Limitations of Small-Signal Modeling of Grid-Connected VSCs under Phase Jumps and Frequency Deviations

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Abstract—The stability of grid-connected voltage source converters (VSCs) is commonly studied by the small-signal model in the synchronous reference (dq) frame. With phase-locked loop dynamic from VSCs, it is inevitable to convert the small-signal model to a common dq frame. When linearizing the frame transformation, the trigonometric functions of the angle are approximated by first-order terms. In this letter, the limitation of such linearization on the dq -transformation is discussed. The accuracy of the model, in presence of phase jumps and frequency variations, are examined. It is found that even under small phase jumps, mismatches are found between the small-signal model (SSM) and a nonlinear model (NM). It is further revealed that during the dynamics of frequency variation, significant mismatches can occur between the SSM and the NM.

Index Terms—Synchronous reference (dq) frame, frame transformation, voltage source converters (VSCs), small-signal model (SSM), nonlinear model (NM)

I. INTRODUCTION

STABILITY of a three-phase grid-connected voltage source converter (VSC) configuration is often analyzed by using a synchronous reference (dq)-frame small-signal model. In the linearization process, the nonlinearities in the trigonometric functions of dq transformation and also within the power calculations are approximated by first-order functions around a steady-state operating point [1].

The dq -frame small-signal modeling usually considers that the VSC is connected to a stiff source with fixed amplitude and frequency [1], [2]. However, to model the power system disturbances such as phase jumps and frequency variations, the phase of the source voltage would also change. Phase jumps are normal power system phenomena which can have many causes [3]. Moreover, in a VSC dominated grid, occurrence of frequency deviations tends to be more frequent, than in the conventional grid governed by synchronous generators [4].

In this letter, the limitations of the dq -frame small-signal modeling are revealed in the presence of phase jumps and frequency variations. The small-signal models are constructed by state-space modeling and two sets of reference systems are used to respectively reflect the issues under phase jumps and frequency variations. In [5], the accuracy problems of the dq -frame small-signal model, in presence of phase jump is discussed. It is stated that the model accuracy is preserved when the phase variation is below 7° . However, as shown in this letter the inaccuracy of the small-signal model always exists under phase-jump events, regardless of the size of phase jump. Moreover, it is found that the small-signal model is

no longer accurate when the system frequency deviates. To effectively evaluate the system stability, the small-signal model should be adapted to a particular system frequency. Numerical simulations confirm the theoretical findings.

II. SMALL-SIGNAL MODELING

In this letter, the vectors that represent current and voltage variables are expressed, respectively, by i and u . For instance, the source voltage vector is expressed as u_s^s , where the superscript ‘ s ’ indicates that the variable is on the $\alpha\beta$ frame. On the other hand, the dq -frame variables are denoted without the superscript ‘ s ’¹. As shown in [1], the dynamic impact of the phase-locked loop (PLL) introduces a so-called converter dq frame. In the converter dq frame, a superscript ‘ c ’ is added for distinction from the grid dq frame (which is the dq frame with no PLL impact), e.g., u_s^c is on the converter dq frame and u_s is on the grid dq frame. The transformation from $\alpha\beta$ frame to dq frame yields

$$\begin{aligned} u_s^c &= e^{-j(\omega t + \theta)} u_s^s = e^{-j(\omega t + \theta)} u_s e^{j(\omega_s t + \theta_s)} \\ &= u_s e^{-j[(\omega - \omega_s)t + (\theta - \theta_s)]} \end{aligned} \quad (1)$$

where ω and θ are the angular velocity and angle of the PLL respectively. ω_s and θ_s are respectively the nominal angular velocity and the angle of u_s^s . From (1), we note that when $\omega = \omega_s = \omega_1$ ($\omega_1 = 2\pi f_1$, where f_1 is the nominal system frequency), (1) is reduced as²

$$u_s^c = u_s e^{-j(\theta - \theta_s)} = u_s e^{-j\Delta\theta} \quad (2)$$

where $\Delta\theta$ is the angle difference between u_s^c and u_s . For a small $\Delta\theta$, $\cos\Delta\theta \approx 1$, $\sin\Delta\theta \approx \Delta\theta$, (2) can be linearized as

$$u_s^c = u_s (1 - j\Delta\theta). \quad (3)$$

Expanding (3) yields

$$u_{sd}^c = u_{sd} + \Delta\theta u_{sq} \quad u_{sq}^c = u_{sq} - \Delta\theta u_{sd}. \quad (4)$$

Further linearizing (4) yields

$$\Delta u_{sd}^c = \Delta u_{sd} + u_{sq0} \Delta\theta \quad \Delta u_{sq}^c = \Delta u_{sq} - u_{sd0} \Delta\theta. \quad (5)$$

¹In this letter, the subscript ‘ s ’, ‘ pc ’, ‘ c ’ stand for ‘source’, ‘points of common coupling’, ‘converter’, respectively.

²To represent perturbation variables, a prefix Δ is used, e.g., Δu_s . A constant representing a steady-state operating point contains a ‘0’ in the end of its subscript, e.g., u_{s0} .

Reference system 1

Fig. 1. Reference system 1. (a) A grid-following VSC connected to a stiff source. (b) The small-signal model of (a).

A. Reference System 1 with Phase Jump

Reference system 1 is shown in Fig. 1(a) and the layout of its linearized system is illustrated in Fig. 1(b). As shown in Fig. 1(a), a grid-following VSC is connected to a source via the phase reactor impedance Z_r^s , the transformer impedance Z_t^s , and the grid impedance Z_g^s . To facilitate the analysis, the VSC dc-bus dynamics are not modeled by assuming that an infinite energy source is connected on the dc side. The CC and PLL are considered. The CC contains the PI regulator (F_{CC}), current-reference feedforward filter (H_{iFF}), and the converter-bus voltage feedforward filter (H_{uFF}) [2].

$$F_{CC} = \begin{bmatrix} k_{pCC}(1 + \frac{k_{iCC}}{s}) & 0 \\ 0 & k_{pCC}(1 + \frac{k_{iCC}}{s}) \end{bmatrix} \quad (6)$$

$$H_{iFF} = \begin{bmatrix} \frac{s}{1+s\tau_{iFF}} & -\omega_1 \\ \omega_1 & \frac{s}{1+s\tau_{iFF}} \end{bmatrix} L_r \quad H_{uFF} = \begin{bmatrix} \frac{1}{1+s\tau_{uFF}} & 0 \\ 0 & \frac{1}{1+s\tau_{uFF}} \end{bmatrix} \quad (7)$$

where τ_{iFF} , τ_{uFF} are the filter time constants, L_r is the phase reactor inductance in p.u.

The linearized closed-loop transfer function of the PLL can be derived as [1]

$$\Delta\theta = \frac{h_{PLL}f_{PLL}}{s + h_{PLL}f_{PLL}} \Delta u_{pccq} \quad (8)$$

where $h_{PLL} = \frac{1}{1+s\tau_{PLL}}$ and $f_{PLL} = k_{pPLL}(1 + \frac{k_{iPLL}}{s})$. In Fig. 1(b), the frame-transformation dynamics between the variables in the two dq frames are modeled by the transfer matrices T_{trans0} , T_{trans1} , T_{trans2} , and T_{trans3} . For instance, the inputs for T_{trans0} are: Δu_{sd} , Δu_{sq} , and $\Delta\theta$ and its outputs are: Δu_{sd}^c , Δu_{sq}^c [2]

$$T_{trans0} = \begin{bmatrix} 1 & 0 & u_{sq0} \\ 0 & 1 & -u_{sd0} \end{bmatrix} \quad T_{trans1} = \begin{bmatrix} 1 & 0 & u_{cq0} \\ 0 & 1 & -u_{cd0} \end{bmatrix} \quad (9)$$

$$T_{trans2} = \begin{bmatrix} 1 & 0 & i_{cq0} \\ 0 & 1 & -i_{cd0} \end{bmatrix} \quad T_{trans3} = \begin{bmatrix} 1 & 0 & -u_{refq0} \\ 0 & 1 & u_{refd0} \end{bmatrix}. \quad (10)$$

The passive circuit components are expressed as

$$Y_r = Z_r^{-1} = \begin{bmatrix} s & -\omega_1 \\ \omega_1 & s \end{bmatrix} L_r \quad Z_g = \begin{bmatrix} s & -\omega_1 \\ \omega_1 & s \end{bmatrix} L_g \quad (11)$$

$$Z_{gt} = \begin{bmatrix} s & -\omega_1 \\ \omega_1 & s \end{bmatrix} L_{gt}. \quad (12)$$

where Y_r is the phase-reactor admittance matrix, L_g represents the grid inductance in p.u., L_{gt} refers to the sum of the grid inductance and transformer inductance in p.u.

The closed-loop dynamics of the system in Fig. 1(b) can be summarized as the reference tracking ($\Delta i_c^c \rightarrow \Delta i_{ref}^c$) and disturbance rejection ($\Delta i_c^c \rightarrow 0$) transfer matrices:

$$\Delta i_c^c = T_{iU} \Delta i_{ref}^c + T_{iDU} \Delta u_s \quad (13)$$

where T_{iU} and T_{iDU} are the corresponding closed-loop transfer matrices.

B. Reference System 2 with Frequency Deviation

To study the impact of system frequency deviation, reference system 2 is created, see Fig. 2(a). Its corresponding small-signal model is constructed and shown in Fig. 2(b). In contrast to the reference system 1, the swing dynamic of synchronous generator (SG) is included in the source frequency, and the outer power-control loop dynamics of the VSC are considered. Consequently, the whole system consists of two parts: the first part represents the swing equations and frequency-regulation dynamics from the governor and turbine of the SG; the second part models the closed-loop dynamics of the grid-following converter. The two parts of the models are integrated by u_s on the grid dq frame. In the first part of the model, the governor and turbine dynamics [6], as well as swing dynamics are modeled as

$$H_{gov} = \frac{k_{pfs}}{1+sT_G} \quad H_{turbine} = \frac{1+sF_{HP}T_{RH}}{(1+sT_{CH})(1+sT_{RH})} \quad (14)$$

$$\Delta \dot{\omega}_s = \omega_1 \Delta \omega_s \quad (15)$$

$$\Delta \dot{\omega}_s = \frac{1}{M} [\Delta p_{gov} + \Delta p_m - \underbrace{(\Delta p_L + \Delta p_f)}_{\Delta p_e}]$$

Reference system 2

Fig. 2. Reference system 2. (a) A grid-following VSC connected to a source with swing dynamic. (b) The small-signal model of (a).

where k_{pfs} is the droop gain for the governor, T_G , F_{HP} , T_{RH} , T_{CH} , and T_{RH} are the filter time constants, $\Delta\theta_s$ and $\Delta\omega_s$ are the rotor angle and angular velocity deviations respectively. Δp_{gov} , Δp_m , Δp_e , Δp_L , and Δp_f are the deviation variables for the governor power, mechanical power, electrical power, load power, and converter output power respectively. $\Delta p_{gov} = H_{gov} H_{turbine} \Delta\omega_{sref}$, $\Delta\omega_{sref}$ is the reference angular velocity for the governor. $\Delta p_f = u_{sd0} \Delta i_{cd} + u_{sq0} \Delta i_{cq} + i_{cd0} \Delta u_{sd} + i_{cq0} \Delta u_{sq}$. Δp_m and Δp_L are treated as external inputs.

An inverse dq transformation is applied on Δu_s^r , i.e., a frame transformation from the rotor dq frame (with superscript 'r') to the grid dq frame. A transfer function matrix T_{trans5} is applied for a transformation of inputs Δu_{sd}^r , Δu_{sq}^r , and $\Delta\theta_s$ to the outputs Δu_{sd} and Δu_{sq} . To regulate the voltage $|u_{pcc}|$ and angular velocity ω_{pcc} at the PCC, the ac-bus-voltage controller (AVC) and frequency controller (FC) are implemented to generate i_{ref}^c . ω_{pcc} is estimated by the PLL ($\omega = \omega_1 + \Delta\omega = \omega_1 + \Delta\theta/\omega_1$) and ω is regulated to ω_{ref} (see Fig. 2). The AVC regulator F_{AVC} contains only the integral controller. T_{trans4} represents the frame transformation dynamics from Δu_{pcc} to Δu_{pcc}^c . The FC consists of proportional (droop) and derivative (virtual inertia) controllers F_{FC}

$$F_{AVC} = \frac{k_{iAVC}}{s} \quad F_{FC} = -k_{pf} - k_{pfd}s \quad (16)$$

$$T_{trans4} = \begin{bmatrix} 1 & 0 & u_{pccq0} \\ 0 & 1 & -u_{pccd0} \end{bmatrix} \quad T_{trans5} = \begin{bmatrix} 1 & 0 & -u_{sq0} \\ 0 & 1 & u_{sd0} \end{bmatrix}. \quad (17)$$

where k_{iAVC} is the integral gain of the AVC, k_{pf} and k_{pfd} are the proportional and derivative gain of the FC respectively.

III. LIMITATIONS OF SMALL-SIGNAL MODELS

A. Phase Jumps

The purpose of reference system 1 is to check the accuracy of the small-signal model when a phase jump $\Delta\theta_s$ occurs at the source. To represent $\Delta\theta_s$ in terms of Δu_s , we have $\Delta u_{sd} = \cos(\Delta\theta_s) - u_{sd0}$, $\Delta u_{sq} = \sin(\Delta\theta_s)$.

With $i_{c0} = j0.5$ p.u., $L_g = 0.1$ p.u., and $u_{s0} = 1$ p.u., Fig. 3 (a)-(d) respectively show the response of i_{cd}^c , i_{cq}^c , u_{sd}^c , and u_{sq}^c under $\Delta\theta_s = 0.0628$ rad. It is straightforward to analyze the response of Δu_s^c since its dynamic is only impacted by the

size of phase jump in the system ($\Delta u_s = -0.002 + j0.0628$ p.u.) and the PLL dynamics from $\Delta\theta$. In Fig. 3 (a) and (b), it can be observed that after the phase jump, the CC regulates the current at $i_{ref} = j0.5$ p.u. A close matching between the nonlinear model (NM) (solid curves) and the small-signal model (SSM) (dashed curves) is observed for the response of i_c^c . However, a difference between the models is found in Fig. 3(c) where $\Delta u_{sd}^c = -0.002$ p.u. in the SSM while u_{sd}^c is restored after the disturbance in the NM. This mismatch can be intuitively checked by T_{trans0} in (9): with $u_{sq0} = 0$ p.u., $\Delta u_{sd}^c = \Delta u_{sd} = -0.002$ p.u. The PLL dynamics on Δu_{sd}^c are thus overlooked in the SSM compared to the NM. On the other hand, the PLL dynamic on Δu_{sq}^c is correctly modeled in the SSM since $u_{sd0} = 1$ p.u. The limitation for the first-order approximation of the trigonometric functions in (9) can be proved by performing Maclaurin series expansion of the trigonometric functions up to their second-order terms:

$$\Delta u_{sd}^c = \Delta u_{sd} + u_{sq0} \Delta\theta - u_{sd0} \frac{\Delta\theta^2}{2} + \Delta u_{sq} \Delta\theta \quad (18)$$

$$\Delta u_{sq}^c = \Delta u_{sq} - u_{sd0} \Delta\theta - u_{sq0} \frac{\Delta\theta^2}{2} - \Delta u_{sd} \Delta\theta. \quad (19)$$

Using (18) and (19), where the $\Delta\theta$ is the outputs of the SSM, the time-domain response (dotted curves) is overlapped with the NM, see Fig. 3(c) and (d)³. This fact shows that the existing small-signal model fails to accurately model VSC dynamics during phase jump events, even when the phase jump is smaller than 7° (0.0628 rad $\approx 3.6^\circ$). In [5], it is discussed to use a nonlinear model for the frame transformation by considering the first three terms of (18) and (19). Yet overlooking relevant second-order terms $\Delta u_{sq} \Delta\theta$ and $\Delta u_{sd} \Delta\theta$ in (18) and (19) would not match the response to the NM, see the dash-dotted curves in Fig. 3(c) and (d).

B. System Frequency Deviations

A step $\Delta p_L = 0.1$ p.u. is applied in the reference system 2, the response of ω_s , ω , and θ are shown in Fig. 4 (a), (b), and (c) respectively. The time-domain simulations match between

³A model having second-order terms is already not a small-signal model. Thus, calculating of (18) and (19) is just for verification purpose.

Fig. 3. $\Delta\theta_s = 0.0628$ rad. (a) Response of i_{cd}^c in the NM and SSM, (b) Response of i_{cq}^c in the NM and SSM, (c) Response of u_{cd} in the NM, SSM, calculation of (18), and calculation of (18) with first three terms, (d) Response of u_{cq} in the NM, SSM, calculation of (19), and calculation of (19) with first three terms, (e) Response of $\Delta\theta$ in the NM and SSM.

the NM and SSM. The steady-state frequency deviation yields -0.004 p.u., with $k_{pfs} = 20$ p.u. and $k_{pf} = 5$ p.u. However, with even a small deviation of $\Delta\omega$ from 0 p.u., $\Delta\theta$ keeps growing [see Fig. 4(c)] since $\Delta\theta = \omega_1 \int \Delta\omega dt$.

With the frequency deviation, the large $\Delta\theta$ creates fundamental problems for the dq frame small-signal modeling since $\omega \neq \omega_1$ in (1) and thus the linearization in (2) can not be achieved. The response of i_{cd}^c and i_{cq}^c are respectively shown in Fig. 4(d) and (e) for the NM and SSM. Good accuracy is found for the dynamic of i_{cd}^c , however, not for i_{cq}^c . This significant mismatch on the dq frame current can be explained by the limitation of small-signal modeling. With $u_{acref} = 1.05$ p.u., $u_{s0} = 1.0$ p.u., and $L_g = 0.1$ p.u., we have $i_{c0} = j0.5$ p.u. It is noticed that at Time = 6 s, $\Delta\theta = -10$ rad in Fig. 4(c). According to the input-output relations of T_{trans2} in (10), $\Delta i_{cq}^c = \Delta i_{cq}$ and Δi_{cd}^c deviates as large as 5 p.u. from Δi_{cd} due to $\Delta\theta = -10$ rad, see Fig. 4(f). To summarize, when $\Delta\omega \neq 0$ in the steady state, $\Delta\theta$ does not settle to a constant value, the time-domain response of the SSM thus does not match the NM since a new steady-state solution of the SSM can not be achieved. This also implies that when the system frequency deviates, the small-signal stability analysis of VSC should always be evaluated at a steady-state frequency f_0 .

IV. CONCLUSIONS

In this letter, the limitations of the small-signal model of grid-connected VSCs is discussed in the presence of phase jumps and frequency variations. Inaccurate dynamics of the

Fig. 4. (a) Response of $\Delta\omega_s$ in the NM and SSM, (b) Response of $\Delta\omega$ in the NM and SSM, (c) Response of $\Delta\theta$ in the NM and SSM, (d) Response of i_{cd}^c in the NM and SSM, (e) Response of i_{cq}^c in the NM and SSM, (f) Response of i_{cd} and i_{cd}^c in the SSM, (g) Response of i_{cq} and i_{cq}^c in the SSM.

VSC are observed even under small phase jumps. Moreover, the dq -frame variables in the SSM show large mismatches from the NM, when system frequency deviates. Under events of system frequency deviation, the small-signal stability analysis of VSC should be evaluated at a steady-state frequency.

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