Investigation into Magnetic Control of Hard-Switching DC-DC Converters

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Abstract - In this paper an investigation into magnetic control (MC) of hard-switching (HS) DC-DC converters is carried out. The proposed control method is based on the modulation of the effective filter inductance of the converter when operating in discontinuous conduction mode (DCM). It is well known that the output characteristic of a HS DC-DC converter operating in DCM is dependent on the effective inductance of the output filter. This way, by using a variable inductance the output of the converter can be controlled. The proposed control method can be applied to any converter topology, namely buck, boost, buck-boost, flyback, forward, and so on. In this paper, the operation of the buck converter with MC is investigated in detail as a case study. This work proves that the proposed control method can be effectively used to control DC-DC converters on its own or by combination with other control parameters as duty cycle and/or frequency. An experimental prototype has been built to test the proposed control method and modeling process and to demonstrate its feasibility and possibilities.

Keywords - magnetic devices, variable inductors, DC-DC converters, magnetic control, DCM operation.

I. INTRODUCTION

Switching DC-DC converters are used in a vast range of applications, including battery chargers, LED drivers, uninterruptible power supplies (UPS), housekeeping power supplies, point-of-load converters, etc. For low power applications, hard-switching (HS) DC-DC converters are the preferred option because of their simplicity, good efficiency, reliability and low component count. The most common and well-known topologies for low power applications are the buck, boost, buck-boost, flyback and forward converters. These converters can be controlled in a great variety of control methods, like voltage mode control and current-programmed control, which are primarily used. However, control methods proposed until now are based on directly or indirectly changing the switch duty cycle and/or switching frequency [1]. Magnetic regulators have been presented in previous literature as a means to improve the operation of power converters [2]-[23]. They have successfully been employed in controlling resonant inverters for electronic ballast [4]-[9], DC-DC converters [10]-[14], power converters for photovoltaic (PV) applications [15], power factor correction (PFC) converters [16][17], LED drivers [18]-[20], smart-grid applications [21], and battery charging [22][23].

This paper looks into the possibility of controlling HS DC-DC converters based on the modulation of the inductance used in the converter output filter. The goal of this work is the evaluation of the possibilities of this control method, which could be used in some niche applications on its own or by combination with duty cycle and/or frequency control. Some advantages of the proposed magnetic control can be identified as follows: i) constant frequency operation, ii) constant duty cycle operation, iii) implementation of multi-output converters with a reduced number of power switches, iv) additional control parameter for different control strategies, and v) simple dynamics, easy to stabilize in closed loop.

In order to introduce the proposed control method, which will be referred to as magnetic control (MC) of DC-DC converters, Fig. 1 illustrates a buck converter and its operating waveforms when it is working under continuous conduction mode (CCM) and discontinuous conduction mode (DCM).

Fig. 1a shows the current waveform through the inductor when the converter operates in CCM for two different values of the inductance L_1 and L_2 higher than L_1 . As can be seen, when operating in CCM any change of the inductance will produce just a change on the inductor current ripple, while the average output current and voltage will keep the same. Therefore, it is not possible to use the inductance as control parameter for the output voltage and current when the converter operates in CCM. Nevertheless, in CCM the change in the inductance will produce a change on the dynamic response of the converter. The lower the inductance, the quicker the dynamic response of the converter. Thus, in CCM MC can be used to act upon the dynamic response of the converter to be adapted to the load or input voltage changes. This idea has previously been investigated in [12].

Fig. 1. Buck converter and operating waveforms under MC.

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On the other hand, Fig. 1b shows the inductor current waveforms when the converter is operating in DCM. In this case, the inductor has a direct effect on the average output current and voltage. As the value of the inductance increases, the average output current will decrease and so will do the average output voltage.

It is also possible to observe in Fig. 1b that as long as the inductance is increasing, the converter operation is approaching to CCM operation. Therefore, for a given switch cycle and switching frequency, there will be a maximum value of the inductance beyond which the buck converter will enter in CCM mode, and the output voltage and current will be fixed to that of the CCM characteristic, which as it is well known will depend exclusively on the switch duty cycle in the ideal case.

In the following, Section II presents the MC of DC-DC converters operating in DCM. Section III focuses on the study of the buck converter with MC, which has been selected as a case study to investigate the proposed control method. Section IV presents the implementation of a laboratory prototype. Section V shows the experimental verification. Finally, Section VI presents the conclusions that can be extracted from this work.

II. MAGNETIC CONTROL OF DC-DC CONVERTERS UNDER DCM OPERATION

A. DC-DC DCM Converters with Magnetic Control

Fig. 1a and Fig. 2 show the schematic diagrams of the five basic DC-DC converters operating under MC, namely buck, boost, buck-boost, flyback and forward converters.

DC-DC converters have extensively been studied in CCM and DCM operation. From the studies presented in [1], it is possible to obtain the output characteristics of the different converters operating in DCM under MC. For this purpose, the normalized inductance L_n of the converter is defined as follows:

$$L_n = \frac{L}{R/2f} \quad (1)$$

where L is the inductance of the converter, and in the case of the flyback converter is the inductance seen from the primary side of the coupled inductors, R represents the load resistance and f is the converter switching frequency. Thus, Table I provides the output characteristic of the five converters and the condition of DCM operation from the point of view of the value of the controllable normalized inductance. In the equations of Table I, D represents the switch duty cycle, and $n = N_s/N_p$ is the turn ratio of the flyback coupled inductors or the forward transformer, N_s and N_p being the turn numbers of secondary and primary windings respectively.

As can be seen from Table I, the voltage gain of the converters in DCM is a direct function of the inductance. This makes it possible to control the converter output voltage or current by means of inductance variation, i.e. by MC.

In order to provide a clearer insight on the static behavior of the different converters under MC, the output characteristics of Table I have been plotted in Fig. 3 and Fig. 4. Fig. 3 shows the output characteristic of the buck, boost and buck-boost converter. Fig. 4 illustrates the output characteristics of the flyback converter for a turn ratio $n = 0.5$ and of the forward converter for a turn ratio $n = 2$. As can be seen, in all

characteristics there is a range corresponding to the DCM operation of the converter within which it is possible to control the output by changing the converter inductance. On the other hand, in the range corresponding to CCM operation, the voltage gain will remain constant; the change of the inductance has no effect on the average value of the output, affecting only to the converter filter cut-off frequency and thus to the current and voltage ripple. For the flyback and forward converters the effect of the turn ratio n is to increase the DCM range of operation for the same voltage gain range, which is good because it is a manner of extending the range of output regulation of the converter.

Fig. 2. DC-DC converters with MC: (a) boost, (b) buck-boost, (c) flyback and (d) forward.

TABLE I. CHARACTERISTICS OF DCM DC-DC CONVERTERS WITH MC.

Converter	Voltage Gain in DCM $M = V_o/V_i$	Voltage Gain in CCM $M = V_o/V_i$	DCM Condition
Buck	$\frac{2}{1 + \sqrt{1 + \frac{4}{D^2} L_n}}$	D	$L_n \leq 1 - D$
Boost	$\frac{1}{\frac{1}{2} + \frac{1}{2} \sqrt{1 + 4D^2 \frac{1}{L_n}}}$	$\frac{1}{1 - D}$	$L_n \leq D(1 - D)^2$
Buck-boost	$D \sqrt{\frac{1}{L_n}}$	$\frac{D}{1 - D}$	$L_n \leq (1 - D)^2$
Flyback $n = N_s/N_p$	$D \sqrt{\frac{1}{L_n}}$	$\frac{nD}{1 - D}$	$L_n \leq \frac{1}{n^2} (1 - D)^2$
Forward $n = N_s/N_p$	$\frac{2n}{1 + \sqrt{1 + \frac{4}{D^2} L_n}}$	nD	$L_n \leq (1 - D)$
$L_n = \frac{L}{R/2f}$

Fig. 3. Voltage ratio characteristics of the three basic non-isolated DC-DC converters with MC: (a) buck converter, (b) boost converter, and (c) buck-boost converter.

B. Implementation of Variable Inductors

Variable inductors (VI) have been studied in previous literature [24]-[27]. These devices are capable of providing an inductance that can be controlled by a DC current injected in one or more auxiliary windings. The DC current generates a biasing magnetic flux that changes the DC operating point of the AC magnetic flux inside the core. This way, by moving the DC operating point along the B-H curve of the material close to the saturation knee, the permeability is changed, thus providing a variation of the effective inductance of the AC winding. Fig. 5 shows the scheme of two common VI; the double-E VI and the quad-U VI. These structures have previously been studied in [24]-[27], where it can be found more information on how to analyze, design and simulate these VI structures.

Fig. 4. Voltage ratio characteristics of the two basic isolated DC-DC converters with MC: (a) flyback converter, and (b) forward converter.

Fig. 5. Schematic of two VI structures: (a) double-E VI, (b) quad-U VI.

III. CASE STUDY: BUCK CONVERTER WITH MAGNETIC CONTROL

In order to evaluate the possibilities of applying MC to DCM-operated DC-DC converters, a case study for the buck converter will be carried out in this section.

A. Static Analysis of the Buck Converter with Magnetic Control

The static characteristics of the buck converter operating in DCM with MC can be obtained from the normalized characteristics shown in Table I. By substituting the normalized inductance given by (1), the following expression can be obtained for the converter voltage gain M :

$$M = \frac{V_o}{V_i} = \frac{2}{1 + \sqrt{1 + \frac{8fL}{D^2R}}} \quad \text{if } L \leq (1-D) \frac{R}{2f} \quad (2)$$

$$M = \frac{V_o}{V_i} = D \quad \text{if } L \geq (1-D) \frac{R}{2f}$$

As an example, Fig. 6 shows the voltage gain characteristics of the buck converter for a particular case with duty cycle $D = 0.4$, switching frequency $f = 100 \text{ kHz}$ and inductance up to $25 \mu\text{H}$, which have been obtained by plotting (2). As can be seen, for a given load resistance, DCM operation is achieved for the lower values of the converter inductance. When a critical inductance value is reached, the converter enters in CCM operation and the voltage gain becomes constant and equal to the converter duty cycle, D . Voltage regulation can only be achieved in DCM operation, while in CCM mode any change of the inductance will affect only to the current and voltage ripple and to the converters dynamics but not to the average output voltage.

Fig. 6. Voltage gain of a buck converter as a function of the converter inductance for different load resistances.

B. Proposed Control Scheme

The proposed control scheme of the buck converter operating in DCM mode with MC is illustrated in Fig. 7. As can be seen, the circuit is quite simple. The output voltage is measured by a voltage divider and fed into a lag-lead compensator, which drives the bipolar transistor Q in order to generate the required bias current i_b for the variable inductor. For a proper design of the compensator parameters, the dynamic model of the buck converter with MC needs to be obtained. This analysis will be presented in the next section.

Fig. 7. Proposed control scheme of the DCM operated buck converter with MC.

C. Dynamic Model of the Buck Converter with Magnetic Control

In this section, the basic dynamic analysis of the buck converter with MC will be presented. With reference to the control scheme previously presented in Fig. 7, the objective of the analysis is to obtain the transfer function $G_{ol}(s)$ that relates the inductance perturbation $l(s)$ with the output voltage perturbation $v_o(s)$, this is:

$$G_{ol}(s) = \frac{v_o(s)}{l(s)} \quad (3)$$

Fig. 8 shows the equivalent circuits of the buck converter operating in DCM in the averaged time domain, Fig. 8a, and in the Laplace variable domain, Fig. 8b. The effect of the series resistance of the filter capacitor have been neglected for simplicity assuming that a film capacitor with very low series resistance will be used; nevertheless, it could easily be taken into account. The inductor average current of the buck converter in DCM mode can be calculated as follows:

$$\langle i_L \rangle = \frac{D^2 V_i}{2f} \left(\frac{V_i}{V_o} - 1 \right) \frac{1}{L} \quad (4)$$

By taking partial derivative with respect to L and using the Laplace transform, the following relationship is obtained between the inductor current and inductor perturbations:

$$i_L(s) = k_{il} l(s) \quad (5)$$

where

$$k_{il} = -\frac{D^2 V_i}{2f} \left(\frac{V_i}{V_o} - 1 \right) \frac{1}{L^2} \quad (6)$$

Now, solving the circuit shown in Fig. 8b for the output voltage, the buck converter dynamics between the output voltage and inductor can be obtained, as follows:

$$G_{ol}(s) = \frac{v_o(s)}{l(s)} = k_{ol} \frac{1}{1 + s/\omega_p} \quad (7)$$

where

$$k_{ol} = k_{il} R \quad (8)$$

$$\omega_p = \frac{1}{RC} \quad (9)$$

As stated in [20], it is a good approximation to neglect the dynamics between VI bias flux perturbations $B_b(s)$ and VI

inductance perturbations $l(s)$ because this dynamics will be of a higher order of magnitude in comparison with the other dynamics of the VI. Therefore, the following expression can be written:

$$l(s) \approx k_{lb} B_b(s) \quad (10)$$

where k_{lb} is the proportionality constant that relates inductance and bias flux perturbations. The procedure to derive this constant is presented in Appendix I.

Also, the bias flux and bias current $i_b(s)$ perturbation can be related by the following expression:

$$B_b(s) = \frac{L_{b,eff}}{2N_b A_b} i_b(s) \quad (11)$$

Fig. 8. Equivalent circuits of the buck converter with MC: (a) averaged time domain, and (b) Laplace variable domain.

where $L_{b,eff}$, N_b and A_b are the effective inductance [20], turn number and effective section of the auxiliary bias winding, respectively.

Note that in (11) the factor 2 before N_b is necessary because the total bias inductance is given by two bias windings, each one with a number of turns equal to N_b .

Using (11) in (10), a direct relationship between inductance and bias current perturbations can be obtained:

$$l(s) \approx k_{li} i_b(s) \quad (12)$$

where:

$$k_{li} = \frac{k_{lb} L_{b,eff}}{2N_b A_b} \quad (13)$$

It must be noted that the constant k_{li} corresponds to the slope of the characteristic of inductance versus bias current of the VI at the given operating point, which can be obtained theoretically following the process shown in [24][27].

With regard to the dynamics between the compensator output signal v_c and the bias winding current i_b , it has already been studied in [20]. It can be expressed as follows:

$$G_{bc}(s) = \frac{i_b(s)}{v_c(s)} = \frac{k_{bc}}{1 + s/\omega_c} \quad (14)$$

where

$$k_{bc} = \frac{h_{fe}}{R_c + h_{ie}} \cdot \frac{1}{1 + h_{oe} R_b} \quad (15)$$

$$\omega_c = \frac{h_{oe}^{-1} + R_b}{L_{b,eff}} \quad (16)$$

where h_{ie} , h_{fe} and h_{oe} are the h parameters of the small-signal model of the bipolar transistor used in the bias winding, namely input impedance, forward current gain and output admittance respectively. These parameters can be obtained from the transistor datasheet if available. Other option is to extract the parameters from its SPICE model by computer simulation. The latter has been the option used in this work.

Fig. 9 shows the block diagram of the derived dynamic model of the buck converter operating with MC. The following transfer function can be defined to represent the dynamic behavior from the control voltage v_c to the output voltage v_o :

$$G_o(s) = \frac{v_o(s)}{v_c(s)} = k_{li} G_{bc}(s) G_{ol}(s) \quad (17)$$

which corresponds to a transfer function with two poles given by ω_p and ω_c in (9) and (16) respectively.

Fig. 9. Block diagram of the dynamic model of the buck converter with MC.

The gain of the feedback network is given by R_1 and R_2 in Fig. 7, as follows:

$$k_s = \frac{R_2}{R_1 + R_2} \quad (18)$$

IV. IMPLEMENTED PROTOTYPE

As an example of implementation, a buck converter operating at a switching frequency of 100 kHz and with a duty cycle of 0.4 (on time, $t_{on} = 4 \mu s$) will be presented and studied in this section. According to the characteristics of Fig. 6, a VI ranging from $5 \mu H$ to $20 \mu H$ has been implemented with the parameters shown in Table II. Fig. 10 shows the characteristic of inductance versus bias current of the implemented VI.

The buck converter has been supplied with 10 V input voltage and nominal load resistance of 5Ω . The output filter capacitance is $33 \mu F$, implemented with a film capacitor so that its series resistance can be neglected. Table III summarizes the parameters of the implemented prototype. The h parameters of the BD139 bipolar transistor used in the bias circuit have been obtained from the SPICE model of the transistor by simulation under LTspice. Appendix II shows the two LTspice circuits that were used for this purpose.

Based on the analysis presented in Section II.C, the dynamic response of the converter with MC has been obtained. Fig. 11 shows this response, corresponding to the bode diagram of amplitude and phase of output voltage $v_o(s)$ over control voltage $v_c(s)$, which has been designated as $G_o(s)$ in Section II.C. Appendix III shows a WinPython 3.7.7 script that can be used to generate this response and to calculate the other transfer functions and poles of the system. From this theoretical analysis, the pole corresponding to the buck output filter as given by (9) is obtained at 964 Hz, while the pole due to the VI response, given by (16), appears at 35.2 kHz. The DC gain of the system is 8.96 dB.

TABLE II. PARAMETERS OF THE IMPLEMENTED VI.

Parameter	Type/Value
Structure	Double E
Core	EFD25/13/9, N87
Main winding	$N=12$, 66×0.08 mm (litz)
Central arm cross-sectional area	$A_c = 59.3$ mm ²
Central arm length	$l_c = 24.4$ mm
Middle arm gap	$l_{gc} = 0.6$ mm
Lateral arms gap	$l_{gl} = 0$ mm
Bias windings	$2 \times N_b = 65$, 1×0.2 mm
Outer arms cross-sectional area	$A_e = 28.7$ mm ²
Outer arms length	$l_e = 43.6$ mm
Main inductance range	$5 \mu\text{H} - 20 \mu\text{H}$
Bias winding effective series inductance and resistance	1.2 mH / 6.2Ω @ 100 Hz

Fig. 10. Measured characteristic of inductance versus bias current of the implemented VI.

TABLE III. PARAMETERS OF THE IMPLEMENTED BUCK CONVERTER WITH MC.

Parameter	Type/Value
Input voltage	10 V
Output voltage	5 V
Duty cycle	0.4
Switching frequency	100 kHz
Inductor	$5 \mu\text{H} - 20 \mu\text{H}$
Capacitor	$33 \mu\text{F}$
Nominal load resistance	5Ω
Power switch	IRFZ48
Diode	11DQ10
Bias winding transistor model parameters	BD139 / $h_{fe} = 43.8$, $h_{ie} = 28.2 \Omega$, $h_{oe} = 0.0079 \Omega^{-1}$
Bias winding transistor base resistance	$R_c = 100 \Omega$
Voltage sensing resistances	$R_1, R_2 = 10 \text{ k}\Omega$

Fig. 11. Theoretical bode diagram of the converter in open loop, corresponding to the transfer function $G_o(s)$ in the block diagram of Fig. 9.

V. EXPERIMENTAL VERIFICATION

Firstly, the characteristic of output voltage v_o versus control voltage v_c has been measured at the laboratory operating with duty cycle of 0.4 and load resistance of 5Ω . Fig. 12 shows the obtained result. The squares show the experimental measurement, while the dashed line shows an approximation obtained by exponential regression. As can be seen, the output voltage can be controlled within a reasonably wide range by means of the control voltage v_c , which changes the effective value of the converter inductance.

As previously explained, the slope of the output control curve shown in Fig. 12 can be used to obtain the total DC gain of the converter control function $G_o(s)$. For example, around the nominal operating point of 5 V at the output, the slope can be approximated to 3.61 (11.1 dB).

Fig. 13 shows the buck converter filter input voltage and VI current in two different situations of output voltage and current. Fig. 13a corresponds to an output voltage and current of 5 V and 1 A respectively. As can be seen, the converter is operating at the DCM-CCM mode. Fig. 13b shows the waveforms for an output voltage and current of 5.5 V and 1.1 A. As can be seen, in this case the converter is clearly operating in DCM mode. This is because to increase the output voltage, the VI inductance must be decreased, what according to the characteristics in Fig. 3a and Fig. 6, forces the operation to go further into DCM mode.

Fig. 12. Characteristic of output voltage versus control voltage of the buck converter with MC. Squares: experimental measurement; dashed line: approximation by exponential curve fitting.

Fig. 13. Buck filter input voltage and VI current in two different situations: (a) $V_o = 5V, I_o = 1 A$; (b) $V_o = 5.5 V, I_o = 1.1 A$. Scales: 10 V/div, 1 A/div, 5 μ s/div.

The second test performed on the prototype was to measure the system dynamics by acquiring the bode diagram of the system transfer function $G_o(s)$. An Omicron Bode 100 has been used for this operation. Fig. 14 shows the obtained results. As can be seen, the response matches with quite good accuracy the theoretical response shown previously in Fig. 11, which validates the developed dynamic model.

In order to test the system regulation in closed loop, a lag-lead compensator as the one illustrated in Fig. 7 has been designed. Table IV shows the parameters of the implemented compensator.

Fig. 14. Experimental response of the control transfer function $G_o(s)$ corresponding to the implemented buck converter with MC.

TABLE IV. PARAMETERS OF THE IMPLEMENTED LAG-LEAD COMPENSATOR.

Parameter	Type/Value
High frequency gain	0 dB
Zero's frequency	1 kHz
Operational amplifier	LM358
Resistances R_3, R_4	16 k Ω
Capacitance C_1	10 nF

Fig. 15 shows the experimental loop gain magnitude and phase of the buck converter with MC when using the compensator shown in Table IV. The measurement was taken using the voltage injection method, with an injection transformer B-WIT 100 from Omicron. As can be seen, the loop gain response is in accordance with the theoretical analysis. The system is well stabilized with a phase margin of around 110 degrees and a gain margin of around 40 dB.

Fig. 15. Experimental loop gain and phase of the buck converter operating with a lag-lead compensator and MC.

Fig. 16 shows experimental responses of the buck converter operating in closed loop with the proposed MC. Fig. 16a shows the output voltage response for a load step between 5 Ω and 10 Ω . The output voltage response is in accordance with the expected one since it corresponds to a first order response with a settling time around 500 μ s. Fig. 16b shows the output voltage response for an input voltage change between 8 V and 11 V. A first order response with a settling time around 500 μ s is also obtained, which is in accordance with the system closed-loop design.

Table V and VI shows the experimental measurements corresponding to the load regulation (Table V) and line regulation (Table VI). The values of the control signal v_c generated by the compensator and of the dc current through the bias winding i_b are also provided to show the operation of the closed-loop regulation. As can be seen in both tables, even though the control range is not very wide for this particular

design, the output voltage is well regulated for both load and line changes, thus proving the feasibility of the proposed control methodology.

VI. CONCLUSIONS

The magnetic control of DC-DC converters operating in DCM has been investigated in this paper. The important control characteristics of the most relevant converters operating with MC have been presented, demonstrating that it is possible to use the converter filter inductance as control parameter to regulate output voltage or current.

The buck converter has been selected as case study to evaluate the possibilities of MC of DC-DC converters. The dynamic model of the MC DCM buck converter proves to be similar to that resulting from other control methods, as voltage-mode, current-programmed or frequency-sweep mode control. Since the converter operates in DCM, it keeps its first order behavior, exhibiting a main pole given by the filter capacitance and load resistance. The MC introduces a second pole defined by the effective inductance of the VI bias winding and by the addition of the output resistance of the current source used to drive the bias winding and the series resistance of the bias winding itself.

The obtained experimental results have proven the theoretical model of the MC buck converter. It has also been demonstrated how the converter can be closed-loop controlled with a simple lag-lead controller that can be designed based on conventional design methods. The experimental measurement of the converter loop gain and the load and input-voltage step responses are in full agreement with the closed-loop design of the converter.

As an additional conclusion, MC DC-DC converters can be used in other applications, such as in LED drivers, electric vehicle charging, DC motor control, PV converters, smart-grid converters, PFC converters, wireless power transfer, etc.

Fig. 16. Experimental closed-loop response of the magnetically controlled buck converter with a lag-lead compensator: (a) Output voltage transient for a load step change between 5 Ω and 10 Ω , (b) Output voltage transient for an input voltage step between 8 V and 11 V. CH1, CH2 and time scales are as labelled in the figures.

TABLE V. EXPERIMENTAL RESULTS: LOAD REGULATION FOR 10 V INPUT VOLTAGE

Output Current (A), I_o	Output Voltage (V), V_o	Control Voltage (V), v_c	VI Bias Current (A), i_b
1.15	5.01	1.10	0.38
0.98	5.01	0.93	0.24
0.83	5.02	0.87	0.17
0.71	5.02	0.82	0.13
0.62	5.02	0.79	0.09
0.52	5.03	0.74	0.06
0.48	5.03	0.73	0.04

TABLE VI. EXPERIMENTAL RESULTS: LINE REGULATION FOR 5 Ω LOAD RESISTANCE.

Input Voltage (V), V_i	Output Voltage (V), V_o	Output Current (A), I_o	Control Voltage (V), v_c	VI Bias Current (A), i_b
8.5	4.83	1.00	2.86	0.85
9.0	4.99	1.03	1.27	0.47
9.5	5.00	1.04	1.04	0.35
10.0	5.00	1.04	0.93	0.25
10.5	5.01	1.04	0.90	0.21
11.0	5.02	1.04	0.80	0.09

APPENDIX I

DERIVATION OF CONSTANT k_{lb} RELATING VI INDUCTANCE AND BIAS INDUCTION PERTURBATIONS

In this appendix the derivation of the constant k_{lb} that relates the VI inductance and the bias induction perturbation will be presented for the case of the double E variable inductor.

As studied in [24], the main winding flux of the double-E variable inductor can be expressed as a function of the small-signal reluctances as follows:

$$\hat{\phi}(B_b) = \frac{N\hat{i}}{r_t(B_b)} \quad (I.1)$$

where B_b is the DC bias induction in the magnetic material, $\hat{i}$ is the small-signal current through the main winding, and r_t is the total small-signal reluctance seen by the main flux, which can be expressed as:

$$r_t(B_b) = r_c + r_{gc} + \frac{1}{2} r_e(B_b) + \frac{1}{2} r_{ge} \quad (I.2)$$

r_c being the central arm reluctance, r_{gc} the central air gap reluctance, r_e the outer arms reluctance, and r_{ge} the outer arms air gap reluctance. These reluctances are defined as follows:

$$r_c = \frac{l_c}{\mu_d(0)A_c} \quad (I.3)$$

$$r_{gc} = \frac{l_{gc}}{\mu_0 A_c} \quad (I.4)$$

$$r_e(B_b) = \frac{l_e}{\mu_d(B_b)A_e} \quad (I.5)$$

$$r_{ge} = \frac{l_{ge}}{\mu_0 A_e} \quad (I.6)$$

where l_c, A_c and l_e, A_e are the lengths and cross-sectional areas of the central and outer arms respectively, l_{gc} and l_{ge} are the central arm and outer arms air gap lengths respectively, μ_0 is the permeability of free space, and μ_d is the differential permeability of the magnetic material, which can be expressed as a function of the bias induction using the Brauer's fitting [24], as follows

$$\mu_d(B_b) = \frac{1}{k_1(1 + 2k_2B_b^2)e^{k_2B_b^2} + k_3} \quad (I.7)$$

where k_1, k_2, k_3 are constants of the magnetic material.

Now, the main winding inductance can be derived from (I.1) as

$$L(B_b) = \frac{N\hat{\phi}(B_b)}{\hat{i}} = \frac{N^2}{r_t(B_b)} \quad (I.8)$$

Taking derivatives on (I.8):

$$dL(B_b) = -\frac{N^2}{r_t^2(B_b)} \frac{dr_t(B_b)}{dB_b} dB_b \quad (I.9)$$

Applying the Laplace transform on (I.9), the relationship between inductance and bias induction perturbation is obtained as desired:

$$l(s) = k_{lb}B_b(s) \quad (I.10)$$

where the constant k_{lb} is defined as follows:

$$k_{lb} = -\frac{N^2}{r_t^2(B_b)} \frac{dr_t(B_b)}{dB_b} \quad (I.11)$$

In order to obtain an expression for k_{lb} the derivative of the total reluctance is first obtained:

$$\frac{dr_t(B_b)}{dB_b} = \frac{1}{2} \frac{dr_e(B_b)}{dB_b} = -\frac{l_e}{2A_e\mu_d^2(B_b)} \frac{d\mu_d(B_b)}{dB_b} \quad (I.12)$$

By deriving (I.7), substituting in (I.12), and then in (I.11), the constant k_{lb} can be obtained as a function of the bias induction at a given operating point:

$$k_{lb} = -\frac{N^2}{r_t^2(B_b)} \frac{l_e}{A_e} k_1 k_2 B_b [3 + 2k_2 B_b^2] e^{k_2 B_b^2} \quad (I.13)$$

In order to evaluate k_{lb} at a given operating point it is necessary to obtain the value of the bias induction B_b for a given dc bias current I_b . As analyzed in [24], the dc bias flux in the double-E structure is given by the following expression:

$$\phi_b = \frac{N_b I_b}{\mathcal{R}_e + \mathcal{R}_{ge}} \quad (I.14)$$

$\mathcal{R}_e$ and $\mathcal{R}_{ge}$ being the great-signal reluctances of the outer arms and outer arm air gap respectively:

$$\mathcal{R}_e = \frac{l_e}{\mu_t(B_b) A_e} \quad (I.15)$$

$$\mathcal{R}_{ge} = \frac{l_{ge}}{\mu_0 A_e} \quad (I.16)$$

where $\mu_t(B_b)$ is the total permeability of the magnetic material, which using the Brauer's fitting can be expressed as:

$$\mu_t(B) = \frac{1}{k_1 e^{k_2 B^2} + k_3} \quad (I.17)$$

Using (I.14)-(I.16), the following equation relating B_b and I_b is obtained:

$$B_b - \frac{N_b I_b}{\frac{l_e}{\mu_t(B_b)} + \frac{l_{ge}}{\mu_0}} = 0 \quad (I.18)$$

By solving (I.18) the value of the bias induction B_b can be obtained for a given value of the bias winding current I_b . The obtained value of B_b can then be used to calculate k_{lb} using (I.13) for a determined operating point.

APPENDIX II. DERIVATION OF THE H PARAMETERS OF TRANSISTOR BD139 FROM SPICE MODEL

Fig. A.1. LTspice circuit used for obtaining h_{ie} and h_{fe} parameters of transistor BD139.

Fig. A.2. LTspice circuit used for obtaining h_{oe} parameter of transistor BD139.

APPENDIX III WINPYTHON 3.7.7 SCRIPT FOR OBTAINING THE DYNAMIC MODEL OF THE LABORATORY PROTOTYPE

```

import numpy as np
from math import pi, exp
from scipy.optimize import fsolve
from control import tf, bode_plot
s = tf('s')
# Constants
u0=4*pi*1e-7
# B-H curve parameters, N87 material
k1= 0.062; k2= 42.995; k3= 302.904
# Input data
Vi=10; Vo=5; D=0.4; f=100e3
L= 12.5e-6; Lb_eff= 1.2e-3; Ib= 0.14
R= 5; C= 33e-6
N= 12; Nb= 65
hfe= 43.8; hoe= 0.0079; hie= 28.2;
Rc= 120; Rb= 6.2
lc= 24.4e-3; Ac= 59.3e-6; le= 43.6e-3; Ae= 28.7e-6
lge= 0.6e-3; lge= 0
# Total permeability function
def ut(B): return 1/(k1*exp(k2*B**2)+k3)
# Differential permeability function
def ud(B): return 1/( k1*(1+2*k2*B**2)*exp(k2*B**2) + k3 )
# Small-signal reluctances
rc= lc/(ud(0)*Ac); rgc=lge/(u0*Ac); rge=lge/(u0*Ae)
def re(B): return lc/(ud(B)*Ac)
def rt(B): return rc + rgc + re(B)/2 + rge/2
# DC flux density function
def fB(B): return B - Nb*Ib/(le/ut(B) + lge/u0 )
# Solving for dc flux density
Bb = fsolve(fB, 0); print("DC bias induction= ", Bb)
# Output voltage over inductance TF: G=vo/vi
kil= -D**2*Vi*(Vi/Vo-1)/(2*f*L**2)
kol= kil*R; wp= 1/(R*C)
Gol= kol/(1 + s/wp)
# Inductance over bias current TF: kli=1/ib
klb= -(N**2/rt(Bb)**2)*(le/Ae)*k1*k2*Bb*\
(3+2*k2*Bb**2)*exp(k2*Bb**2)

```

```

kli= klb*Lb_eff/(2*Nb*Ae)
# Bias current over control voltage TF: Gbc=ib/vc
kbc= hfe/(Rc+hie)/(1+hoe*Rb)
wc= (1/hoe + Rb)/Lb_eff
Gbc= kbc/(1 + s/wc)
# Output voltage over control voltage TF: Go= Vo/Vc
Go= kli*Gbc*Gol
# Printing results and plotting Go Bode diagram
print("Constant k_lb= ", klb)
print("Constant k_li= ", kli)
print("DCM buck pole (Hz): fp= ", wp/2/pi)
print("VI control pole (Hz): fc= ", wc/2/pi)
print("Transfer functions: ", Gbc, Gol, Go)
bode_plot(Go, dB=True, Hz=True, omega_limits=(10, 1e5), \
          omega_num=500)

```

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