Low-Power, Low-Voltage Complex Gm-C Filter Structure With Self-Common-Mode Control

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Low-Power, Low-Voltage Complex G_m -C Filter Structure With Self-Common-Mode Control

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Abstract— This work develops a low-power and low-voltage differential G_m -C filter structure that effectively achieves self-common-mode control (SCC), including DC stabilization and common-mode (CM) rejection, without employing extra control circuitry. The structure relies upon an incorporation of voltage inverting amplifiers to make it inherently contain no CM positive feedback loops for DC stabilization, and to enable splitting of the core transconductors into pairs for CM signal rejection. A DC CM stability analysis reveals that stabilization of the SCC structure can be reached without any dedicated CM control circuitry. An analytical comparison on power consumption of a high-order lowpass G_m -C filter implemented using an inverter-based transconductor for the SCC structure and the same transconductor with a CM control network (the Nauta's technique) for the conventional structure indicates theoretical overhead power saving by over 50%. Furthermore, an even higher overhead power saving at over 70% can be achieved in the complex SCC G_m -C filter because no additional inverting amplifiers are required to eliminate CM positive feedback loops in the crossing transconductors for complexification. The impact of the inverting amplifiers on the noise and frequency characteristics as well as the compensation technique are outlined to enable design optimization. The SCC filter was verified via extensive simulations of a 5th-order 1.1-MHz elliptic complex filter in a 0.18- μ m CMOS process. As compared to the conventional filter counterpart with similar SNR (~ 63 dB) and in-band/out-of-band SFDRs (~ 52 dB/56dB), the proposed structure yields an overhead power saving by 70% with an improved figure-of-merit over 40% under a 1-V supply.

Index Terms— G_m -C filter, transconductor, Nauta, complex filter, CMOS, common-mode, stability, positive feedback

I. INTRODUCTION

A complex filter is one of the essential building blocks in modern wireless Low-IF receivers [1]–[13]. Its main function is to remove any image signal interference coming from adjacent radio channels after an RF-to-IF down conversion. The transconductor-capacitor (G_m -C) complex filter has been an attractive filtering solution due to its suitability for high-frequency and low-voltage operation stemming from the transconductor's simplicity and compactness [14]–[16]. Under a very low-voltage supply V_{DD} , it becomes increasingly challenging to implement a MOS transconductor with the capability to reject common-mode (CM) signals and prevents

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(a) Differential circuit, G = differential-mode (DM) transconductance

(b) CM equivalent circuit indicating an inherent positive feedback loop, G_c = CM transconductance, g_{oc} = CM output conductance

Fig. 1. Conventional differential G_m -C filter (convention: output current flow into the transconductor with a positive voltage swing)

instability arising from CM positive feedback loops inside the conventional differential G_m -C filter structure as shown in Fig. 1(a) (there are four head-to-tail CM positive feedback loops shown in Fig. 1(b)).

A low-power, low-voltage filter design typically makes use of a pseudo-differential transconductor with CM control circuitry based upon common-mode feedback (CMFB), common-mode feedforward (CMFF) or common-mode/differential-mode feedforward (CM/DM-FF) techniques. In [17], a CMFB technique was used to increase the transconductor's CM output conductance, g_{oc} , to lower the CM loop gain so as to prevent instability as well as to set up a DC bias level of the source-degenerated *pseudo*-differential transconductor. This CMFB technique usually demands high power consumption and silicon area due to the deployment of error feedback amplifiers. Alternatively, the filters in [6], [18–21] utilize CMFF and CM/DM-FF techniques which essentially minimize the CM loop gain by making the CM transconductance, $G_c \approx 0$. The transconductor with CMFF can double power consumption and transistor area [18–20], making it rather inefficient in terms of transconductance-power ratio. Moreover, such CMFF technique also requires another CMFB loop for DC bias setup. A more power-efficient solution presented in [6] utilizes both CMFB and CMFF only at the biquad's integrating nodes (outside the transconductor cell). The transconductors with CM/DM-FF in [21–23] further improve efficiency since their cross-forward structure simultaneously provides CM control and doubles DM transconductance. However, its operation is heavily dependent on transistor matching.

Fig. 2. Proposed differential G_m -C filters with basic self-CM-control (SCC) mechanism which inherently contain no CM positive feedback loop.

The recent current-recycling technique [24–25] offers a compact and low power G_m -C complex filter with a very competitive figure-of-merit. This technique, however, might not be applicable to a general G_m -C filter such as that realized from a LC -ladder prototype. Alternatively, the Nauta's inverter-based transconductor has been widely employed in G_m -C filters intended for high-frequency, highly linear and large-dynamic range applications under a low-voltage supply [7], [26–37]. However, the CM control network inside the Nauta's transconductor, which equivalently sets $g_{oc} = G_c$ (Fig. 1(b)), is responsible for nearly half of the filter's total power consumption and transconductor area.

To alleviate the aforementioned problems, a power-efficient low-voltage LC -ladder-based differential G_m -C complex filter structure is proposed. The structure is stabilized systematically by removing all the CM positive feedback loops typically found in the conventional structure. Without employing any CMFB or CMFF technique to maximize g_{oc} or minimize G_c , the filter can be inherently CM stabilized, thereby achieving significant power reduction.

Section II.A introduces the low-voltage G_m -C filter with basic self-CM control (SCC) that can be directly realized from a LC -ladder lowpass filter structure. The complete SCC filter structure with CM rejection ability is developed in Section II.B as a more general filter structure. An analysis on the loop gains and nodal conductances is carried out in Section II.C, D for the filter's DC CM stability and CM gain inspection. The filter's power efficiency is assessed in Section II.E with an overhead-power ratio between the proposed and the conventional based on the Nauta's transconductor. Section III.A illustrates a low-power 5th-order SCC complex elliptic filter. An overhead-power comparison of the complex filter is estimated in Section III.B. Noise performance and frequency response influenced by non-idealities of the proposed structure are also studied with a frequency compensation method presented. The practical feasibility of the SCC filter is verified

in Section IV with extensive simulations of the 5th-order 1.1-MHz complex elliptic filter in a standard 0.18- μ m CMOS process.

II. G_m -C FILTER STRUCTURE WITH SELF COMMON-MODE CONTROL (SCC)

A. Basic SCC Structure with DC Stabilization

Without loss of generality, the basic self-CM-control (SCC) structure will be developed from the LC -ladder-based 5th-order G_m -C filter of Fig. 1(a). This is illustrated with the differential G_m -C filter in Fig. 2(a). Without disturbing the transfer function of the conventional filter, inverting voltage amplifiers are inserted only at the *even*th nodes (even type) or only at the *odd*th nodes (odd type). The inverting voltage amplifier can be simply constructed from two transconductors (the bottom of Fig. 2(a)). Since there are sign alterations of $-V_3$, $-V_4$ and $-V_2$, $-V_3$ in Fig. 2(a) as compared to the node voltages in the conventional filter, this basic SCC filter thus necessitates cross-over connections of C_{13} and C_{35} .

The key purpose for the inclusion of the inverting amplifiers is to eliminate CM positive feedback loops normally existing in the conventional differential filter structure. This can be explained by considering the CM equivalent circuits of the SCC G_m -C filter structures. As shown in Fig. 2(b), the associated CM feedback loops (dash circles in the figures) are all in a negative feedback manner. Thus, the proposed structure requires no additional circuitry to stabilize the CM voltages inside each of the transconductors. This significantly helps save power consumption and silicon area compared to the conventional structure. The extra power and area consumptions added to the core transconductors, G , are mainly from the inverting voltage amplifiers.

With the inclusion of the inverting amplifiers, this SCC structure might look similar to the biquadratic-based G_m -C filter in [38]. However in this case, there is no requirement to load the output of each transconductor with a small adjustable

(a) Differential implementation with $G_{c\pm} = (1 \pm \varepsilon)G/2$, $G_{c\mp} = (1 \pm \delta)G/2$ (b) CM equivalent circuit of (a) with $G_{c\pm} = (1 \pm \varepsilon)G_c/2$, $G_{c\mp} = (1 \pm \delta)G_c/2$ Fig. 3. Proposed self-CM-control (SCC) G_m -C filter with CM Rejection.

1/G resistor in order to avoid using a CMFB circuit and maintain CM stability. Moreover, the basic SCC structure can be readily applied to more general G_m -C filters – especially those realized from well-established, least-sensitive, doubly terminated LC-ladder filters.

B. SCC Filter with CM Rejection

By inspecting the circuits in Fig. 2, it is clear that the DC CM gain of the basic SCC lowpass filter is identical to that of the DC DM gain at 1/2, and this inevitably renders a DC common-mode rejection ratio (CMRR) of unity. In order to maintain the DC-stabilizing feature and simultaneously allow CM rejection, the basic structures in Fig. 2(a) are modified into the *complete* SCC structure in Fig. 3(a) without disturbing the original DM response with the modified transconductors $G_{c\pm} = (1 \pm \varepsilon)G/2$ and $G_{c\mp} = (1 \pm \delta)G/2$. Note that ε and δ are of the same polarity and $0 < (\varepsilon, \delta) \leq 1$ or $-1 \leq (\varepsilon, \delta) < 0$. This complete SCC structure, still preserves the original low-power property where the total transconductance value of the core transconductors and the inverting amplifiers remains unchanged.

When $(\varepsilon, \delta) \neq 0$, it can be seen from the corresponding CM equivalent circuit in Fig. 3(b) that the associated CM feedback loops still remain in a negative feedback manner. Moreover, there is a certain degree of CM current cancellation between the shaded transconductor pairs, $G_{\delta+}$ and $G_{\delta-}$ as well as the non-shaded pairs, between $G_{\varepsilon+}$ and $G_{\varepsilon-}$, due to the presence of the inverting amplifiers. By inspecting the DM and CM signal excursions inside the $G_{c\pm}$ pairs and the $G_{c\mp}$ pairs, it suggests

that these transconductor pairs basically resembles CM/DM-FF operation (with shared inverting amplifiers) at the structural level as compared to the circuit-level implementation found in [21-23]. Such similarity renders the forward CM transconductance from j^{th} node to $(j+1)^{\text{th}}$ node, $G_{c\text{fwd}} = \delta G_c$, and the backward CM transconductance from j^{th} to $(j-1)^{\text{th}}$ node, $G_{c\text{bkwd}} = \varepsilon G_c$.

If $\varepsilon = \delta = 0$, one may hope to enjoy a perfect CM rejection. However, since all the filter's internal nodes (except the first and last nodes) becomes a DC CM open circuit (zero conductance) and the filter stays right at the borderline between DC stable and unstable regions. This issue will become clearer with the CM analysis presented in the following subsection. Specifically for $\varepsilon = \delta = +1$ (-1), the transconductors $G_{\varepsilon-}$, $G_{\delta-}$ ($G_{\varepsilon+}$, $G_{\delta+}$) would vanish and the inverting amplifiers associated with them become redundant. Thus the structure is simply reduced to the original *even*-type (*odd*-type) basic SCC filter of Fig. 2(a). Therefore the complete SCC filter is a more general structure that covers both *odd*- and *even*-type basic SCC filters.

C. DC and Common-Mode Stability Inspection

Fig. 4(a) shows the DC CM equivalent circuit of the proposed SCC filter structure for DC stability verification via an analysis of the DC loop gains. To enable the use of the analysis for DC stability discussion on the conventional filters with other CM control techniques, the CM output conductance g_{oc} of each G_c (except the input G_{ci} for ease of analysis) is also included. The factor γ represents the inverting amplifier's voltage gain which may be different from -1 . The parameters G_j and $\hat{G}_j$ are the DC CM conductance *looking from the j^{th} node to the right side and the left side, respectively*. According to these conductance definitions, it follows that $\hat{G}_1 = G_N = 2g_{oc} + G_c$, and G_j for $j = N-1, N-2, \dots, 1$ can be calculated recursively as

$$G_j = 2g_{oc} - \frac{G_c^2}{4G_{j+1}} \left[(1+\gamma) \pm \delta(1-\gamma) \right] \left[(1+\gamma) \mp \varepsilon(1-\gamma) \right]. \quad (1)$$

Similarly, $\hat{G}_j$ for $j = 2, 3, \dots, N$ can be expressed as

$$\hat{G}_j = 2g_{oc} - \frac{G_c^2}{4G_{j-1}} \left[(1+\gamma) \pm \varepsilon(1-\gamma) \right] \left[(1+\gamma) \mp \delta(1-\gamma) \right]. \quad (2)$$

Similar to the analysis in [39], the *effective* DC loop gain LG_j at the j^{th} node, for $j = 1, 2, \dots, N$ in Fig. 4(a), can be derived in terms of the j^{th} conductances from the two possible feedback formations in Fig. 4(b), and their corresponding current feedback system in Fig. 4(c) as

$$LG_j = 2 - (G_j + \hat{G}_j) / 2g_{oc}. \quad (4)$$

It is known that a negative feedback loop gain indicates stability regardless of its magnitude. On the other hand, a positive loop gain can indicate stability only if its magnitude is less than unity. For the case of multiple loops such as the CM DC equivalent circuit of Fig. 4(a), all the loop gains LG_j must be less than unity to indicate overall DC stability, i.e., $LG_j < +1$ for all $j = 1$ to N .

Fig. 4. DC CM equivalent circuit of the SCC filter and loop gain calculation at node j for $j = 1, 2, \dots, N$.

It is also worth exploring the *total* grounded DC CM conductance, G_{Tj} at the j^{th} node as an alternative to LG_j for stability inspection. From the feedback system of Fig. 4(c), the loop gain, LG_j can be directly linked with G_{Tj} by

$$G_{Tj} = i / V_{cj} = 2g_{oc}(1 - LG_j) \quad (5)$$

for $j = 1, 2, \dots, N$. This total conductance G_{Tj} of Fig. 4(a)–(b) can also be seen from the circuit perspective, as the sum of two conductances G_j and $\hat{G}_j$ subtracted by the excess (common) output conductance $2g_{oc}$, i.e.

$$G_{Tj} = G_j + \hat{G}_j - 2g_{oc} \quad (6)$$

for $j = 1, 2, \dots, N$. With the LG_j expression in (4), the conductance G_{Tj} relations in (5) and (6) are directly related. The filter is DC stable if this total conductance is positive, i.e., $G_{Tj} > 0$, which is the same condition as the stable feedback loop condition of $LG_j < 1$ and this can also be seen from (5). Thus, the condition “ $LG_j < 1$ ” or “ $G_{Tj} > 0$ ” are equivalent for stability inspection. In the following discussion, only G_{Tj} will be used for stability assessment. The nodal conductance G_j and $\hat{G}_j$ from (1) and (2) can be summarized in Table I with $\gamma = +1$ for the conventional G_m - C filter structure of Fig. 1 and with $\gamma = -1$ for the SCC G_m - C filter structure of Fig. 3. Note also that if $\varepsilon = \delta = \pm 1$ for $\gamma = -1$, the SCC structure simply converges to the basic SCC G_m - C filter structures in Fig. 2.

Fig. 5. A complete Nauta's inverter transconductor with a common-mode control network (in gray) [26]–[28]

TABLE I CM nodal conductances G_j and $\hat{G}_j$ used for G_{Tj} calculation, $G_{Tj} = G_j + \hat{G}_j - 2g_{oc}$ for $j = 1, 2, \dots, N$

$\gamma = +1$ (conventional)	$G_j = 2g_{oc} - \frac{G_c^2}{G_{j+1}}$ for $j = 1, 2, \dots, N-1$ $\hat{G}_j = 2g_{oc} - \frac{G_c^2}{\hat{G}_{j-1}}$ for $j = 2, 3, \dots, N$
$\gamma = -1$ (SCC)	$G_j = 2g_{oc} + \frac{\varepsilon\delta G_c^2}{G_{j+1}}$ for $j = 1, 2, \dots, N-1$ $\hat{G}_j = 2g_{oc} + \frac{\varepsilon\delta G_c^2}{\hat{G}_{j-1}}$ for $j = 2, 3, \dots, N$

$\hat{G}_1 = G_N = 2g_{oc} + G_c$. If $\varepsilon = \delta = +1$ (-1), the SCC becomes the *even-* (*odd-*) type basic SCC.

Since the pseudo-differential inverter-based transconductor (M_1 – M_4 in Fig. 5) will be the transconductor choice for the implementation and verification of the SCC filters in Sections II and III, it is instructive to use the CM analysis to examine DC stability in the conventional G_m - C filters of Fig. 1 based on such a transconductor. This complementary inverter-based transconductor is a core part of the complete Nauta's transconductor [26]–[28] in Fig. 5 where a CM control network is also included. By using Table I, under the condition $\gamma = +1$ (“ γ ” amplifiers associated with G_{c-} and $G_{c\delta}$ in Fig. 4 become redundant), $\varepsilon = \delta = +1$ and $g_{oc} \approx 0$ as no CM control network is added and the transconductor ideally possesses a zero output conductance, with $N = \text{odd}$ this renders $\hat{G}_j = G_j \approx -G_c$ and $+G_c$ for *even*th and *odd*th nodes respectively. This consequently produces $G_{Tj} = -2G_c$ (*even*th nodes) and $+2G_c$ (*odd*th nodes) indicating an unstable filter. It is also important to note that for $\gamma = +1$ and $g_{oc} \approx 0$, even a perfect CM-rejection transconductor (i.e., $G_c = 0$) could only give $G_{Tj} = 0$ where DC stability cannot be guaranteed. The conventional filter can become stable by adopting the CM control network of the Nauta's transconductor (Fig. 5), which makes $g_{oc} = G_c$. Note that the Nauta transconductor's core possesses the same value of DM and CM transconductances, i.e., $G = G_c$ while its CM control network provides a zero DM transconductance with a CM transconductance of G_c . To simplify the analysis description, we firstly choose to omit the CM control network at the first and last nodes, i.e., by setting $g_{oc} = 0$ for $j = 1$ and N , while still keeping $g_{oc} = G_c$ for $j = 2, 3, \dots, N-1$. Since $\hat{G}_1 = G_N \approx +G_c$, the first row of Table I gives $\hat{G}_j = G_j \approx +G_c$ for $j = 2, 3, \dots, N-1$, and $G_1 = \hat{G}_N = -G_c$ and $G_{Tj} = 0$ for all $j = 1, 2, \dots, N$. This indicates that the above g_{oc} setting yields a borderline condition between DC stable and unstable

filter. By also setting $g_{oc} = G_c$ for $j = 1$ and N as employed in the original Nauta's G_m -C filter [7], [26-29], the DC stability can thus be guaranteed. Although this DC stabilization technique has been demonstrated to be very effective, especially under a very low-voltage supply, it is at the expense of high power and large silicon area.

In case of the SCC filter with no extra CM control circuitry in Fig. 3(a) with $\gamma = -1$, applying the second row of Table I to (6) renders $G_{Tj} = 2g_{oc} + \varepsilon\delta G_c^2 (G_{j+1}^{-1} + \hat{G}_{j-1}^{-1})$ for $j = 2, \dots, N-1$ where $G_{T1} = G_1 + G_c$ and $G_{TN} = \hat{G}_N + G_c$. Under a typical scenario where $g_{oc} \approx 0$, this simplifies G_j and $\hat{G}_j$ in Table I to $G_j \approx +\varepsilon\delta G_c^2 G_{j+1}^{-1}$ for $j = 1, 2, \dots, N-1$ and $\hat{G}_j \approx +\varepsilon\delta G_c^2 \hat{G}_{j-1}^{-1}$ for $j = 2, 3, \dots, N$. Since $\hat{G}_1 = G_N \approx +G_c$, this further yields G_j and $\hat{G}_j$ with $N = \text{odd}$, to $G_j = \hat{G}_j \approx +\varepsilon\delta G_c$ with $G_{Tj} \approx +2\varepsilon\delta G_c$ for $j = \text{even}$ and $G_j = \hat{G}_j \approx +G_c$ with $G_{Tj} \approx +2G_c$ for $j = \text{odd}$. However if $N = \text{even}$, $G_j = \hat{G}_k \approx +\varepsilon\delta G_c$ for $j = \text{odd}$, $k = \text{even}$ and $G_j = \hat{G}_k \approx +G_c$ for $j = \text{even}$, $k = \text{odd}$ and this simply renders $G_{Tj} \approx (1 + \varepsilon\delta)G_c$ for all j . Therefore as long as ε and δ are non zero and of the same polarity, i.e., $0 < (\varepsilon, \delta) \leq +1$ or $-1 \leq (\varepsilon, \delta) < 0$, all of the conductances G_j , $\hat{G}_j$ and G_{Tj} are positive ensuring that the SCC filter is always stable. As previously noted with $\varepsilon = \delta = +1$ or -1 , the SCC filters simply turn into the basic SCC filters without any CM rejection as in Fig. 2(a). Moreover, with the condition $g_{oc} \approx 0$, these basic SCC filters possess $G_1 = \hat{G}_N \approx +G_c$, $G_j = +G_c$ for $j = 2, \dots, N$, and $\hat{G}_j = +G_c$ for $j = 1, \dots, N-1$ resulting in $G_{Tj} \approx +2G_c$ for all j which is higher than those from the SCC filters at $j = \text{even}$ implying a more stable condition offered by the basic SCC filters. Conclusively, the inclusion of the inverting amplifiers in the SCC filters fundamentally makes all of the total node conductances positive. As a result, these SCC filter structures are inherently DC stable.

When ε, δ are close to zero with $N = \text{odd}$, then G_{Tj} also approaches zero for $j = \text{even}$. This simply pushes the filter very close to the unstable region. Although a CM control network can be included (i.e. to set $g_{oc} \gg 0$) to guarantee stability, this may inevitably incur significant cost to power consumption. A more practical approach is to keep the condition $0 < (\varepsilon, \delta) \leq +1$ or $-1 \leq (\varepsilon, \delta) < 0$ and find a suitable trade-off between the stability and CM rejection for a particular filter implementation. This will be discussed in the next subsection.

D. DC CM Gain Analysis and Design Trade-off

The DC CM voltage at each stage, V_{Cj} of the SCC Filter in Fig. 4(a) for $j = 2, 3, \dots, N$ can be expressed for $N = \text{odd}$ (even) as

$$V_{Cj} = V_{Cj-1} \frac{G_c}{2} (\gamma(1-\delta) + (1+\delta)) G_j^{-1}, \quad j = \text{even (odd)} \quad (7a)$$

$$V_{Cj} = V_{Cj-1} \frac{G_c}{2} ((1-\delta) + \gamma(1+\delta)) G_j^{-1}, \quad j = \text{odd (even)}. \quad (7b)$$

If a perfect inverting amplifier is assumed, i.e., $\gamma = -1$, the magnitude of these DC CM voltages become

$$|V_{Cj}| = |V_{Cj-1}| \delta G_c G_j^{-1}. \quad (7c)$$

for $j = 2, 3, \dots, N$ and $|V_{C1}| = |V_{ic}| G_{ci} G_{T1}^{-1}$. Following the G_j and $\hat{G}_j$ values of the SCC filter with $g_{oc} \approx 0$ in Section II.C, we have $G_{T1} = 2G_c$ for $N = \text{odd}$ (only an *odd*th-order filter is considered due to its popularity over its *even*th-order counterpart). If $G_{ci} = G_c$, the DC CM voltage at first node V_{C1} from the DC CM input voltage V_{ic} then becomes $V_{C1} = V_{ic}/2$ for $N = \text{odd}$. Therefore the N^{th} -order filter's CM DC gain, A_{cm} can be found by using (7c) recursively as

$$A_{cm} = \frac{V_{oc}}{V_{ic}} = \frac{1}{2} \left(\frac{\delta}{\varepsilon} \right)^{\left(\frac{N-1}{2} \right)} \quad \text{for } N = \text{odd}, N \geq 3 \quad (8)$$

The above equation (8) indicates that a small δ/ε ratio is required for a good CM rejection. As already shown in Section II.C, with $N = \text{odd}$, the product of $\varepsilon\delta$ should be sufficiently large to obtain a positively large G_{Tj} ($\approx +2\varepsilon\delta G_c$) at $j = \text{even}$, for a high degree of DC stability. As a consequence, there is a design trade-off for the *odd*th-order SCC filter between the stability (i.e., G_{Tj} values) and the DC CM gain (A_{cm}), by balancing ε and δ to provide a stable filter with CM rejection that meets the required specifications.

E. Overhead Power Comparison: Inverter-based Pseudo-Differential Transconductor Case

In order to quantify the power saving offered by the SCC structures, a comparison between the *overhead* power consumption required for CM stabilization in an N^{th} -order lowpass G_m -C filter using the proposed and conventional structures is given. Due to its class-AB operation and versatility for a very low supply voltage, the complementary inverter-based pseudo-differential pair is chosen as the core transconductor G of the filters. In the conventional structure, each of these individual transconductors is equipped with the CM control network which resembles a complete Nauta's transconductor [7], [26]–[30]. In the SCC structures of Fig. 2(a) and Fig. 3(a), the same core transconductor is also employed to implement the inverting amplifiers.

Since the relation between the quiescent supply current and transconductance is approximately linear in the inverter-based transconductor, the *overhead* power comparison can be directly related to the *added* transconductance for CM stabilization and control. In the conventional N^{th} -order lowpass G_m -C filter using the Nauta's type CM network, it is straightforward to show that the added transconductance is $G_{Coh} = (2N+1)G$. In the SCC N^{th} -order filter counterparts, the added transconductance is from the inverting amplifier, which requires 2α basic units (G or $G/2$) of the pseudo-differential transconductors for each implementation, e.g. $\alpha = 1$ as in Fig. 2(a) and Fig. 3(a). For the basic *even*-type SCC filter structure similar to the left one in Fig. 2(a), its G_{Coh} is equal to $\alpha N \cdot G$ and $\alpha(N-1)G$ for $N = \text{even}$ number and *odd* number, respectively. While for the basic *odd*-type SCC structure (the right one in Fig. 2(a)), the G_{Coh} becomes $\alpha N \cdot G$ and $\alpha(N+1)G$

Fig. 6. Overhead power ratio (OPR) between the conventional and the proposed self-stabilized for the N^{th} -order elliptic lowpass filter with $\alpha = 1$.

respectively for $N = \text{even}$ and odd . Whereas for the SCC structure as in Fig. 3(a), the G_{Coh} is $\alpha N \cdot G$ for even and odd N . Therefore, the overhead power ratio, OPR , between the conventional and the basic SCC or the SCC filters is given by

$$OPR = (2N + 1) / (\alpha(N + c)), \quad (9)$$

where $c = 0$ for the SCC structure with $N = \text{even}$ and for both the basic *even*- and *odd*-type SCC structures. But for the basic SCC structures with $N = \text{odd}$, c becomes -1 and $+1$ for the *even*- and *odd*-type configurations, respectively. Specifically for a 5th-order lowpass filter and $\alpha = 1$, equation (9) gives $OPR = 11/4$ ($11/6$) = 2.75 (1.83) for the *even*- (*odd*-) type basic SCC filter in Fig. 2(a) and $OPR = 11/5 = 2.2$ for the SCC filter in Fig. 3(a). The equation also suggests that for a very high order filter, $N \gg 1$, the OPR converges to 2 which is equivalent to 50% overhead-power saving. This implies that, by using the inverter-based pseudo-differential circuit as the core transconductor, the conventional filter structure essentially requires twice as much overhead power as that required in the SCC structures for CM stabilization. Plot of the OPR 's versus N using (9) is displayed in Fig. 6.

III. LOW-POWER DIFFERENTIAL COMPLEX GM-C FILTER USING SELF-CM-CONTROL STRUCTURE

A. Self-CM-Controlled (SCC) Differential Complex G_m -C Filter with Common-Mode Rejection

In the conventional complexification technique for differential G_m -C filters, appropriate pairs of crossing transconductors are introduced between I and Q lowpass filter (LPF) sections. These include one crossing transconductor pair, G_{xj} s ($= \omega_0 C_j$), for each grounded capacitor C_j (as also shown in Fig. 7(a)), and four crossing transconductor pairs, G_{xij} s ($= \omega_0 C_{ij}$), for each floating capacitor C_{ij} , where ω_0 is the center radian frequency of the complex filter [5], [7], [29-30], [40]. Since these G_{xi} s and G_{xij} s are connected in a head-to-tail fashion similar to those core transconductors, G_s , inside the I and Q LPFs, the complexification structure inevitably introduces more CM positive feedback loops. In order to suppress the resulting CM positive feedback effect and hence stabilize the complex filter, each of these G_{xi} s and G_{xij} s must require a CM control network in addition to those already needed in the I and Q LPF sections. This, as a result, significantly increases power and silicon area.

Fig. 7. Implementation for complexification with G_{xj} : (a) conventional (b) employing the SCC feedback structure from Fig. 3(a) with $G_{xj\pm} = (1 \pm \varepsilon_x)G_{xj}/2$, $G_{xj\delta\pm} = (1 \pm \delta_x)G_{xj}/2$.

Fig. 7(b) shows a differential implementation example for the crossing G_{xj} at any node j with grounded C_j where the I and Q sections utilize the proposed SCC filter structure from Fig. 3(a) and each G_{xj} in Fig. 7(a) has been split into a pair of $G_{xj\pm}$ and $G_{xj\delta\pm}$ as similar to the SCC structure in Fig. 3(a) with $G_{xj\pm} = (1 \pm \varepsilon_x)G_{xj}/2$, $G_{xj\delta\pm} = (1 \pm \delta_x)G_{xj}/2$. Co-operation with two half-sized inverting amplifiers already existed in the I and Q sections gives two necessary differential negative feedback loops for complexification. Since each of these loops essentially involves one inverting amplifier, this thus prevents CM positive feedback loops inevitably introduced in the conventional differential complex filter (Fig. 7(a)). Regarding to a CM signal, the second row of Table I can also be applied to Fig. 7(b) for calculating the CM conductance $G_{jx,I(Q)}$ looking into the complexification $G_{xj\pm}$, ($G_{xj\delta\pm}$) network from the I (Q) section at j^{th} node as influenced by the $G_{Txj,Q(I)}$ from the opposite Q (I) section, i.e., $G_{jx,I(Q)} \approx +\varepsilon_x \delta_x G_{xj}^2 G_{Txj,Q(I)}^{-1}$. The offsets ε_x and δ_x can thus be assigned to modify the resultant total CM nodal conductance, G_{Tj} on both I and Q sections – namely $G_{Tj,I}$ and $G_{Tj,Q}$. This complexification technique for G_{xj} 's can be directly applied for G_{xij} 's (split into $G_{xij\pm}$ and $G_{xij\delta\pm}$ with $G_{xij\pm} = (1 \pm \varepsilon_x)G_{xij}/2$, $G_{xij\delta\pm} = (1 \pm \delta_x)G_{xij}/2$) and this eventually renders a fully-functional low-power 5th-order SCC complex filter as shown in Fig. 8 (illustrated with $\varepsilon_x, \delta_x = 0$).

The ideal inverting amplifiers with perfect matching among $G_{xj}/2$'s and $G_{xij}/2$'s ($\varepsilon_x, \delta_x = 0$) would result in a complete CM signal cancellation from I section to Q section and vice versa, i.e. $G_{xj}/2$'s and $G_{xij}/2$'s become open circuit for CM signal. This would basically isolate I and Q sections from each other yielding the complex filter's CM response and stability characteristic that closely converges to those of the two separate stable SCC LPFs. Thus, it follows that the complex filter of Fig. 8 requires neither extra CM control network nor additional inverting amplifiers, apart from the existing ones in the I and Q LPF sections. As a consequence, the proposed SCC filter structure of Fig. 3(a) is even more efficient in terms of power and area consumptions when utilized for complex filter implementations.

An impact by the inverting amplifier's finite bandwidth on the imbalances of the I and Q signals from the required quadrature phase shift may be investigated by considering only the *even* nodes for simplicity. A differential-mode voltage at the node j (for $j = 2, 4 \dots$) of the combined I - Q sections can be given by

$$V_{j,I} = \left[(-V_{j-1,I}) (-G_{\delta+} + \gamma G_{\delta-}) + V_{j+1,I} (-G_{\delta+} + \gamma G_{\delta-}) \right] / s(C_j/2) + \dots + (-G_{xj\delta-} + \gamma G_{xj\delta+}) (+V_{j,Q}) / s(C_j/2) \quad (10a)$$

$$V_{j,Q} = \left[(-V_{j-1,Q}) (-G_{\delta+} + \gamma G_{\delta-}) + V_{j+1,Q} (-G_{\delta+} + \gamma G_{\delta-}) \right] / s(C_j/2) + \dots + (-G_{xj\delta+} + \gamma G_{xj\delta-}) (-V_{j,I}) / s(C_j/2). \quad (10b)$$

The above equations suggest that the perfect quadrature phase relations between the I and Q voltages can be realized by setting $G_{xj\delta+(-)} = G_{xj\delta+(-)}$ with $\varepsilon_x = -\delta_x$ (including $\varepsilon_x, \delta_x = 0$ as in Fig. 8) because the factors γ associated with the inverting amplifier's response are perfectly balanced within the I and Q sections. Since the same conclusion applies for the *odd* nodes, it can be deduced that the complex filter's balanced characteristic between I and Q sections can be maintained regardless of the inclusion of the inverting amplifiers in the differential SCC complex filter. The crossing $G_{xj\delta\pm}$, $G_{xj\delta\pm}$ network renders a negative CM conductance $G_{jx,I(Q)}$ for $\varepsilon_x = -\delta_x$ (non-zero values). A similar recursive calculation on the second row of Table I suggests that if this negative $G_{jx,I(Q)}$ is applied to the *odd* nodes only, it would effectively modify the original G_{Tj} 's of the *simple* SCC complex filter (with $\varepsilon_x = \delta_x =$

0) such that the G_{Tj} 's at $j = \text{odd}$ (*even*) is decreased (increased) from $+2G_c$ ($+2\varepsilon\delta G_c$). This in turn helps reduce the difference of the *resultant* G_{Tj} within the SCC complex filter and bring the quiescent CM DC voltage levels (including the bias voltages) among all the nodes closer to each other. However, to ensure the SCC complex filter's stability with this $\varepsilon_x = -\delta_x$ condition, the magnitude of $\varepsilon_x \cdot \delta_x$ has to be sufficiently small so that all the modified G_{Tj} 's remained positive.

B. Overhead Power Consumption: An Inverter-Based Pseudo-Differential Transconductor Study Case

By using the overhead power ratio OPR , the complex G_m - C filters based on the conventional and SCC structures are compared in terms of the added power consumption necessary to achieve the CM DC stabilization. Since the complex filter with the elliptic characteristic is the main focus in this work, only the ratio analysis for an odd-order filter is given. In the conventional structure, it requires that the CM stabilization network of the Nauta's type be employed for all the core and the crossing transconductors G 's, G_{xi} 's, G_{xij} 's [7], [29-30]. By contrast, no additional circuit is required in the SCC filter, since the embedded inverting amplifiers readily provide CM negative feedbacks for both internal and cross-coupling loops.

Fig. 8. Proposed low-power SCC G_m - C complex filter structure with CM rejection where $G_{\delta\pm} = (1 \pm \varepsilon)G/2$, $G_{\delta\pm} = (1 \pm \delta)G/2$, $\alpha = 1$ and $\varepsilon_x, \delta_x = 0$.

Fig. 9. Overhead power ratio by the SCC N^{th} -order elliptic complex filter from (28) (N = order of the I , Q lowpass sections) with $\alpha = 1.0, 1.5$.

Similar to the analysis in Section II.E, the overhead power comparison is determined by the added transconductance for CM stabilization. For complexification, since each of the N grounded capacitors requires two crossing transconductors G_{xi} and each of the $(N-1)/2$ floating capacitors needs eight crossing transconductors G_{xij} , the OPR can be written as

$$OPR = \left(2(2N+1) + \left(\sum_{i=1}^N 2 \frac{G_{xi}}{G} + \sum_{j=1}^{(N-1)/2} 8 \frac{G_{x(2j-1)(2j+1)}}{G} \right) \right) / (\alpha 2N) \quad (11)$$

with $G_{xi} = \omega_b C_i$ ($i = 1, \dots, N$), $G_{x(2j-1)(2j+1)} = \omega_b C_{(2j-1)(2j+1)}$ ($j = 1, \dots, (N-1)/2$). By putting the average values of G_{xi} and $G_{x(2j-1)(2j+1)}$ as $\bar{G}_x$ and $\bar{G}_{xf}$, respectively, (11) is simplified to

$$OPR = \left(2(2N+1) + \left(2N \frac{\bar{G}_x}{G} + \frac{(N-1)}{2} 8 \frac{\bar{G}_{xf}}{G} \right) \right) / (\alpha 2N) \quad (12)$$

To investigate how the OPR is dependent on the complex filter's bandwidth ($2\omega_b$) and center frequency (ω_0), the I/Q LPF section's bandwidth (ω_b) can be approximately related to the core transconductance, G , and the average grounded capacitance, $\bar{C} (= \bar{G}_x / \omega_0)$ by $\omega_b \cong G / \bar{C}$. Together with the average value of the floating capacitance $\bar{C}_{xf} (\cong \bar{G}_{xf} / \omega_0)$, the OPR in (11) can alternatively be expressed as

$$OPR \cong \left(2(2N+1) + (\omega_b / \omega_0)^{-1} \left(2N + \frac{(N-1)}{2} 8 (\bar{C}_{xf} / \bar{C}_x) \right) \right) / (\alpha 2N) \quad (13)$$

For a special case of the elliptic complex filter with $\omega_b/\omega_0 = 1/2$ and $\bar{C}_{xf}/\bar{C}_x = 1/4$ as in the design in Section IV, both (12) and (13) yield $OPR = 5/\alpha$, e.g., the OPR s are 5.0 and 3.3 for $\alpha = 1.0$ and 1.5, respectively. Note that, the factor α plays a vital role in the filter's noise performances as will be analyzed in Section III.D. This specifically indicates that based on the inverter-based pseudo-differential transconductors, the

Fig. 10. Single-ended biquadratic structure for noise investigation as impacted by the inverting amplifier with $G_{F\delta\pm} = (1 \pm \delta)G_F/2$, $G_{F\delta\pm} = (1 \pm \delta)G_F/2$ and $G_{inv} = \alpha_F G_F$, $i_{ninv(A,B)}^2 = \alpha_F^2 i_{nG_F}^2 / 2$

conventional complex filter requires at least three times as much overhead power as that required in the SCC complex filter for CM stabilization. The significant power saving in the proposed complex filter simply stems from the fact that there is no overhead power required for those crossing transconductors G_{xi} , G_{xij} .

Plots of the OPR s using (13) are illustrated in Fig. 9 showing how OPR is related to the filter's order N ($=$ odd) and the bandwidth/center frequency ratio ω_b/ω_0 for $\alpha = 1.0$ and 1.5. As evident from the plots, the OPR is almost constant against the filter's order (Fig. 9(a)). Also for a high-order filter ($N \geq 5$), the OPR is inversely proportional to ω_b/ω_0 (Fig. 9(b)). This is mainly because of the relation $\omega_b \propto G$ where a larger ω_b results in a larger G , making the effect of no added power in G_{xi} and G_{xij} in the SCC filter less significant.

C. Effect of Inverting Amplifier on Noise Performance

The impact of the inverting amplifiers on the noise characteristic of the SCC filter is analyzed using a second-order circuit, which serves as the core structure of both higher-order and complex filters, as shown in Fig. 10 where $G_{F\delta\pm} = (1 \pm \delta)G_F/2$, $G_{F\delta\pm} = (1 \pm \delta)G_F/2$ with $G_F = G$ for the lowpass sections' transconductors, and $G_F = G_{xi}$ and G_{xij} of the crossing transconductors. The inverting amplifier comprises a pair of pseudo-differential transconductors, with $G_{inv} = \alpha_F G_F$. The conductor-capacitor networks, $(g_A + sC_A)$ and $(g_B + sC_B)$ have been included to model adjacent second-order stage loading. Also shown in Fig. 10 are noise sources, which model the thermal noise associated with the transconductors $G_{F\delta\pm}$, $G_{F\delta\pm}$ and $G_{inv}/2$, where $i_{nA\delta\pm}^2 = (1 \pm \delta)^2 i_{nG_F}^2 / 2$ and $i_{nB\delta\pm}^2 = (1 \pm \delta)^2 i_{nG_F}^2 / 2$ with $i_{nG_F}^2 = \gamma_G k T G_F$ and $i_{ninv(A,B)}^2 = \gamma_G k T G_{inv} / 2 = \alpha_F^2 i_{nG_F}^2 / 2$. The conductors' noise may also be included but this is omitted for simplicity. The mean-square noise voltages $\bar{v}_{nA}^2$ and $\bar{v}_{nB}^2$ generated from these uncorrelated current noise sources were analyzed as summarized in Table II.

For the mean-square noise voltages due to i_{nA}^2 , i_{nB}^2 in Table II, they are the same as the noise transfers in the second-order filter without the inverting amplifier. This thus indicates no difference between the noise characteristics of the SCC and conventional filters due to these sources. On the other hand, the mean-square noise voltages due to $i_{ninv(A,B)}^2 (= \alpha_F^2 i_{nG_F}^2 / 2)$ indicate that the noise contribution from the inverting amplifier's transconductor is dependent on the factor $\alpha_F (=$

TABLE II NOISE TRANSFER FROM VARIOUS NOISE SOURCES IN FIG. 10

$\overline{i_{nA}^2} \rightarrow \overline{v_{nA}^2}, \overline{v_{nB}^2}$	$\overline{v_{nA}^2} = \overline{i_{nA}^2} \frac{ g_B + j\omega C_B ^2}{D(j\omega)}, \overline{v_{nB}^2} = \overline{i_{nA}^2} \frac{ G_F ^2}{D(j\omega)}$
$\overline{i_{nB}^2} \rightarrow \overline{v_{nA}^2}, \overline{v_{nB}^2}$	$\overline{v_{nA}^2} = \overline{i_{nB}^2} \frac{ G_F ^2}{D(j\omega)}, \overline{v_{nB}^2} = \overline{i_{nB}^2} \frac{ g_A + j\omega C_A ^2}{D(j\omega)}$
$\overline{i_{nimA}^2} \rightarrow \overline{v_{nA}^2}$	$\overline{v_{nA}^2} = \overline{i_{nimA}^2} \frac{(1-\delta)^2 g_A + j\omega C_A ^2}{\alpha_F^2 D(j\omega)} = \overline{i_{nB\delta-}^2} \frac{ g_A + j\omega C_A ^2}{\alpha_F D(j\omega)}$
$\overline{i_{nimA}^2} \rightarrow \overline{v_{nB}^2}$	$\overline{v_{nB}^2} = \overline{i_{nimA}^2} \frac{(1-\delta)^2 G_F^2}{\alpha_F^2 D(j\omega)} = \overline{i_{nB\delta-}^2} \frac{G_F^2}{\alpha_F D(j\omega)}$
$\overline{i_{nimB}^2} \rightarrow \overline{v_{nA}^2}$	$\overline{v_{nA}^2} = \overline{i_{nimB}^2} \frac{(1+\varepsilon)^2 g_A + j\omega C_A ^2}{\alpha_F^2 D(j\omega)} = \overline{i_{nA\varepsilon+}^2} \frac{ g_A + j\omega C_A ^2}{\alpha_F D(j\omega)}$
$\overline{i_{nimB}^2} \rightarrow \overline{v_{nB}^2}$	$\overline{v_{nB}^2} = \overline{i_{nimB}^2} \frac{(1+\varepsilon)^2 G_F^2}{\alpha_F^2 D(j\omega)} = \overline{i_{nA\varepsilon+}^2} \frac{G_F^2}{\alpha_F D(j\omega)}$

$$\overline{i_{nA}^2} = \overline{i_{nA\varepsilon+}^2}, \overline{i_{nB}^2} = \overline{i_{nB\delta-}^2}, \alpha_F = \frac{G_{inv}}{G_F}, D(j\omega) = |(g_A + j\omega C_A)(g_B + j\omega C_B) + G_F^2|$$

G_{inv}/G_F). In case of a lowpass filter design using the SCC structure with small ε , δ and $\alpha_F = 1$, i.e., $G_{inv} = G_F = G$, the mean-square noise transfers due to $\overline{i_{nim(A,B)}^2}$ is similar to those due to $\overline{i_{nG_F}^2}$'s. However, because the total transconductance is smaller, the SCC filter structure therefore exhibits less total noise as compared to the conventional filter with CM control networks. Specifically at low frequencies, the SCC lowpass filter can provide total thermal noise reduction by a similar degree to the power saving factor as discussed in Section II.E.

In case of a complex filter design, the crossing transconductors G_{xi} 's and G_{xij} 's can be treated as G_F in the second-order circuit of Fig. 10. The values of G_{xi} 's are typically larger than G in the lowpass I , Q sections depending on the ω_b/ω_c ratio. As a consequence, with $G_{inv} = G$, the factor $\alpha_F (=G_{inv}/G_{xi}=G/G_{xi})$ could be much less than unity. As indicated in Table II, the noise contribution from $\overline{i_{nim(A,B)}^2}$ is essentially increased by the factor $1/\alpha_F^2$. This adverse effect must be taken into account when designing a SCC complex filter. As adopted in the prototype filter in Section IV, a general design guideline is to select the transconductance $G_{inv} > G$ so that the factor α_F is increased and the SCC complex filter exhibits an overall noise performance comparable to that of its conventional counterpart, while still enjoying a significant amount of power saving.

D. Effect of Inverting Amplifier's Parasitic Capacitance on Frequency Response

The non-ideal effect due to the inverting amplifier's parasitic capacitor C_{inv} on the filter's frequency response can be investigated from the single-ended 2nd-order SCC circuit as shown in Fig. 11. The impedance looking into node X, Z_X can be expressed as

$$Z_X = \frac{sC_L(G_F/2)^{-2}}{\left[(1+\delta) + \frac{(1-\delta)}{1+sC_{inv}/G_{inv}} \right] \left[(1-\varepsilon) + \frac{(1+\varepsilon)}{1+sC_{inv}/G_{inv}} \right]}. \quad (14)$$

To provide insight and, at the same time, simplify the analysis, this SCC structure can be reduced to a simple *even*- or *odd*-type basic SCC structure with $\varepsilon = \delta = +1$ or -1 , respectively where the second-order circuit is transformed into an

Fig. 11. DM single-ended biquad for study on the impact of the inverting amplifier's C_{inv} on frequency response with $G_{F\delta\pm} = (1\pm\delta)G_F/2$, $G_{F\delta\pm} = (1\pm\delta)G_F/2$. Resistor R_Z in series with C_L is for compensation.

equivalent parallel RLC circuit as also provided in Fig. 11. The equivalent series inductor L_S and resistor r_S are given by

$$L_S = C_L / G_F^2, r_S = -\omega^2 (C_L / G_F^2) (C_{inv} / G_{inv}). \quad (14b)$$

As indicated by (14b), the capacitance C_{inv} has no impact on the inductance L_S . On the contrary, it gives rise to r_S , which is proportional to C_{inv} and is of a frequency-dependent *negative* resistance (FDNR) type. This thus results in a higher quality factor of the parallel RLC circuit than the designed value, and the effect is more pronounced at higher frequencies. Another non-ideal effect is on the resonant frequency ω_r , defined as the frequency that yields zero imaginary part in the admittance of the RLC circuit. It is given by

$$\omega_r^2 = \frac{\sqrt{1+4(C_{inv}/G_{inv})^2(G_F^2/C_L C_L)}-1}{2(C_{inv}/G_{inv})^2} = \frac{\sqrt{1+4(C_{inv}/G_{inv})^2\omega_{r-id}^2}-1}{2(C_{inv}/G_{inv})^2} \quad (15)$$

when C_{inv} approaches zero, by using the approximation $(1+x)^{1/2} \approx 1+(x/2)$, we have the ideal resonant frequency at $\omega_{r-id} = G_F / \sqrt{C_L C_L}$. By rearranging (15) as follows:

$$\omega_{r-id}^2 / \omega_r^2 = 1 + (C_{inv}/G_{inv})^2 \omega_r^2, \quad (16)$$

and since all the variables are positive, it can be deduced that $\omega_{r-id} > \omega_r$. Thus, the effect of C_{inv} results in a lower resonant frequency than the designed value. Both the increase in the quality factor and the decrease in the resonant frequency due to C_{inv} invariably change the frequency response of the biquad circuit, and hence distort the overall frequency response of the filter. Therefore the parasitic C_{inv} must be kept sufficiently small to maintain the desired response, unless a suitable compensation technique is introduced as described next.

E. Series-Resistor Technique for Parasitic Pole Cancellation in SCC Filter

As widely employed in integrated continuous-time filters, and extensively explored in [41], the technique employing a passive resistor R_Z in series with the capacitor C_L can be used to compensate for the effect of C_{inv} from the inverting amplifier associated with C_L in the 2nd-order SCC filter of Fig. 11. The resistor R_Z effectively introduces a zero at $-1/R_Z C_L$ at the inverting amplifier's input voltage. This helps cancel out the parasitic pole at $-G_{inv}/C_{inv}$ at its output voltage, thereby enabling the associated $G_{F\delta+} (= (1+\varepsilon)G_F)$ to produce the output current with no magnitude or phase distortion as compared to the ideal case when $C_{inv} = 0$.

This simple series-resistor technique can be extended to compensate for the effect of C_{inv} from any inverting amplifier in a general SCC filter structure. Similar to the case of Fig. 11,

Fig. 12. A DM single-end I section illustrating a pole/zero compensation using R_z in series with capacitors for the proposed SCC filters at the j^{th} node.

the underlying compensation principle is to make sure that all the transconductors and the floating capacitors in Fig. 3(a) and Fig. 8 supply the current signals into the grounded integrating capacitors C_j with no frequency-response distortion. Fig. 12 shows the general schematic at the j^{th} node of the SCC complex filter's I section (with $\varepsilon_x, \delta_x = 0$). Also included in the schematic are the compensation resistors R_z 's in series with the grounded capacitors C_j and the floating capacitors $C_{(j-2)j}$, $C_{(j+2)j}$. Following this, the voltage $\tilde{v}_j$ at the inverting amplifier's output can be related to the voltage v''_j at its input, and the associated current signals injected into C_j , as given by

$$\tilde{v}_j = \frac{v''_j}{1 + s(C_{inv}/G_{inv})} = \frac{R_z + (1/sC_j)}{1 + s(C_{inv}/G_{inv})} \left\{ \sum_k i_{jk} + i_{C(j-2)j} + i_{C(j+2)j} \right\} \quad (17)$$

where $\sum i_{jk}$ represents the sum of the currents from $G_{\delta+}$, $G_{\delta-}$, $\pm G_{xj}/2$, $\pm G_{x(j-2)j}/2$ and $\pm G_{x(j+2)j}/2$; $i_{C(j-2)j}$ and $i_{C(j+2)j}$ are the currents from the floating capacitors $C_{(j-2)j}$ and $C_{(j+2)j}$. Since $C_{(j-2)j}$ and $C_{(j+2)j}$ are connected from the adjacent nodes, $(j-2)^{\text{th}}$, and $(j+2)^{\text{th}}$ to the j^{th} node, (17) can be rewritten as

$$\tilde{v}_j = \frac{R_z + (1/sC_j)}{1 + s(C_{inv}/G_{inv})} \left\{ \sum_k i_{jk} + \frac{(v''_{j-2} - v''_j)}{(R_{Z(j-2)j} + (1/sC_{(j-2)j}))} + \frac{(v''_{j+2} - v''_j)}{(R_{Z(j+2)j} + (1/sC_{(j+2)j}))} \right\}, \quad (18a)$$

$$\tilde{v}_j = \frac{R_z + (1/sC_j)}{1 + s(C_{inv}/G_{inv})} \left\{ \sum_k i_{jk} + \frac{(1 + s(C_{inv}/G_{inv}))(\tilde{v}_{j-2} - \tilde{v}_j)}{(R_{Z(j-2)j} + (1/sC_{(j-2)j}))} + \frac{(1 + s(C_{inv}/G_{inv}))(\tilde{v}_{j+2} - \tilde{v}_j)}{(R_{Z(j+2)j} + (1/sC_{(j+2)j}))} \right\}. \quad (18b)$$

Based on the above equations, it requires that the following conditions be satisfied to achieve perfect compensation:

$$R_{Zj}C_j = R_{Z(j-2)j}C_{(j-2)j} = R_{Z(j+2)j}C_{(j+2)j} = C_{inv}/G_{inv}. \quad (19)$$

The ideal relation between the input voltages and output currents of the transconductors at j^{th} node is restored by applying the condition in (19) to (18) where the voltage $\tilde{v}_j$ of the associated inverting amplifiers can now be expressed as

$$\tilde{v}_j = \frac{1}{sC_j} \left\{ \sum_k i_{jk} + \frac{\text{with floating capacitors only}}{sC_{(j-2)j}} (\tilde{v}_{j-2} - \tilde{v}_j) + sC_{(j+2)j} (\tilde{v}_{j+2} - \tilde{v}_j) \right\}. \quad (20)$$

It is important to note that only the correctly compensated voltage v_j or its inverted version $\tilde{v}_j$ (not its over-compensated

Fig. 13. Inverter-based transconductor (i.e. Nauta) as a basic G cell

counterpart, v''_j) are used by various transconductors, e.g. $v_{j,I}$ and $\tilde{v}_{j,I}$ are used by $G_{\delta-}$ and $G_{\delta+}$, respectively in Fig. 12. Thus, the compensation resistors R_z 's are required only for the grounded capacitors C_j and the floating capacitors C_{ij} at the input of the inverting amplifiers.

IV. CIRCUIT VERIFICATIONS

The proposed SCC complex filter structure in Fig. 8 has been verified via simulation in a standard $0.18\text{-}\mu\text{m}$ CMOS process employing an inverter-based pseudo-differential transconductor from Fig. 5 as the core transconductor cell with minor modification as depicted in Fig. 13(a). In this design, α is selected to be 1.5 to achieve the output noise and the dynamic range (DR) close to that of the conventional design. Since the common-mode G_c value is identical to the differential-mode G , the value of the DC CM nodal conductance G_{Tj} is well-defined where $G_{Tj} \approx +2\varepsilon\delta G_c$ for $j=\text{even}$ and $G_{Tj} \approx +2G_c$ for $j=\text{odd}$ with $\varepsilon_x, \delta_x = 0$ as discussed in Section II.A. Therefore, similar to the technique in [7], [26–29], at a fixed body voltage of PMOS, V_{bp} , NMOS and PMOS of the transconductor cell can be appropriately sized to set their gate and drain DC bias voltages to $V_{DD}/2$ for the entire filter without extra circuitry. However, to counter process, temperature and supply voltage variations, a *master-slave* scheme (e.g. in [23]) is utilized to automatically adjust V_{bp} and accurately set up the required DC bias voltage. This V_{bp} is generated from the *master* bias set-up circuit of Fig. 13(b) comprising the diode-connected transconductor's DC half circuit in a negative-feedback servo loop. This transconductor's half-circuit replicates the desired DC voltage level of the transconductors working within the filter.

The filter operates at $V_{DD} = 1\text{V}$ with 1.1-MHz center frequency and 0.8-MHz bandwidth. A stabilization of the SCC filter (with $\varepsilon = 1.0$, $\delta = 0.25$) is verified in Fig. 14(a) depicting Monte-Carlo (MC) simulations of transient response of the output voltage converge to a stable value of $V_{DD}/2 = 0.5\text{V}$ after the supply voltage being subjected to a step change from 0 to 1V (without input signal). Moreover, the histogram plots in Fig. 14(b) and (c) compare the filter's nodal bias voltages for $\varepsilon_x, \delta_x = 0$ and $\varepsilon_x = -\delta_x = 0.075$ (applied to $G_{xj\delta\pm}$, $G_{xj\delta\pm}$ at $j = 1, 3, 5$) with the respective means (μ) and standard deviations (σ). As predicted in Section III.A, having $\varepsilon_x = -\delta_x$ renders the bias voltage levels closer to $V_{DD}/2$ (especially those at nodes 4). Fig. 15(a) shows the filter's frequency responses with $\varepsilon = 0.75, 1.0$ and $\delta = 0.25$. The CM frequency response is also included in Fig. 15(a). The in-band CMRR level is at $\sim 43\text{dB}$ and it is clear that the DC CM gain decreases with the δ/ε ratio where its value is as closely predicted by (8). Fig. 15(b)

(a) Output voltage response to the V_{DD} step change (b) Bias voltage histograms for $\epsilon_x, \delta_x = 0$ (c) Bias voltage histograms for $\epsilon_x = -\delta_x = 0.075$

Fig. 14. Monte Carlo simulations (100 runs) to monitor the complex filter's bias voltages

depicts the MC frequency response indicating that component mismatch plays role in the image-band distorted response.

An two-tone test was carried out with two input frequencies at 1.05 and 1.15 MHz. Fig. 16 shows an in-band intermodulation (IM) level plot where the IIP3 values are at $+3\text{dBV}_{\text{rms}}$ for the SCC filter and $+1\text{dBV}_{\text{rms}}$ for the conventional filter. The output noise power spectral density is shown in Fig. 17 with the output noise levels (integrated from 10 kHz to 10 MHz) of $6.4 \times 10^{-9} \text{V}^2$ and $6.1 \times 10^{-9} \text{V}^2$ for the SCC and conventional filters, respectively. With the filter's passband gain of 0.5, these output noise could be respectively translated to an input-referred noise of $-75.92\text{dBV}_{\text{rms}}$ and $-76.13\text{dBV}_{\text{rms}}$. The IM test also indicates a linear gain up to an input level of $-13.0\text{dBV}_{\text{rms}}$ for both filters – with the corresponding signal-to-noise ratios (SNR) of 62.9 dB and 63.1 dB. Following the calculation in [2], [7], where the spurious-free dynamic range, $\text{SFDR}_{\text{dB}} = (2/3)(\text{IIP3}_{\text{dBV}_{\text{rms}}} - \text{InputNoise}_{\text{dBV}_{\text{rms}}})$, the in-band SFDRs have been calculated at 52.6 dB (the SCC) and 51.4 dB (the conventional). The out-of-band IM was also simulated with two blocker frequencies at 3 and 4 MHz where the corresponding SFDRs have been found at 56.3 dB (the SCC) and 56.8 dB (the conventional).

The conventional and proposed complex filters consume static power of $851\mu\text{W}$ and $549\mu\text{W}$, respectively. The SCC filter prototype thus renders $\text{OPR} \approx 3.4$ or 70.5% saving on the overhead power consumption with no significant deterioration on the DR. This OPR value is closely agreed with the calculated of 3.7 from (13) using $\omega_b/\omega_0 = (0.88/2)/1.1 = 2.5$, $\bar{C}_{sf}/\bar{C}_x \approx 0.23$, and $\alpha = 1.5$ for this particular design. As summarized in Table III, the filters have been compared using a figure-of-merit, $\text{FoM} = (\text{Power Consumption})/(N \cdot f_c \cdot \text{DR})$ where f_c is the centre operating frequency. The calculation suggests that the SCC filter renders a significant improvement by 40% and 30% on the FoM respectively with the in-band and out-of-band SFDR.

V. CONCLUSION

A low-power G_m -C filter structure with self CM stabilization and CM rejection has been proposed. Through the use of inverting amplifiers, the proposed structure removes all the CM positive feedback loops and achieves stabilization without employing a dedicated, power-hungry CM control network usually utilized in the conventional design. The filter's CM conductance, CM gain, frequency and noise responses have been analyzed to assist the design. With no performance degradation in terms of in-band and out-of-band SFDR or SNR, the self-CM-controlled 5th-order elliptic complex filter offers over 70% saving on the overhead power compared to the conventional design. Although the proposed

(a) DM and CM frequency responses

(b) DM frequency response with Monte-Carlo analysis

Fig. 15. The SCC complex filter's frequency response

G_m -C filter has been demonstrated with the inverter-based pseudo-differential transconductor, the concept can be applied to a vast range of low-voltage transconductors – especially those without intrinsic capability to reject CM signals.

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TABLE III PERFORMANCE COMPARISON IN 0.18- μ m CMOS

Filter type, V_{DD}	Complex 5 th -order elliptic, 1V	
	Conventional	SCC
Total integrating capacitance	151.74 pF	151.74 pF
Total transconductance	4198.29 μ S	2649.14 μ S
Power consumption, OPR	851 μ W, 1	549 μ W, 3.4
Input-referred noise	-76.13 dBV _{rms}	-75.92 dBV _{rms}
Signal-to-noise ratio (SNR)	63.1 dB	62.9 dB
In-band DR: SFDR	51.4 dB	52.6 dB
IIP3-Input Noise	77.1 dB	78.9 dB
Out-of-band DR: SFDR	56.8 dB	56.3 dB
IIP3 - Input Noise	85.1 dB	84.4 dB
FoM based on: In-band SFDR	0.21 pJ	0.12 pJ
Out-of-band SFDR	0.11 pJ	0.078 pJ

Fig. 16. In-band two-tone intermodulation test

Fig. 17. Output noise power spectral density

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