Modeling the MOSFET's Inversion Layer and Its Universal Mobility: A New Experimental Method

This paper was downloaded from TechRxiv (<https://www.techrxiv.org>).

LICENSE

CC BY 4.0

SUBMISSION DATE / POSTED DATE

24-12-2021 / 29-12-2021

CITATION

Shoucair, F. (2021): Modeling the MOSFET's Inversion Layer and Its Universal Mobility: A New Experimental Method. TechRxiv. Preprint. <https://doi.org/10.36227/techrxiv.17571509.v1>

DOI

[10.36227/techrxiv.17571509.v1](https://doi.org/10.36227/techrxiv.17571509.v1)

Modeling the MOSFET's Inversion Layer and its Universal Mobility: A New Experimental Method¹

F. S. Shoucair

Abstract—We formulate a simple, yet accurate, model for a non-uniform mobile charge density $\rho(z)$ giving rise to a mean potential ψ^* across an inversion layer of finite extent, which we measure by means of a novel, sensitive, experimental method involving nulls of harmonic distortion components ($D_2 \approx D_3 \approx 0$) of the drain current under sinusoidal excitation below saturation. We thus establish analytically and experimentally, that the low-field, “universal” effective mobility μ_{eff} varies as $\sim(E^*_T)^{-1/3}$ for transversal fields

$$E^*_T = -\frac{1}{\epsilon_{sl}} [\eta Q_i + Q_b] \leq 0.5 \text{ MV/cm},$$

wherein η varies continuously between 1/2 and 1/3. We also establish and observe that the higher order, derivative, parameter θ_T quantifying μ_{eff} 's modulation by E^*_T varies as $\sim(E^*_T)^{-5/3}$ under laminar flow conditions, thereby further corroborating the foregoing effects and interpretations thereof.

I. INTRODUCTION

The well-known *charge sheet* and constant effective mobility, μ_{eff} , approximations, which simplify the MOSFET's (Metal Oxide Semiconductor Field Effect Transistor) drain current computations, forsake consequential information about the charge distribution $\rho(z)$ within, and mean potential ψ^* across, the inversion layer, both of which are critical to the conceptual modeling of channel conductivity, and of the modulation thereof by transverse electric fields E^*_T . The limitations and drawbacks entailed by these approximations are set forth

in section II, along with their relevance to our results, which indicate that (i) the inversion layer's detailed characteristics remain consequential even as this layer shrinks with process downscaling and, thus, (ii) the conventional assumption of a “negligibly” thin inversion layer may not warrant its outright neglect, absent contrary and/or contextual justification.

In section III, we apply an experimental method originally conceived to suppress harmonic distortion components in analog integrated filters [1,2], to *measure the mean potential ψ^* across the inversion layer*, a novel technique in this context, which yields the corresponding mean transverse E^*_T , (including η), as well as the ratio of mean to surface potentials (ψ^*/ψ_s) from which we deduce, and subsequently model $\rho(z)$ analytically in section IV. The accuracy of this model is confirmed, in section V, by further, sensitive, measurements of the *modulation of μ_{eff} by E^*_T* , namely the parameter $\theta_T = (1/\mu)(d\mu/dE^*_T)$, which are in agreement with our model's analytical predictions that θ_T should vary as $\sim(E^*_T)^{-5/3}$. Our overall results are summarized in section VI.

II. CONDUCTIVITY, MOBILITY, AND THE CHARGE SHEET MODEL

Under *laminar flow* conditions, the *conductivity* of a thin sheet of charge at depth z inside the inversion layer in a (W - x) plane, parallel to that of the gate insulator, is $\sigma(x,z) = [q\rho(x,z)\mu(z)]$, where q is the electronic charge, $\rho(x,z)$ the charge density at x along the channel, and $\mu(z)$ the average *mobility* of charge carriers

¹ This is a self-contained, abbreviated, version of sections III through VI (only) of the manuscript entitled “**Conservation Laws at Physical Origins of Universal Mobility in MOSFET Inversion Layers in Consilience with River Flow in a Gravitational Field**” (<https://doi.org/10.36227/techrxiv.17088419.v1>), recast in standalone form. Sections I and II are similarly archived under the title “**Physical Origins of Universal Mobility in MOSFET Inversion Layers: Conservation Laws**”.

along the x direction, which is related to the drift velocity of (9) in [3]:

$$v_{drift} = \mu(z) \frac{dV}{dx} = \mu(z) E_{II} \quad (1)$$

where E_{II} is the longitudinal electric field, along the x (or channel length) direction. The resulting drain current is:

$$\begin{aligned} I_D &= \left(\frac{W}{L}\right) \int_0^L \int_0^{z_{max}} \sigma(x, z) dz dx \\ &= \left(\frac{W}{L}\right) \int_0^L \int_0^{z_{max}} q[\rho(x, z) \mu(z)] dz dx \quad (2) \end{aligned}$$

wherein the double integral is customarily simplified by assuming (a) *constant mobility* along the z axis, $\mu(z) = \mu_{eff}$ thereby neglecting the effects of transverse fields E^*_{T} on mobility and, (b) an inversion layer of *zero thickness* — the “charge-sheet” approximation — such that $\rho(x, z) = \rho(x)$ is a function of x only, or of the potential $V(x)$, since $(dV/dx) = E_{II}$. Under these simplifying assumptions, the drain current may be expressed in closed form, by integration along the channel length:

$$\begin{aligned} I_D &= \left(\frac{W}{L}\right) \mu_{eff} \int_0^{V_D} q \rho(V) dV = \\ &\mu_{eff} C'_{ox} \left(\frac{W}{L}\right) [a_1 V_D + a_2 V_D^2 + a_3 V_D^3] \\ &+ \dots \text{higher order terms} \quad (3) \end{aligned}$$

wherein $\rho(V)$ is a known, explicit, function of the terminal voltages applied to the transistor, $V_S = 0$ and V_D are the potentials at its source ($x = 0$) and drain ($x = L$) terminals respectively, C'_{ox} the gate oxide capacitance per unit area, $a_1, a_2, \dots, a_n$ are constants determined by terminal bias and signal potentials, and the higher order terms are negligibly small in this context. For a given value of I_D , μ_{eff} is then the only unknown quantity in (3), which is thereby deduced readily from standard measurements. We note that, as simplification (a) above sets $\mu_{eff}(E^*_{T}) = \mu_{eff}$ in (3), the value of E^*_{T} corresponding to a given I_D , which is essential to the determination of the function $\mu_{eff}(E^*_{T})$, is variously *assumed* to be an “average” [4], or an “effective” [5],[7], value $E^*_{T} = -(1/\epsilon_{Si})[\eta Q_i + Q_b]$, with $\eta = 1/2$ for

electrons [4],[5],[7], $\eta = 1/3$ for holes [5], with Q_i (corrected [8]), and Q_b , the total inversion and bulk charges, respectively.

A. Limitations of the charge-sheet and constant mobility models

In view of Poisson’s potential field equation associated with a given charge distribution, the assumption of an inversion layer of zero thickness — entailing the tacit neglect of its spatial density profile — results in the *loss of information* about both the electric field $E(z) = (-q/\epsilon_{Si}) \int \rho(x, z) dz$, and the potential $\psi(z) = -\int E(z) dz$ within the inversion layer, with ϵ_{Si} the permittivity of silicon, since these integrals are taken between identical limits $[0, z_{max}(x)]$ when $z_{max} = 0$, as indeed the potential difference across any spatial region of zero extent is zero. The surface potential ψ_s at the Si/insulator interface, may nonetheless be calculated accurately from knowledge of the *total* inversion layer charge Q_i , per (5) below, which is represented by $q\rho(V)$ in (3) above [9],[10].

If $\mu(z)$ is not assumed constant in (2), and an inversion layer of finite thickness $z_{max} \neq 0$ is assumed, then $E(z)$ and $\psi(z)$ are, in principle, known functions given $\rho(x, z)$, but the drain current in (2) may no longer be expressed in analytical closed form as in (3), because $\mu(z)$ may not properly be removed from the integrand of (2). Consequently, and crucially for purposes of quantifying the dependence of mobility on transverse fields (E^*_{T}) from observations of I_D , the function $\mu_{eff}(E^*_{T})$ may no longer be “disentangled” from the result of the integration in (2), nor, thereby, extracted unambiguously from standard measurements of I_D , unless the function $\rho(x, z)$ is known to a sufficient degree of accuracy. The following section is devoted to the description of a method for achieving the latter objective.

III. AN ACCURATE METHOD FOR MEASURING THE MEAN POTENTIAL (ψ^*) AND TRANSVERSE ELECTRIC FIELD (E^*_T) ACROSS THE INVERSION LAYER

The function $\rho(z)$ in (2) may be deduced from such bias conditions as result in low/null harmonic distortion components ($D2 \approx D3 \approx 0$) for I_D , when sinusoidal signals of suitable amplitudes are applied to the transistor's terminals simultaneously, drain and body especially, thereby modulating E^*_T , as set forth in [1],[2]: under these experimental conditions, *nulls of $D3$ occur uniquely because substrate effects on mobility counteract those of the gate (by increasing the threshold voltage)*, I_D is then (nearly) proportional to V_D , hence the channel conductance constant, and the conducting channel obeys Ohm's Law, which corresponds to *laminar flow* since constant $(dz/dx) = (dz/dV)(dV/dx) = (dz/dV)E_{//}$, per II.B.(d) in [3], yields proportionality between $z(x)$ and $V(x)$, hence between I_D and V_D .

By contrast, traditional methods [4], [5],[7], entail an extrapolation of the drain current characteristics at low drain-source potential, to a “threshold” voltage whose definition is neither well defined nor unique [9], thereby resulting in various inconsistencies. These, and other, derivative methods [8],[11],[12], have thus left unresolved the *physical origins* of the “universality” of mobility curves, among various related observations.

The method implemented below side-steps such inaccuracies and inconsistencies as incurred by the latter methods, by setting forth the *objective criterion* ($D2 \approx D3 \approx 0$) for *determining the mean electric field E^*_T across the inversion layer from measurements of ψ^** . This method furthermore yields $\rho(z)$, $\mu_{eff}(E^*_T)$, and the latter' derivative function (θ_T) accurately, thereby rendering these quantities in a manner that is mutually consistent, and verifiable by direct observation, as will be shown in the sequel.

A. Measuring the mean potential ψ^* across the inversion layer

In the ensuing, we use standard MOSFET terminology for an n channel device, and denote $V_B = (V_{SB} + 2\phi_F + b^*\phi_t)$, wherein ϕ_F and $\phi_t = kT/q$ are the Fermi and thermal potentials respectively. The mean potential ψ^* across the inversion layer is:

$$\psi^* = b^*\phi_t \ln \left[\left(\frac{2}{\gamma} \right) \sqrt{V_B} \left(\frac{1}{r} - 1 \right) \right] \quad (4)$$

where $r = \Delta V_B / \Delta V_{GS}$ is the ratio of increments of applied V_B and V_{GS} bias voltages which result in observed nulls of $D3$, per (3) in [1], and γ is the body effect coefficient [9]. The mean, observed, value of r over the range of our data is 0.39. The corresponding, observed, values of $\psi^* = (b^*\phi_t)$ range between $(1.45\phi_t)$ and $(1.65\phi_t)$, (see Fig.1), and the analytical values of the surface potential $\psi_s = (b_s\phi_t)$ range between $(4.5\phi_t)$ and $(6.7\phi_t)$, as obtained by solving the transcendental [9]:

$$\psi_s = V_{GS} - V_{T0} + \gamma \sqrt{2\phi_F} - \gamma \sqrt{\psi_s + \phi_t [e^{(\psi_s - V_B + V)/\phi_t} - 1]} \quad (5)$$

for $(\psi_s - V_B) = (b_s\phi_t)$ to 0.1% accuracy for the observed bias conditions, with $V = 0$. The ratio of derivatives,

$$\left(\frac{\partial \psi_s}{\partial V_{GS}} \right) / \left(\frac{\partial \psi^*}{\partial V_B} \right) = r$$

yields the right-hand side of (4), which relates r to b^* , thereby determining b^* and ψ^* . The observed variations of ψ^* , and our analytical predictions for these (section IV below), are shown in Fig.1 for ready comparison with the empirical Schwarz–Russek model [7].

The observed ratio (ψ^*/ψ_s) thus ranges between $(1.45)/(4.5) \approx (1/3)$ near to the onset of strong inversion, and $(1.65)/(6.7) \approx (1/4)$ near to the saturation of ψ_s (see Fig. 2). A ratio of $(1/3)$ is expected from Poisson's equation for a spatially *uniform* charge distribution, whereas a ratio of $(1/4)$ corresponds to a spatially *linear* distribution $\rho(z) \sim z$. These

results are suggestive of the analytical model to follow, which must account for the foregoing observations.

Fig. 1 Observed mean potential ψ^* across inversion layer and predictions of (13).

Fig. 2 Observed ratio (ψ^*/ψ_s) and carrier concentration of commensurate charge sheet model ($N_i = Q_i/q$). Model curve is per (13).

B. Determining the mean transverse electric field E^*_T across the inversion layer

The mean electric field E^*_T (V), which is modulated by applied, sinusoidal, drain-source and body-source potentials [1,2], is obtained from $Q_i(\psi_s)$ and $Q_b(\psi_s)$ [9, p.79]:

$$Q_i(\psi_s) \approx -\gamma C'_{ox} \left[\sqrt{\psi_s + \Phi_t} e^{(\psi_s - V_B + V)/\Phi_t} - \sqrt{\psi_s} \right] \quad (6)$$

$$Q_b(\psi_s) \approx -\gamma C'_{ox} \sqrt{\psi_s + V} \quad (7)$$

wherein C'_{ox} is the gate oxide capacitance per unit area. The mean transverse electric field is then:

$$E^*_T \approx -\frac{1}{\epsilon_{si}} \left[\frac{Q_i(\psi_s)}{m(E^*_T)^2} + Q_b(\psi_s) \right] = -\frac{1}{\epsilon_{si}} [\eta(E^*_T) Q_i(\psi_s) + Q_b(\psi_s)] \quad (8)$$

wherein $0 \leq m(E^*_T) \leq 1$ is the power of z in $\rho(z) \sim z^m$ per (7) in [3], and to which corresponds $(1/3) \leq \eta(E^*_T) \leq (1/2)$, per (8) above. The correspondence between the values of m and η extracted from our measurements of (ψ^*/ψ_s) , and the mean values of z resulting for the inversion layer are shown in Fig.3 below, wherein:

- (a) $m(E^*_T)$ varies approximately linearly with (Q_i/Q_b) near to $(Q_i/Q_b) \approx 1$; i.e., for $E^*_T \approx 0.2$ MV/cm.
- (b) $\eta(E^*_T) = 1/[m(E^*_T) + 2]$ is a related function of inversion level (Q_i/Q_b) .
- (c) the ratio (ψ^*/ψ_s) varies between $(1/3)$ and $(1/4)$ (as per Fig.2 above).
- (d) the mean inversion layer thickness $\langle z \rangle = (\psi^*/E^*_T)$ resulting from our observations of ψ^* and E^*_T (per Fig.1) is commensurate with the predictions of Stern and Howard [13], and Schwarz and Russek [7].

Fig. 3 Observed values of m , (ψ^*/ψ_s) , and $\langle z \rangle$.

IV. ANALYTICAL MODEL OF THE INVERSION LAYER'S CHARGE DENSITY PROFILE DEDUCED FROM EXPERIMENTAL OBSERVATIONS

In view of the observations described in section III.A above we posit, for an

inversion layer of extent z_{max} supporting a given total charge $Q_c = Q_i + Q_b$:

$$b(z) \approx b^* \left(\frac{z}{z_{max}} \right)^m ; 0 \leq z \leq z_{max} \quad (9)$$

such that $\psi^* = b^* \phi_t$ represents the mean potential across z_{max} , since $b(z_{max}) = b^*$ when $z = z_{max}$ and, ignoring minority holes,

$$\rho(z) \approx N_a \left[e^{b^* \left(\frac{z}{z_{max}} \right)^m} - 1 \right] \quad (10)$$

for the corresponding *mobile charge density within the inversion layer*, pending experimental corroboration. Equation (10) is equivalent to a linear approximation $\rho(z) \sim z$ when $m \approx 1$ and $(z/z_{max}) \ll 1$ (see Figs.4(b) and (c) below), which meets the critical boundary condition $\rho(0) = 0$.

(a)

(b)

(c)

Fig. 4(a) Uniform inversion layer charge density; $|Q_i/Q_b| \approx 0.4$, $\eta = (1/2)$, $(\psi^*/\psi_s) = (1/3)$, $\psi^* = b^* \phi_t \approx 1.45 \phi_t \approx 37 \text{ mV}$, $\psi_s \approx 111 \text{ mV}$ (see Figs.1 and 2) observed at low values of $|Q_i/Q_b| < 1$ where the “flat” depletion charge Q_b is dominant. The standard “flat” inversion charge profile between 0 and z_{max} becomes physically unrealistic as $|Q_i/Q_b|$ increases, because there must be a finite transition region near to the oxide interface wherein $\rho(z)$ drops towards zero, which is a critical boundary condition of the system.

(b) $|Q_i/Q_b| \approx 1.8$, $\eta = (1/3)$, $(\psi^*/\psi_s) = (1/4)$, $\psi^* = b^* \phi_t \approx 1.65 \phi_t \approx 43 \text{ mV}$, $\psi_s \approx 172 \text{ mV}$; unlike in (a), the boundary condition $\rho(z=0) = 0$ holds for our model which is here a valid approximation of:

(c) Stern–Howard model [13] for an inversion layer charge density $\rho(z) \sim (1/2)\beta^3 z^2 e^{-\beta z}$, $\beta \approx (1/15)$, whose $\langle z \rangle \approx (3/2\beta) \approx 22.5 \text{ \AA}$, and $z_{max} \approx 45 \text{ \AA}$. The areas, between 0 and z_{max} , under the Stern–Howard curve and our model’s are nominally equal, and correspond to the total inversion charge Q_i . The potential values indicated correspond to condition (b).

Equating the inversion charge of the sheet model, $Q_i(\psi_s)$ per (6), to that posited by (10) for an inversion layer of finite extent z_{max} ,

$$Q_i = \int_0^{z_{max}} q \rho(z) dz \approx \int_0^{z_{max}} q N_a \left[e^{b^* \left(\frac{z}{z_{max}} \right)^m} - 1 \right] dz \quad (11)$$

establishes an *implicit* analytical relationship between the surface potential $\psi_s = b_s \phi_t$ of the sheet model, per (5), and the desired mean potential $\psi^* = b^* \phi_t$ derived from (11). Integrating Poisson’s equation with $\rho(z)$ thus chosen, twice to obtain the surface potential $\psi_s = (b_s \phi_t)$, once more to obtain the mean potential $\psi^* = (b^* \phi_t)$ over $[0, z_{max}]$, thus yields:

$$\frac{\psi^*}{\psi_s} = \frac{b^*}{b_s} = \frac{1}{m+3} \quad (12)$$

whereby $\eta = 1/(m+2)$ for E^*_T in (8), which is obtained as an intermediate result.

For $0 \leq m(E^*_T) \leq 1$, the parameter η is thus expected to vary continuously between $(1/2)$ and $(1/3)$, per (8), as observed for standard devices with $\langle 100 \rangle$ surfaces, and predictably as a function of inversion level (see Fig.3). Because this transition is especially prominent in the range of bias conditions whereby neither Q_i nor Q_b is overwhelmingly dominant over the other, our

measurements are concentrated on this particular range, namely $0.4 \leq (Q_i/Q_b) \leq 1.8$ (see Figs. 1–3,5), which corresponds, approximately, to $0.15 \text{ MV/cm} \leq E^*_{\rho} \leq 0.25 \text{ MV/cm}$, where laminar flow obtains, and over which we observe D_3 nulls.

Proceeding thus, we derive in closed form:

$$\frac{b^*}{b_s} = \frac{e^{b^*} - 1 - b^* - \frac{(b^*)^2}{2!} - \frac{(b^*)^3}{3!}}{b^* \left(e^{b^*} - 1 - b^* - \frac{(b^*)^2}{2!} \right)} \quad (13)$$

which yields b^* readily, given $b_s = (\psi_s/\phi_i)$ as obtained from (5). The ratio (13) thus reduces to the expected value $(3!)/(4!) = (1/4)$ in the charge-sheet limit $b^* \rightarrow 0$, and matches observations of $(b^*/b_s) = (\psi^*/\psi_s)$ nearly perfectly for $(Q_i/Q_b) \geq 1$, where $m \approx 1$ as observed in Fig.2, and otherwise to within $\pm 5\%$, on an average, over the full observation span. The left-hand side of (13) indicates that the ratio $(b^*/b_s) = (\psi^*/\psi_s)$ decreases as ψ_s increases, i.e., as inversion level increases: physically, this means that as E^*_{ρ} increases, the increasing electron charge contributes mainly to an inversion layer's mobile Q_i concentrated nearer to the oxide, while the fixed depletion charge Q_b remains relatively unaltered. These trends are indicated by the arrows attached to the various quantities depicted in Fig.4(c).

The ratio of mean electric field (E^*_{ρ}) across the inversion layer corresponding to $\rho(z)$ per (10), to that (E^*_u) of a uniform charge distribution supporting the same total charge Q_c , and obtained as an intermediate result is:

$$\frac{E^*_{\rho}}{E^*_u} = \left(\frac{2}{b^*} \right) \left[\frac{e^{b^*} - 1 - b^* - \frac{(b^*)^2}{2!}}{e^{b^*} - 1 - b^*} \right] \quad (14)$$

whose charge-sheet limit is indeed $(2/3)$, as expected when b^* approaches 0:

$$\lim_{b^* \rightarrow 0} \left(\frac{E^*_{\rho}}{E^*_u} \right) = \left(\frac{2}{b^*} \right) \left[\frac{\frac{(b^*)^3}{3!}}{\frac{(b^*)^2}{2!}} \right] = \frac{2}{3} \quad (15)$$

in which case the mean transverse electric field E^*_{ρ} , per (8), is:

$$-\varepsilon_{si} E^*_{\rho} = \frac{Q_i}{2} \left(\frac{E^*_{\rho}}{E^*_u} \right) + Q_b \quad (16)$$

wherein the ratio $(E^*_{\rho}/2E^*_u) = \eta$ yields the expected result $(1/2) \cdot (2/3) = (1/3)$ for an inversion layer varying linearly with z , or $m = 1$ in (9)–(12), as indicated above. The traditional assumption of a value $\eta = (1/2)$, fixed *a priori* (as illustrated in Fig.4(a), and corresponding to constant $\rho(z)$), which does not account for these effects, and fails to meet the boundary condition $\rho(0) = 0$, may thereby be expected to give rise to observable discrepancies left wanting of clarification and reconciliation, as indeed noted in [4],[5],[7].

V. TRANSVERSE-FIELD MODULATION PARAMETER (θ_T)

Conceptually, θ_T represents a *first order Taylor series expansion* of the expression for the mobility $\mu_{eff}(E^*_{\rho})$ associated with the MOSFET's drain current: the constant μ_{eff} in (3) is replaced by $\mu_{eff} [1 + \theta_T \cdot V(x)]$, where $V(x)$ is the potential along the channel's direction (x), and $[\theta_T \cdot V(x)] \ll 1$. The drain current I_D is then obtained by integrating the function $[1 + \theta_T \cdot V(x)] \cdot \rho[V(x)]$ from 0 to L , in the conventional manner [1],[2].

A. Experimental conditions and results

All measurements were made on long n-channel devices ($L \approx 100\mu\text{m}$) fabricated in a standard, industrial, CMOS process. The range of E^*_{ρ} for our data is centered near to 0.2 MV/cm , where universal mobility behavior prevails, as reported in the literature [4],[5].

θ_T was extracted from nulls of third harmonic current distortion components ($D_2 \approx 0$, $D_3 = 0$)² in the triode region of operation, with applied $V_{DS} = V_{peak} \sin(2\pi ft)$, where $V_{peak} < V_{DSsat}$, $f = 1 \text{ kHz}$, and $(E^*_{\rho}/E_{//}) \geq 2000$, [1]:

² Although θ_T may likewise be obtained from nulls of D_2 , those of D_3 are much more sensitive to bias and

signal amplitude conditions (see II.B in [1]), which is an advantage for the purpose at hand.

$$\theta_T(E_T^*) = \left(\frac{\gamma}{8} \right) \left[\frac{V_B^{-\frac{3}{2}}}{1 - \frac{\gamma}{4\sqrt{V_B}}} \right] \quad (17)$$

Fig.5 below compares these observations to the predictions of the foregoing model, as set forth in section III.B, whereby E_T^* is as per (8):

$$\begin{aligned} \theta_T(V) \big|_{V=0} &= \left(\frac{1}{\mu} \right) \frac{d\mu}{dV} \big|_{V=0} = \left(\frac{1}{\mu} \right) \frac{\partial \mu}{\partial E_T^*} \frac{dE_T^*}{dV} \big|_{V=0} \\ &\approx \frac{1}{3} \left[\frac{1 - \frac{\gamma}{2\sqrt{V_B}} \left(\frac{1}{\eta} - 1 \right)}{V_{GS} - V_{T0} + \gamma(\sqrt{V_B} + \sqrt{2\Phi_F})} \right] \end{aligned} \quad (18)$$

While the agreement of (18) is within $\approx \pm 5\%$ throughout the observation range, the qualitative effect of varying values of m (hence of η) is most conspicuous, which illustrates the inadequacy of a uniform inversion layer model (the curve labeled $m = 0$, or fixed $\eta = 1/2$): the data deviate from the latter, increasingly as m increases between 0.2 and 1.05 (i.e., η decreasing between $(1/2)$ and $(1/3)$) approximately, as also illustrated in Fig.5), whereas (18) accounts for these effects while rendering θ_T explicitly in terms of terminal potentials and device parameters.

Fig. 5. Parameter θ_T , defined by (18), quantifying mobility modulation by E_T^* . Our data are in good agreement with the expected behavior per (19), whereas the model with fixed $\eta = 1/2$ is not.

A further, crucial, check on the consistency of the foregoing results, and confirmation of laminar flow conditions, are

now at hand: since, under such conditions, μ_{eff} and $\langle z \rangle$ vary as $\sim (E_T^*)^{-1/3}$, $(dz/dx) \approx -(E_{II}/E_T^*)$ per (11) in [3], ψ^* is a weak function of z (Fig.1), and $(dx/dV) = (1/E_{II})$, we expect:

$$\begin{aligned} \theta_T(V) &= \left(\frac{1}{\mu} \right) \frac{d\mu}{dV} = \left(\frac{1}{\mu} \frac{\partial \mu}{\partial E_T^*} \right) \frac{\partial E_T^*}{\partial \psi^*} \left(\frac{\partial \psi^*}{\partial z} \right) \frac{\partial z}{\partial x} \frac{dx}{dV} \\ &\sim \left(\frac{-1}{3E_T^*} \right) \frac{1}{\langle z \rangle} \left(\frac{-E_{II}}{E_T^*} \right) \frac{1}{E_{II}} \sim \frac{(E_T^*)^{1/3}}{(E_T^*)^2} \sim (E_T^*)^{-5/3} \end{aligned} \quad (19)$$

which is indeed observed in Fig. 5. In view of our analogy with a river's flow [3], the same result may be deduced from (4) and/or (9) in [3], wherein $v dv = -z dz$, with $v = (\mu E_{II})$ from (1) above, such that:

$$\begin{aligned} \frac{1}{\mu} \frac{d\mu}{dE_T^*} &= \frac{1}{v} \frac{dv}{dE_T^*} = -z \frac{dz}{dE_T^*} \\ &\sim - (E_T^*)^{-1/3} \cdot \frac{d}{dE_T^*} \left[(E_T^*)^{-1/3} \right] \sim - (E_T^*)^{-1/3} \cdot \left[- (E_T^*)^{-4/3} \right] \end{aligned}$$

VI. SUMMARY

A sensitive method for measuring the mean potential (ψ^*) across the MOSFET's inversion layer was applied under conditions of harmonic distortion component nulls ($D2 \approx D3 \approx 0$) of the drain current under sinusoidal excitation, below saturation. We showed, thereby, how the inversion charge density profile $\rho(z)$ may be deduced accurately from the ratio (ψ^*/ψ_s) , as the surface potential (ψ_s) increases from onset of strong inversion towards saturation, and how such information underpins the physical origins of disparate observations of long standing. We found and established:

- (1) Non-uniform, or non-constant, $\rho(z)$ causes variations of $1/3 \leq \eta \leq 1/2$, which determine the mean transverse electric field $E_T^* = -(1/\epsilon_{Si})[\eta Q_i + Q_b]$ in terms of the inversion Q_i , and bulk Q_b , charges.
- (2) The higher order, derivative, parameter θ_T quantifying the modulation of μ_{eff} by E_T^*

varies as $\sim(E^*_T)^{-5/3}$ under laminar flow conditions.

ACKNOWLEDGMENT

The author is indebted to Mrs. V. Vigil, Berkeley Public Library, Berkeley, CA, and to R. Rice, for valuable technical discussions and assistance.

REFERENCES

- [1] F. S. Shoucair and W. R. Patterson, "Analysis and Modeling of nonlinearities in VLSI MOSFETs including substrate effects", *IEEE Trans. Electron Devices*, vol. ED-40, no.10, pp. 1760–1767, October 1993.
- [2] F. S. Shoucair, "Modeling, decoupling and suppression of MOSFET distortion components", *IEE Proceedings, Circuits, Devices and Systems*, vol. ED-146, no.1, pp. 37–43, February 1999.
- [3] F. S. Shoucair, "Physical Origins of Universal Mobility in MOSFET Inversion Layers: Conservation Laws", unpublished. [Online]. Available: <https://doi.org/10.36227/techrxiv.17088419.v1>.
- [4] S. C. Sun and J.D. Plummer, "Electron mobility in inversion and accumulation layers on thermally oxidized silicon surfaces", *IEEE Trans. Electron Devices*, vol. ED-27, no.8, pp. 1497–1508, August 1980.
- [5] S. Takagi, A. Toriumi, M. Iwase, and H. Tango, "On the universality of inversion layer mobility in Si MOSFET's: Part – I Effects of substrate impurity concentration, Part – II, Effects of surface orientation", *IEEE Trans. Electron Devices*, vol. ED-41, no.12, pp. 2357–2368, December 1994.
- [6] S.-W. Lee, "Extraction of MOSFET carrier mobility characteristics and calibration of a mobility model for numerical device simulation", *Solid-State electronics*, vol. 33, no. 6, pp. 719–726, 1990.
- [7] S. A. Schwarz and S. E. Russek, "Semi-empirical equations for electron velocity in Silicon: Part II — MOS inversion layers", *IEEE Trans. Electron Devices*, vol. ED-30, no.12, pp. 1634–1639, December 1983.
- [8] W. Zhu, J.-P. Han, and T.P. Ma, "Mobility measurement and degradation mechanisms of MOSFETs made with ultrathin high-k dielectrics", *IEEE Trans. Electron Devices*, vol. ED-51, no.1, pp.98–105, January 2004.
- [9] Y. P. Tsividis, *Operation and Modeling of the MOS Transistor*, McGraw-Hill, 1987, p.79.
- [10] S. M. Sze, *Physics of Semiconductor Devices*, John Wiley & Sons, 1981, p.850.
- [11] K. Remachan, N.A. Wong, K. Chan, S.P. Sim and C.Y. Yang, "Modeling inversion-layer carrier mobilities in all regions of MOSFET operation", *Solid-State electronics*, vol. 46, pp. 153–156, 2002.
- [12] P.-M.D. Chow and K.-L. Wang, "A new AC technique for accurate determination of channel charge and mobility in very thin gate MOSFET's", *IEEE Trans. Electron Devices*, vol. ED-33, no.9, pp.1299–1304, September 1986.
- [13] F. Stern and W.E. Howard, "Properties of semiconductor surface inversion layers in the electric quantum limit", *Physical Review*, vol. 163, no.3, pp. 816–835, November 1967.