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Linear Precoder Design for PAPR Reduction of GFDM Signals Using Gradient Descent Methods

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Abstract—This paper addresses linear precoder design for Peak-to-Average Power Ratio (PAPR) reduction of Generalized Frequency Division Multiplexing (GFDM). A general framework, which is composed of four different scenarios and utilizes Gradient-based iterative methods to reduce PAPR through minimizing statistical parameters of the instantaneous power of GFDM signal including variance, power, and third moment, is suggested. Numerical results confirm when the step-size of the Gradient method is dynamically computed using the Wolf line search rule, the suggested algorithm circumvents drawbacks of existing studies and converges to a precoder providing advantages in design speed, obtained PAPR, symbol error rate, and out-of-band emission.

Index Terms—GFDM, linear precoding, PAPR reduction, Steepest descent and Conjugate-Gradient iterative methods, Wolf line search rule.

I. INTRODUCTION

IN recent two decades, generalized frequency division multiplexing (GFDM) has been received increasing attention as the most suitable waveform supporting the requirements of next-generation wireless systems, which achieves lower out-of-band (OOB) emission, higher spectral efficiency, and stronger robustness against time and frequency offsets than orthogonal frequency division multiplexing (OFDM) [1], [2]. However, GFDM systems still suffer from the high peak-to-average power ratio (PAPR) due to the randomly coherent superposition of the modulated subcarriers [3], [4].

Several PAPR reduction approaches have been proposed to overcome this difficulty, including clipping with iterative reception [5], companding transforms [6], [7], and linear precoding [8], [9]. Among them, the latter one is a data and channel-independent technique employing a linear precoder before the GFDM prototype filter. In [8], a block diagonal DFT matrix has been considered as the precoder. To provide a more optimal design, [9] has proposed an iterative algorithm based on the steepest descent (SD) method to improve the block diagonal DFT matrix across iterations by decreasing the variance of instantaneous power (IP) of GFDM signal subject to the restriction of unaltered average IP. However, this algorithm suffers from the lack of convergence, and the objective function, i.e., IP variance, does not experience decreasing behavior as expected in successive iterations of any

optimization methods. To overcome this flaw, the proposed approach of [9] included a stopping constraint, which terminates the algorithm in each iteration the objective function begins to diverge. In [9], it is also emphasized that this constraint practically occurs in numerical simulations.

In this paper, to avoid this limitation and provide a more efficient design, we present a comprehensive framework for precoder design, which relies on the minimization of different statistical parameters of GFDM signal using various Gradient-based optimization methods. Specifically, in addition to the variance, we first consider the second and third moments of IP as two new statistical parameters whose minimization leading to a reduction in PAPR. Then, we suggest an algorithm that exploits two different gradient-based iterative methods to minimize these parameters and converge to the optimal precoder design. In this regard, two constraints are considered in the suggested algorithm, resulting in four different scenarios depending on how and where these constraints are employed. Moreover, in each scenario, the step-size of the Gradient-based methods is updated using two different approaches, where the first one considers fixed step-size similar to [9] and the second of which uses the Wolf line search rule to dynamically adjust step-size. Numerical results confirm the fixed approach does not converge in some scenarios, resulting in a non-optimal design of precoder in terms of achievable PAPR, symbol error rate (SER), and OOB emission. However, the latter approach converges in all circumstances and leads to a design providing advantages in design-speed, achievable PAPR, and OOB emission.

Notations: in a respective order, lowercase, boldfaced lowercase and boldfaced uppercase letters denote scalar number, vector and matrix; $(\cdot)^*$, $(\cdot)^T$, and $(\cdot)^H$ represent conjugate, transpose, and Hermitian operations; $|\cdot|$, $\|\cdot\|$, and $\mathbb{E}\{\cdot\}$ stand for the absolute value, norm, and statistical expectation.

II. SIGNAL MODEL

We consider a linear precoded GFDM system designed to transmit a $M \times K$ block of zero-mean, i.i.d. symbols of M-ray QAM constellation, where M and K are the numbers of GFDM time instances and subcarriers, respectively. In matrix representation, the $MN \times 1$ output vector of the GFDM prototype filter, s , can be formalized as below:

$$s = A Q d = X d \quad (1)$$

where $N \geq K$ is the upsampling rate. The $MK \times 1$ input vector $d = [d_0^T, \dots, d_{M-1}^T]^T$ is a rearranged version of the input block, where $d_m = [d_{0,m}, \dots, d_{K-1,m}]^T$ and $d_{k,m}$

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denotes the data symbol of k th subcarrier at m th time instance. This symbol is transmitted using its corresponding pulse shape $g_{k,m}[n] = g[(n-mN) \bmod MN]e^{j2\pi kn/N}$, where $g(n)$ is the GFDM prototype filter. In the above matrix representation, these pulse shape coefficients are collected in $MN \times MK$ GFDM matrix $\mathbf{A} = [\mathbf{g}_{0,0}, \dots, \mathbf{g}_{K-1,0}, \mathbf{g}_{0,1}, \dots, \mathbf{g}_{K-1,M-1}]$, where $\mathbf{g}_{k,m} = [g_{k,m}[0], \dots, g_{k,m}[MN-1]]^T$. Finally, $\mathbf{Q}$ is a $MK \times MK$ precoder matrix, and $\mathbf{X} = \mathbf{A}\mathbf{Q}$.

A scalar representation of the above model can be as below:

$$s(n) = \sum_{u \in \mathcal{U}} \mathbf{a}_n \mathbf{q}_u d_u = \sum_{u \in \mathcal{U}} x_{nu} d_u, \quad n \in \mathcal{N} \quad (2)$$

in which $\mathcal{U} = \{0, \dots, MK-1\}$, and $s(n)$ is the n th element of $\mathbf{s}$, for every $n \in \mathcal{N} = \{0, \dots, MN-1\}$. $\mathbf{a}_n$ is the n th row of $\mathbf{A}$, $\mathbf{q}_u$ and d_u are the u th column and entity of $\mathbf{d}$ and $\mathbf{Q}$, and $x_{nu} = \mathbf{a}_n \mathbf{q}_u$ represents (n, u) element of $\mathbf{X} = \mathbf{A}\mathbf{Q}$.

III. SUGGESTED ALGORITHM

According to (2), PAPR of the GFDM system, as a function of its IP $|s(n)|^2$, can be defined as $PAPR = \max_{0 \leq n \leq MN-1} |s(n)|^2 / \mathbb{E}\{|s(n)|^2\}$. Since our algorithm for PAPR reduction relies on minimizing different statistical parameters of IP, these parameters are first formulated in the following, and the suggested algorithm is then provided.

The statistical expectation and variance of IP are $\mathbb{E}\{|s(n)|^2\}$ and $\mathbb{E}\{|s(n)|^4\} - (\mathbb{E}\{|s(n)|^2\})^2$. By averaging over a GFDM block, the averaged expectation and variance are as below:

$$\bar{\mu}_1 = \frac{1}{MN} \sum_{n \in \mathcal{N}} \mathbb{E}\{|s(n)|^2\} = \frac{\sigma_d^2}{MN} \sum_{n \in \mathcal{N}} \sum_{u \in \mathcal{U}} |x_{nu}|^2 \quad (3)$$

$$\begin{aligned} \bar{\sigma}^2 &= \frac{1}{MN} \sum_{n \in \mathcal{N}} \mathbb{E}\{|s(n)|^4\} - \frac{1}{MN} \sum_{n \in \mathcal{N}} (\mathbb{E}\{|s(n)|^2\})^2 \\ &= \frac{1}{MN} \sum_{n \in \mathcal{N}} [(\sigma_d^4 - 2(\sigma_d^2)^2) \sum_{u \in \mathcal{U}} |x_{nu}|^4 + (\sigma_d^2)^2 (\sum_{u \in \mathcal{U}} |x_{nu}|^2)^2] \end{aligned} \quad (4)$$

where $\sigma_d^2 = \mathbb{E}\{|d_u|^2\}$, $\sigma_d^4 = \mathbb{E}\{|d_u|^4\}$, and the IP moments $\mathbb{E}\{|s(n)|^2\}$ and $\mathbb{E}\{|s(n)|^4\}$ are formalized in the Appendix A. Note that, in [9], the average variance of $\bar{\sigma}^2$ has been defined with a minor difference as $P_{var} \triangleq \frac{1}{MN} \sum_{n \in \mathcal{N}} \mathbb{E}\{|s(n)|^4\} - (\bar{\mu}_1)^2$, and minimized subject to the constraint of unchanged $\bar{\mu}_1$. However, when $\bar{\mu}_1$ is considered constant, both $\bar{\sigma}^2$ and P_{var} are shifted versions of the average power of IP given in (5). The following two IP moments are also considered for PAPR reduction to clarify this and confirm the adaptability of the proposed method with other statistical parameters.

$$\bar{\mu}_2 = \frac{1}{MN} \sum_{n \in \mathcal{N}} \mathbb{E}\{(|s(n)|^2)^2\} = \frac{1}{MN} \sum_{n \in \mathcal{N}} \mathbb{E}\{|s(n)|^4\} \quad (5)$$

$$\bar{\mu}_3 = \frac{1}{MN} \sum_{n \in \mathcal{N}} \mathbb{E}\{(|s(n)|^2)^3\} = \frac{1}{MN} \sum_{n \in \mathcal{N}} \mathbb{E}\{|s(n)|^6\} \quad (6)$$

The suggested algorithm for designing optimal precoder via minimizing the above statistical parameters using Gradient-based iterative methods is generally presented in Algorithm 1. In step 2, the initial precoder $\mathbf{Q}_0$ is considered as a DFT block diagonal matrix similar to [9]. This is because a unitary precoder matrix is needed for the detection procedure. In

Algorithm 1 : Suggested algorithm for precoder design

- 1: assign number of iterations to $(m = 0)$
 - 2: set initial value of precoder $\mathbf{Q}_0$
 - 3: **While** termination criterion is not satisfied, **Do**
 - 4: calculate the descent direction matrix $\mathbf{D}_m$, (according to consider status for the $\bar{\mu}_1$)
 - 5: calculate the step-size value α_m
 - 6: set $m = m + 1$
 - 7: update the precoder matrix using $\mathbf{Q}_{m+1} = \mathbf{Q}_m + \alpha_m \mathbf{D}_m$
 - 8: employ SER criterion, (in this step or step 10)
 - 9: **EndWhile**
 - 10: employ SER criterion (if does not employed in step 8)
-

step 3, based on the selected objective function f among $\bar{\sigma}^2$, $\bar{\mu}_2$ and $\bar{\mu}_3$, its normalized increment over consecutive iterations $\epsilon_{m+1} = \frac{|f_{m+1} - f_m|}{|f_0|}$ is considered as the termination criterion; however, since the algorithm may not converge in some scenarios, the maximum number of iterations, it_{max} , is also assumed as the other stopping criterion.

In step 4, the search direction of the Gradient-based optimization method, i.e., $\mathbf{D}_m$, is computed for the updating process of $\mathbf{Q}_{m+1} = \mathbf{Q}_m + \alpha_m \mathbf{D}_m$. Herein, we employ two approaches of SD and Conjugate-Gradient (CG). In the SD method, the search direction is $\mathbf{D}_m = -\mathbf{G}_m$, where $\mathbf{G}_m = \nabla f(\mathbf{Q}_m)$ denotes the Gradient matrix of the objective function with respect to precoder matrix at m th iteration. However, for the case of CG method, this will be as below [10]:

$$\mathbf{D}_m = \begin{cases} -\mathbf{G}_0, & m = 0 \\ -\mathbf{G}_m + \frac{\|\mathbf{G}_m\|^2}{\|\mathbf{G}_{m-1}\|^2} \mathbf{G}_{m-1}, & m \geq 1 \end{cases} \quad (7)$$

Note that the computations of step 4 are conducted in two different modes depending on the expected status for the $\bar{\mu}_1$, where the constraint of unaltered $\bar{\mu}_1$ being assumed for the first mode, similar to [9], and this restriction is ignored as a new approach in the second mode. Among the three objective functions, only $\bar{\sigma}^2$ depends on $\bar{\mu}_1$. When the constraint of unaltered $\bar{\mu}_1$ is assumed, the gradient vector of $\bar{\sigma}^2$ with respect to the u th column of $\mathbf{Q}_m$ is given in (8). However, when $\bar{\mu}_1$ is considered constant, this gradient is equal to that of $\bar{\mu}_2$ and is computed from (8) by substituting the coefficient of the second term with the $2(\sigma_d^2)^2$. Moreover, this gradient for $\bar{\mu}_3$ is presented in (9).

$$\mathbf{g}_{q_u}(\bar{\sigma}^2) = 2 \frac{\partial \bar{\sigma}^2}{\partial q_u^*} = \frac{4}{MN} \sum_{n \in \mathcal{N}} [(\sigma_d^4 - 2(\sigma_d^2)^2) |x_{nu}|^2 + (\sigma_d^2)^2 \sum_{u \in \mathcal{U}} |x_{nu}|^2] x_{nu} \mathbf{a}_n^H \quad (8)$$

$$\begin{aligned} \mathbf{g}_{q_u}(\bar{\mu}_3) = 2 \frac{\partial \bar{\mu}_3}{\partial q_u^*} &= \frac{2}{MN} \sum_{n \in \mathcal{N}} [12(\sigma_d^2)^3 (\sum_{u \in \mathcal{U}} |x_{nu}|^2)^2 + \\ &3(\sigma_d^6 - 4(\sigma_d^2)^3) |x_{nu}|^4 + (9\sigma_d^4 \sigma_d^2 - 12(\sigma_d^2)^3) \\ &(|x_{nu}|^4 + 2|x_{nu}|^2 (\sum_{u \in \mathcal{U}} |x_{nu}|^2 - 1))] x_{nu} \mathbf{a}_n^H \end{aligned} \quad (9)$$

In step 5, the step-size of the Gradient method is calculated using two different approaches, where the $\alpha_m = 1$ is the fixed

approach similar to that of [9]. Moreover, to ensure algorithm convergence, dynamical step-sizing using Wolf line search rule is used as another approach, in which α_m must satisfied the following equation for $0 < \mu < 0.5$, and $\mu < \beta < 1$ [10]:

$$\begin{cases} f(\mathbf{Q}_m + \alpha_m \mathbf{D}_m) \leq f(\mathbf{Q}_m) + \mu \alpha_m [\bar{\nabla} f(\mathbf{Q}_m)]^T \mathbf{D}_m \\ \text{Tr}[(\bar{\nabla} f(\mathbf{Q}_m + \alpha_m \mathbf{D}_m))^T \mathbf{D}_m] \geq \text{Tr}[\beta (\bar{\nabla} f(\mathbf{Q}_m))^T \mathbf{D}_m] \end{cases}$$

where $\text{Tr}[\cdot]$ denotes the Trace of the matrix.

Eventually, the precoder matrix is updated in step 7, and the SER criterion of $\mathbf{Q}_{m+1} = \mathbf{Q}_{m+1} \sqrt{\frac{\bar{\mu}_1(0)}{\bar{\mu}_1(m+1)}}$ is also employed in step 8 or 10 to avoid SER failure, where $\bar{\mu}_1(m)$ denotes average IP at the m th iteration and computes from (3).

According to Algorithm 1, the suggested method can be categorized into the following four scenarios, where the performances are investigated in the next section:

- First scenario: the constraint of unchanged $\bar{\mu}_1$ is taken into account, and the SER criterion is used in step 8.
- Second scenario: the constraint of unchanged $\bar{\mu}_1$ is taken into account, and the SER criterion is used in step 10.
- Third scenario: the constraint of unchanged $\bar{\mu}_1$ is ignored, and the SER criterion is used in step 8.
- Forth scenario: the constraint of unchanged $\bar{\mu}_1$ is ignored, and the SER criterion is used in step 10.

IV. SIMULATION RESULTS

A GFDM system of $N = 128$, $M = 5$, and $K = 42$, which exploits root-raised cosine filter with the roll-off factor of 0.7, is considered. Simulation results for four different systems of (i) original GFDM, (ii) DFT-based precoded GFDM [8], (iii) iterative-based precoded GFDM with fixed step-sizing, and (iv) iterative-based precoded GFDM with dynamical step-sizing are provided using 64-QAM modulation and a Rayleigh channel of 15 taps. In the simulations, the terminating criterion of $\epsilon_{m+1} \leq 10^{-15}$ and $it_{max} = 1000$ are assumed.

The outcomes of simulations for the above four scenarios are shown in Fig. 1 when the variance of IP given in (4) is used as the objective function. At the left side of Fig. 1, the upper and lower parts represent the normalized variance increment over consecutive iterations, i.e., ϵ_{m+1} , for the iterative precoded GFDM with fixed and dynamical step-sizing, respectively. Further, the PSD, SER, and CCDF of PAPR are provided in respective order from right side to left. In the case of IP moments, since there is no dependent on $\bar{\mu}_1$, only the first two scenarios must be evaluated. Fig. 2 presents the simulation results for these two scenarios when the power of IP given in (5) is the objective function. Finally, to investigate the results for higher-order statistics, in Fig. 3, the performance of the suggested method in the terms of PAPR is compared in the second scenario for the second and third moments of IP. Note that the SD algorithm is considered in the case of Fig. 1, and the results of Fig. 2 and Fig. 3 are based on the CG method.

Eventually, regarding the results of simulations, the following points are important to be noticed:

- First, it is worth to be noted that the results provided for the DFT-based precoded GFDM are indeed those of the previous study in [8], and the results of iterative-based precoded GFDM with fixed step-sizing in the first

scenario of Fig. 1 are identical to those of [9]. Moreover, as outlined earlier, if $\bar{\mu}_1$ is considered constant, then the variance of (4) will be a shifted version of the power in (5). Therefore, as expected, similar results are obtained in the first and second scenarios of Fig. 1 and Fig. 2 for the power and variance.

- In the case of algorithm convergence, as seen, when the SER criterion is considered inside the iterative loop in the first and third scenarios, the decreasing behavior of ϵ_{m+1} for the iterative-based precoded system with the fixed step-sizing will stop at a specific iteration and the algorithm does not converge. In addition, this system does not meet the terminating criterion of $\epsilon_{m+1} \leq 10^{-15}$ in any of the scenarios, and the algorithm terminates by the constraint of $it_{max} = 1000$. However, when the Wolf line rule is used to dynamically adjust the step size, the algorithm will converge in all cases and the criterion of $\epsilon_{m+1} \leq 10^{-15}$ is satisfied before reaching the $it_{max} = 1000$. Indeed, using the Wolf line rule not only provides algorithm convergence in all scenarios, but also leads to lower design time over the fixed step-sizing approach due to convergence in lower iterations.
- Moreover, as seen from the results of the first scenario, the iterative precoding with fixed step-sizing experiences a remarkable performance drop in the terms of PAPR parameter, which is weaker than the DFT-based method of [8] and similar to that of the original GFDM. Overall, although the PAPR performance of this method improves in the other three scenarios; however, the method of iterative precoding with dynamical step-sizing outperforms in all scenarios and has superiority over the fixed approach. In addition, as demonstrated in Fig. 3, the results confirm that the proposed algorithm is valid and usable for the higher-order statistical parameters of the IP, and even the third moment of IP provides a slight improvement in PAPR compared to the second one.
- Besides, due to taking the constraint SER into account, the SER performance of both iterative precoding methods with fixed and dynamical step-sizing is desirable and within the range of the original GFDM system. In addition, in the case of OOB emissions, although the method of iterative precoding with fixed step-sizing experiences a little performance drop in the first scenario, however, both methods generally perform well and have PSD similar to the unprecoded and DFT-based precoded GFDM systems. Consequently, it is concluded that both methods of iterative precoding with fixed and dynamical step-sizing can design a linear precoder without any adverse effect on the SER and OOB emissions. However, the latter one provides advantages in algorithm convergence, high speed of precoder designing, and lower PAPR.

V. CONCLUSION

In this paper, an optimal design of linear precoder for PARP reduction of GFDM systems was addressed. A new algorithm, which consisted of four different scenarios and is based on minimizing various statistical parameters of the

Fig. 1. The simulation results for the variance of IP, (from the left to right: the curves of normalized variance increment, CCDF of PAPR, SER, and PSD).

instantaneous power using the Gradient-based optimization methods, was suggested. In each scenario, the step-size of the Gradient method is updated dynamically using the Wolf rule or constantly. The results confirm that the Wolf approach overcomes the non-convergence problem of existing studies and provides advantages in design speed, PAPR, SER, and OOB emission.

APPENDIX A

The n th moment of IP, $|s(n)|^2 = s(n)s^*(n)$, is as below:

$$\mathbb{E}\{|s(n)|^2\} = \mathbb{E}\left\{\left(\sum_{u \in \mathcal{U}} \sum_{u' \in \mathcal{U}} x_{nu} x_{nu'}^* d_u d_{u'}^*\right)^n\right\} \quad (10)$$

$$\begin{aligned} &= \mathbb{E}\left\{\left(\sum_{u \in \mathcal{U}} |x_{nu}|^2 |d_u|^2 + \sum_{\substack{u \in \mathcal{U} \\ u' \in \mathcal{U} \\ u' \neq u}} x_{nu} x_{nu'}^* d_u d_{u'}^*\right)^n\right\} \\ &= \mathbb{E}\{(A + B)^n\} \end{aligned}$$

Since the data symbols $\{d_u, u \in \mathcal{U}\}$ are zero-mean, i.i.d. complex numbers belonging to a M-ray symmetric modulation, $\mathbb{E}\{(d_u)^n\}$ will be zero for every n . Using this, it is concluded that $\mathbb{E}\{B\} = \mathbb{E}\{AB\} = \mathbb{E}\{A^2B\} = \mathbb{E}\{B^3\} = 0$. Consequently, the moments of IP are as below, where $\sigma_d^6 = \mathbb{E}\{|d_u|^6\}$.

$$\mathbb{E}\{|s(n)|^2\} = \mathbb{E}\{(A + B)^1\} = \mathbb{E}\{A\} = \sigma_d^2 \sum_{u \in \mathcal{U}} |x_{nu}|^2 \quad (11)$$

Fig. 2. The simulation results for the power of IP, (from the left to right: the curves of normalized power increment, CCDF of PAPR, SER, and PSD).

Fig. 3. PAPR comparison for the second and third moments of IP.

$$\begin{aligned} \mathbb{E}\{|s(n)|^4\} &= \mathbb{E}\{(A+B)^2\} = \mathbb{E}\{A^2\} + \mathbb{E}\{B^2\} \\ &= (\sigma_d^4 - 2(\sigma_d^2)^2) \sum_{u \in \mathcal{U}} |x_{nu}|^4 + 2(\sigma_d^2)^2 \left(\sum_{u \in \mathcal{U}} |x_{nu}|^2 \right)^2 \end{aligned} \quad (12)$$

$$\begin{aligned} \mathbb{E}\{|s(n)|^6\} &= \mathbb{E}\{(A+B)^3\} = \mathbb{E}\{A^3\} + 3\mathbb{E}\{AB^2\} \\ &= 4(\sigma_d^2)^3 \left(\sum_{u \in \mathcal{U}} |x_{nu}|^2 \right)^3 + (\sigma_d^6 - 4(\sigma_d^2)^3) \sum_{u \in \mathcal{U}} |x_{nu}|^6 \\ &\quad + (9\sigma_d^4\sigma_d^2 - 12(\sigma_d^2)^3) \sum_{u \in \mathcal{U}} |x_{nu}|^4 \left(\sum_{u' \in \mathcal{U}} |x_{nu'}|^2 - 1 \right) \end{aligned} \quad (13)$$

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