Likelihood Partitioned Bayesian Filtering for Lithium Sulfur Battery State Estimation

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Abstract—This paper introduces a novel Bayesian filtering design framework for estimating the state-of-charge (SoC) and state-of-energy (SoE) of Lithium-sulfur (Li-S) batteries suitable for direct deployment in real-time battery management systems (BMSs). In Li-S batteries, the open-circuit voltage has two regions: a steeply-descending ‘high-plateau’ and a flat ‘low-plateau’. There is a sharp transition between the two. Because of this, battery parameter are segmented into two regions; high- and low-plateau parameters. In contrast with traditionally Bayesian approaches, each partitioned parameter set is given its own probability density functions (PDFs) for a *priori* state and likelihood and propagates any one of them at a time. The proposed strategy has been implemented with a simple nonlinear Bayesian filter, namely the extended Kalman filter (EKF). The performance of both SoC and SoE estimators is demonstrated with experimental test data for a Li-S battery cell designed for aerospace application with a pulse discharge duty cycle. The cell’s experiment was considered under two different temperature conditions. The proposed likelihood partitioned EKF was validated under two different initial estimated state conditions. The proposed estimators gave good accuracy, achieving 97 percent.

Index Terms—battery management systems, Lithium sulfur battery, extended Kalman filter, nonlinear equations, nonlinear filters, parameterization, state estimation, state-of-charge, state-of-energy.

I. INTRODUCTION

IN recent years, research in the area of lithium-sulfur (Li-S) battery technology has accelerated the advances in chemistry [1-3], modeling [4] and battery management system [5-9]. The studies on the chemistry behaviors show that Li-S batteries are environmentally friendly, safe, and low cost as they use toxin-free

and abundant sulfur cathode. In addition to these, the high energy density and cycle life of Li-S batteries are evidence that the Li-S batteries may be an alternative choice to lithium-ion (Li-ion) batteries for aerospace and automotive applications. Battery management systems (BMS) is an essential component of the Li-S battery powered propulsion system.

In BMS of Li-S batteries, the estimation of Li-S battery’s performance metrics accurately is a challenging and complex problem as the electrical characteristics of Li-S batteries are highly nonlinear and dynamic. Particularly, the discharge open-circuit voltage of Li-S is a highly nonlinear function, which has a deep-dive high-plateau, range from 2.6 to 2.1 V and in flat low-plateau which has poor observability [7], range from is 2.0 to 1.4 V. In addition to the slope of high plateau of voltage changes, the joining point and equivalent circuit network (ECN) model parameters are sensitive to the temperature [8,14]. Propp et.al [6] used Bayesian filtering techniques, including extended Kalman filter (EKF), unscented Kalman filter (UKF) and particle filter (PF) for state-of-charge (SoC) estimation. In [9] the dual EKF proposed for based SoC, the state of available Power (SoAP) and state of health (SoH). On other hand, Abbas et.al [7] presented Li-S battery SoC estimation using adaptive neuro-fuzzy inference system (ANFIS) with PEM based ECN parameterization, and [4] presented support vector machine (SVM) based SoC estimation of Li-S batteries with weighted least-square ECN parameterization.

Results of the above literature were promising However, further research on Li-S battery management (BMS) needs to overcome the limitations. For example, training and mapping with the Coulomb counting approach in ANFIS and SVM, lead to a significant computational issue. The accuracy of Li-S battery’s SoC estimation by both ANFIS [7] and SVM [4] improved when their response was merged to the Coulomb counting SoC, which is impractical to evaluate accurately when the initial SoC is unknown. Compared to machine learning algorithms, the

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nonlinear filters are very simple two-step recursive processes, estimating the state based on the measurement and easy to implement for real-world applications. Nevertheless, the conventional Bayesian approaches [13] used a partial sinusoidal function (PSF) for smooth transition of high to low-plateau of Li-S battery parameters. To eliminate PSF, in conference paper [11] a modified filtering strategy, with likelihood-partition, was proposed for SoC and SoE estimation of Li-S battery. Hereafter the likelihood-partitioned filtering approach is referred as a likelihood-partitioned Bayesian filter (LPBF).

This paper is an extended version of [11]. Contrast to conference, this article briefs the concept and theory of the LPBF for SoC and SoE estimation of Li-S battery cell. This paper also addresses the two additional issues such as surface temperature variation and initial state variation as [11] studied only SoC and SoE estimation of this same Li-S cell at 20°C temperature under a single initial estimated state condition. The considered temperature conditions are 20°C and 30°C. In addition, this version investigates reliability of LPBF by assuming two different initial estimated state condition. The presented results of SoC and SoE estimation are much more accurate than results of [11] as the design parameters are changed appropriately. Having experimental pulse drive cycle test data set, the SoC and SoE estimation of Li-S by proposed LPBF is validated. Results show that the proposed method outperforms and accurately estimates SoC and SoE of Li-S battery cell, without large training and additional computation burden.

II. BATTERY EXPERIMENT AND MODELLING

This section presents experimental test data (a link for data access is give in the acknowledgement) and explains how such data can be modelled and used to estimate the SoC and SoE of Li-S batteries.

A. Experimental Battery Sensor Data

The battery experiment test rig-measurement system provides the current, voltage and surface temperature. Fig 1 shows a high-level block diagram for test rig, sensor data flow and their integration to the filter based SoC and SoE estimators.

Fig. 1. Block diagram of experimental battery rig with input current and output voltage that used by proposed Li-S battery state estimation.

This test rig integrates source and load, where computer control system controls and emulates the real-world operating conditions of battery cell. A small unmanned air vehicles (UAV) application designated type of Li-S battery cell is used for experiment. Table 1 shows the important specifications the cell. As shown in the first

column of Table 1, the capacity of Type A cell is 1.9Ah. From the total energy and mass density given in the table I, it is an evident that the energy density is about 120.25Wh/Kg.

TABLE I
SPECIFICATION OF LI-S BATTERY CELLS USED FOR EXPERIMENT

Specification with unit	specifications
Capacity (Ah)	1.9
Total energy (Wh)	5
Maximum voltage (V)	2.6
Minimum voltage (V)	1.4
Nominal voltage (V)	2.1
Maximum discharge current (A)	6
Maximum charge current (A)	1
Cell mass (kg)	0.0158

Fig. 2. Test data of type A cell; (a) Battery terminal voltage, (b) load current (negative magnitude indicates discharge current).

Fig. 3. Change in response of Li-S battery cell to short-period current pulses as temperature changes

Fig. 4. Voltage response of Li-S battery cell to different C-rate

Fig 2 shows test data of Li-S cell discharge cycles test at two different temperature. For instant, Fig 2a shows the measured battery voltage at temperature 20°C, and 30°C for current pulses

of mixed C-rate. In Fig 2a, the black solid line denotes the voltage response at 20°C whereas the red-dotted line shows the response at 30°C. We have conducted experiment for 50°C, voltage response of Li-S cell shown by blue-dotted line of fig 3., but it is not discussed and used in this paper for the sake of simplicity. Fig 2b shows the mixed C-rate current pulses that applied to the Li-S cell. It is noted that duration of pulses is fixed and short. This may be best fit to drive cycle of UAV application [10]. Test results show that the voltage of Li-S battery is nonlinear in nature, that is, deep-dive high-plateau and flat low-plateau. Furthermore, the voltage characteristic of Li-S cell varies as the temperature varies, for instance, the voltage at 30°C is slightly lower than the voltage at 20°C as shown in Fig 3. For study the dynamic characteristic such polarization voltage, we have conducted experiment with C/15 rate current pulses 200 s length and observed the voltage response, which is shown as Fig 4. Fig 4 shows that Li-S cells are highly dynamic [8] to C-rate of current level. These are challenging to model and estimate battery state. The battery state and dynamic model of Li-S cells are briefed in the next section.

B. Battery Modeling

The battery output voltage is a function of the OCV, the voltage across the internal resistance and dynamic polarization voltage. Since the measured voltage is not only function of current and temperature, it is also a function of state of battery. therefore, the ECN model for SoC estimation problem is a function of SoC as shown Fig 5. Similarly, the ECN model of Li-S cell for SoE problem is expressed as function of SoE as shown in Fig 6.

B. Battery states

This subsection describes the SoC and SoE of battery, and how each parameter of 1RC ECN models is expressed as a function of each such state. Because [7] defined the integration of current is SoC whereas [15] defined the integration of power as SoC, this study explores the significant difference, if any, between SoC and SoE, their challenges and advantages in Li-S BMS by analysing and estimating SoC in Ampere-hour (Ah) and SoE in Watt-hour (Wh) and their normalization. The estimation of SoE in Wh can be directly used for optimize the energy and predict the remaining time of the battery use [12]. It is assumed that both state and parameters have no parameter uncertainty caused by temperature change.

The SoC can be evaluated by integrating the current. After converting the integration into summation, the SoC in Ampere-hour (Ah), is expressed as

$$x_{soc}(k) = x_{soc}(k-1) - I_L(k)\Delta t + \Delta x_{soc}(k) \quad (1a)$$

or the SoC in normalization is expressed as

$$x_{soc}(k) = x_{soc}(k-1) - \frac{I_L(k)\Delta t}{C_B} + \Delta x_{soc}(k) \quad (1a)$$

Where

$$x_{soc}(k) = f_{c,1}(x_c(k-1), I_L(k)) + \Delta x_{soc}(k) \quad (2)$$

is the SoC at instant k , C_B is the total capacity, unit Ah, of the battery, $\Delta x_{soc}(k) \sim N(0, Q_x)$ is a Gaussian noise, which is an error due unknown modelling or due integral approximation, $\Delta t = T_s$ denotes the sampling time in seconds.

Like SoC, the SoE can be evaluated by integrating power as [11, 12]. Therefore, the SoE in Wh with unknown modelling error can be expressed as

$$x_{soe}(k) = x_{soe}(k-1) + V_{ocv,soe}(k)I_L(k)\Delta t + \Delta x_{soe}(k) \quad (3a)$$

where $V_{ocv,soe}(k) = V_{ocv}(x_{soe})$ is the open-circuit voltage, which is a function of SoE.

The SoE in normalization with unknown modeling error can be written as

$$x_{soe}(k) = x_{soe}(k-1) + \frac{V_{ocv,soe}(k)I_L(k)\Delta t}{E_B} + \Delta x_{soe}(k) \quad (3b)$$

This can be expressed as

$$x_{soe}(k) = f_{e,1}(x_e(k-1), V_{ocv,soe}(k), I_L(k)) + \Delta x_{soe}(k) \quad (4)$$

where $\Delta x_{soe}(k) \sim N(0, Q_x)$ is a Gaussian noise with mean zero and variance Q_x . where E_B is the energy capacity of the battery cell, this is not same as total capacity of battery cell C_B .

Polarization Voltage

The polarization voltage across RC network is given as [1-3]

$$\frac{dV_P(t)}{dt} = -\Omega(\cdot)V_P(t) + \rho(\cdot)R_{int}(\cdot)\Omega(\cdot)I_L \quad (5)$$

where

$$\Omega(\cdot) = \frac{1}{R_P(\cdot)C_P(\cdot)} \quad (6)$$

$$R_{int}(\cdot) = R_i(\cdot) + R_P(\cdot) \quad (7)$$

$$\rho(\cdot) = \frac{R_P(\cdot)}{R_{int}(\cdot)} \quad (8)$$

The parameters form vector $\theta(\cdot) = [V_{oc}(\cdot), R_{int}(\cdot), \Omega(\cdot), \rho(\cdot)]'$. These are related the actual ECN parameters that given in fig 5 of SoC and Fig 6 of SoE characterization. The dynamics characteristics of Li-S battery cell are studied by polarization voltage.

The dynamic of polarisation voltage $V_P(t)$ in the discrete-time domain can be written as

$$V_{P,c}(k) = f_{c,2}(x_{soc}, \theta(x_{soc}), V_{P,c}(k), I_L, T_s) + \Delta V_{P,c}(k) \quad (9)$$

$$V_{P,c}(k) = f_{c,2}(R_{int}(x_{soc}), \rho(x_{soc}), \omega(x_{soc}), V_{P,c}(k), I_L, T_s) \quad (10)$$

Similarly, the polarization voltage as a function of SoE is expressed as

$$V_{P,e}(k) = f_{e,2}(x_{soe}, \theta(x_{soe}), V_{P,e}(k), I_L, T_s) + \Delta V_{P,e}(k) \quad (11)$$

$$V_{P,e}(k) = f_{e,2}(R_{int}(x_{soe}), \rho(x_{soe}), \omega(x_{soe}), V_{P,e}(k), I_L, T_s) \quad (12)$$

Fig. 5. Battery first-order ECN model in terms of the SoC

Fig. 6. Battery 1-RC ECN model in terms of the SoE

Output Voltage Model

Considering the IRC network given in fig 5 as a closed circuit, the output voltage of battery can be written as a function of $V_{oc}(\cdot)$, $V_p(\cdot)$, and $V_i(\cdot)$. Since the open circuit voltage $V_{oc}(\cdot)$ is a function of the state of the battery, the terminal voltage for SoC estimation problem is expressed in terms of the SoC as

$$g_c(\cdot) = V_{T,c} = V_{ocv}(x_{soc}(k)) - V_p(k) - V_i(k) \quad (13)$$

For simplicity, the notation for L, T is dropped in (31)-(32) and hereafter. Considering voltage sensor noise, the measurement model for SoC as

$$y_c(k) = g_c(k) + v_s(k) \quad (14)$$

Similarly, the terminal voltage for SoE estimation problem is written as

$$y_e(k) = g_e(k) + v_s(k) \quad (15)$$

where

$$g_e(x_e, \theta_e) = V_{T,e} = V_{ocv}(x_{soe}(k)) - V_p(k) - V_i(k) \quad (16)$$

$v_s(k) \sim N(0, R_s)$ is a Gaussian noise. The term $v(k)$ denotes a Gaussian noise with zero mean and known covariance, $R_c(k)$.

The SoE estimation could be a redundancy to SoC because the SoE equals SoC when battery open-circuit voltage equals to its terminal voltage [24]. and SoC is 0, the battery do not have energy (SoE is 0) and when SoC is 100% the battery has maximum energy, i.e. SoE 100% [13].

III. ESTIMATION PROBLEM AND BAYESIAN FILTERING

A. The State Estimation Problem

The probabilistic state transition model for SoC estimation is

$$x_c(k) \sim p(x_c(k)|x_c(k-1), \theta(x_{c,k-1}), u_k, w_k) \quad (17)$$

where $x_c(k) \in \mathbb{R}^{n_x} = \begin{pmatrix} x_{soc,k} \\ V_{p,soc,k} \end{pmatrix} = \begin{pmatrix} x_{c,1}(k) \\ x_{c,2}(k) \end{pmatrix}$ denotes the state vector of the battery system, $p(x_c(k)|x_c(k-1))$ is the probability density function of the state sequence $\{x_c(k), k \in \mathbb{N}\}$ that represent a state transition model with nonlinear function $f_c(k): \mathbb{R}^{n_x} \times \mathbb{R}^{n_\theta} \times \mathbb{R}^{n_u} \times \mathbb{R}^{n_w} \rightarrow \mathbb{R}^{n_x}$ as

$$x_c(k) = f_c(x_c(k-1), \theta_c(k-1), u(k)) + w(k), \quad (18)$$

and $\{w(k), k \in \mathbb{N}\}$ is the independent and identically distributed (i.i.d) process noise. The probabilistic measurement model for SoC estimation is written as

$$y_{ck} \sim p(y_{ck}|x_{c,k}, \theta(x_{c,k-1}), u_{k-1}, v_k) \quad (19)$$

where $y_c(k) \in \mathbb{R}^{n_y}$ is the measurement at time instant k , and $p(y_c(k)|x_c(k))$ is the probability density function of measurement sequence $\{y_c(k), k \in \mathbb{N}\}$ that represent a measurement model with nonlinear function $g_c(k): \mathbb{R}^{n_y} \times \mathbb{R}^{n_\theta} \times \mathbb{R}^{n_u} \times \mathbb{R}^{n_v} \rightarrow \mathbb{R}^{n_y}$ as

$$y_c(k) = g_c(x_c(k), \theta_c(k), u(k)) + v(k), \quad (20)$$

and $\{v(k), k \in \mathbb{N}\}$ is i.i.d measurement noise. In () and (), $u(k)$ is the inputs such as the load current and temperature, and

$$\theta_c(k) = [V_{oc}(x_{c,1}(k)), R_{int}(x_{c,1}(k)), \Omega(x_{c,1}(k)), \rho(x_{c,1}(k))]'$$

The $\theta_c(k)$ is estimated by the PEM [11]. It is also assume that the initial pdf $p(x_c(0)|y_c(0))$, of state vector $x_c(k)$ is known though no measurement is made available $y_c(0)$, the covariance of processes is given as

$$Q = Q_c = Q_e = \begin{bmatrix} Q_x & 0 \\ 0 & Q_v \end{bmatrix} \quad (21)$$

where Q_x denotes variance in the states such as SoC and SoE, Q_v denotes variance in the polarization voltage. Note that Q_v does not have the effect of parameter variation due to temperature change. Similarly, the covariance of measurement is given as

$$R = R_c = R_e = R_s \quad (22)$$

where R_s is variance of voltage sensor noise.

B. The Traditional Bayesian Filtering Framework

According to the general Bayesian filtering [13], the posteriori pdf $\hat{x}_c(k|k)$ of state vector $x_c(k)$ from the available measurement $y_c(1:k) = \{y_c(i), i = 1, \dots, k\}$ can be evaluated as

$$p(x_c(k)|y_c(1:k)) = \frac{p(y_c(k)|x_c(k))p(x_c(k)|y_c(1:k-1))}{p(y_c(k)|y_c(1:k-1))} \quad (23)$$

where

$$p(x_c(k)|y_c(1:k-1)) =$$

$$\int p(x_c(k)|x_c(k-1))p(x_c(k-1)|y_c(k-1))dx_c(k-1) \quad (24)$$

is the priori pdf of state vector is also known as prediction state vector, and $p(y_c(k)|x_c(k))$ is the likelihood function evaluated by the measurement model with additive noise $v(k) \sim N(0, R)$ as

$$p(y_c(k)|y_c(1:k-1)) =$$

$$\int p(y_c(k)|x_c(k), \theta_c(k-1))p(x_c(k)|y_c(1:k-1))dx_c(k) \quad (25)$$

is a normalized constant.

Approximating $p(x_c(k)|y_c(1:k))$ by Gaussian and linearizing the nonlinear function at local operating point, the conventional extended Kalman filter [13], which is one of the nonlinear Bayesian filters, propagates the priori and posteriori estimated state and error covariance of $x_c(k)$ as follow:

$$p(x_c(k-1)|y_c(1:k-1))$$

$$\approx N(x_c(k-1); \hat{x}_c(k-1|k-1), P_c(k-1|k-1)) \quad (26)$$

$$p(x_c(k)|y_c(1:k-1)) \approx N(x_c(k); \hat{x}_c(k|k-1), P(k|k-1)) \quad (27)$$

$$p(x_c(k)|y_c(1:k)) \approx N(x_c(k); \hat{x}_c(k|k), P(k|k)) \quad (28)$$

The conventional EKF algorithm [9] diverges for both SoC and SoE estimation due to highly nonlinear behavior of ECN parameters. Using other nonlinear estimators may converges, but we interest to use EKF as it is computationally efficient and easy to implement. Therefore, a novel strategy is introduced for EKF based SoC and SoE estimators of Li-S batteries follow.

IV. PROPOSED BAYESIAN STATE ESTIMATION STRATEGY

The typical high- and low-plateau characteristics of Li-S battery state estimation problem dealt by partitioning the ECN parameters into two segments and then evaluating priori state and likelihood accordingly. o smoothly dealt highly nonlinearity of Li-S battery SoC and SoE estimation, the ECN parameters are partitioned as

$$\theta_c(k) = \begin{cases} \theta_{c,h}(k), & 1 \geq x_{soc}(k) \geq x_j \\ \theta_{c,l}(k), & x_j \geq x_{soc}(k) \geq 0 \end{cases} \quad (29)$$

where

$$\theta_{c,h}(k) = [V_{oc,h}(\cdot) \quad R_{int,h}(\cdot) \quad \Omega_h(\cdot) \quad \rho_h(\cdot)]' \quad (30)$$

Is high-plateau fitting models

$$\theta_{c,l}(k) = [V_{oc,l}(\cdot) \quad R_{int,l}(\cdot) \quad \Omega_l(\cdot) \quad \rho_l(\cdot)]' \quad (31)$$

is low-plateau fitting models of ECN parameters as given in the table II. Based on these parameters, the proposed strategy propagates any one of two priori pdfs and likelihood functions as follow:

$$p(x_c(k)|y_c(1:k-1)) = \begin{cases} p(x_c(k)|y_c(1:k-1), \theta_{c,h}(k-1)), & 1 \geq x_{soc}(k) \geq x_j \\ p(x_c(k)|y_c(1:k-1), \theta_{c,l}(k-1)), & x_j \geq x_{soc}(k) \geq 0 \end{cases} \quad (32)$$

$$p(y_c(k)|x_c(k)) = \begin{cases} p(y_c(k)|x_c(1:k-1), \theta_{c,h}(k-1)), & 1 \geq x_{soc}(k) \geq x_j \\ p(y_c(k)|x_c(1:k-1), \theta_{c,l}(k-1)), & x_j \geq x_{soc}(k) \geq 0 \end{cases} \quad (33)$$

The ECN parameter vector is parameter vector is given as

$$\theta_c(k) = [V_{oc}(x_{c,1}(k)), R_{int}(x_{c,1}(k)), \Omega(x_{c,1}(k)), \rho(x_{c,1}(k))]'$$

Similarly, the state vector of SoE comprises the two components; the state-of-energy $x_{soe}(k)$, and the polarization voltage, $V_{p,soe}(k)$ which is a function of SoE. The following state vector is used in SoE estimation problem

$$x(k) = x_e(k) = \begin{pmatrix} x_{soe}(k) \\ V_{p,soe}(k) \end{pmatrix} = \begin{pmatrix} x_{e,1}(k) \\ x_{e,2}(k) \end{pmatrix} \quad (35)$$

$$\theta_e(k) = \begin{cases} \theta_{e,h}(k), & 1 \geq x_{soe}(k) \geq x_j \\ \theta_{e,l}(k), & x_j \geq x_{soe}(k) \geq 0 \end{cases} \quad (36)$$

where

$$\theta_{e,h}(k) = [V_{oc,h}(\cdot) \quad R_{int,h}(\cdot) \quad \Omega_h(\cdot) \quad \rho_h(\cdot)]' \quad (37)$$

Is high-plateau fitting models

$$\theta_{e,l}(k) = [V_{oc,l}(\cdot) \quad R_{int,l}(\cdot) \quad \Omega_l(\cdot) \quad \rho_l(\cdot)]' \quad (38)$$

is low-plateau fitting models of ECN parameters as given in the table II. Based on these parameters, the proposed strategy propagates any one of two priori pdfs and likelihood functions as follow:

$$p(x_e(k)|y_e(1:k-1)) = \begin{cases} p(x_e(k)|y_e(1:k-1), \theta_{e,h}(k-1)), & 1 \geq x_{soe}(k) \geq x_j \\ p(x_e(k)|y_e(1:k-1), \theta_{e,l}(k-1)), & x_j \geq x_{soe}(k) \geq 0 \end{cases} \quad (39)$$

$$p(y_e(k)|x_e(k)) = \begin{cases} p(y_e(k)|x_e(1:k-1), \theta_{e,h}(k-1)), & 1 \geq x_{soe}(k) \geq x_j \\ p(y_e(k)|x_e(1:k-1), \theta_{e,l}(k-1)), & x_j \geq x_{soe}(k) \geq 0 \end{cases} \quad (40)$$

The last term of right side of (1) and (2) are the Gaussian noise components $w(k) \sim N(0, Q)$ and that represent the uncertainty in process and measurement, respectively, and they are identically independent distribution (i.i.d).

According to (39-40), in high-plateau segment the priori state, approximation of nonlinear functions and likelihood are evaluated. If the SoC is in the high-plateau region, the proposed EKF based SoC and SoE estimators propagate the priori state vectors $\hat{x}_c(k|k-1)$ and $\hat{x}_e(k|k-1)$ of Li-S battery cells as follow:

$$[\hat{x}_c(k+1|k), \hat{x}_e(k+1|k)] = \hat{f}_{x,h}(\hat{x}_c(k|k), \hat{x}_e(k|k), \hat{\theta}_{c,h}(\hat{x}_c(k|k)), \hat{\theta}_{e,h}(\hat{x}_e(k|k)), I_L, T_s, T) \quad (41)$$

where

$$\hat{\theta}_{c,h}(\hat{x}_c(k|k)) = [0 \quad \hat{R}_{int,c,h} \quad \hat{\omega}_{c,h} \quad \hat{\rho}_{c,h}]' \quad (42)$$

$$\hat{\theta}_{e,h}(\hat{x}_e(k|k)) = [\hat{V}_{oc,e,h} \quad \hat{R}_{int,e,h} \quad \hat{\omega}_{e,h} \quad \hat{\rho}_{e,h}]' \quad (43)$$

The following steps are used in (\$\$)

$$\hat{R}_{int,c,h} = \hat{R}_{int,h,1}\hat{x}_{c,1}^n(k|k) + \hat{R}_{int,h,2}\hat{x}_{c,1}^{n-1}(k|k) + \dots + \hat{R}_{int,h,n}\hat{x}_{c,1}(k|k) + \hat{R}_{int,h,n+1} \quad (44)$$

$$\hat{R}_{int,e,h} = \hat{R}_{int,h,1}\hat{x}_{e,1}^n(k|k) + \hat{R}_{int,h,2}\hat{x}_{e,1}^{n-1}(k|k) + \dots + \hat{R}_{int,h,n}\hat{x}_{e,1}(k|k) + \hat{R}_{int,h,n+1} \quad (45)$$

$$\hat{\omega}_{c,h} = \hat{\omega}_{c,h,1}\hat{x}_{c,1}^n(k|k) + \hat{\omega}_{c,h,2}\hat{x}_{c,1}^{n-1}(k|k) + \dots + \hat{\omega}_{c,h,n}\hat{x}_{c,1}(k|k) + \hat{\omega}_{c,h,n+1} \quad (46)$$

$$\hat{\omega}_{e,h} = \hat{\omega}_{h,1}\hat{x}_{e,1}^n(k|k) + \hat{\omega}_{h,2}\hat{x}_{e,1}^{n-1}(k|k) + \dots + \hat{\omega}_{h,n}\hat{x}_{e,1}(k|k) + \hat{\omega}_{h,n+1} \quad (47)$$

$$\hat{\rho}_{c,h} = \hat{\rho}_{h,1}\hat{x}_{c,1}^n(k|k) + \hat{\rho}_{h,2}\hat{x}_{c,1}^{n-1}(k|k) + \dots + \hat{\rho}_{h,n}\hat{x}_{c,1}(k|k) + \hat{\rho}_{h,n+1} \quad (48)$$

$$\hat{\rho}_{e,h} = \hat{\rho}_{h,1}\hat{x}_{e,1}^n(k|k) + \hat{\rho}_{h,2}\hat{x}_{e,1}^{n-1}(k|k) + \dots + \hat{\rho}_{h,n}\hat{x}_{e,1}(k|k) + \hat{\rho}_{h,n+1} \quad (49)$$

$$\hat{x}_c(k+1|k) = [\hat{x}_{c,1}(k|k) - T_s \frac{I_L}{C_B}; \hat{x}_{c,2}(k|k) - T_s \hat{\omega}_{c,h} \hat{x}_{c,2}(k|k) + T_s \hat{\rho}_{c,h} \hat{R}_{int,c,h} \hat{\omega}_{c,h} I_L] \quad (50)$$

$$\hat{V}_{oc,e,h} = \hat{V}_{oc,h,1}\hat{x}_{e,1}^n(k|k) + \hat{V}_{oc,h,2}\hat{x}_{e,1}^{n-1}(k|k) + \dots + \hat{V}_{oc,h,n}\hat{x}_{e,1}(k|k) + \hat{V}_{oc,h,n+1} \quad (51)$$

$$\hat{x}_e(k+1|k) = [\hat{x}_{e,1}(k|k) - T_s \frac{I_L \hat{V}_{oc,e,h}}{E_B}; \hat{x}_{e,2}(k|k) - T_s \hat{\omega}_{e,h} \hat{x}_{e,2}(k|k) + T_s \hat{\rho}_{e,h} \hat{R}_{int,e,h} \hat{\omega}_{e,h} I_L] \quad (52)$$

The $F_c(k)$ and $F_e(k)$ are Jacobian for SoC and SoE respectively. The $\hat{V}_c(k+1)$ and $\hat{V}_e(k+1)$ are likelihood of SoC and SoE estimation, respectively, and the function $g_h(x_c(k), \theta_c(k), u(k))$ returns them by the following steps

$$\hat{\theta}_{c,h}(\hat{x}_c(k+1|k)) = [\hat{V}_{oc,c,h} \quad \hat{R}_{int,c,h} \quad 0 \quad \hat{\rho}_{c,h}]' \quad (53)$$

$$\hat{\theta}_{e,h}(\hat{x}_e(k+1|k)) = [\hat{V}_{oc,e,h} \quad \hat{R}_{int,e,h} \quad 0 \quad \hat{\rho}_{e,h}]' \quad (54)$$

$$[\hat{V}_c(k+1), \hat{V}_e(k+1)] = g_h(\hat{x}_c(k+1|k), \hat{x}_e(k+1|k), \hat{\theta}_{c,h}, \hat{\theta}_{e,h}, I_L, T_s, T) \quad (55)$$

$$\hat{V}_{oc,c}(\hat{x}_{c,1}(k+1|k)) = V_{oc,h,1}\hat{x}_{c,1}^n(k+1|k) + V_{oc,h,2}\hat{x}_{c,1}^{n-1}(k+1|k) + \dots + V_{oc,h,n}\hat{x}_{c,1}(k+1|k) + V_{oc,h,n+1} \quad (56)$$

$$\hat{V}_{oc,e}(\hat{x}_{e,1}(k+1|k)) = V_{oc,h,1}\hat{x}_{e,1}^n(k+1|k) + V_{oc,h,2}\hat{x}_{e,1}^{n-1}(k+1|k) + \dots + V_{oc,h,n}\hat{x}_{e,1}(k+1|k) + V_{oc,h,n+1} \quad (57)$$

$$\hat{R}_{int,c}(\hat{x}_{c,1}(k+1|k)) = R_{int,h,1}\hat{x}_{c,1}^n(k+1|k) + R_{int,h,2}\hat{x}_{c,1}^{n-1}(k+1|k) + \dots + R_{int,h,n}\hat{x}_{c,1}(k+1|k) + R_{int,h,n+1} \quad (58)$$

$$\hat{R}_{int,e}(\hat{x}_{e,1}(k+1|k)) = R_{int,h,1}\hat{x}_{e,1}^n(k+1|k) + R_{int,h,2}\hat{x}_{e,1}^{n-1}(k+1|k) + \dots + R_{int,h,n}\hat{x}_{e,1}(k+1|k) + R_{int,h,n+1} \quad (59)$$

$$\hat{\rho}_{c,h}(\hat{x}_{c,1}(k+1|k)) = \rho_{h,1}\hat{x}_{c,1}^n(k+1|k) + \rho_{h,2}\hat{x}_{c,1}^{n-1}(k+1|k) + \dots + \rho_{h,n}\hat{x}_{c,1}(k+1|k) + \rho_{h,n+1} \quad (60)$$

$$\hat{\rho}_{e,h}(\hat{x}_{e,1}(k+1|k)) = \rho_{h,1}\hat{x}_{e,1}^n(k+1|k) + \rho_{h,2}\hat{x}_{e,1}^{n-1}(k+1|k) + \dots + \rho_{h,n}\hat{x}_{e,1}(k+1|k) + \rho_{h,n+1} \quad (61)$$

$$\hat{V}_{t,c}(k+1) = \hat{V}_{oc,c} - \hat{x}_{c,2}(k+1|k) - (1 - \hat{\rho}_{c,h}(\cdot))\hat{R}_{int,c,h}I_L \quad (62)$$

$$\hat{V}_{t,e}(k+1) = \hat{V}_{oc,e} - \hat{x}_{e,2}(k+1|k) - (1 - \hat{\rho}_{e,h}(\cdot))\hat{R}_{int,e,h}I_L \quad (63)$$

Using the above priori, the nonlinear measurement functions are approximated. In EKF the Jacobians H_c and H_e are nonlinear approximation of voltage for SoC and SoE state estimation respectively. These priori state and likelihood propagate as same as strategy given in fig 6 of [11], which is a novel strategy.

$$H_c = \left[\frac{\partial g_{c,h}(\cdot)}{\partial x_{c,1}} \quad \frac{\partial g_{c,h}(\cdot)}{\partial x_{c,2}} \right]_{|x(k)=\hat{x}_c(k+1|k)} \quad (64)$$

$$H_e = \left[\frac{\partial g_{e,h}(\cdot)}{\partial x_{e,1}} \quad \frac{\partial g_{e,h}(\cdot)}{\partial x_{e,2}} \right]_{|x(k)=\hat{x}_e(k+1|k)} \quad (65)$$

Like high-plateau, the proposed strategy evaluates the priori state, approximation of system function, likelihood and approximation of nonlinear measurement for SoC and SoE estimation in the low-plateau region, as follow:

$$[\hat{x}_c(k+1|k), \hat{x}_e(k+1|k)] = \hat{f}_{x,l}(\hat{x}_c(k|k), \hat{x}_e(k|k), \hat{\theta}_{c,l}(\hat{x}_c(k|k)), \hat{\theta}_{e,l}(\hat{x}_e(k|k)), I_L, T_s, T) \quad (66)$$

where

$$\hat{\theta}_{c,l}(\hat{x}_c(k|k)) = [0 \quad \hat{R}_{int,c,l} \quad \hat{\omega}_{c,l} \quad \hat{\rho}_{c,l}]' \quad (67)$$

$$\hat{\theta}_{e,l}(\hat{x}_e(k|k)) = [\hat{V}_{oc,e,l} \quad \hat{R}_{int,e,l} \quad \hat{\omega}_{e,l} \quad \hat{\rho}_{e,l}]' \quad (68)$$

The following steps are used in (\$\$)

$$\hat{R}_{int,c,l} = \hat{R}_{int,l,1}\hat{x}_{c,1}^n(k|k) + \hat{R}_{int,l,2}\hat{x}_{c,1}^{n-1}(k|k) + \dots + \hat{R}_{int,l,n}\hat{x}_{c,1}(k|k) + \hat{R}_{int,l,n+1} \quad (69)$$

$$\hat{R}_{int,e,l} = \hat{R}_{int,l,1}\hat{x}_{e,1}^n(k|k) + \hat{R}_{int,l,2}\hat{x}_{e,1}^{n-1}(k|k) + \dots + \hat{R}_{int,e,l,n}\hat{x}_{e,1}(k|k) + \hat{R}_{int,c,l,n+1} \quad (70)$$

$$\hat{\omega}_{c,l} = \hat{\omega}_{l,1}\hat{x}_{c,1}^n(k|k) + \hat{\omega}_{l,2}\hat{x}_{c,1}^{n-1}(k|k) + \dots + \hat{\omega}_{l,n}\hat{x}_{c,1}(k|k) + \hat{\omega}_{l,n+1} \quad (71)$$

$$\hat{\omega}_{e,l} = \hat{\omega}_{l,1}\hat{x}_{e,1}^n(k|k) + \hat{\omega}_{l,2}\hat{x}_{e,1}^{n-1}(k|k) + \dots + \hat{\omega}_{l,n}\hat{x}_{e,1}(k|k) + \hat{\omega}_{l,n+1} \quad (72)$$

$$\hat{\rho}_{c,l} = \hat{\rho}_{l,1}\hat{x}_{c,1}^n(k|k) + \hat{\rho}_{l,2}\hat{x}_{c,1}^{n-1}(k|k) + \dots + \hat{\rho}_{l,n}\hat{x}_{c,1}(k|k) + \hat{\rho}_{l,n+1} \quad (73)$$

$$\hat{\rho}_{e,l} = \hat{\rho}_{l,1}\hat{x}_{e,1}^n(k|k) + \hat{\rho}_{l,2}\hat{x}_{e,1}^{n-1}(k|k) + \dots + \hat{\rho}_{l,n}\hat{x}_{e,1}(k|k) + \hat{\rho}_{l,n+1} \quad (74)$$

$$\hat{x}_c(k+1|k) = [\hat{x}_{c,1}(k|k) - T_s \frac{I_{L1}}{C_B}; \hat{x}_{c,2}(k|k) - T_s \hat{\omega}_{c,l} \hat{x}_{c,2}(k|k) + T_s \hat{\rho}_{c,l} \hat{R}_{int,c,l} \hat{\omega}_{c,l} I_L] \quad (75)$$

$$\hat{V}_{oc,e,l} = \hat{V}_{oc,l,1}\hat{x}_{e,1}^n(k|k) + \hat{V}_{oc,l,2}\hat{x}_{e,1}^{n-1}(k|k) + \dots + \hat{V}_{oc,l,n}\hat{x}_{e,1}(k|k) + \hat{V}_{oc,l,n+1} \quad (76)$$

$$\hat{x}_e(k+1|k) = [\hat{x}_{e,1}(k|k) - T_s \frac{I_L \hat{V}_{oc,e,l}}{E_B}; \hat{x}_{e,2}(k|k) - T_s \hat{\omega}_{e,l} \hat{x}_{e,2}(k|k) + T_s \hat{\rho}_{e,l} \hat{R}_{int,e,l} \hat{\omega}_{e,l} I_L] \quad (77)$$

Using the low plateau priori estimate SoC and SoE, the Jacobian $F_c(k)$ and $F_e(k)$, of nonlinear system equations are evaluated. These Jacobians are used to evaluate the likelihood.

Let the $\hat{V}_c(k+1)$ and $\hat{V}_e(k+1)$ are likelihood of SoC and SoE estimation, respectively. The function $g_l(x_c(k), \theta_c(k), u(k))$ returns them by the following steps

$$\hat{\theta}_{c,l}(\hat{x}_c(k+1|k)) = [\hat{V}_{oc,c,l} \quad \hat{R}_{int,c,l} \quad 0 \quad \hat{\rho}_{c,l}]' \quad (78)$$

$$\hat{\theta}_{e,l}(\hat{x}_e(k+1|k)) = [\hat{V}_{oc,e,l} \quad \hat{R}_{int,e,l} \quad 0 \quad \hat{\rho}_{e,l}]' \quad (79)$$

$$[\hat{V}_c(k+1), \hat{V}_e(k+1)] = g_l(\hat{x}_c(k+1|k), \hat{x}_e(k+1|k), \hat{\theta}_{c,l}, \hat{\theta}_{e,l}, I_L, T_s, T) \quad (80)$$

$$\hat{V}_{oc,c,l}(\hat{x}_{c,1}(k+1|k)) = V_{oc,l,1}\hat{x}_{c,1}^n(k+1|k) + V_{oc,l,2}\hat{x}_{c,1}^{n-1}(k+1|k) + \dots + V_{oc,l,n}\hat{x}_{c,1}(k+1|k) + V_{oc,l,n+1} \quad (81)$$

$$\hat{V}_{oc,e,l}(\hat{x}_{e,1}(k+1|k)) = V_{oc,l,1}\hat{x}_{e,1}^n(k+1|k) + V_{oc,l,2}\hat{x}_{e,1}^{n-1}(k+1|k) + \dots + V_{oc,l,n}\hat{x}_{e,1}(k+1|k) + V_{oc,l,n+1} \quad (82)$$

$$\hat{R}_{int,c,l}(\hat{x}_{c,1}(k+1|k)) = R_{int,l,1}\hat{x}_{c,1}^n(k+1|k) + R_{int,l,2}\hat{x}_{c,1}^{n-1}(k+1|k) + \dots + R_{int,l,n}\hat{x}_{c,1}(k+1|k) + R_{int,l,n+1} \quad (83)$$

$$\hat{R}_{int,e,l}(\hat{x}_{e,1}(k+1|k)) = R_{int,l,1}\hat{x}_{e,1}^n(k+1|k) + R_{int,l,2}\hat{x}_{e,1}^{n-1}(k+1|k) + \dots + R_{int,l,n}\hat{x}_{e,1}(k+1|k) + R_{int,l,n+1} \quad (84)$$

$$\hat{\rho}_{c,l}(\hat{x}_{c,1}(k+1|k)) = \rho_{l,1}\hat{x}_{c,1}^n(k+1|k) + \rho_{l,2}\hat{x}_{c,1}^{n-1}(k+1|k) + \dots + \rho_{l,n}\hat{x}_{c,1}(k+1|k) + \rho_{l,n+1} \quad (85)$$

$$\hat{\rho}_{e,l}(\hat{x}_{e,1}(k+1|k)) = \rho_{l,1}\hat{x}_{e,1}^n(k+1|k) + \rho_{l,2}\hat{x}_{e,1}^{n-1}(k+1|k) + \dots + \rho_{l,n}\hat{x}_{e,1}(k+1|k) + \rho_{l,n+1} \quad (86)$$

$$\hat{V}_{t,c}(k+1) = \hat{V}_{oc,c,l} - \hat{x}_{c,2}(k+1|k) - (1 - \hat{\rho}_{c,l}(\cdot))\hat{R}_{int,c,l}I_L \quad (87)$$

$$\hat{V}_{t,e}(k+1) = \hat{V}_{oc,e,l} - \hat{x}_{e,2}(k+1|k) - (1 - \hat{\rho}_{e,l}(\cdot))\hat{R}_{int,e,l}I_L \quad (88)$$

The approximation of nonlinear measurement functions of SoC and SoE estimators are evaluated as

$$H_c = \left[\frac{\partial g_{c,l}(\cdot)}{\partial x_{c,1}} \quad \frac{\partial g_{c,l}(\cdot)}{\partial x_{c,2}} \right]_{|x(k)=\hat{x}_c(k+1|k)} \quad (89)$$

$$H_e = \left[\frac{\partial g_{e,l}(\cdot)}{\partial x_{e,1}} \quad \frac{\partial g_{e,l}(\cdot)}{\partial x_{e,2}} \right]_{|x(k)=\hat{x}_e(k+1|k)} \quad (90)$$

As (89) is different from (64), the proposed Bayesian framework evaluates the likelihood differently than (25). This is likelihood partitioned EKF for SoC estimation and it is due to (32) and (33). Similarly (19) is different from (65), this means the likelihood partitioned in the SoE estimation as well because of (39) and (40). In this same way, the Jacobians for nonlinear system equations. The proposed strategy propagates the priori and posteriori estimation of Li-S battery's SoC and SoE, as fig 6 of [11]. The proposed approach works well for given experiment data as shown in the next section.

V. DEMONSTRATION USING EXPERIMENTAL TEST DATA

This section presents the validated results of the proposed EKF based SoC and SoE estimators given in section III using experimental test data given in section II.

For both SoC and SoE estimation, the EKF sets the following initial conditions. The initial estimated error covariance is set to $P_c(0|0) = P_e(0|0) = I_{2 \times 2}$. Two different initial estimated state conditions are considered to evaluate the performance of the proposed EKF under two different typical problems. For first case, the initial state vector is set to $\hat{x}_c(0|0) = \hat{x}_e(0|0) = [1, 0]'$. This means the EKF has no error in the initial SoC and SoE. For second case, $\hat{x}_c(0|0) = \hat{x}_e(0|0) = [0.6, 0]'$. This means there is 40 percent error in the initial states. As $x_j = 0.7$ is used in (32)-(33) and (39)-(40) for partition the likelihood of Bayesian method, validation by two different initial conditions, ensures the reliability of the proposed method. Former case is a straightforward, the algorithm recursively propagates state and estimate error covariance of high plateau to low-plateau state of the Li-S batteries. Later case is a nontrivial problem as actual state is high-plateau, but the initial estimated state is in low-plateau. The validation of proposed method under second initial condition ensures the reliability of the algorithm.

Two temperature conditions such as 20°C and 30°C are considered. Table II and III show parameters of tested Li-S cell at 20°C and 30°C, respectively. These parameters were obtained by PEM as [8,11].

a. Results at 20°C

For simulate the SoC and SoE estimation of cell 20°C, the proposed EKF uses the fitted ECN parameters of Li-S cell at 20°C given in the Table III.

Fig.7 compares the estimated state of Li-S cell at 20°C to corresponding theoretical or true state. For instance, Fig 7(a) compares the estimated SoC in Ah to that of true SoC of cell at 20°C. It is observed that the considered Li-S cell has 2.72Ah when it was fully charged. Fig.7(b) shows the normalized SoC, which has no unit. In these, the black dotted-line indicated true SoC, whereas the red-dotted line and blue-dashed line denote estimated SoC when initial SoC is 1 and 0.6, respectively. In the case of known initial SoC case, the performance of EKF is straightforward, the estimated SoC by the EKF immediately follows the true SoC. When the EKF starts with initial SoC is 0.6, the estimated SoC takes couple of minutes to reach its true SoC. It is interesting to note here that the proposed method starts converging from next second. This means that proposed LPB method of EKF when initial state is known and initial state is

unknown. The accuracy in the SoC estimation is very good, less than five percent error between actual SoC and estimated SoC.

TABLE II
THE ECN PARAMETER FITTING MODEL OF A-CELL AT 20°C

Plateau	V_{oc} (V)	R_{int} (Ω)	ρ (no unit)	ω (rad/s)
High	$[V_{oc,h}]' =$ [108.1 -361.13 444.73 -238.18 47.03 1.88]	$[R_{int,h}]' =$ [1.07 -2.445 1.457]	$[\rho_h]' =$ [-0.0451 0.4892]	$[\omega_h]' =$ [0.415 -0.7441 0.3469]
low	$[V_{oc,l}]' =$ [-752.62 2085.66 -2392.87 1466.98 -517.42 105.21 -11.69 0.62 2.1]	$[R_{int,l}]' =$ [3.792 -8.212 6.428 -1.56 0.07346 0.1458]	$[\rho_l]' =$ [0.5087 -0.6572 0.6815]	$[\omega_l]' =$ [6.8270 -7.4220 1.6850 0.1407 -0.1544 0.1441]

Fig. 7. Estimated vs True states of Li-S cell at 20°C under two different state; (a) SoC in Ah (b) SoC normalized (c) SoE in Wh (d) SoE-normalized.

Like SoC estimation, the SoE estimation by the proposed EKF is accurate as shown in the Fig 7(c) and 7(d). In both these, the red-dotted line and blue-dashed line denote the estimated SoE of Li-S type A cell at 20°C when initial state is 1 and 0.6, respectively. Fig8(c) compares the estimated SoE in Wh to that of true SoE. It is an evident that when considered Li-S type A is fully charged, it has 5.1Wh energy, and that estimated accurately by the EKF as shown in Fig 7(c). The estimation of SoE in Wh can be directly used for energy/power optimization [16] of Li-S battery powered air vehicles. In Fig 7(d) the normalized SoE estimation is compared. The performance is good as the accuracy

of SoE estimation is good, less than five percent error. While comparing SoE to the SoE estimation, the SoC estimation is more accurate. Particularly, at the end of discharge and transition of high-and low-plateau regions, the SoE estimation has slight deviation from the actual SoE. This may be the impact of voltage in (3a)-3(b). Though there is slight difference between SoC and SoE, in overall, the proposed EKF performs well for both SoC and SoE estimation of experimented Li-S cell at 20°C

b. Results at 30°C .

When temperature of Li-S cell is 30°C, the proposed EKF uses the ECN parametrization of given in Table III. Using pulse test data acquired at temperature 30°C given in Fig 2, the PEM based ECN parameterization is used here. The simulation results of the novel framework for SoC and SoE estimation of experimented cell at 30°C are observed.

TABLE III
THE ECN PARAMETER FITTING MODEL OF A-CELL AT 30°C

Plateau	V_{oc} (V)	R_{int} (Ω)	ρ (no unit)	ω (rad/s)
High	$[V_{oc,h}]' =$ [72.9503 -259.6135 341.4807 -195.5125 43.1183]	$[R_{int,h}]' =$ [2.001 -3.759 1.853]	$[\rho_h]' =$ [-0.1295 0.5687]	$[\omega_h]' =$ [1.0395 -0.7603]
low	$[V_{oc,l}]' =$ [-31.2310 69.3765 -60.1029 25.6103 -5.6128 0.5928 2.0974]	$[R_{int,l}]' =$ [173.7228 -515.0199 616.6895 -381.7758 129.5985 -23.1084 1.6510 0.1450]	$[\rho_l]' =$ [-2.1590 2.2718 -23.1084 1.6510]	$[\omega_l]' =$ [-1.3532 2.0927 -1.0609 0.1907]

Fig. 8. Estimated vs True states of Li-S cell at 30°C; (a) SoC in Ah (b) SoC normalized (c) SoE in Wh (d) SoE normalized.

Fig 8 illustrates a comparison between the SoC and SoE estimation and their corresponding true state of given cell at 30°C.

The results show that the EKF's perform well at cell at 30°C temperature condition. In all subplots of fig 8, the red-dotted line denotes the estimate state and black-solid line denotes the true state. For instance, the red-dotted line and black-dotted line of Fig 8(a) represent the estimated SoC in Ah and true SoC, respectively, in Ah of type A cell at 30°C. From the beginning of discharge to depletion state of cell, the estimated state by EKF closely follows the true state. The similar trends follow in normalization of SoC as well. Fig 8(b) compares SoC normalised to its normalised true SoC. Both Fig 8(a) and Fig 8(b) show that the estimated SoC by the EKF is almost superimposed the true SoC from the beginning to end of discharge. Similar trend is followed by SoE estimation as well. For instance, Fig 8(c) shows that the estimated SoE in Wh is very close to that true SoE. Similarly, Fig 8(d) shows the normalized SoE estimation is accurate as close to the true SoE. The results are clear evidence that the performance of proposed EKF for both SoC and SoE estimation is promising as their accuracy is good.

The accuracy of both SoC and SoE estimation by proposed EKF evaluated in terms of RMSE error and absolute error. The degree of observability [11] analysis is used for RMSE study. In addition, the absolute and squared error between estimated state and its true state are evaluated. As algorithm recalibrated here for improve the smoothness and accuracy, the error is much lower than the fig 12 of [11]. The error plots are not presented here due to page limitation. However, the proposed EKF's accuracy for SoC and SoE estimation of given cell at both temperature conditions is numerically evaluated, and it is about 97 percent.

VI. CONCLUSIONS AND FUTURE WORKS

A novel likelihood-partitioned Bayesian filter has been developed for estimate the SoC and SoE of lithium-sulfur battery cells. The battery cells are modelled as ECN models, and its parameters are estimated using a prediction error minimization (PEM) method based estimator. the proposed Bayesian filtering strategy is implemented on a simple nonlinear Bayesian filter namely extended Kalman filter (EKF). The proposed EKF strategy has been validated with experimental test data of a particular type of Li-S battery cell. The EKF is calibrated by mixed-C-rate pulse test data at each temperature points. The performance of proposed method for estimating both SoC and SoE of both cell types is quite acceptable, estimation accuracy is more than 95 percent for given test data under two different temperature and initial conditions. As this novel filter based SoC and SoE estimation does not use additional functions or training sets, it is computationally efficient for coping with the highly nonlinear and complex behaviors of Li-S batteries. By exploring the definitions SoC and SoE in terms of their unit, and their estimation of both Li-S cell series the physical significant difference between the estimated SoC in Ah and SoE in Wh at different point of times has been studied. The estimated SoC in Ah and SoE in Wh can be directly used to the optimization by closed loop energy/or power management control system of electric vehicles. Future research includes to apply the proposed strategy to other applications such as road vehicles, and to explore the robustness against different uncertainties.

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