On Beamforming Gain Models for Performance Evaluation and Analysis of Narrowband and Wideband Wireless Networks

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Abstract

Directional antennas and beamforming techniques that bring promising transmission gain to the wireless links are widely incorporated in the system-level analysis of wireless networks. In many existing studies, the beamforming gain model to calculate the beamforming gain considers beamforming pattern and aligned as well as misaligned cases. However, the channel properties, e.g., the K factor and the spatial distribution of multi-paths, are neglected, which could significantly influence the beamforming gain. In this paper, a general beamforming gain model is appropriately defined, while the traditional beamforming gain model is proved to be only a special case in the proposed general model by considering an oversimplified channel with no angular spread. In light of this, expressions of the received signal amplitude and the beamforming gain are rigorously derived for narrowband fading and wideband statistical mmWave channels, respectively. Thorough comparison between the proposed beamforming gain model and the traditional beamforming gain is provided, which demonstrates and validates that the traditional model incorrectly captures the beamforming gain and thereby, leads to inaccurate system-level network analysis. To this end, the effectiveness and importance of the proposed general beamforming gain model are revealed, particularly for millimeter-wave wideband systems.

Index Terms

Beamforming, Channel Model, Millimeter-wave, Stochastic geometry, Wireless Communication Networks.

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I. INTRODUCTION

System-level performance evaluation and analysis are widely recognized to be critical in design of wireless networks [1]. In the past few decades, stochastic geometry has emerged as a powerful approach due to its analytical tractability in the performance evaluation and analysis of wireless networks for ad hoc networks, cellular networks, and local and personal area networks [2]–[5]. With directional antennas and MIMO techniques, directional beam demonstrates the potential of combating fading, increasing spectral efficiency and potentially reducing interference. Recently, it has been incorporated in the stochastic-geometry-based network evaluation and analysis as an additional beamforming gain [2]–[16]. For instance, the authors in [16] propose use finite homogeneous Poisson point process, which is suitable for finite regions with a random number of nodes, and comprehensively address the modeling and analysis of finite mmWave wireless networks using the properties of the PPP. Moreover, the transmitter selection strategy that chooses maximum received power among LoS and NLoS transmitters is considered. In particular, the beamforming gain model utilized in those studies to calculate the statistics of the beamforming gain takes the beamforming pattern and beamforming case into account. However, the critical channel properties, e.g., the K factor and direction distribution of the multi-paths, are missing in the *traditional beamforming gain model* that we refer to in the rest of this paper. Specifically, the traditional model is based on a sectorized beamforming pattern and developed by assuming two cases of the beamforming, i.e., the aligned beamforming and the misaligned beamforming case. In the aligned case, the beamforming gain is assumed as the gain of the main lobe of the beamforming pattern. In the misaligned beamforming case, the main lobe is randomly directed, and thus the beamforming gain is a discrete random variable, taking the value of the gain of each lobe with the probability of the normalized beamwidth of the lobe.

In line with this, the authors in [12] recently attempt to address the inappropriate modeling of the channel gain and beamforming gain by considering the realistic channel model and the realistic antenna radiation model in the antenna array for mmWave network analysis. They focus on replacing the isotropic antenna radiation model with a realistic antenna radiation model (3GPP model) and adopting a measurement-based channel model to generate a wireless channel. Unfortunately, they do not provide any analytical derivation on the channel gain and the beamforming gain. Instead, they use a curve-fitting method to approximate the statistics of the channel gain and the beamforming gain based on the simulation results.

The objectives of this paper are twofold. First, we address the incorrect modeling of the beamforming gain in the system-level analysis of the wireless networks in the literature [2]–[16]. Second, we demonstrate the traditional beamforming gain model leads to an incorrect evaluation of the signal power and hence system performance. To reach these goals, we first introduce the system model for wireless communications with beamforming and traditional beamforming gain model utilized in the system-level analysis. Moreover, we present the general definition of beamforming gain deriving from the perspective of signal power, which accounts for the beamforming pattern, beamforming case as well as channel parameters. Furthermore, we rigorously derive the true beamforming gain models for the most common narrowband fading channel models, i.e., Rayleigh fading and Rician fading model, in sub-6 GHz band, as well as the wideband statistical channel models in mmWave band under aligned and misaligned beamforming cases. Finally, through extensive numerical analysis, we investigate the impact of the beamforming gain model on the distribution of the received power, under different channel models and beamforming cases. To further show the impact of the proposed beamforming gain model on the system-level performance analysis and evaluation of wireless networks, we analyze the distributions of signal-to-interference-noise-ratio (SINR) and the consequent coverage probability based on the stochastic geometry method, with varying beamforming gain models under the wideband NYU channel model.

The contributions of this paper are summarized as follows.

- **We present a proper definition of the beamforming gain that incorporates the beamforming pattern, beamforming case as well as spatial channel properties.** Indeed, it is proved that the traditional beamforming gain model in [2]–[16] is a special case of the proposed beamforming gain model when the channel energy is directed in one direction.
- **We develop the beamforming gain model in a narrowband fading channel, i.e., Rayleigh fading and Rician fading channels in a scattering environment.** We further derive the closed-form expressions of the received signal amplitude with the beamforming gain in the two fading channels in comparison with the traditional beamforming model (**Remarks 1-4**).
- **We develop the semi-closed-form expression of the beamforming gain in the wideband statistical channel models.** The boundaries of the beamforming gain and the probabilities of the boundaries are analytically investigated (**Remarks 5-6**).
- Extensive comparison between the proposed beamforming gain model and the traditional beamforming gain model is carried out in both narrowband and wideband channels. **We**

validate and show that the traditional beamforming gain model incorrectly evaluates the beamforming gain. The inaccuracy of the traditional model increases with a large number of antennas and the low K factor.

- **The impact of the channel parameters and antenna array parameters on the received signal amplitude and the beamforming gain are thoroughly analyzed.** We reveal that the channel parameters significantly influence the boundaries and the distribution of beamforming gain, which is not captured in the traditional beamforming gain model. Numerical results demonstrate that the impact of the channel parameters as well as the number of antennas are inappropriately estimated under the traditional beamforming gain model.
- **System-level analysis by the stochastic geometry method with different beamforming gain models is studied.** We observe that the traditional model overestimates the SINR as well as ergodic capacity with massive antennas and a small channel K factor, since it underestimates the interference links. In addition, the larger number of antennas steadily brings significant improvement of SINR in the traditional model, which is however incorrect. Instead, the proposed models show that the increment of SINR decreases as the number of antennas increases.

The remainder of this paper is organized as follows. In Sec. II, the traditional beamforming gain model is introduced. Then the general definition of the beamforming gain is provided and compared with the traditional model. The beamforming gain in the narrowband fading channel is analyzed in Sec. III. The beamforming gain in the wideband statistical channel is investigated in Sec. IV. Extensive numerical results and analysis are presented in Sec. V. Finally, this paper is concluded in Sec. VI.

II. SYSTEM MODEL AND BEAMFORMING GAIN MODEL

In this section, we introduce a system model for wireless communication networks with beamforming techniques. In wireless networks, a base station (BS) is the transmitter (Tx) side that is equipped with multiple antennas and conducts beamforming and beam-tracking technique, while the receiver (Rx) is the user equipment (UE), as shown in Fig. 1. The locations of both BSs and UEs follow infinite homogeneous Poisson point process (HPPP), which is the basic assumption of stochastic-geometry-based studies. The beamforming gain model and analysis can be extended for the wireless communication networks where both Tx and Rx are equipped with multiple antennas.

Fig. 1. Stochastic geometry of wireless communication networks.

(a) Multi-cone model of the beamforming pattern. (b) Cone-plus-sphere model of the beamforming pattern.

Fig. 2. Traditional beamforming pattern models.

A. System Model for System-level Analysis

In the system-level analysis of the wireless communication networks, the received power, P_r , is regarded as a superposition of the beamforming gain at Tx, G_t , channel gain, H , transmit power, P_t with additional noise power, N_0 , given by,

$$P_r = G_t H P_t + N_0 \quad (1)$$

where channel gain H is determined by the channel model. In the system-level analysis of wireless communication networks, a *narrowband fading model* is commonly used, in which the channel gain consists of two parts, i.e., *large-scale fading* and *small-scale fading* [17].

On one hand, the large-scale fading usually comprises of path loss and shadowing. On the other hand, the small-scale fading is described by a complex random variable. The distribution of the complex random variable is different in different narrowband small-scale fading models. Commonly-used small-scale fading models are Rayleigh fading, Rician fading, and Nakagami fading models [9], [18]. For example, in a Rayleigh fading model, one assumes the real and imaginary parts are two Gaussian with zero mean and same variance, and thereby, the envelope of the channel is Rayleigh distributed, and its phase is uniformly distributed. This model is usually utilized in the rich scattering environment with no dominant path, e.g., a blockage of the line-of-sight (LoS) path.

B. Beamforming Pattern

Before we introduce the beamforming gain model utilized in the literature, we need to introduce the beamforming pattern. The beamforming pattern, $A_p(\theta)$, describes the amplitude gain of the transmitted waves at the angular direction θ , which is determined by the configuration

of the antenna array or the directional antenna at the base station. Without loss of generality, we assume that the max amplitude gain is at the direction of $\theta = 0^\circ$, i.e., $A_{\max} = A_p(0)$. The beamforming pattern after beamforming and beam-tracking that directs the main lobe to the direction of θ_{main} , $A(\theta; \theta_{\text{main}})$, can be expressed by, $A(\theta; \theta_{\text{main}}) = A_p(\theta - \theta_{\text{main}})$.

As the beamforming pattern usually consists of a main lobe and multiple side lobes, the beamforming pattern is generally modeled by a multi-cone model, $A_p^{\text{MC}}(\theta)$, as illustrated in Fig. 2(a), given by [13],

$$A_p^{\text{MC}}(\theta) = g_i, \theta \in \Theta_i \text{ for } i = 1, 2, \dots, N_c \quad (2)$$

where N_c stands for the number of all the beam lobes, and g_i refers to the amplitude gains of the i^{th} beam lobe respectively, which are all non-negative real values. In addition, Θ_i denotes the angular interval of the i^{th} beam lobe, and $|\Theta_i|$ describes the width of the i^{th} beam lobe. We emphasize that the multi-cone model is a general model for beamforming pattern and it can be regarded error-free if N_c is large enough. Therefore, we adopt multi-cone model in the rest of this paper for derivations.

Specifically, as demonstrated in Fig. 2(b), when N_c equals to 2, the multi-cone model boils down to a cone-plus-sphere (CPS) model, $A_p^{\text{CPS}}(\theta)$. We denote g_1 as the amplitude gain of the main lobe, while g_2 describes the amplitude gain of the side lobes. By further defining the parameter k as the ratio between the energy of the main lobe and the one leaked to side lobes, we obtain $g_1^2 = 2[(1 - \cos(|\Theta_1|/2)) + k(1 + \cos(|\Theta_1|/2))]$, and $g_2^2 = kg_1^2$.

C. Traditional Beamforming Gain Models

In the traditional beamforming gain models adopted in [2]–[14], the beamforming gain is regarded to be dependent on the beamforming pattern and the beamforming cases, however, independent on the spatial channel properties. Next, we elaborate on the beamforming gain models in aligned and misaligned beamforming cases, respectively.

1) Aligned Beamforming Case: When we consider the link between the BS and target UE (redpoint and black point in Fig. 1), the main lobe of the base station with the highest amplitude gain is directed to the LoS path with the strongest power, i.e., θ_{main} is a determined constant. The resulting beamforming gain in the traditional beamforming gain model is then regarded as a constant and equals to the square of the maximum amplitude gain, as

$$G' = A_{\max}^2. \quad (3)$$

2) *Misaligned Beamforming Case*: The misaligned beamforming case occurs in the interference links. The main lobe of the interference BS (blue point in Fig. 1) is assumed to be uniformly distributed in $[-\pi, \pi]$. This can be explained that BS directs its main lobe to its target UE so that the main lobe direction follows uniform distribution from the sight of the other UEs since the location of the target UE follows a infinite HPPP [6]. It should be noted that in some studies [11], the link between BS and its target UE is regarded to be a misaligned beamforming case when the link is non-line-of-sight (NLoS). The reason is that as there is no significantly strong LoS path, the beamforming strategy decides to randomly direct the main lobe. As a result, the beamforming gain, G_t , is a random variable as,

$$G'_t = A_p^2(\theta_u) \quad (4)$$

where θ_u is a random variable following uniform distribution in $[-\pi, \pi]$. When applying the multi-cone model according to (2), the PDF of G'_t , $P_{G'_t}(x)$, is given by,

$$P_{G'_t}(x) = \sum_{i=1}^{N_c} \frac{|\Theta_i|}{2\pi} \delta(x - g_i^2), \quad (5)$$

where $\delta(\cdot)$ is the Dirac delta function.

Remarkably, it should be noted that in the work that considers finite point process [15], the direction of the main lobes of the transmitter, θ_u , is uniformly distributed in a range which depends on the location of the transmitter instead of the entire $[-\pi, \pi]$ with infinite HPPP. This difference does not violate the proof of Lemma 1, which is the key contribution of this work. In the remainder of this paper, we adopt infinite HPPP without loss of generality.

D. True Beamforming Gain Models

We first denote the transmit signal $u(t)$ that is a baseband complex signal, and the baseband-equivalent complex channel impulse response as $h(t, \theta)$, respectively. Then, the received signals with beamforming and an isotropic beam pattern, $r^{\text{BF}}(t)$ and $r^{\text{ISO}}(t)$, are given by, $r^{\text{BF}}(t) = \int_{-\pi}^{\pi} A(\theta; \theta_{\text{main}})(h(t, \theta) * u(t)) d\theta$, and $r^{\text{ISO}}(t) = \int_{-\pi}^{\pi} h(t, \theta) * u(t) d\theta$ where $*$ denotes a convolution operator with respect to time t .

Based on the above system model, we define the beamforming gain G_t from the perspective of power, which is obtained as a ratio between the received signal power with beamforming, P_r^{BF} , and the received signal power with an isotropic beam pattern, P_r^{ISO} , as

$$\text{Key result: } G_t = \frac{P_r^{\text{BF}}}{P_r^{\text{ISO}}} = \frac{|r^{\text{BF}}|^2}{|r^{\text{ISO}}|^2} = \frac{|\int_{-\pi}^{\pi} A(\theta; \theta_{\text{main}})(h(t, \theta) * u(t)) d\theta|^2}{|\int_{-\pi}^{\pi} h(t, \theta) * u(t) d\theta|^2} \quad (6)$$

From (6), we notice that the beamforming gain is dependent on the beamforming pattern, beamforming case as well as the spatial channel properties. A remark is drawn here that the channel property, $h(t, \theta)$, is incorrectly ignored in the traditional beamforming gain models in Sec. II-C.

Lemma 1: The traditional model is equivalent to a special case of the general beamforming gain model in (6), if the channel has not angular spread (the energy of the channel is concentrated to one direction).

Proof: We consider a channel consisting of N_p ($N_p \geq 1$) paths. As a result, the spatial channel impulse response, $h(t, \theta) = h_R(t, \theta)$, is described as,

$$h_R(t, \theta) = \sum_{k=1}^{N_p} h_k e^{j2\pi f_c \tau_k + \phi_k} \delta(t - \tau_k) \delta(\theta - \theta_k) \quad (7)$$

where h_k , τ_k , θ_k denote the complex amplitude gain, delay and direction of departure (DoD) of the k^{th} path, respectively, which are independent. Without loss of generality, we consider that the paths are sorted in an ascending order with respect to the amplitude gain. Next, we proceed with the proof by separating the aligned and misaligned beamforming cases.

1) *Aligned Beamforming Case:* The main lobe is directed to the DoD of the first path, i.e., $\theta_{\text{main}} = \theta_1$.

By substituting (7) into (6), we have

$$G_t = \frac{|\sum_k A(\theta_k; \theta_1) h_k u(t - \tau_k)|^2}{|\sum_k h_k u(t - \tau_k)|^2} \quad (8)$$

If all the DoDs equal to the DoD of the first path, i.e., $\theta_k = \theta_1$, then we have,

$$G_t = \frac{|A(\theta_1; \theta_1) \sum_k h_k u(t - \tau_k)|^2}{|\sum_k h_k u(t - \tau_k)|^2} = A_{\text{max}}^2 \quad (9)$$

which is the same as (3).

2) *Misaligned Beamforming Case:* The main lobe is directed to a random direction, i.e., $\theta_{\text{main}} = \theta_u$, which leads to

$$G_t = \frac{|\sum_k A(\theta_k; \theta_u) h_k u(t - \tau_k)|^2}{|\sum_k h_k u(t - \tau_k)|^2} \quad (10)$$

If all the DoDs equal to the DoD of the first path, i.e., $\theta_k = \theta_1$, the above expression boils down to

$$G_t = \frac{|\sum_k A(\theta_1; \theta_u) h_k u(t - \tau_1)|^2}{|\sum_k h_k u(t - \tau_1)|^2} = |A_p(\theta_u - \theta_1)|^2 \stackrel{(a)}{=} |A_p(\theta_u)|^2 \quad (11)$$

which demonstrates the same expression as (4). Note that (a) holds since θ_u is uniformly distributed in the circular domain.

Furthermore, we observe that the distributions of h_k , θ_k and τ_k as well as the transmit signal $u(t)$ have no impact on the aforementioned results. Therefore, this proof holds under any channel condition as long as the energy of the channel is concentrated in one direction. ■

III. BEAMFORMING GAIN MODELS IN NARROWBAND FADING CHANNEL

In this section, we analyze the beamforming gain based on the narrowband channel models, i.e., Rayleigh and Rician fading.

A. Rayleigh Fading Model

In the Rayleigh fading model, the spatial channel impulse response, $h(t, \theta) = h_{RL}(t, \theta)$, is given by [19],

$$h_{RL}(t, \theta) = \sum_{k=1}^{N_p} h_k e^{j2\pi f_c \tau_k + \phi_k} \delta(t - \tau_k) \delta(\theta - \theta_k), \quad (12)$$

where N_p indicates the number of multi-path components (MPCs) and f_c is the carrier frequency. Additionally, h_k represents the amplitude gain of k^{th} MPC, which is an i.i.d. random variable satisfying $E\{|h_k|^2\} = 2\sigma_h^2$. ϕ_k is the phase shift caused by reflection, scattering, etc, which is also an i.i.d random variable following a uniform distribution. Moreover, τ_k and θ_k denote the delay and DoD of the k^{th} MPC, respectively. As f_c is large, a small variation of τ_k can cause very large shift of the phase term of $2\pi f_c \tau_k$, which leads that $\Phi_k = 2\pi f_c \tau_k + \phi_k$ is regarded to be i.i.d. uniform distributed. Without loss of generality, θ_k is assumed to be i.i.d von Mises distribution with PDF [20]–[22],

$$f_v(\theta) = \exp(k_v \cos(\theta - \mu_v)) / (2\pi I_0(k_v)) \quad (13)$$

where $I_0(\cdot)$ stands for the zero-order modified Bessel function of the first kind, μ_k accounts for the mean value and k_v is a real parameter controlling angle spread. For $k_v = 0$, the von Mises distribution is equivalent to a uniform distribution that denotes isotropic scattering environment. By contrast, $k_v > 0$ describes a non-isotropic scattering environment.

Moreover, in the narrowband channel, the transmit signal, $u(t)$, is a time-invariant random variable with normalized power, i.e., $E\{|u(t)|^2\} = 1$. For simplicity, we remove (t) in the rest

of this section. Hence, the received signal with isotropic beam pattern under the Rayleigh fading channel, $r^{ISO}(t)$, is expressed as,

$$r^{ISO} = u \sum_{k=1}^{N_p} A(\theta_k; \theta_{\text{main}}) h_k e^{j\Phi_k} = r_I^{ISO} + jr_Q^{ISO}, \quad (14)$$

where the time dependence is omitted since the narrowband channel is time-invariant. Moreover, r_I^{ISO} and r_Q^{ISO} refer to the in-phase and quadrature components of the received signal $r^{ISO}(t)$, respectively. Since the terms in r_I^{ISO} and r_Q^{ISO} are i.i.d, according to the Central Limit Theorem (CLT), when N_p is sufficiently large, r_I^{BF} and r_Q^{BF} are two independent Gaussian variables with zero mean and variance of σ_h^2 . As a result, the received signal with isotropic beam pattern follows a Rayleigh distribution with the scale parameter of σ_h .

By contrast, the received signal with beamforming under the Rayleigh fading channel, $r^{BF}(t)$, is expressed as,

$$r^{BF}(t) = u \sum_{k=1}^{N_p} A(\theta_k; \theta_{\text{main}}) h_k e^{j\Phi_k} = r_I^{BF} + jr_Q^{BF}, \quad (15)$$

where r_I^{BF} and r_Q^{BF} are the in-phase and quadrature components of the received signal r^{BF} , given by

$$r_I^{BF} = u \sum_{k=1}^{N_p} A(\theta_k; \theta_{\text{main}}) h_k \cos \Phi_k \quad (16a)$$

$$r_Q^{BF} = u \sum_{k=1}^{N_p} A(\theta_k; \theta_{\text{main}}) h_k \sin \Phi_k \quad (16b)$$

Next, we investigate the received signals in the Rayleigh fading channel with aligned and misaligned beamforming separately.

1) Aligned Beamforming Case: The main lobe is directed to a deterministic direction θ_d , i.e., $\theta_{\text{main}} = \theta_d$, since we do not know the optimal direction of the main beam. Being aware that the terms in r_I^{BF} and r_Q^{BF} are i.i.d, by invoking the CLT, when N_p is sufficiently large, r_I^{BF} and r_Q^{BF} are two independent Gaussian variables with zero mean and variance of $\text{E}\{A^2(\theta_k; \theta_d)\}\sigma_h^2$. Hence, the received signal with beamforming also follows a Rayleigh distribution with a scaling factor of $\sqrt{\text{E}\{A^2(\theta_k; \theta_d)\}}\sigma_h$.

By using the property of the Rayleigh distribution, we can derive that the beamforming gain G_t defined in (6) is a constant and equals to $\text{E}\{A^2(\theta_k; \theta_d)\}$ in this case, which is given by,

$$\text{E}\{A^2(\theta_k; \theta_d)\} = \int_{-\pi}^{\pi} A_p^2(\theta_k - \theta_d) \frac{\exp(k_v \cos(\theta_k - \mu_v))}{2\pi I_0(k_v)} d\theta_k. \quad (17)$$

It can be seen that $E\{A^2(\theta_k; \theta_d)\}$ is a function of θ_d with given parameters k_v and μ_v , if $k_v > 0$. There actually exists an optimal direction θ_{opt} in non-isotropic scattering environment to maximize the beamforming gain. This optimal direction is determined by the beam pattern $A(\theta)$ and the distribution of DoD, θ_k .

Remark 1: In the aligned beamforming case under Rayleigh fading channel, the beamforming gain G_t is equal to $\max\{E\{A^2(\theta_k; \theta_d)\}\}$, instead of the result $G_t = A_{\text{max}}^2$ in traditional beamforming gain models.

2) *Misaligned Beamforming Case:* The main lobe points to a random direction, i.e., $\theta_{\text{main}} = \theta_u$. Similarly, since the terms in r_I^{BF} and r_Q^{BF} are i.i.d., r_I^{BF} and r_Q^{BF} are two independent Gaussian variables with zero mean and variance of $E\{A_p^2(\theta_u)\}\sigma_h^2$ where the expectation operator is with respect to θ_u . We observe that the received signal with beamforming also follows a Rayleigh distribution with a scaling factor $\sqrt{E\{A_p^2(\theta_u)\}\sigma_h}$, given by

$$E\{A_p^2(\theta_u)\} = \int_{-\pi}^{\pi} A_p^2(\theta_u) d\theta_u \quad (18)$$

Remark 2: In the misaligned beamforming case under Rayleigh fading channel, the beamforming gain G_t using the definition in (6) is equal to $E\{A_p^2(\theta_u)\}$ that yields a constant. However, in the traditional beamforming gain models, $G'_t = A_p^2(\theta_u)$ is a random variable.

B. Rician Fading Model

In a Rician fading model, the channel can be regarded as a Rayleigh fading channel plus a dominant path. Then the spatial channel impulse response, $h(t, \theta) = h_{Ri}(t, \theta)$, boils down to [23],

$$h_{Ri}(t, \theta) = \sum_{k=1}^{N_p} h_k e^{j(2\pi f_c \tau_k + \phi_k)} \delta(t - \tau_k) \delta(\theta - \theta_k) + h_0 e^{j\Phi_0} \delta(t - \tau_0) \delta(\theta - \theta_d) \quad (19)$$

where h_0 , Φ_0 , τ_0 and θ_d denote the amplitude, phase, delay and direction of departure (DoD) of the dominant path, respectively. According to the CLT, the in-phase and quadrature components are independent Gaussian random variables with the same mean and variance, i.e., $r_I^{ISO}, r_Q^{ISO} \sim N(h_0, \sigma_h^2)$. Hence, the received signal with isotropic beam pattern follows a Rician distribution, of which the resulting PDF of the amplitude is given by, $p_R(z) = \frac{2(K+1)z}{\Omega} e^{-K - \frac{(K+1)z^2}{\Omega}} I_0(2\sqrt{\frac{K(K+1)}{\Omega}}z)$ where the K -factor $K = K^{\text{ISO}} = h_0^2/2\sigma_h^2$ is the ratio between the power in the direct path and total received power, and $\Omega = \Omega^{\text{ISO}} = h_0^2 + 2\sigma_h^2$ denotes the total received power. The in-phase

and quadrature components of the received signal with beamforming under the Rician fading channel are given by,

$$r_I^{BF} = u \sum_{k=1}^{N_p} A(\theta_k; \theta_{\text{main}}) h_k \cos \Phi_k + A(\theta_0; \theta_{\text{main}}) h_0 \cos \Phi_0 \quad (20a)$$

$$r_Q^{BF} = u \sum_{k=1}^{N_p} A(\theta_k; \theta_{\text{main}}) h_k \sin \Phi_k + A(\theta_0; \theta_{\text{main}}) h_0 \sin \Phi_0 \quad (20b)$$

Next, we investigate the received signals in the Rician fading channel with aligned and misaligned beamforming separately.

1) *Aligned Beamforming Case:* The main lobe is directed to the DoD of the dominant path θ_0 , i.e., $\theta_{\text{main}} = \theta_0$. Then the received signals are,

$$r_I^{BF} = u \sum_{k=1}^{N_p} A(\theta_k; \theta_0) h_k \cos \Phi_k + A_{\text{max}} h_0 \cos \Phi_0 \quad (21a)$$

$$r_Q^{BF} = u \sum_{k=1}^{N_p} A(\theta_k; \theta_0) h_k \sin \Phi_k + A_{\text{max}} h_0 \sin \Phi_0 \quad (21b)$$

By observing (21), we find that the received signal $r(t)$ consists of a Rayleigh variable with a scaling factor $\sqrt{\mathbb{E}\{A^2(\theta_k; \theta_0)\}} \sigma_h$ and a LoS component $A_{\text{max}} h_0 e^{j\Phi_0}$. Since $\mathbb{E}\{A^2(\theta_k; \theta_0)\}$ is a constant as θ_0 is fixed, the received signal $r(t)$ also follows a Rician distribution with parameters

$$K = \frac{A_{\text{max}}^2 h_0^2}{\mathbb{E}\{A^2(\theta_k; \theta_0)\} 2\sigma_h^2} = \frac{A_{\text{max}}^2}{\mathbb{E}\{A^2(\theta_k; \theta_0)\}} K^{\text{ISO}} \quad (22a)$$

$$\Omega = A_{\text{max}}^2 h_0^2 + \mathbb{E}\{A^2(\theta_k; \theta_0)\} 2\sigma_h^2 = \frac{\Omega^{\text{ISO}}}{K^{\text{ISO}} + 1} (A_{\text{max}}^2 K^{\text{ISO}} + \mathbb{E}\{A^2(\theta_k; \theta_0)\}) \quad (22b)$$

Remark 3: In the aligned beamforming case under Rician fading channel, the beamforming gain G_t in this case is a random variable that is expressed as a ratio of two Rician random variables with different parameters. Inaccurately, the traditional beamforming gain G_t is regarded as a constant $|G_{\text{max}}|^2$, which makes the received signal as a Rician random variable with $K' = K^{\text{ISO}}$ and $\Omega' = |A_{\text{max}}|^2 \Omega^{\text{ISO}}$.

2) *Misaligned Beamforming Case:* The main lobe is randomly directed, i.e. $\theta_{\text{main}} = \theta_u$. Then the received signals are

$$r_I^{BF} = u \sum_{k=1}^{N_p} A_k(\theta_0) h_k \cos \Phi_k + A_0(\theta_u) h_0 \cos \Phi_0 \quad (23a)$$

$$r_Q^{BF} = u \sum_{k=1}^{N_p} A_k(\theta_0) h_k \sin \Phi_k + A_0(\theta_u) h_0 \sin \Phi_0 \quad (23b)$$

We observe that r_I^{BF} and r_Q^{BF} are not Gaussian random variables since the LoS component $A_0(\theta_u)h_0$ is also a random variable rather than a constant. Therefore, when performing beamforming and beam-tracking, the received signal cannot be considered as a Rician random variable, although the channel is under Rician fading. By denoting the PDF of $A(\theta_u)$ as $p_A(x)$, and the PDF of Gaussian random variable with zero mean and variance of $\sigma_r^2 = E\{|A(\theta_u)|^2\}\sigma_h^2$ as $p_G(x)$, the PDFs of r_I^{BF} and r_Q^{BF} are given by $p_I(x) = p_G(x) * \frac{p_A(\frac{x}{h_0 \cos \Phi_0})}{h_0 \cos \Phi_0}$, and $p_Q(x) = p_G(x) * \frac{p_A(\frac{x}{h_0 \sin \Phi_0})}{h_0 \sin \Phi_0}$. Then the joint PDF of r_I^{BF} and r_Q^{BF} is calculated as, $p_{I,Q}(x, y) = p_I(x)p_Q(y)$. By using a Jacobi transform, the joint PDF of the amplitude $|r^{BF}| = \sqrt{(r_I^{BF})^2 + (r_Q^{BF})^2}$ and the phase $\Phi = \arctan \frac{r_Q^{BF}}{r_I^{BF}}$ is expressed as, $p_{|r^{BF}|, \Phi}(r, \Phi) = rp_{I,Q}(r \cos \Phi, r \sin \Phi)$. Finally, the PDF of the amplitude $|r^{BF}|$ is derived as,

$$p_{|r^{BF}|}(r) = \int_{-\pi}^{\pi} p_{|r^{BF}|, \Phi}(r, \Phi) d\Phi \quad (24)$$

Remark 4: In the misaligned case under Rician fading channel, the beamforming gain under random beamforming is a random variable, which is the ratio between a random variable whose PDF given in (24) and a Rician random variable. This brings a noticeable difference from the beamforming gain in the traditional beamforming as a random variable with PDF of $p_A(x)$.

When applying the multi-cone model according to (2), we let $\Phi_0 = 0$ for further derivation, which yields the PDF of the amplitude $|r^{BF}|$ under the Rician fading channel and random beamforming strategy, as

$$p_{|r^{BF}|}(r) = \sum_{i=1}^{N_c} \frac{|\Theta_i|}{2\pi\sigma_r^2} e^{-\frac{r^2 + h_0^2 g_i^2}{2\sigma_r^2}} I_0\left(\frac{h_0 g_i r}{\sigma_r^2}\right), r \geq 0 \quad (25)$$

By comparison, the amplitude $|r^{BF}|$ in a traditional beamforming gain model is $|A(\theta_u)|r^{ISO}$ and its PDF is calculated as,

$$p_{|r^{BF}|}(r) = \sum_{i=1}^{N_c} \frac{|\Theta_i|}{\pi g_i^2 2\sigma_h^2} e^{-\frac{r^2 + h_0^2 g_i^2}{g_i^2 2\sigma_h^2}} I_0\left(\frac{h_0 r}{g_i \sigma_h^2}\right), r \geq 0 \quad (26)$$

C. Summary of Beamforming Gain in Narrowband Channels

In light of the received signal amplitude, the comparison between the proposed beamforming gain model and the traditional beamforming gain model in narrowband channels are summarized in Tab. I. To put in a nutshell, the traditional beamforming gain model is independent of the channel, while the beamforming gain is highly related to the channel fading and direction distribution of multi-paths. To be concrete, the traditional beamforming gain model assumes that

the beamforming gain is a constant in aligned beamforming case and a random variable in the misaligned beamforming case. Instead, the proposed true model suggests that the beamforming gain is constant in the Rayleigh fading channel, while the beamforming gain is a random variable in Rician fading channel, in both aligned and misaligned cases. Therefore, the mistreated beamforming gain models in both aligned and misaligned cases make substantial impact on the distribution of the received signal and consequently, leading significantly inaccurate system-level analysis.

TABLE I
RECEIVED SIGNAL AMPLITUDE AND BEAMFORMING GAIN MODELS IN NARROWBAND CHANNELS.

		Rayleigh fading channel		Rician fading channel	
		Aligned case	Misaligned case	Aligned case	Misaligned case
Beamforming gain	<i>Traditional model</i>	Constant value	Random variable	Constant value	Random variable
	<i>Proposed model</i>	Constant value		Random variable	
Received signal amplitude	<i>Traditional model</i>	Rayleigh fading with scaling parameter $A_{\max}\sigma_h$	$A_p(\theta_u)$ multiplied by Rayleigh fading with scaling parameter σ_h	Rician fading with parameters $K = K^{ISO}$ $\Omega = A_{\max}\Omega^{ISO}$	PDF given in (30)
	<i>Proposed model</i>	Rayleigh fading with scaling parameter $\sqrt{E\{A^2(\theta_k; \theta_d)\}}\sigma_h$	Rayleigh fading with scaling parameter $\sqrt{E\{A^2(\theta_u)\}}\sigma_h$	Rician fading with parameters given in (25)	PDF given in (29)

IV. BEAMFORMING GAIN MODELS FOR WIDEBAND STATISTICAL CHANNEL

In this section, we proceed the analysis on beamforming gain models in wideband channels, which are common in the mmWave and THz bands.

A. Wideband Channel Framework

In a general wideband space-time statistical model, the channel consists of a LoS path and several NLoS paths. The resulting spatial channel impulse response with the signal power of P_s is given by,

$$\begin{aligned}
 h(\tau, \theta) = & \sqrt{\frac{P_s K}{K+1}} h_{LoS} e^{j\phi_{LoS}} \cdot \delta(\tau - \tau_{LoS}) \delta(\theta - \theta_{LoS}) \\
 & + \sqrt{\frac{P_s}{K+1}} \sum_{m=1}^{N_m} h_m e^{j\phi_m} \cdot \delta(\tau - \tau_m) \delta(\theta - \theta_m),
 \end{aligned} \tag{27}$$

where h_{LoS} and h_m refer to the path gain amplitudes for the LoS path and m^{th} path, respectively. Moreover, τ_{LoS} and τ_m are the ToA for the LoS path and m^{th} path. θ_{LoS} and θ_m are the DoD for the LoS path and m^{th} path. P_s is the received power. Additionally, K is the power ratio between the LoS path and NLoS paths, which is sensitive to the frequency and propagation environment. It is reported to have a mean value of 3.8 dB in the indoor environment and 6 dB in the urban environment at 28 GHz [24], [25]. Nokia Bell Lab shows that 80% of the positions have the K factor less than 0 dB at 28 GHz in outdoor small cell environment [26]. Also, its mean value ranges from -5 dB to 2.75 dB at 5.2 GHz in urban macro-cell scenario [27]. And at 800 MHz and 2.6 GHz in urban environment, the K factor decreases to 0.22 and 0.68, respectively [28].

We adopt the NYU channel model framework proposed for mmWave communication networks [29], in which the assumptions about the statistics of the channel parameters are commonly accepted in many mmWave and THz channel models [30]–[34]. The signal power P_s is calculated by $\alpha - \beta$ path loss model as,

$$P_s[\text{dB}] = P_t[\text{dB}] - \alpha - 10\beta \log_{10}(d) - \xi, \quad (28)$$

where P_t is the transmitted power. α is simply taken as the free-space path loss at distance of 1 m while β is taken as 2 for LoS channel given in [29]. Moreover, ξ describes the shadowing fading and is Gaussian distributed with standard deviation of σ_{SF} .

In the NYU channel model framework, the number of NLoS paths, N_m , follows a cut-off Poisson distribution with parameter λ_m , expressed as $N_m \sim \max\{1, \text{Poisson}(\lambda_m)\}$. In addition, the DoDs of all the paths follow a uniform distribution. The ToA of the m^{th} NLoS path, τ_m , follows an exponential distribution, $\tau_m \sim E(r_\tau)$. The amplitude of the m^{th} NLoS path, h_k , is calculated as, $h_m =$ where $h'_m = \exp(-\sigma_\tau \tau_m)$, which ensures the power of all the NLoS paths is normalized. Next, by accounting for non-coherent power combination of the multipath components, the beamforming gain G_t according to (6) is rearranged as,

$$G_t = \frac{1}{K+1} (K A^2(\theta_{LoS}; \theta_{\text{main}}) + \sum_m A^2(\theta_m; \theta_{\text{main}}) h_m^2) \quad (29)$$

In particular, the distribution of the received power with the true beamforming gain model is calculated as,

$$p_{P_r}(x) = \int p_{P_s}(\tau - x) p_{G_t}(x) d\tau, \quad (30)$$

where p_{P_s} is the PDF of P_s that follows 10-based log-normal distribution while $p_{G_t}(x)$ is the PDF of the true beamforming gain model that will be given in Sec. IV-B and Sec. IV-C.

In comparison, the distribution of the received power with traditional beamforming gain model for aligned beamforming case is given as,

$$p_{P_r}(x) = \frac{p_{P_s}(x/A_{\max})}{A_{\max}}. \quad (31)$$

Moreover, the distribution of the received power with traditional beamforming gain model for misaligned beamforming case is stated as,

$$p_{P_r}(x) = \sum_{i=1}^{N_c} \frac{|\Theta_i|}{2\pi} \frac{p_{P_s}(x/g_i)}{g_i}. \quad (32)$$

B. Aligned Beamforming Case

In the aligned beamforming case, the main lobe is directed to the LoS path, i.e., $\theta_{\text{main}} = \theta_{\text{LoS}}$. Therefore, the beamforming gain G_t is expressed as,

$$G_t = \frac{1}{K+1} (K A_{\max}^2 + \sum_{m=1}^{N_m} A_p^2(\theta_u^k) h_m^2). \quad (33)$$

Since, G_t is a function of N_m which is a discrete random variable, the PDF of G_t is calculated as,

$$p_{G_t}(x) = \sum_{l=1}^{\infty} P(N_m = l) p_{G_t|N_m=l}(x). \quad (34)$$

where $P(N_m = l) = \frac{1}{1-e^{-\lambda_m}} \frac{\lambda_m^l}{l!}$. $p_{G_t|N_m=l}(x)$ is given as

$$p_{G_t|N_m=l}(x) = p_{G_{\text{Ali}}^{(l)}}(x - \frac{K}{K+1} A_{\max}^2) \quad (35)$$

where $p_{G_{\text{Ali}}^{(l)}}(x)$ is the PDF of the variable of $\sum_{m=1}^{N_m} A_p^2(\theta_u^k) h'_m / ((K+1) \sum_{j=1}^{N_m} h'_j)$.

Finally, we have

$$p_{G_t}(x) = \frac{1}{1-e^{-\lambda_m}} \sum_{l=1}^{\infty} \frac{\lambda_m^l}{l!} e^{-\lambda_m} p_{G_{\text{Ali}}^{(l)}}(x - \frac{K}{K+1} A_{\max}^2) \quad (36)$$

If we consider the multi-cone model for beamforming pattern, $p_{G_{\text{Ali}}^{(m)}}$ is derived by introducing slack variables $Y_1 = \sum_{m=1}^{N_m} A_p^2(\theta_u^k) h'_m / ((K+1) \sum_{j=1}^{N_m} h'_j)$, $Y_2 = h'_2, \dots, Y_l = h'_l$, as

$$\begin{aligned} p_{G_{\text{Ali}}^{(l)}}(x) &= p_{Y_1} = \int \cdots \int p_{Y_1, Y_2, \dots, Y_l}(x, x_2, \dots, x_l) dx_2 \cdots dx_l \\ &= \sum_{I_1=1}^{N_c} \cdots \sum_{I_l=1}^{N_c} \int_0^1 \cdots \int_0^1 \frac{\sum_{i=2}^l x_i ((k+1)x - g_{I_i}^2) |\sum_{j=2}^l x_j (g_{I_1}^2 - g_{I_j}^2)|}{(g_{I_1}^2 - (K+1)x)^3} \\ &\quad \cdot \left(\frac{r_\tau}{\sigma_\tau}\right)^{l-1} (x_2 \cdots x_l)^{r_\tau/\sigma_\tau-1} \frac{|\Theta_{I_1}| \cdots |\Theta_{I_l}|}{(2\pi)^l} dx_2 \cdots dx_l \end{aligned} \quad (37)$$

We reveal that some conditions where the denominator of (37) is zero that deserves special considerations. If $g_{I_1}^2 - (K + 1)x = 0$, we have $x = g_{I_1}^2/(K + 1)$, which corresponds to $G_t = \frac{g_{I_1}^2 + Kg_{I_1}^2}{K+1}$, $I_1 = 1, \dots, N_c$. These conditions occur when all the NLoS paths are within the I_1^{th} lobe of the beamforming pattern, which thereby cause non-continuous components in the PDF of G_t , $p_{G_t}(x)$. Therefore, the probability of these conditions are given as,

$$P[G_t = \frac{g_{I_1}^2 + Kg_{I_1}^2}{K+1}] = \frac{1}{1 - e^{-\lambda_m}} \sum_{l=1}^{\infty} \frac{\lambda_m^l}{l!} e^{-\lambda_m} \left(\frac{|\Theta_{I_1}|}{2\pi}\right)^l = \frac{e^{\lambda_m(|\Theta_{I_1}|/(2\pi)-1)} - e^{-\lambda_m}}{1 - e^{-\lambda_m}}, \quad (38)$$

where $I_1 = 1, 2, \dots, N_c$. The lower bound and upper bound of the beamforming gain are obtained by taking I_1 as N_c and 1, which yields $G_{\text{lower}} = g_{N_c}^2 + Kg_1^2/(K + 1)$ and $G_{\text{upper}} = g_1^2$.

Remark 5: In the aligned beamforming case under wideband statistical channel, the boundaries of the beamforming gain is determined by the K value, and the PDF of beamforming gain consists of continuous and discrete components. By contrast, the beamforming gain in the traditional model is inaccurately assumed as a constant g_1^2 .

C. Misaligned Beamforming Case

In the misaligned beamforming case, the main lobe is uniformly and randomly directed, i.e., $\theta_{\text{main}} = \theta_u$. Therefore, the beamforming gain G_t is expressed as,

$$G_t = \frac{1}{K+1} (KA_p^2(\theta_u^0) + \sum_{m=1}^{N_m} A_p^2(\theta_u^k) h_m^2). \quad (39)$$

As the number of NLoS paths, N_m , follows a cut-off Poisson distribution, the PDF of G_t , p_{G_t} , is expressed as,

$$p_{G_t}(x) = \frac{1}{1 - e^{-\lambda_m}} \sum_{l=1}^{\infty} \frac{\lambda_m^l}{l!} e^{-\lambda_m} p_{G_{\text{Mis}}^{(l)}}(x), \quad (40)$$

where $p_{G_{\text{Mis}}^{(l)}}$ is the PDF of the beamforming gain in (39) when the total number of NLoS paths is l . If we consider the multi-cone model for beamforming pattern, $p_{G_{\text{Mis}}^{(m)}}$ is given by,

$$p_{G_{\text{Mis}}^{(l)}}(x) = \sum_{I_0=1}^{N_c} \cdots \sum_{I_m=1}^{N_c} \int_0^1 \cdots \int_0^1 \frac{\sum_{i=2}^l x_i((k+1)x - Kg_{I_0}^2 - g_{I_i}^2) |\sum_{j=2}^l x_j(g_{I_1}^2 - g_{I_j}^2)|}{(g_{I_1}^2 + Kg_{I_0}^2 - (K+1)x)^3} \cdot \left(\frac{r_\tau}{\sigma_\tau}\right)^{l-1} (x_2 \cdots x_l)^{r_\tau/\sigma_\tau-1} \frac{|\Theta_{I_0}| \cdots |\Theta_{I_l}|}{(2\pi)^{l+1}} dx_2 \cdots dx_l. \quad (41)$$

We observe that some conditions where the denominator of (41) is zero need special considerations, given by, $G_t = \frac{g_{I_1}^2 + Kg_{I_0}^2}{K+1}$, $I_1, I_0 = 1, \dots, N_c$. These conditions occur when all the NLoS

paths are within the I_1^{th} lobe of the beamforming pattern while the LoS path is within the I_0^{th} lobe. The probability of these conditions are,

$$P[G_t = \frac{g_{I_1}^2 + K g_{I_0}^2}{K + 1}] = \frac{|\Theta_{I_0}| e^{\lambda_m(|\Theta_{I_1}|/(2\pi)-1)} - e^{-\lambda_m}}{2\pi(1 - e^{-\lambda_m})}, \quad (42)$$

where $I_0, I_1 = 1, 2, \dots, N_c$. The boundaries of the beamforming gain are obtained by taking I_0 and I_1 as N_c and 1, which yields $G_{\text{lower}} = g_{N_c}^2$ and $G_{\text{upper}} = g_1^2$.

Remark 6: In the misaligned beamforming case under wideband statistical channel, the PDF of beamforming gain consists of continuous and discrete components. On the contrary, the beamforming gain in the traditional model is assumed as a discrete random variable with PDF given in (5).

V. NUMERICAL RESULTS AND ANALYSIS

In this section, we validate our theoretical beamforming gain models with simulations for narrowband channels, including Rayleigh fading channel and Rayleigh channel. In addition, we compare the distributions of the received signal amplitudes under the proposed beamforming gain model and the traditional beamforming gain models in the literature for both narrowband fading channels. Furthermore, we compare and analyze the distribution of the beamforming gain of the proposed beamforming gain model and the traditional beamforming gain model in the wideband statistical channel.

The beamforming pattern model utilized in this section is the cone-plus-sphere model introduced in Sec. II-B. The amplitude gain of the main lobe and that outside the main lobe are g_1 and g_2 , respectively. The relations among g_1 , g_2 and the beamwidth of the main lobe $|\Theta_1|$ are given in II-B. For simplicity, we further assume that the g_1 and $|\Theta_1|$ are related to the number of the antennas, N_{ant} , as $g_1 = N_{\text{ant}}$, and $\sin \frac{|\Theta_1|}{2} = \frac{1}{N_{\text{ant}}}$.

To validate the theoretical distribution of the received signal amplitude in narrowband fading channels, i.e., Rayleigh channel fading and Rician fading channel, we run Monte Carlo simulation to generate 10000 received signals for each case introduced in Sec. II-A and B and then compare the cumulative density function (CDF) of the received signal amplitudes with the theoretical and analytical distributions.

A. Narrowband Rayleigh Fading Channel

The CDF of the received signal amplitude under the proposed beamforming gain model in a Rayleigh fading channel is validated, compared with the traditional model.

Fig. 3. Comparison of the proposed beamforming gain model and the traditional model in Rayleigh fading channel in aligned beamforming case.

1) *Aligned Beamforming Case:* Fig. 3 shows the CDF of the received signal amplitude by varying channel parameters and the number of antennas in aligned beamforming case for Rayleigh fading channel. In Fig. 3(a), we observe that k_v (the parameter of the distribution of AoD) influences the distribution of the received signal amplitude of Rayleigh fading channel in the aligned beamforming case. However, the traditional beamforming gain model implicitly puts forward that channel parameters have no influence on the beamforming gain and thus the received signal amplitude. This is explained that the traditional beamforming gain model ignores the spatial channel properties, as we discussed in Sec. II. Moreover, a small k_v leads to a larger derivation from the traditional beamforming gain model, since k_v controls the concentration degree of the angular power distribution. A larger k_v denotes that the power is less dispersive in the angular domain while concentrating the power in certain direction, which is exactly the assumption in the traditional beamforming gain models proved in Sec. II-D. Therefore, the traditional beamforming gain model coincides with the proposed beamforming gain model with a large value $k_v = 10$.

Fig. 3(b) shows the number of antennas N_{ant} has impact on the received signal amplitude distribution. First, a larger number of antennas leads to higher inaccuracy of the traditional beamforming gain model. For example, the difference between the true model proposed in this work and the traditional model is nearly 10 dB in the case of 16 antennas, while this difference drops to 5 dB in the case of 4 antennas. This can be explained that more antennas make the beamforming pattern more concentrative. The traditional beamforming gain model is the same as the proposed channel model when the beamforming pattern is omnidirectional. Therefore, the

traditional beamforming gain model is substantially inaccurate if there are a massive number of antennas or equivalently, extremely narrow beamforming patterns. Second, the traditional beamforming gain model overestimates the impact of the number of antennas, or equivalently the main lobe gain g_1 , on the received signal amplitude. We observe that the x-axis difference among the three solid lines is much smaller than that of three dashed lines. This suggests that in the true beamforming gain model, the increase of the number of antennas does not dramatically enhance the received signal strength as the traditional beamforming gain model proposes. This is due to the fact that the traditional beamforming gain model assumes that in the aligned beamforming case, the power in all directions can be enhanced equally, which is far from reality since only a small fraction of power is actually confined in the main lobe.

Fig. 4. Comparison of the proposed beamforming gain model and the traditional model in misaligned beamforming case.

2) *Misaligned Beamforming Case:* In Fig. 4(a), we observe that the received signal amplitude does not vary with different k_v , which is consistent with (18), since the beamforming gain only depends on the beamforming pattern $A_p(\theta)$. Moreover, the deviation between the received signal amplitude in the true beamforming gain model and the traditional model can easily reach 10 dB. Specifically, at a low signal amplitude, the traditional model shows a much higher cumulative probability than the true model, while at a high signal amplitude, the traditional model yields a slightly smaller cumulative probability.

In Fig. 4(b), the increase of the number of antennas tremendously degrades the accuracy of the traditional models in the misaligned beamforming case, similarly in the aligned beamforming case. The difference between the signal amplitude exceeds 15 dB with 32 antennas. Additionally, the traditional beamforming gain model underestimates the impact of the number of antennas (or equivalently the main lobe gain g_1) on the signal amplitude, which is opposite to the aligned

beamforming case. To be concrete, the differences among the dashed lines are fairly small, especially at low signal amplitudes. However, the average difference among the solid lines is 3 dB, which suggests doubling the number of antennas results in doubled signal power.

Fig. 5. Comparison of the proposed beamforming gain model and the traditional model in aligned case in Rician fading channel.

B. Narrowband Rician Fading Channel

The CDF of the received signal amplitude under the proposed beamforming gain model in the Rician fading channel is validated, compared with the traditional model and analyzed as follows. In the Rician fading channel, we consider the AoA of the LoS path component is at 0° without loss of generality.

1) *Aligned Beamforming Case:* In the aligned beamforming case, the main lobe of the beamforming pattern points to the LoS path component of Rician fading channel. As shown in Fig. 5, the CDF of the received signal amplitude with varying channel parameters, k_v , K factor, and number of antennas N_{ant} . In Fig. 5(a), first, varying k_v indeed changes the received signal amplitude in the proposed beamforming gain model for the aligned beamforming case. By contrast, the traditional beamforming gain models wrongly consider that channel parameters have no influence on the signal amplitude. Second, the impact of k_v is related to u_v , which denotes the mean AoA of the Rayleigh scattering components. Compared to the red line with $k_v = 0$ that describes isotropic scattering environment around a base station, the black line ($k_v = 1$ and $u_v = 0$) is closer to the dashed line (traditional model) while the blue line with the same $k_v = 1$ yet different $u_v = \pi$ is far from the dashed line. The reason is that $u_v = 0$ indicates that the power of the Rayleigh scattering components is concentrated in the same direction as the LoS path component in the Rician fading channel, which is contradictory to the case of $u_v = \pi$ where Rayleigh scattering components are in the opposite direction of the LoS path.

Therefore, $u_v = 0$ makes the power distributed in a narrow direction, which is consistent with the assumption made in the traditional beamforming gain model.

In Fig. 5(b), we further investigate the impact of K factor of the Rician channel on the received signal amplitude. First, the traditional model underestimates the impact of K . We notice that the differences among solid lines (proposed model) reach 5 dB, which are much larger than that among the dashed lines (traditional model). This is due to the fact that the traditional model assumes that the power of all the path components is enhanced by the main lobe. However, in reality, only the LoS path and a small fraction of the Rayleigh scattering components benefit from the power improvement of the main lobe. Therefore, K , which denotes the power ratio between the direct path components and Rayleigh scattering components, demonstrates a sound impact on the received signal amplitude. Second, the traditional model wrongly estimates the distribution behavior of the signal amplitude, as discussed in Sec. III-B. We notice that the CDF of the signal amplitude with large K is almost vertical, which implies that the fading vanishes after beamforming, and the channel tends to be deterministic. However, this phenomenon is not observed in the traditional model, where the CDF of signal amplitude is still relatively smooth and gentle. This can be explained that the traditional model assumes that the deterministic component (the LoS path) and fading components (Rayleigh scattering paths) are equally strengthened by the main lobe. Nevertheless, in realistic beamforming radiation, the deterministic component absorbs substantially higher gain than other fading components.

The impact of the number of antennas N_{ant} is evaluated in Fig. 5(c). First, more antennas cause a steeper CDF of the signal amplitude in the proposed model, compared to that of the traditional model. Second, a more severe inaccuracy of the traditional model is observed with massive antennas. For example, the difference between the two models is 7 dB with 4 antennas, which, however, reaches 12.5 dB with 32 antennas. Third, at a low signal amplitude, the cumulative probability of the traditional model is higher than the proposed model. By contrast, at a high signal amplitude, the cumulative probability of the proposed model becomes higher in turn.

2) *Misaligned Beamforming Case*: In this case, the main lobe is randomly directed. Fig. 6 depicts the impact of the channel parameters and the number of antennas on the distribution of the signal amplitude. In Fig. 6(a), we observe that firstly, there are generally two cross-points of the CDFs of the signal amplitude under the proposed model and traditional model. Therefore, in the range of low amplitudes, the cumulative probability in the traditional model is lower than that in the proposed model. In the middle amplitude range, the cumulative probability in

the traditional model is slightly higher. Then, when the signal amplitude rises up, while the cumulative probability based on the traditional model is below that of the proposed model. Second, the traditional model underestimates the impact of K as in the aligned beamforming case, for the same reason, as explained in Sec. IV-B-(2). Third, a low K value can lead to indispensable inaccuracy of the traditional model as the solid black line, and the black dashed line show the largest difference, which reaches 7 dB.

Fig 6(b) illustrates the number of antennas has a great impact on the signal amplitude distribution. When N_{ant} increases from 4 to 32, the difference between the proposed model and the traditional model accordingly increases from 5 dB to 11 dB. Moreover, the traditional model obviously underestimates the impact of the number of antennas N_{ant} . To be concrete, in our proposed model, twice the number of antennas allows for an averagely 2.5 dB increment of the signal amplitude, which, however, is totally missed in the traditional model.

Fig. 6. Comparison of the proposed beamforming gain model and the traditional model in misaligned case in Rician fading channel.

Fig. 7. Comparison of the proposed beamforming gain model and the traditional model in the aligned case in NYU channel model.

C. Beamforming gain in Wideband Statistical Channel

Next, we adopt the wideband statistical channel model to analyze the beamforming gain, based on the proposed model in Sec. IV, in contrast with the traditional beamforming gain model.

1) *Aligned Beamforming Case:* In this case, the main lobe points towards the LoS path, which has the lowest path loss among all MPCs of the channel. In Fig. 7, we evaluate the beamforming gain with varying parameters, including K , λ_m , and the number of antennas, N_{ant} . We observe that the beamforming gain is bounded in the region $[G_{\text{lower}}, G_{\text{upper}}]$. Specifically, the lower bound, G_{lower} , satisfies

$$G_{\text{lower}} = \frac{g_2^2 + K g_1^2}{K + 1} > \frac{K}{K + 1} \cdot g_1^2 = \frac{K}{K + 1} \cdot N_{\text{ant}}^2. \quad (43)$$

with the probability,

$$P[G_t = G_{\text{lower}}] = \frac{e^{\lambda_m(|\Theta_2|/(2\pi)-1)} - e^{-\lambda_m}}{1 - e^{-\lambda_m}}. \quad (44)$$

In addition, the upper bound G_{upper} is equal to $g_1^2 = N_{\text{ant}}^2$ with the probability calculated as,

$$P[G_t = G_{\text{upper}}] = \frac{e^{\lambda_m(|\Theta_1|/(2\pi)-1)} - e^{-\lambda_m}}{1 - e^{-\lambda_m}}. \quad (45)$$

The explanations of the boundaries of G_t are as follows. The LoS path component contains $K/(K + 1)$ portion of the received power, which is enhanced by a factor of $|g_1|^2$. Therefore, the beamforming gain is at least $K/(K + 1) \cdot |g_1|^2$. However, the beamforming gain cannot exceed $|g_1|^2$, since the maximum beamforming gain in any direction is $|g_1|^2$. By contrast, the traditional beamforming gain model superficially assumes that the beamforming gain is equal to a constant value $|g_1|^2$, which is generally overestimated. As the beamwidth of the main lobe $|\Theta_1|$ is generally smaller than the beamwidth of the side lobe $|\Theta_2|$, $P[G_t = G_{\text{lower}}]$ is larger than $P[G_t = G_{\text{upper}}]$. Therefore, the traditional model by considering the beamforming gain stays at the upper bound, which indeed incorrectly estimates the beamforming gain.

As demonstrated in Fig. 7(a), we observe that the beamforming gain is highly related to K value. Moreover, the CDF at the lower bound of beamforming gain is very high, and is larger than 0.6 for any K values in our proposed model, which is consistent with our calculations.

We further demonstrate that the number of NLoS paths, λ_m , affects the beamforming gain in Fig. 7(b). In particular, an increasing value of λ_m causes a large difference between the proposed model and the traditional model. The reason is that a large λ_m stands for a large number of NLoS paths and consequently, the NLoS power turns to be dispersive in the angular domain.

By incorrectly assuming all the power concentrates in a certain direction, the traditional model degrades the accuracy for a large λ_m .

Fig. 7(c) presents that massive antennas make a large difference between the beamforming gain in the proposed model and the traditional model. Still, we emphasize that the beamforming gain lies in the region $(K/(K+1)N_{\text{ant}}^2, N_{\text{ant}}^2]$. Therefore, the ratio between the upper bound and the lower bound is determined only by K value. If we depict the x-axis in a log scale, the difference between the two models maintains at $10 * \log_{10}(K/(K+1))$ dB constantly, which is independent of the number of antennas.

(a) Varying K with $\lambda_m = 2$ and 4 (b) Varying λ_m with $K = 1$ and 4 (c) Varying number of antennas with antennas. $K = 1$ and $\lambda_m = 2$.

Fig. 8. Comparison of the proposed beamforming gain model and the traditional model in misaligned case in NYU channel model.

2) *Misaligned Beamforming Case:* In this case, the main lobe is directed to a random direction. Fig. 8 illustrates the comparison of the beamforming gain in our proposed model and the traditional model. It can be seen that the traditional model regards that the beamforming gain is a discrete random variable with the probabilities, $P[G'_t = g_2^2] = \frac{|\Theta_2|}{2\pi}$, and $P[G'_t = g_1^2] = \frac{|\Theta_1|}{2\pi}$.

In comparison, the CDF of the proposed beamforming model is a continuous function along with some discrete components with the beamforming gain bounded by $[|g_2|^2, |g_1|^2]$. This is consistent with the bounds of that of the traditional model, although being different from the bounds in the aligned case. The reason is that the power in any direction provides a beamforming gain over the range between $|g_1|^2$ and $|g_2|^2$. The probabilities of the lower bound and the upper bound in the proposed beamforming gain model are calculated according to (42) as,

$$P[G_t = G_{\text{lower}}] = \frac{|\Theta_2|(e^{\lambda_m(|\Theta_2|/(2\pi)-1)} - e^{-\lambda_m})}{2\pi(1 - e^{-\lambda_m})}, \quad (46)$$

$$P[G_t = G_{\text{upper}}] = \frac{|\Theta_1|(e^{\lambda_m(|\Theta_1|/(2\pi)-1)} - e^{-\lambda_m})}{2\pi(1 - e^{-\lambda_m})}. \quad (47)$$

We verify that the result in the aligned beamforming case, $P[G_t = G_{\text{lower}}] > P[G_t = G_{\text{upper}}]$, still holds.

As shown in Fig. 8(a), we observe that a higher K value leads to a larger difference between the proposed model and the traditional model. The reason is the same as we discussed in the narrowband fading channels in Sec. V-B. A high K indicates the most of the power concentrates in a direction, which owes to the assumption of the traditional beamforming gain model. Moreover, a sudden rise of the CDF is observed near the position of beamforming gain of $K/(K+1)|g_1|^2$, or equivalently, $|g_1|^2 = 16$ as the number of antennas is 4. For example, the sudden rise occurs at the beamforming gain of 8 and 13.3, for $K = 1$ and $K = 5$, respectively, which can be explained as follows. As the LoS path contains $K/(K+1)$ energy while the NLoS paths carry the rest $1/(K+1)$, if the LoS path is within the main lobe and the NLoS paths are located in the side lobe, the beamforming gain equals to $\frac{Kg_1^2 + g_2^2}{K+1}$. As g_2 is much smaller than g_1 , the beamforming gain approximates to $K/(K+1)|g_1|^2$.

Similar to the aligned beamforming case, we further analyze that a higher λ_m value with severe power dispersion in the angular domain leads to smaller accuracy of the traditional model, as demonstrated in Fig. 8(b). Moreover, the sudden rise of the CDF does not change with varying λ_m , which shows that the sudden rise depends only on K and g_1 . In Fig. 8(c), we can observe that a large number of antennas cause severe inaccuracy of the traditional model. For example, the largest difference between the two models exceeds 20 dB with $N_{\text{ant}} = 16$, while the difference is 10 dB with $N_{\text{ant}} = 4$. In addition, we notice that the sudden rise vanishes as the number of antennas increases. The explanation is that the vertical length of the sudden rise is proportional to the beamwidth of the main lobe as, $P[G_t = \frac{Kg_1^2 + g_2^2}{K+1}] = \frac{|\Theta_1|e^{\lambda_m(|\Theta_2|/(2\pi)-1)} - e^{-\lambda_m}}{2\pi(1-e^{-\lambda_m})} \leq \frac{|\Theta_1|}{2\pi}$. Therefore, the vertical length decreases with the number of antennas.

D. System-level SINR Analysis with Wideband NYU channel model

To further evaluate our analysis of the beamforming gain model, we investigate the distribution of SINR by a stochastic-geometry method with the wideband NYU channel model [29], at 60 GHz with 1 GHz bandwidth and 15 dBm transmit power. The

The system geometry for simulation is the same as described in our previous work [35]. We first choose an UE and assign it as the receiver of interest, Rx_0 . Then, we consider random deployment of BSs modeled by a homogeneous Poisson point process with intensity of $\lambda_a = 0.1$ (averagely 10 BSs in an area of 100 m²). As a typical UE, Rx_0 is located at the origin O

Fig. 9. CCDF of SINR with varying K and N_{ant} and compared with the traditional beamforming gain model using the wideband NYU channel model.

without loss of generality, which is a standard approach using the stochastic geometry method. The associated BS with which Rx_0 communicates is the nearest one to Rx_0 , while the other BSs are regarded as interference source. The channel between the associated BS and UE is the aligned beamforming case, while the channel between the interference BS and the UE is the misaligned beamforming case. The probability of coverage is interpreted as the probability that an arbitrarily UE can achieve a target SINR, T , as

$$P_c(T) = P(\text{SINR} > T), \quad (48)$$

which boils down to the CCDF of SINR as, $P_c(T) = 1 - P(\text{SINR} \leq T) = \text{CCDF}_{\text{SINR}}(T)$.

The probability of coverage is computed in Fig. 9, as a function the target SINR, T (i.e., CCDF of SINR) with varying K and N_{ant} values. First, we observe from Fig. 9(a) and 9(b) that the traditional model significantly overestimates SINR with larger N_{ant} and smaller K values. We take the target SINR T corresponding to 90% coverage probability, i.e., $T_{0.9}$ ($P_c(T_{0.9}) = 90\%$), as an example. When $N_{ant} = 16$, $T_{0.9}$ equals to 1.87 dB in the traditional model for different K values. To compare, $T_{0.9}$ in the proposed model is -1 dB, 0.83 dB, and 1.45 dB for $K = 1, 5, 100$, respectively. In addition, the difference of $T_{0.9}$ between the proposed model and traditional model increases from 2 dB, 2.56 dB, 5.86 dB, to 12.35 dB for $N_{ant} = 4, 16, 32, 128$. The reason is that the traditional model overestimates the beamforming gain in the aligned case while underestimates the beamforming gain in the misaligned case. More importantly, the traditional model shows that increasing N_{ant} improves SINR without limit in 9(b), which can be also observed in [5]. On average, doubled N_{ant} leads to 3.3 dB increment of $T_{0.9}$ in the traditional model. However, the increment of SINR indeed decreases as N_{ant} increases in our proposed model. This upper bound is decided by K value as shown in Fig. 9(c), where a larger K value

leads to a higher level of SINR. To be concrete, $T_{0.9}$ is 0.75 dB with $K = 1$ and 1.41 dB with $K = 10$ and $N_{ant} = 128$, respectively. The reason is given in our previous explanation in Sec. V-A that the traditional model underestimates the impact of increasing large N_{ant} on beamforming gain in the misaligned beamforming case and thereby the interference link power.

Fig. 10. Ergodic capacity of mmWave wireless networks with varying K and number of antennas. $\lambda_m = 100$.

In Fig. 10, we further evaluate the ergodic capacity of the mmWave wireless networks with varying K and number of antennas. Numerical results show that the traditional model overestimates the ergodic capacity of the wireless networks. The difference in ergodic capacity between the proposed beamforming gain model and the traditional beamforming gain model is significant with a large number of antennas and small K values, which is similar to the observations of SINR.

E. Complexity of the beamforming gain models

As a benchmark, one realization of the traditional beamforming gain model costs 0.5 ns and 25 ns for aligned and misaligned beamforming cases, respectively. Then, we evaluate the ratio of the computation times of the proposed beamforming gain model and the traditional beamforming gain model for different beamforming cases and channel models in Table II. Overall, the proposed beamforming gain model introduces additional computational complexity compared with the traditional model. To be concrete, the computational complexity of the proposed beamforming model in aligned beamforming case under narrowband small-scale fading channel is over 100 times of the traditional model. In addition, the computational complexity of

the proposed beamforming gain model is observed to further increase under wideband NYU channel.

TABLE II

THE RATIO OF COMPUTATION TIMES OF TRUE BEAMFORMING GAIN MODEL AND TRADITIONAL MODEL

	Aligned beamforming case	Misaligned beamforming case
Rayleigh fading model	136.36	2.87
Rician fading model	147.27	2.90
NYU model	222363.63	4133.98

VI. CONCLUSION

In this paper, we first point out that the traditional beamforming gain model in the literature ignores the channel properties. To be concrete, in the narrowband fading channel, the channel property is the spatial distribution of the scattering path and Rician K factor. In the wideband statistical channel, the ignored channel properties include K factor and the parameters of the number of NLoS paths. Instead, we properly present the general definition of the beamforming gain, and thereby propose a new beamforming gain model that incorporates the beamforming pattern, beamforming cases, and spatial channel properties. We further prove that the traditional beamforming gain model is indeed a special case of the proposed model when the energy of the channel is directed to one direction.

Furthermore, we derive the beamforming gain and the distribution of the signal amplitude in both aligned and misaligned beamforming cases for narrowband fading channels, i.e., Rayleigh fading channel and Rician fading channel by using the proposed beamforming gain model. By comparison, we reveal that the traditional model wrongly estimates the distribution of the signal amplitude in the narrowband fading channels. Moreover, we derive the beamforming gain model in the wideband statistical channel model. The PDF of the proposed model is a continuous function with some discrete components, while that of the traditional model is a discrete function. In addition, the boundaries and distribution of the beamforming gain are highly related to the channel parameters, which are ignored in the traditional model.

In the numerical analysis, we thoroughly compare and discuss the proposed beamforming gain model and the traditional model for both narrowband and wideband channels. The extensive results demonstrate that the traditional model causes several to 20 dB inaccuracy in the estimation

of the signal amplitude and beamforming gain. The traditional model overestimates the impact of the number of antennas in the aligned beamforming case in the Rayleigh fading channel. However, it underestimates the impact of the number of antennas in the misaligned beamforming case in both the narrowband fading and wideband statistical channels. In addition, the traditional model underestimates the effect of K factor in both aligned and misaligned beamforming cases for narrowband fading channels and wideband statistical channels.

The lessons for the system-level performance analysis of the wireless networks are as follows. First, the beamforming gain is highly related to the K factor according to the proposed model, and the traditional model is less accurate with a small K factor. Therefore, in the analysis of those environments with a low K factor, the traditional model is not suggested. Second, the traditional mode greatly differs from the proposed model with a narrower beam. The traditional model is not suggested in the cases of massive antennas or high-gain directional antenna. Third, we notice that in the misaligned beamforming case, which is mostly the case of interference links, the traditional model underestimates the beamforming gain as well as the impact of the increasing number of antennas on the beamforming gain.

Remarkably, the traditional model overestimates the SINR as well as ergodic capacity with massive antennas, which is validated in our analysis by the stochastic geometry approach. In addition, the larger number of antennas steadily brings significant improvement of SINR and ergodic capacity in the traditional model, which is however incorrect. Instead, the increment of SINR and ergodic capacity in our proposed model decreases as the number of antennas increases. We conclude that missing consideration of the channel characteristics in the beamforming gain model leads to incorrect system-level performance analysis of wireless networks.

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