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Multi-criteria performance assessment based on closed-loop system identification

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Abstract

A method to assess the performance of control loops, based on closed-loop system identification is proposed. This method allows to take into account the trade-off between process variable and manipulated variable energy, thus overcoming one of the most important criticisms to Harris'index. To illustrate the proposed approach, a numerical example is given, for which the proposed index is 8% greater than Harris'index.

Keywords: Control Loop, Performance Assessment

1. Introduction

2 In the process industry sector, companies need to extract actionable information from sensor-based data collected on a daily basis through their integrated
4 IT/OT systems [1]. It is estimated that each operator typically needs to assess the performance of 90-180 control loops [2], and therefore the interest in
6 developing computer-aided tools to facilitate their work. In this framework, timely and accurate assessment may enable the implementation of appropriate
8 corrective actions as soon as required.

A common approach for control loop performance assessment (CPLA) is
10 based on selecting an *ideal benchmark* to which the actual loop can be compared [3]. As an example, if the Minimum Variance (MV) control loop is selected, it
12 is possible to determine the well-known *Harris' index* (HI), using only routine

data, collected during normal operation, without the need of *ad hoc* invasive ex-
14 periments [4]. This index has been extensively used in commercial applications
and extended to assess more complex processes [5].

16 Although numerous of similar techniques are available [6], due to real-world
phenomena like non-linearity, unknown disturbances and dynamics, unstable or
18 multivariable loops, etc., this can still be a challenging problem. In this paper
we propose a multi-criteria approach based on the following rationales:

- 20 • It may be necessary to consider different aspects related to loop perfor-
mance, possibly in contradiction, which is not possible using the classic HI.
22 It is well-known that this index is not realistic because the MV controller
is too aggressive. Therefore, in practice it may largely underestimate loop
24 performance.
- Companies are currently implementing, or willing to implement, comput-
26 ing infrastructure able to run more complex analysis algorithms, compared
to past situation.

28 The structure of this paper is as follows. In section 2, problem statement is
presented. In section 3, the proposed approach is described and in section 4 a
30 simple case study is presented.

2. Problem statement

32 Let $\mathcal{S}$ be a commissioned stable single-input single-output closed-loop LTI
system, as in Fig. 1, where $\varepsilon(k)$ is a disturbance assumed to be unit *white noise*.
34 Based on the data collected during the lapse $k = 0, 1, 2, \dots, N - 1$, we define the
loop performance $\mathcal{P}$ to be a discrete variable

$$\mathcal{P} \in \{\text{Good, Fair, Poor}\} \quad (1)$$

36 Thus, the plant under control is described by the following model

$$y(k) = \frac{G_p(z^{-1})}{1 + G_c(z^{-1})G_p(z^{-1})} \varepsilon(k) \quad (2)$$

$$G_p(z^{-1}) = \frac{B(z^{-1})}{A(z^{-1})} \quad (3)$$

Figure 1: Commissioned control system

We assume that $A(z^{-1})$, $B(z^{-1})$ are unknown polynomials, and n , m their
 38 corresponding degrees. The controller model $G_c(z^{-1})$ is assumed known. We
 are interested in the following problem: *given $\mathcal{S}$ determine its performance $\mathcal{P}$,*
 40 *by using only the available measurements*

$$y(-N_H), \dots, y(-2), y(-1), y(0), y(1), y(2), \dots, y(N-1) \quad (4)$$

where $N_H + N$ is the total number of available measurements, with $N_H \gg N$.

42 As discussed in the previous section, the classical approach to solve this
 problem is to determine the *Harris' index* (HI), denoted hereafter η_{MV} . If we
 44 denote σ_{MV}^2 the minimum variance achievable by any linear controller, η_{MV}
 will be given by the ratio:

$$\eta_{MV} = \frac{\sigma_{MV}^2}{s_y^2} \quad (5)$$

46 where s_y^2 is the measured output variance

$$s_y^2 = \frac{1}{N-1} \sum_{k=0}^{N-1} [y(k) - m_y]^2 \quad (6)$$

$$m_y = \frac{1}{N} \sum_{k=0}^{N-1} y(k) \quad (7)$$

48 The estimation of minimum variance σ_{MV}^2 requires the identification of an
 ARMA model (see for example [3])

$$\hat{A}_{cl}(z^{-1})y(k) = \hat{B}_{cl}(z^{-1})\varepsilon(k) \quad (8)$$

50 from historical data $y(-N_H), \dots, y(-2), y(-1), y(0)$, which allows also to estimate the closed-loop transfer function, given by

$$\hat{G}_{cl}(z^{-1}) = \frac{\hat{B}_{cl}(z^{-1})}{\hat{A}_{cl}(z^{-1})} \quad (9)$$

52 Knowing σ_{MV}^2 and η_{MV} (see equation 5) the loop performance $\mathcal{P}$ is given by

$$\mathcal{P} = \begin{cases} \text{Good, if } \eta_{MV} \geq \eta_G \\ \text{Fair, if } \eta_P < \eta_{MV} < \eta_G \\ \text{Poor, if } \eta_{MV} \leq \eta_P \end{cases} \quad (10)$$

54 where η_G, η_P define performance ranges, usually determined by human experts according to their past experience and *a priori* information.

56 The previous approach is well-known and statistical confidence intervals can be calculated for η_{MV} [7]. However, as discussed in the previous section, in
58 practice we may be largely underestimating loop performance.

3. Proposed approach

60 We focus on the trade-off between process variable $y(k)$ and manipulated variable $u(k)$ energy. For this, let's denote

$$m_u = \frac{1}{N} \sum_{k=0}^{N-1} u(k) \quad (11)$$

62

$$s_u^2 = \frac{1}{N-1} \sum_{k=0}^{N-1} [u(k) - m_u]^2 \quad (12)$$

$$s_{y,u} = (s_y^2, s_u^2) \quad (13)$$

As in the previous section, we start by estimating $\hat{A}_{cl}(z^{-1}), \hat{B}_{cl}(z^{-1})$ (see equation 9) from which we obtain the open-loop transfer function

$$\hat{G}_p(z^{-1}) = \frac{\hat{G}_{cl}(z^{-1})}{1 - G_c(z^{-1})\hat{G}_{cl}(z^{-1})} = \frac{\hat{B}(z^{-1})}{\hat{A}(z^{-1})}$$

64 Note that in practice, the order of polynomials $\hat{A}(z^{-1})$, $\hat{B}(z^{-1})$, namely $\hat{n}$, $\hat{m}$, needs to be assumed. Then, let $\hat{y}(k+\tau)$ be the one-step-ahead output prediction:

$$\hat{y}(k+1) = \frac{F(z^{-1})}{\hat{B}(z^{-1})} y(k) + E(z^{-1}) u(k) \quad (14)$$

66 where $E(z^{-1})$ and $F(z^{-1})$ are obtained solving the Diophantine equation

$$\hat{B}(z^{-1}) = E(z^{-1})\hat{A}(z^{-1}) + z^{-1}F(z^{-1}) \quad (15)$$

For each value $\alpha = \frac{1}{N_P}, \frac{2}{N_P}, \dots, 1$, with $N_P \gg 1$, we determine the solution
68 $u^*(k)$ to the following optimization problem

$$\min_{u(k)} \left[\alpha \hat{y}(k+1)^2 + (1-\alpha) u(k)^2 \right] \quad (16)$$

Here, it is possible to show that the solution is $u^*(k)$ such that

$$u^*(k) = -K_\alpha^*(z^{-1})y(k) \quad (17)$$

70 with

$$K_\alpha^*(z^{-1}) = \frac{\alpha F(z^{-1})E(z^{-1})}{\alpha E(z^{-1})^2 B(z^{-1}) + (1-\alpha)B(z^{-1})} \quad (18)$$

Note that this result (18) was found independently by the author of this
72 paper, however an equivalent result was previously published in [8]. Now, denote S^* the set of points $(s_{u,\alpha}^{*2}, s_{y,\alpha}^{*2})$

$$s_{u,\alpha}^{*2} = \left\| \frac{G_p(z^{-1})K_\alpha^*(z^{-1})}{1 + K_\alpha^*(z^{-1})G_p(z^{-1})} \right\|_2^2 \quad (19)$$

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$$s_{y,\alpha}^{*2} = \left\| \frac{G_p(z^{-1})}{1 + K_\alpha^*(z^{-1})G_p(z^{-1})} \right\|_2^2 \quad (20)$$

for $\alpha = \frac{1}{N_P}, \frac{2}{N_P}, \dots, 1$. Note that S^* is a numerical *approximation of the Pareto*
76 *front* [9] corresponding to the problem

$$\min_{u(k)} \begin{pmatrix} u(k)^2 \\ \hat{y}(k+1)^2 \end{pmatrix} \quad (21)$$

Now, denote $(p_{u,\alpha}^{*2}, p_{y,\alpha}^{*2})$ the point in S^* which is *nearest to* $s_{y,u} = (s_y^2, s_u^2)$
78 but dominates $s_{y,u}$, which means

$$p_{u,\alpha}^{*2} \leq s_u^2 \quad (22)$$

$$p_{y,\alpha}^{*2} \leq s_y^2 \quad (23)$$

80 The performance will be assessed based on the following indicator:

$$\eta_{MC} = \frac{p_{y,\alpha}^{*2}}{s_y^2} \quad (24)$$

Note immediately that

$$p_{y,\alpha}^{*2} \geq \sigma_{MV}^2$$

and therefore

$$\eta_{MC} \geq \eta_{MV} \quad (25)$$

82 4. Numerical results

Consider the first-order system described by

$$y(k) = \frac{1}{1 + a_1 z^{-1}} [u(k) + \varepsilon(k)] \quad (26)$$

84 with $a_1 = -0.9$ (also considered in [3]) and controller

$$G_c(z^{-1}) = \frac{0.2z^{-1}}{1 - 0.8999z^{-1}} \quad (27)$$

As explained in the previous section, first we generate the data

$$y(-N_H) \dots, y(-2), y(-1) \quad (28)$$

86 and identify the following third-order ARMA model, assuming $\hat{n} = 3$, $\hat{m} = 3$,

$$\hat{G}_{cl}(z^{-1}) = \frac{\hat{B}_{cl}(z^{-1})}{\hat{A}_{cl}(z^{-1})} = \frac{1 - 0.7863z^{-1} - 0.0629z^{-2} - 0.0298z^{-3}}{1 - 1.743z^{-1} + 0.620z^{-2} + 0.09022z^{-3}} \quad (29)$$

and generate S^* showed in Fig. 2, with $N_P = 30$, and

$$K_\alpha^*(z^{-1}) = \alpha \frac{0.904 - 1.504z^{-1} + 0.505z^{-2} + 0.1020z^{-3}}{1 - 0.7863z^{-1} - 0.0629z^{-2} - 0.0298z^{-3}} \quad (30)$$

88 Note that the distribution of points (black dots) is not uniform. The red line cor-
 responds to the interpolation curve corresponding to the Pareto approximation
 90 obtained by using the real polynomials $A(z^{-1})$ and $B(z^{-1})$.

Next, we generate the data

$$y(0), y(1), y(2), \dots, y(N-1)$$

which corresponds to the window we want to assess, obtaining the following
 92 results

$$(s_y^2, s_u^2) = (2.74, 0.5) \quad (31)$$

$$(p_{u,\alpha}^{*2}, p_{y,\alpha}^{*2}) = (1.22, 0.28) \quad (32)$$

$$\eta_{MV}(\%) = 36.5 \quad (33)$$

$$\eta_{MC}(\%) = 44.4 \quad (34)$$

Note, that, as expected, the proposed multi-criteria index is almost 8% greater
 94 than Harris' index. The code to obtain these results is available in the following
 link: https://github.com/multiopiti/research/blob/main/MC_CLPA.ipynb

96 5. Conclusion

A multi-criteria approach for control loop performance assessment was pro-
 98 posed, which allows to take into account the trade-off between process variable
 and manipulated variable energy, thus overcoming one of the most important
 100 criticisms to Harris' index. In future, we plan to extend this method to more
 complex models, including unstable multivariable processes.

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Figure 2: Pareto approximation

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