Network Throughput and Outage Analysis in a Poisson and Matérn Cluster based LTE-Advanced Small Cell Networks

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Abstract

In LTE-A (LTE-Advanced), the access network cell formation is an integrated form of outdoor unit and indoor unit. With the indoor unit extension the access network becomes heterogeneous (HetNet). HetNet is a straightforward way to provide quality of service (QoS) in terms better network coverage and high data rate. Although, due to uncoordinated, densely deployed small cells large interference may occur, particularly in case of operating small cells within the spectrum of macro base stations (MBS). This paper probes the impact of small cell on the outage probability and the average network throughput enhancement. The positions of the small cells are retained random and modelled with homogeneous Poisson Point Process (PPP) and Matérn Cluster process (MCP). The paper provides an analytic form which permits to compute the outage probability, including the mostly applied fast fading channel types. Furthermore, simulations are evaluated in order to calculate the average network throughput for both random processes. Simulation results highlights that the network throughput remarkably grows due to small cell deployment.

Keywords: Femtocell, HetNet, LTE-Advanced, Matérn Cluster process, Network Throughput, Outage Analysis, Small cells

1. Introduction

Nowadays, in many countries LTE (Long-Term Evolution) provides high data rate mobile communication. The LTE network was standardized by 3GPP (3rd Generation Partnership Project) [1]. Nevertheless, evolution and research process are incessantly runs in background, in order to make faster and better networks. The updated version of LTE is the so-called LTE-Advanced (LTE-A)

network. The literature refers LTE-A as a 4.5G mobile network. The standardization of LTE-A is currently in progress. Nevertheless, rigorous requirements (i. e. throughput, latency), have been already defined for 5G networks. The appearance of 5G capable networks are expected around 2020 [2]. The rigorous requirements of throughput in 4.5G and 5G can be satisfied by increasing the level of coverage. The importance of coverage becomes a rather important issue especially in densely populated areas, where due to population, the number of (mobile) users are increasing rapidly. Therefore, the demand for mobile network services (especially high data rate or low latency services – such as data transfer, media stream etc.) increase extremely.

In the network planning phase, the mobile operators should take into account these aspects, in order to serve as many users as possible, with a strictly defined service quality level. In other words, the goal is to fulfil the Quality of service (QoS) requirements. To enhance the coverage and throughput, an essential solution is to extend the conventional, “one-tier” macrocell architecture to a multi-tier architecture [3, 4]. The macrocell tier can be augmented by the small cell. The small cell tier is formed by the combination of several femtocells. Micro- and picocells are already widely used in densely populated areas, such as shopping malls, crowded public transport stations etc. A femtocell is a base station with low power emission. The purpose to use femtocells is to eliminate coverage holes for example in indoor or urban environments where due to the building material the coverage level of outdoor macrocell might not be sufficient to ensure QoS. Although, the size of the femtocell covered area is lower, compared to a macrocell, femtocells can reduce the load in the macrocell tier [5]. The goal is to bring the base stations as close as possible to the users. This potentially facilitates supporting higher data rates. In order to keep the model general, the locations of the small cells are kept random. Two dimensional random processes are suitable for modelling small cell locations. We apply homogeneous Poisson Point process (PPP) and a special Poisson Cluster Process (PCP), the so-called Matérn Cluster process (MCP). The analysis of small cells to reduce number of outage users and also to increase network coverage in PPP based deployment is already have a huge literature. In [6] an optimization of the macro base stations is given the macro cell deployment and the macro eNBs are modelled with PPP.

Authors of [7] investigate the impact of interference on the coverage and connectivity of Ad hoc networks. Kim *et al.* in [8] provide a performance analysis for a two-tier femtocell network. Cognitive femtocells networks were analyzed in [9]. A remarkable survey of stochastic geometry based network modelling is provided in [10]. Authors of [11] provide a simulation tool to gain the perfect small cell deployment scenario. The distribution of the cumulative small cell interference is previously investigated in [12] using the idea of stochastic geometry. Shue Chen *et al.* investigate random and grid topologies for small cell deployment modelling [13]. Authors of [14] provide a PPP based framework for performance analysis of the wireless infrastructure with a high density of access points, Haenggi and Ganti in [15] investigate general wireless networks with the aid of stochastic geometry. In [16] Andrews *et al.* use stochastic geometry in order to investigate the outage probability and the transmission rate in a pure macro cell structure (“one-tier” access network). The results of [16] have been extended to a multi-tier network in [17].

This paper investigates the effect of densely deployed small cells in a two-tier LTE-A network. We calculate the service outage probability relying on interference distribution (given in [12]) assuming a perfectly homogeneous PPP scattered small cell tier. In [16, 17] the users outage probability is calculated with the so-called probability generating functional (PGFL). In this paper in a PPP model we use the interference distribution to get the users’ outage probability. Simulation results are provided for outage probability in a Matérn Cluster based small cell tier. Furthermore the average network throughput is investigated for both point processes in two types of fading channel, such as Nakagami- m fading type and Rayleigh fading type. Matérn cluster process can be utilized by several input parameters, therefore the proposed modelling method might be useful for mobile operators in the network planning phase by, deploying small cells to the desired area.

The rest of the paper is organized as follows. The system model with a brief introduction of Matérn Cluster process and the interference description is given in Section 2. Section 3 provides the calculation of outage probability with the aid of stochastic geometry. The average network throughput performance tests are evaluated in Section 4 and finally Section 5 concludes the paper.

2. System model

Our network model is a determinate square area indicated by $|\mathbb{R}|$. The model is a two dimensional topology, where the first layer is the macro cell layer, meanwhile the second layer is constructed from the small cells. We define middle of the area as outset of the coordinate system. The macrocell Evolved NodeB (eNB) are scattered on $|\mathbb{R}|$ and follows Poisson Point process with intensity parameter λ_m . The set of macrocell eNBs is denoted by Φ_m . The small cells are installed in $|\mathbb{R}|$ as stated by random processes (represented by Φ_s). The emitted constant power from macrocell and small cells eNBs are indicated by P_m and P_s , respectively. It is considered that the macrocell and small cell eNBs have omni directional antennas. The employed scheduling algorithm is round robin (RR). Small cells are operating in licensed downlink spectra with a bandwidth of 20 MHz (E-UTRA Band 23) [18] and the carrier frequency of this band is 2190 MHz. The macro eNBs and the small cell eNBs share the same spectrum, the spectra of macro tier and small tier completely overlaps. In LTE-A the air interface is realized jointly by Orthogonal Frequency Division Multiplexing Access (OFDMA) and advanced antenna techniques (multiple-input and multiple-output-MIMO). It applies adaptive modulation and coding (AMC) in order to enhance the throughput and spectral efficiency. The minimum resource allocation unit (in LTE access network) that can be assigned to the users is the Physical Resource Blocks (PRB). The PRB is a collection of a many adjacent OFDM based OFDM sub-carriers. These sub-carriers are assigned to a user for a predetermined amount of time. Thus PRBs have both a time and frequency dimension. The PRB contains pilot and control signals [19]. The number of allocatable PRBs is denoted by N_{PRB} and the number of PRBs for a bandwidth of 20 MHz is 100. We assume that the small cell tier is independent from the macrocell tier, thus no collaboration is considered (e. g. interference management or control) between the base stations (eNBs). In the macrocell tier, the co-channel interference is neglected, in other words the macrocell eNBs do not interfere each other. However, small cells are allocating resources to potential users (within their coverage zone) independent from each other and the macrocells. The resource allocation between greedy users is fair, thus there is no priority order between the users. We consider a heavily loaded access network, where every user requires the

whole offered resource in the access network. Therefore, if only one user is in a small cell, then resources are given to the user ($N_{\text{PRB}} = 100$). In case of having multiple users in a small cells, entire resources are given to the users. For example, if 4 active users are there in a small cell, then each user get 25 PRBs. If there is no user in the cell, then no resources have been allocated.

The mobile users, known as user equipments (UEs), are scattered in the network area according to PPP. The mean number of users is denoted by N_u . The actual location of a user in the plain is defined by vector $\mathbf{z}$ and the distance from the origin is $\|\mathbf{z}\|$. The mobility of the users are not modelled in this paper, therefore it can be paraphrased as a snapshot view of the network. Although the mobility is not modelled, the positions of the small cells and the users ($\mathbf{z}$) is also random.

The location of the small cell eNBs is modelled with two random processes, namely: the Homogeneous PPP and Matérn Cluster process, respectively. A brief characterization of the aforementioned random processes is as follows:

2.1. Homogeneous Poisson Point process (PPP)

According to the homogeneous PPP model the small cells (and the mobile users) are scatter in $|\mathbb{R}|$ uniformly. The actual number of small cells follows Poisson distribution with density λ . The mean value of the small cells, thus $N_s = \lambda \cdot |\mathbb{R}|$. In this model, we assume that the process is homogeneous, thus the points are scattered uniformly in the plane. In this case, the intensity parameter λ is constant in every part of $|\mathbb{R}|$. The actual position of the small cells is symbolized by vector $\mathbf{x}$. The small cells distance from the origin is represented by the absolute value of the vector $\|\mathbf{x}\|$. Furthermore the distance between a small cell and the appointed user is defined by $\|\mathbf{x} - \mathbf{z}\|$. Note that, if the process is not homogeneous, then the intensity parameter should be given as $\lambda(\mathbf{x})$. The homogeneous version of Poisson point process is a widely used, accepted and popular model. Our model uses homogeneous point process. For the sake of simplicity, we drop the word homogeneous and refer the process simply PPP hereafter.

(a) Illustration of (Homogeneous) Poisson point process

(b) Illustration of Matérn Cluster process

Figure 1: Random processes applied for Small cell deployment modeling

2.2. Matérn Cluster process (MCP)

The Matérn Cluster process belongs to the family of Poisson cluster processes (PCP) [20]. This location model breaks the homogeneous nature and groups the small cells into clusters. Thus in some parts of the field $|\mathbb{R}|$ the intensity of small cell is higher than others. Furthermore, there are a few part of the area ($|\mathbb{R}|$), where no small cells can be found. This cluster based process provides an accurate model compared to PPP, since in real environments some parts of the area might have

higher small cell intensity (e. g. block of flats), than others parts of the area (e. g. public parks). Matérn cluster process stands on a Poisson point process with parameter λ_p . The literature refers to these points as parent points. In our model the actual position of the parent points is defined by vector $\mathbf{x}_p$. The daughter points are spread over the parent points [20]. The daughter points are independent from each other and from the other parent points. Furthermore, they are identically distributed around the parent point. Note that the parent points are not members of the cluster. Furthermore the actual number of daughter points in a cluster is also random, and follows Poisson distribution. It is important to highlight that in a Poisson Cluster process (e. g. MCP) the actual position of the daughter points (around the parent points) and the actual number of the daughter points are also random. The mean value of the daughter points in a cluster is represented by $\bar{c}$. The distance between daughter points and the cluster centres (parent points) is given by vector $\mathbf{y}$. In MCP, the daughter points represents the small cells. Their actual location is defined by vector $\mathbf{x}_p + \mathbf{y}$. The density of the cluster process can be calculated with $\lambda = \lambda_p \cdot \bar{c}$, where $\bar{c}$ represents the average number of small cells in a cluster. The daughter points spread over the parent points uniformly in a circle with radius R . The density function is given by [20]:

$$f(\mathbf{y}) = \begin{cases} \frac{1}{\pi R^2}, & \text{if } \|\mathbf{y}\| \leq R \\ 0, & \text{otherwise} \end{cases} \quad (1)$$

An illustration of Matérn Cluster is given in Figure 1b. Note that, the actual number of small cells in a cluster is still a random variable, only the mean value is given ($\bar{c}$). Furthermore, the actual number of the clusters is also a random variable. The mean number of small cells in this case: $N_s = \lambda_p \cdot |\mathbb{R}| \cdot \bar{c}$.

2.3. Path loss model and Fast fading

The employed path loss for small cells stands on the non-logarithmic version of Stanford University Interim (SUI) channel model [21], however some parameters have been modified due to

network specific requirements e. g. carrier frequency:

$$g(\mathbf{z}) = \frac{1}{K_i} \cdot \|\mathbf{z}\|^{-\alpha}, \quad (2)$$

where $g(\mathbf{z})$ indicates the path loss at the defined position. Path loss due to carrier frequency and other parameters are included to the model via the constant values. The constant parts (e. g. propagation loss due to carrier frequency) are merged into K_i and $K_i = 15.85$. The outdoor path loss exponent is symbolized by $\alpha = 4$. The effect of shadowing and Additive White Gaussian Noise (AWGN) is neglected in this paper. Furthermore, for the sake of simplicity, we ignore the co-tier interference and focus on only cross-tier (between macrocell tier and small cell tier) interference. It is assumed that the MIMO channel paths are independent of one another and the employed transmitter (and receiver) antenna does not change the path loss. Let us assume that the transmitter emits on P_m and the receiver is located at $\mathbf{z}$. Therefore the received power at this location is: $P_m \cdot g(\mathbf{z}) = P_m \|\mathbf{z}\|^{-\alpha} K_i^{-1}$. The effect of fast fading in power domain is indicated by h and it is considered as independent and non-distorting. h follows Gamma distribution with parameters m shape and $1/m$ scale, if the channel has Nakagami- m fading [22, 23]. The sum of i.i.d. (independent and identically distributed) Rayleigh distributed random variables follows Nakagami distribution [22, 24]. The probability density function (p.d.f) of a Gamma random variable can be expressed as follows [23]:

$$f(h) = \frac{h^{m-1} \cdot e^{-h/m}}{\left(\frac{1}{m}\right)^m \Gamma(m)}, \quad (3)$$

where $\Gamma(m)$ indicates the complete Gamma function: $\Gamma(m) = \int_0^\infty x^{m-1} e^{-x} dx$. Point to be noted, $m = 1$ yields Rayleigh fading. Here, h follows exponential distribution with rate 1.

3. Interference and Signal-to-Interference Ratio

The straightforward step to characterize the received (cumulative) interference power in a given random location is given as follows. Firstly, let us assume that a macro cell user is settled at $\mathbf{z}$. In this case, the total received interference power from small cells can be calculated with the following

formula [15]:

$$I^{\text{PPP}}(\mathbf{z}) = \sum_{\mathbf{x} \in \Phi_s} P_s h g(\mathbf{x} - \mathbf{z}), \quad (4)$$

$$I^{\text{MCP}}(\mathbf{z}) = \sum_{\mathbf{x}_p + \mathbf{y} \in \Phi_s} P_s h g(\mathbf{x}_p + \mathbf{y} - \mathbf{z}). \quad (5)$$

Vector $\mathbf{x}_p + \mathbf{y}$ indicates the random position of interference source base stations (from set Φ_s) in case of MCP. The impact of i.i.d fast fading is indicated by h and the path loss between the receiver unit and the interference source is stated as $g(\|\mathbf{x}_p + \mathbf{y} - \mathbf{z}\|)$. Of course LTE is a multi-carrier OFDMA based communication system, where the subcarriers are clamped to PRBs. Nevertheless, we consider a highly loaded networks, where entire possible resources (PRBs) are assigned to potential users, therefore the overall interference is interpretable with (5).

In PPP model (small cell locations are modelled with PPP), if the outdoor path loss exponent is $\alpha = 4$, the distribution of the cummulative interference has a closed form cumulative distribution function (c.d.f.) and probability density function (p.d.f.). Furthermore, the c.d.f is the so-called Lévy distribution [12, 15]:

$$f_{I^{\text{PPP}}}(x) = \sqrt{\frac{c}{2\pi}} \frac{\exp\left(-\frac{c}{2x}\right)}{x^{3/2}}, \quad (6)$$

$$F_{I^{\text{PPP}}}(x) = \mathbf{P}\{I_{\text{UE}} \leq x\} = \text{erfc}\left(\sqrt{\frac{c}{2x}}\right). \quad (7)$$

where $\text{erfc}(x) = 1 - \text{erf}(x) = \frac{2}{\sqrt{\pi}} \int_x^\infty e^{-t^2} dt$ is the complementary error function [25]. The scale- and location parameters are $c = \pi^3 \lambda^2 (\mathbb{E}\{\sqrt{h}\})^2 P_s / 2$ and $\mu = 0$, respectively. Without fast fading $h = 1$, therefore $\mathbb{E}\{\sqrt{h}\} = 1$, evidently. Meanwhile, with Rayleigh fading $\mathbb{E}\{\sqrt{h}\} = \Gamma\left(1 + \frac{2}{\alpha}\right) = \Gamma\left(\frac{3}{2}\right) = \frac{\sqrt{\pi}}{2}$ [15]. For Nakagami- m fading

$$\mathbb{E}\{\sqrt{h}\} = \frac{(2m-1)!!}{2^m(m-1)! \sqrt{m}} \sqrt{\pi},$$

where $m!!$ indicates the double factorial of a positive integer m . The detailed calculation of $\mathbb{E}\{\sqrt{h}\}$

for Nakagami- m fading is given in [12].

The PPP process is stationary and isotropic [20], thus the actual receiver location is irrelevant, the distribution of the interference is valid for every $\mathbf{z}$. Hence, for the outage probability investigation, we fix the mobile user to the centre i. e. distance $\|\mathbf{x} - \mathbf{z}\|$, simplifies to $\|\mathbf{x}\|$, which is actually the distance small cell from the origin. In order to calculate the outage probability for a macro cell user, we assume that the macro cell user connects to the closest macro eNB. Let us introduce a new variables $\mathbf{r}$, which denotes the coordinate of the closest macro eNB.

Now let us calculate the Signal-to-Interference ratio (SIR) distribution a macro user (also known as probability of service outage for a user) with the aid of the interference distribution:

$$\mathbf{P}\{\text{out}\} = \mathbf{P}\{\text{SIR} < T\} = \mathbf{P}\left\{\frac{P_m h \cdot g(\mathbf{r})}{I^i} < T\right\}, \quad (8)$$

where $\mathbf{r}$ represents the coordinates of the closest macro eNB, whilst the interference at this location is indicated by I^i , which is formulated in (4) or (5), depending upon the small cell deployment model. Parameter T denotes the threshold value and it is a network specific parameter.

To quantify the outage probability, initially, we modify the SIR distribution. Note that, we have a closed form c.d.f only for PPP case thus, hereafter the calculations are valid for PPP model.

$$\mathbf{P}\left\{\frac{P_m h \cdot g(\mathbf{r})}{I_o} < T\right\} = 1 - \mathbf{P}\left\{I_o \leq \frac{P_m h \cdot g(\mathbf{r})}{T}\right\}. \quad (9)$$

After substituting (2) to $g(\mathbf{z})$, we get:

$$\mathbf{P}\{\text{out}\}(\mathbf{z}) = 1 - \mathbf{P}\left\{I_o \leq \frac{P_m h \cdot \|\mathbf{r}\|^{-\alpha}}{K_i T}\right\}. \quad (10)$$

Of course P_m , K_i and T are input parameters to the model, meanwhile I_o , $\mathbf{r}$ and h are remain random

variables. Applying the Law of total probability (for continuous distributions) on (10) yields:

$$\mathbf{P}\{\text{out}\} = \int_0^\infty \int_0^\infty \underbrace{\left(1 - \Pr\left\{I \leq \frac{P_m h \|\mathbf{r}\|^{-\alpha}}{K_i T}\right\}\right)}_{\text{c.d.f of Lévy distribution}} \cdot f(h) \cdot f(\mathbf{r}) d\mathbf{r} dh, \quad (11)$$

The p.d.f of h ($f(h)$) is given in (3) and $f(\mathbf{r})$ denotes the probability density function of the closest macro eNB [16]:

$$f(\mathbf{r}) = \exp\left(-\lambda_m \pi \mathbf{r}^2\right) \cdot 2\pi \lambda_m \mathbf{r}. \quad (12)$$

Substituting $f(\mathbf{r})$, $f(h)$ and the c.d.f of the Lévy distribution given in (7) to (11) yields:

$$\begin{aligned} \mathbf{P}\{\text{out}\} = & \int_0^\infty \int_0^\infty \left(1 - \operatorname{erfc}\left(\frac{N_s \pi^{3/2} \sqrt{P_s K_i^{-1}}}{2 \cdot |\mathbb{R}|} \sqrt{\frac{h}{\frac{T}{P_m h \|\mathbf{r}\|^{-\alpha}}}}\right)\right) \\ & \cdot \frac{h^{m-1} \cdot e^{-h \cdot m - \lambda_m \pi \mathbf{r}^2} \cdot 2\pi \lambda_m \mathbf{r}}{\left(\frac{1}{m}\right)^m \Gamma(m)} d\mathbf{r} dh. \end{aligned} \quad (13)$$

4. Performance Analysis

The correctness of (13) is verified with simulations. The Monte Carlo simulations are evaluated with MatLab. This model contains a basic Round robin scheduling algorithm. The simulated network handles the number of macrocell eNBs and small cell eNBs as random values, only their mean value is given with N_c and N_s , respectively. The small cells are deployed outdoor (thus wall penetration loss is considered to be zero, however it can be easily included to parameter K_i). The macrocell and small cell eNBs transmission power is $P_m = 20$ W (43 dBm) and $P_s = 200$ mW (23 dBm), respectively. The nominal network bandwidth is 20 MHz, provides us $N_{\text{PRB}} = 100$ per slot. The threshold value is $T = 0$ dB. The average number of mobile users in the network is $N_u = 100$. The equipment have 2×2 MIMO antenna configuration. The simulation environment has average N_u active mobile terminals. The network is fully loaded i. e. all PRBs are assigned. The network area $|\mathbb{R}|$ is spread over 10^8 square metre. The SIR results (in the simulations) are gained from (8) and the results are averaged value of $N = 10^3$ iterations.

Figure 2: Probability of Outage for Macro-tier vs. mean number of Small cells (PPP)

4.1. Outage probability for PPP deployment scenario

The analytic forms are calculated considering the set of input parameters as P_m, P_s, T etc. The same parameter set is applied for the simulations. The results are presented in Figure 2. The mean number of small cells N_s is indicated on the horizontal axis and vertical axis scales the probability of outage. The analytic results are calculated with (13) and denoted by dashed lines, meanwhile the markers represent the simulation results. We calculated the outage probability for both Rayleigh fading and Nakagami-4 fading, with $\lambda_m = 2 \cdot 10^{-7}$ and $\lambda_m = 5 \cdot 10^{-8}$, respectively. Note that the network area is a 10^4 meter $\times$ 10^4 meter square, thus the mean values of macrocell eNBs are 20 and 5, respectively. The simulation results validate that (13) is an accurate form to calculate the outage probability for a macro cell user.

4.2. Outage probability for MCP deployment scenario

In this section, we provide simulation results for MCP deployment scenario, similarly to the previous case. The radius of the clusters are set to fix $R = 100$ m. According to the concept of MCP the small cells are deployed around the parent points, thus we distinguish two scenarios. In the first scenario, we restrict the average number of small cells in a cluster to $\bar{c} = 5$. In this case, depending upon N_s , we consider clusters containing relatively low number of small cells.

Meanwhile in the next scenario, where $\bar{c} = 20$, we have a few, but large clusters. It is important to highlight the fact that from network point of view, the mean number of small cells are the same for both scenarios, however the deployment of the small cells rather different. The results are presented in Figure 3a and Figure 3b. It is visible that the macro-layer outage probability is lower in Figure 3b, since the small cells are squeezed into a few big cluster, therefore the cumulative interference is lower, compared to the other MCP scenario and PPP. PPP can be interpreted as the worst case scenario, since in this case the small cells are deployed uniformly the whole area, causing massive interference to the macro layer.

(a) MCP Small cell deployment model, $\bar{c} = 5$

(b) MCP Small cell deployment model, $\bar{c} = 20$

Figure 3: Probability of Outage for Macro-tier vs. mean number of Small cells (MCP)

SIR [dB]	Throughput [kbps]/PRB	SIR [dB]	Throughput [kbps]/PRB
-12	0	9	340
-9	15	12	480
-6	38	15	620
-3	70	18	740
0	110	21	850
3	170	24	910
6	250	27	930

Table 1: Measured throughputs in MIMO 2×2 configuration [26]

4.3. Average Network Throughput

The first step of calculating the average network throughput is to get the SIR values. Next, we can map the averaged SIR values with the values of Table 1. In this investigation, every base station is open, therefore every user connects to the eNB that has the highest SIR value. The radius of the clusters are $R = 100$ m and $R = 500$ m. The results are presented in Figure 4. The horizontal axis indicates the mean number of small cells N_s , meanwhile the vertical axis scales the average network throughput. From the results, it is visible that PPP provides the best performance in both

fading types. Of course in this model, there is higher probability that a user can connect to a small cell compared to MCP cases, where a remarkable amount of user have to connect to macro eNBs, due to the cluster and therefore the average network throughput is lower, since the macro eNBs are allocating/sharing resources between more users. In Figure 4b, if we extend the radius of the clusters, then the average system throughput increases, hence small cells (in the cluster) are now scattered on a bigger area and a huge amount of users can connect to these cells, instead of connecting to macro eNBs. Therefore small cells offload macro eNBs.

Afterwards let us investigate the macro user throughput level in contrast to the mean number of small cells. The result are presented in Figure 5. Similarly to Figure 4, horizontal axis scales the mean number of small cells, meanwhile the vertical axis denotes the average macro user throughput. In this situation, we choose a designated macro cell user and investigate the gained throughput. In every iteration the macro users position and the location of interference sources are random. Finally the iteration results are averaged. The averaged values are denoted with the markers in

(a) $R = 100$ m

(b) $R = 500$ m

Figure 4: Average System throughput vs. N_s

Figure 5. As expected the increasing number of small cells remarkably reduces the average macro user throughput. Similarly to the system level throughput the cluster size ($\bar{c}$) has a great influence on the results. The results belong to $\bar{c} = 20$ are illustrated with dashed lines. It is visible, that higher macro user throughput is gained when $\bar{c} = 20$. In this case the interference source small cells are squeezed into a few, but large clusters. Meanwhile in scenario $\bar{c} = 5$ we have more clusters, although the number of small cells in the cluster are lower.

Figure 5: Average Macro user throughput vs. mean number of Small cells ($R=100$ m)

5. Conclusion

With the proposed PPP and MCP random processes, we can model the access network as a part of a two-tier LTE-A cell network. Relying on the mathematics tools offered by stochastic geometry, in PPP model, we have calculated the outage probability for a macrocell user in contrast of the mean value of small cells. This closed form allows to calculate the outage probability, if the channel has Rayleigh fading or Nakagami- m fading. The results showed that the small cells can provide high outage to macrocell users. Another interesting finding of the paper is, that if the access network is modelled with MCP, then the outage probability is lower, compared to PPP. Therefore PPP model can be interpreted as a worst case scenario. In MCP, the outage probability highly depends on the cluster size ($\bar{c}$), thus only the mean value (N_s) is not enough to get reliable results. On the other hand, small cells can increase the overall network throughput by providing better receiving conditions to the users connected to small cells. In MCP, if the cluster size is small (R) i. e. the small cells are grouped into small circle, the average network throughput is lower compared to the scenario, where the cluster size is higher. PPP provides the solution where the average network throughput is maximal.

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