high voltage of several 10^3 to 3×10^6 voltages (the HVEM can be applied to 200 to 1000 kV) between a cathode, as an electron source and an anode, as an acceleration part, going through a vacuum area to illuminate the specimen and create an image. As the wavelength of electron can be up to 100,000 times shorter than that of visible light photons, electron microscopes have higher resolving power than light microscopes and can reveal the ultrastructure of smaller objects.

If the electron beam is accelerated by anode with voltage of $6 \times 10^2 \text{ kV}$, i.e. $V=6 \times 10^5 \text{ V}$, and electron's charge, $e=1.602 \times 10^{-19} \text{ C}$, electron's mass, $m_e=9.109 \times 10^{-31} \text{ kg}$, so the electron's energy accelerated, E_e ,

wavelength, λ_e and group velocity, v_{g_e} , namely the particle's propagation speed v , should equate to:
 $E_e = eV = 9.612 \times 10^{-14} \text{ J}$ (18)

$$V = v_{g_e} = \sqrt{c^2 - \frac{m_0^2 c^6}{E^2}}$$

$$= 1.571 \text{ m.s}^{-1}$$

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$= 1.174 \quad \text{So} \quad (19)$$

$$\lambda = h / (\gamma m_0 V)$$

$$= 3.944 \times 10^{-12} \text{ m} = 3.944 \times 10^{-3} \text{ nm} \quad (20)$$

The minimum energy needed to start propagating is

$$E_s = 8.187 \times 10^{-14} \text{ J} \quad (21)$$

Conclusion

The matter wave's relativistic propagation, i.e. the related particle's propagation and information's convey is not only fast but also adjustable. The velocity is changeable with its energy and related wavelength in its non-dispersive propagation, once the energy is over a minimum value and the wavelength is shorter than certain value, the propagation can be starting. Take electron as an example, if the electron's total energy is higher than $8.187 \times 10^{-14} \text{ J}$ and the related wavelength is shorter than around $5.316 \times 10^{-3} \text{ nm}$, the matter wave can start to propagate.

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