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On the Impact of Probabilistic Shaping on Fiber Nonlinearities

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ABSTRACT

This paper addresses probabilistic shaping (PS) which has been a latest key technique to approach capacity of fiber-optic channels. We investigate the impact of PS on nonlinear interference (NLI), including self channel interference (SCI), cross channel interference (XCI), and multi channel interference (MCI) for a polarization multiplexed 16-ary quadrature amplitude modulation format in a wavelength division multiplexed (WDM) system. To this end, we consider performing PS in two scenarios: (i) Solely on the channel of interest and (ii) over all C-band WDM channels of a fiber-optic link by analyzing the effective signal to noise ratio and symbol error rate.

It is demonstrated that using the enhanced Gaussian noise model with merely 10% overhead in the first scenario, the applied PS scheme increases the SCI and the total experienced NLI by about 19.23%, and 6.6%, respectively. Interestingly, despite enhancing the NLI in this scenario, the simulated PS technique leads to about 47.6% increase in the transmission reach. In the second scenario, the numerical results show increase of the SCI, XCI, and total NLI around 19.8%, 23.34%, and 20.2%, respectively, but resulting in an increase of 32.3% in the transmission reach.

1. Introduction

Today's society relies on fast and reliable exchange of information [1]. The extreme growth in the demand for data usage of numerous emerging services and applications such as cloud computing, social medias, etc. are stretching the needs for high-capacity communications [1]. The increase in traffic demand together with the development of software-defined transceivers that can adapt the transmission parameters to the physical channel have increased the interest in designing networks that utilize the resources more efficiently [2]. High prices of the equipments which are used in the backbone of optical networks motivates usage of the digital signal processing (DSP) techniques with reasonable complexity on the same hardwares. One of these techniques which significantly improves the performance of the network is probabilistic shaping (PS). PS is used in optical communications to increase signal to noise ratio (SNR) per symbol for a fixed launch power that provides a substantial increase in the transmission reach. It also furnishes fine adaptation in the transmission rate.

1.1. Problem Statement

To study the influence of PS on the nonlinear effects in fiber-optic communications, a comprehensive knowledge about the channel impairments is inevitable. Channel models which accurately compute physical layer impairments are useful tools to analytical elaborate on non-linear interference (NLI) noise terms, consisting of self channel interference (SCI), cross channel interference (XCI), and multi channel interference (MCI), instead of computationally complex numerical methods such as split-step Fourier method (SSFM).

Investigating the effect of PS on each nonlinear term of

NLI is beneficial for designing NLI-optimized modulation-formats. Furthermore, analyzing the effect of PS on WDM transmission systems with respect to transmitter and link parameters is crucial. However, these two topics have not been addressed before in literature.

1.2. Related Work

Main recent performance improvement technique for fiber-optical communications extensively discussed in the literature is probabilistic signal shaping. Shaping is performed either with changing position of constellation points to make a Gaussian distribution in complex plane, called geometric shaping (GS), or with changing probability of constellation symbols, called PS. GS has some disadvantages in practice where mentioned thoroughly in [3]: (i) there is no simple solution to finding locations of the constellation points for arbitrary channel conditions; (ii) the irregular constellation points of GS increase the complexity of coherent DSP for robust signal recovery prior to decoding; (iii) the general infeasibility of Gray mapping increases the complexity of demapping symbols to soft-decision bit metrics. Unlike GS, PS typically is used in a fixed architecture where introduced in [4] including PS and channel coding jointly aka probabilistic amplitude shaping architecture. It enables to use different forward error correction schemes with the same PS encoder. It also allows to use the same structure for the receiver and does not need to involve in the receiver design like GS.

A comprehensive review on the evolution of fiber-optical channel modeling can be found in [5], so we only review here some of the well-known models introduced in the literature. Gaussian noise (GN) model was proposed in [6] is capable of estimating uncompensated optical coherent transmission system performance. However its predictions, when used to obtain a detailed picture of NLI noise accumulation along a link, may be affected by a substantial NLI overestimation error, especially in the first spans of the link [6]. Enhanced Gaussian noise (EGN) model of [7] includes well-known impairments made by fiber-optic channel which degrade per-

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Figure 1: An intuitive instance of PS. The bars represent the probability of each constellation symbol. These distributions made by CCDDM on a 16-QAM constellation.

formance of transmission system. Actually the correction terms appeared in it was added to the GN model terms where provide a better range of precision.

The concept of PS was introduced by three independent groups [8, 9, 10] in the form of shell mapping technique in the same time in 1990's. Shell mapping is specified in the ITU-T Recommendation V.34 standard for constellation shaping [11] and it is widely used in wireless modems. Although PS has advantageous applications in wireless communications, it also plays an important role in fiber-optic communications. Lots of research papers studied PS in optical transmission systems in recent years, e.g [12, 13, 14, 3, 15, 16, 17]. More detailed, a pragmatic rate-adaptive probabilistically shaped coded modulation scheme introduced in [12]. A PS scheme proposed for optical wavelength division multiplexed (WDM) systems in [13] and a reach improvement of around 7% reported. It has been experimentally shown in [15] that PS increases transmission reach by more than 40%. It also has been demonstrated using numerical simulations together with GN model and EGN model in [16], [17], respectively that PS reduces effective SNR after fiber propagation in a fixed average launch power, while it increases achievable information rates (AIRs). Rate/reach tradeoff via PS has been investigated in flexible optical transceivers in [14].

All of the aforementioned works investigated PS using information-theoretic approach by measuring MI and AIRs in a coded system. We consider an uncoded system and use digital communications fundamentals for analytical study of the following problems.

1.3. Main Contributions

Considering the nonlinear behavior of optical channels, we investigate the effect of PS on different NLI noise terms of fiber nonlinearities by using the EGN model in a polarization multiplexed (PM) 16-ary quadrature amplitude modulation (QAM) WDM system. We also measure two figures of merit for comparing different schemes: effective SNR and symbol error rate (SER). Moreover, we answer two main questions: (i) How much does PS effect on each term of NLI and which of them is more influenced by PS? (ii) What difference would be obtained by applying PS to all WDM channels compare to the case that PS is applied merely to

the channel of interest (COI)?

The rest of this paper is organized as follows. In Section 2, PS is discussed in detail. In Section 3, EGN model of fiber-optic channel is explained. In Section 4, performance assessment metrics are introduced. After that, numerical results are provided in Section 5 and finally concluding remarks are stated in Section 6.

2. Probabilistic Shaping

PS is used as a technique in fiber-optic systems in order to change the probabilities of modulation symbols that leads to reducing average transmitted power where causes a dramatic improvement of system performance. PS also decreases the gap to the Shannon limit and increases optical channel capacity and transmission reach. The desired distribution in PS is chosen from Maxwell-Boltzmann (MB) family that is a Gaussian like and well known distribution in physics and statistical mechanics. MB probability distribution is given by [8, Section IV]

$$P_X(x) = \frac{e^{-\lambda|x|^2}}{\sum_{i=1}^M e^{-\lambda|x_i|^2}}, \quad (1)$$

where x represents constellation symbols which are the channel inputs, M is the modulation order, and λ is the shaping factor that plays an important role in the trade-off between bit rate and average energy. The scalar λ is chosen such that the mutual information (MI) between a MB-distributed input and the corresponding continuous channel output is maximized for an additive white Gaussian noise (AWGN) channel at a particular SNR [17].

Four different probability distribution of probabilistically shaped 16-QAM are shown in Figure 1. It is obvious that the probability of symbols with lower energy is larger than the probability of symbols with higher energy due to PS. As distributions become more shaped, the entropies decrease.

One of the most used methods of PS is called constant composition distribution matching (CCDDM) where proposed in [18] and belongs to fixed-to-fixed (f2f) length distribution matcher (DM) family. It transforms sequences of k uniformly distributed bits to sequences of n nonuniformly distributed symbols.

The main idea behind CCDDM is minimizing informational divergence that is also known as *Kullback-Leibler divergence* or *relative entropy* between the desired distribution and the distribution made by CCDDM. The name of CCDDM recalls that all of the outputs of this kind of DM is *constant composition* and the sequence is called *n*-type, i.e. the number of occurrences of each output alphabet symbol has a specific value that can be computed using [19, Algorithm 2].

The significant advantage of CCDDM in compare with earlier DMs is the fact that it is asymptotically optimal, i.e. when the length of the output of DM is very large, the rate of DM converges to the entropy of the distribution. In the other words, we have $\frac{k}{n} = H(P)$ when $n \rightarrow \infty$.

Sequences $B^k = B_1 B_2 \dots B_k$ and $A^n = A_1 A_2 \dots A_n$ are input and output blocks of the DM, respectively. The empirical distribution of a sequence $\mathbf{u} = u_1 u_2 \dots u_n$ of length n is defined as [18, Section III]

$$P_{\mathbf{u}}(\ell) = \frac{n_{\ell}(\mathbf{u})}{n}, \quad (2)$$

where $n_{\ell}(\mathbf{u}) = |\{i : u_i = \ell\}|$ is the number of occurrences of symbol ℓ in sequence $\mathbf{u}$. The set of sequences with empirical distribution P of length n is then given by [18, Section III-A]

$$\mathcal{T}_P^n = \left\{ \boldsymbol{\vartheta} \in \mathcal{A}^n \left| \frac{n_{\ell}(\boldsymbol{\vartheta})}{n} = P(\ell) \quad \forall \ell \in \mathcal{A} \right. \right\}, \quad (3)$$

where $\mathcal{A}$ is the alphabet of the output of DM which corresponds to the set of possible signal amplitudes in amplitude shift keying (ASK). The mapping to a set of vectors defined by $\mathcal{T}_P^n$ is done using the function f_{CCDDM} where uses arithmetic coding for indexing sequences efficiently and makes codewords.

$$f_{\text{CCDDM}} : \{0, 1\}^k \rightarrow \mathcal{T}_P^n. \quad (4)$$

To ensure the possibility of inverting transformation functionality of DM, input blocklength of CCDDM can be at most $\log_2 |\mathcal{T}_P^n|$. It set to $k = \lfloor \log_2 |\mathcal{T}_P^n| \rfloor$. The function f_{CCDDM} is invertible and a codebook that is constructed in this manner is stored and the demapper uses it in the receiver. A ready to use MATLAB web demo package of CCDDM where proposed in [20] is used in this work.

3. Estimation of the fiber nonlinearities

To analyze fiber-optic communications, a high accuracy AWGN model including all impairments that recently remarkably has been studied is a promising solution instead of computationally complex numerical methods such as SSFM. EGN model is the most accurate model has been presented. It is used for simulating an optical link with the desired characteristics and its predictions are such accurate where is used in the optical networks design and analysis problems.

The impairments with this model are divided to three segments. 1st segment is the impact of the channel of interest

(COI) on itself which is called SCI. When two or more optical channels are transmitted simultaneously inside an optical fiber using the WDM technique, another nonlinear terms are appeared. 2nd segment is the result of interference between the COI and each of the interfering (INT) channels, namely XCI. 3rd segment is caused by simultaneously interaction of the COI with two or more INT channels which is known as MCI. SCI and XCI make significant degradation in system performance and MCI is often ignored because of its small amount, despite all of them is studied in this paper. EGN model of [5] without considering stimulated raman scattering is used in this work. It is approximated using Monte Carlo method of [21].

NLI can be interpreted as a noise with the following variance

$$\sigma_{\text{NLI}}^2 = \int_{-\frac{R_s}{2}}^{\frac{R_s}{2}} [G_{\text{SCI}}^{\text{EGN}}(f) + G_{\text{XCI}}^{\text{EGN}}(f) + G_{\text{MCI}}^{\text{EGN}}(f)] df, \quad (5)$$

where $G_{\text{SCI}}^{\text{EGN}}(f)$, $G_{\text{XCI}}^{\text{EGN}}(f)$, and $G_{\text{MCI}}^{\text{EGN}}$ are power spectral density of each impairment term (SCI, XCI, & MCI), respectively which can be extracted from [5] or [7]. Since NLI variance is power dependent and it would be better for comparison, if a same situation be in all WDM channels, it is assumed that the input power of both of the COI and INT channels are equal, i.e. $P_{\text{L,COI}} = P_{\text{L,INT}} = P_{\text{L}}$, where $P_{\text{L}} = \mathbb{E}\{|a_x|^2 + |a_y|^2\}$ is the average power of the constellations of dual polarization. Since constellations on both polarizations are the same, it could be interpreted that $P_{\text{L}} = 2 \mathbb{E}\{|a|^2\}$, where $\mathbb{E}\{|a|^2\} = \mathbb{E}\{|a_x|^2\} = \mathbb{E}\{|a_y|^2\}$.

4. Performance metrics

In this section, two main parameters which are used for assessing system performance are discussed. Effective SNR after propagation of the light in the fiber-optic channel is defined as [22, Section IV]

$$\text{SNR}_{\text{eff}} = \frac{P_{\text{L}}}{\sigma_{\text{eff}}^2} = \frac{P_{\text{L}}}{\sigma_{\text{ASE}}^2 + \sigma_{\text{NLI}}^2}, \quad (6)$$

where $\sigma_{\text{ASE}}^2 = N_{\text{Span}} \cdot G \cdot \text{NF} \cdot h \cdot \nu \cdot W$ is amplified spontaneous emission noise per two polarizations which induced by Erbium doped fiber amplifiers (EDFAs) used in the system. In this equation, G and NF are gain and noise figure of each amplifier in each span, respectively, h is Planck's constant, $\nu = \frac{C}{\lambda}$, where C is the speed of the light, and λ is the wavelength of the light in the COI. W is the bandwidth of the transmitted signal. Since pulse shape of transmitted signal is assumed root-raised cosine (RRC) with a small roll-off factor β which approximately is equal to *sinc* function in the time domain or rectangular in the frequency domain, W is equal to $(1 + \beta)R_s$, where R_s is the baud rate of the transmission.

To dispart power dependency of NLI, one may rewrite Equation 6 as [22, Section IV-A]

$$\text{SNR}_{\text{eff}} = \frac{P_{\text{L}}}{\sigma_{\text{ASE}}^2 + P_{\text{L}}^3 \cdot \eta}, \quad (7)$$

Figure 2: NLI terms for various PS schemes of Figure 1 applied to COI. These figures show the effect of PS on each term of NLI. XCI and MCI are also affected by PS, but they are negligible that is approximately the same for all cases $P_1 - P_4$.

Figure 3: NLI, Effective SNR, and SER for various PS schemes of Figure 1 applied to COI.

where η is normalized NLI noise variance which does not depend on power. To find the optimum input power which maximizes the value of SNR_{eff} , $\frac{\partial \text{SNR}_{\text{eff}}}{\partial P_L}$ must be equal to zero which results in

$$P_L^* = \sqrt[3]{\frac{\sigma_{\text{ASE}}^2}{2\eta}}, \quad (8)$$

where P_L^* is the optimum input power that maximizes value of SNR_{eff} and results in $\sigma_{\text{NLI}}^2 = \frac{\sigma_{\text{ASE}}^2}{2}$ which is specially useful for validation of simulations.

We notice here that SNR_{eff} is not a proper tool for measuring transmission reach. In fact, it merely measures effective SNR which is a useful criterion in optical communications in general. But since it does not take into account discrete signal space concepts in SNR calculation, it can not account for the Euclidean distance change resulted by PS which is directly influencing transmission reach. To this end, one may use SER as a metric for measuring and comparing transmission reach of distinct PS schemes. SER of a communication system with uniform M -QAM is calculated as [23, Equation 4.3-30]

$$P_e^{M\text{-QAM}} \approx 4Q\left(\sqrt{\frac{3\varepsilon_{s,\text{avg}}}{(M-1)N_0}}\right), \quad (9)$$

where $Q(\cdot)$ denotes Q-function, i.e. $Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{-\frac{u^2}{2}} du$.

In this paper, PM 16-QAM constellation is used where $\varepsilon_{s,\text{avg}} = \frac{5}{2}d_{\min}^2$ is the average energy of the constellation symbols in each polarization and N_0 is half of AWGN variance, i.e. $N_0 = \frac{\sigma^2}{2}$. Here σ^2 is identical to σ_{eff}^2 which includes both ASE and NLI made by link parameters. So one may rewrite SER as

$$P_e^{16\text{-QAM}} \approx 4Q\left(\sqrt{\frac{d_{\min}^2}{2N_0}}\right) = 4Q\left(\frac{d_{\min}}{\sigma}\right), \quad (10)$$

where $d_{\min}$ is the minimum distance or Euclidean distance between the points in the constellation diagram. Since symbols are transmitted simultaneously on both polarizations, we must compute SER for PM system in the form of

$$\begin{aligned} P_{e,\text{PM}}^{M\text{-QAM}} &= 1 - P_{C,\text{PM}}^{M\text{-QAM}} = 1 - (P_C^{M\text{-QAM}})^2 \\ &= 1 - (1 - P_e^{M\text{-QAM}})^2 \end{aligned} \quad (11)$$

The same formula for the shaped scheme is obtained in section Appendix.

Figure 4: NLI terms for various PS schemes of Figure 1 applied to all WDM channels. These figures show the effect of PS on each term of NLI. MCI is also affected by PS, but it is negligible that is approximately the same for all cases $P_1 - P_4$.

Figure 5: NLI, effective SNR, and SER for various PS schemes of Figure 1 applied to all WDM channels.

5. Numerical Results

Comprehensive details of the simulation parameters and their values are available in Table 1. The parameter values are chosen to be consistent with practical use. We then partition the results into two items where each of them corresponds to a different scenario of PS usage in WDM channels.

5.1. Scenario I: PS in COI

In this scenario, PS is solely applied to COI. It is shown that PS increases SCI and almost does not change XCI and MCI where can be seen in Figure 2. It is also revealed in Figure 3a, Figure 3b that for a fixed average power, PS increases NLI and decreases SNR_{eff} a bit which is consistent with the theory and is expected according to Equation 7. It is also evident that PS reduces SER because increasing minimum distance of the constellation due to PS overcomes increasing NLI due to it which is depicted in Figure 3c. Indeed, PS causes an increase of up to 47.6 % in the transmission reach.

5.2. Scenario II: PS in all WDM channels

In this scenario, PS is applied to all WDM channels. Same changes due to PS is happened, but it also increases XCI like as SCI and almost does not change MCI which is depicted in Figure 4. It also leads to more increase in NLI

Table 1

SYSTEM AND SIMULATION PARAMETERS

Parameter	Value
Modulation	16-QAM
Input PMF	uniform and nonuniform
Blocklength of CCDD	21600
Input launch power	5 dBm
Polarization	Dual
Symbol rate	32 GBaud
Pulse shape	RRC
RRC roll-Off	0.01
WDM channel number	9
WDM channel spacing	1.05 × Symbol Rate
Nonlinear coefficient (γ)	1.3 1/W/km
Dispersion	17 ps/km.nm
Attenuation (α)	0.22 dB/km
Length per span	80 km
Amplification	EDFA
EDFA noise figure	5.5 dB
Laser wavelength (λ)	1550 nm

and more decrease in effective SNR where can be seen in Figure 5a, Figure 5b, respectively, nevertheless PS still reduces SER because increasing minimum distance of the constellation due to PS is more prominent than increasing NLI

due to it which is depicted in Figure 5c. In fact, an increase of up to 32.3 % in the transmission reach is obtained.

It might be strange for the reader that NLI increases by PS. In a fixed average power, NLI increases by PS, because power is assumed to be fixed and the normalized value of NLI increases, so does NLI. It is worth mentioning that the NLI increase due to PS is almost not dependent to channel spacing value.

6. Conclusions

We have studied the effect of PS on NLI terms which are SCI, XCI, and MCI. Effective SNR and SER has been assessed in a multi-span WDM system in both cases of PS in the COI and PS in all WDM channels. It has been demonstrated that in the first case, NLI has 6.6 % increase which is due to the effect of PS on SCI, while effective SNR has 0.27 dB decrease, and up to 47.6 % reach increase is observed. In the second case, PS has more prominent role in increasing NLI due to its effect on SCI and XCI. It also leads to a 0.77 dB decrease in the effective SNR and up to 32.3 % increase in the transmission reach. It has been manifested that the second case has more decrease in the effective SNR and it has less gain in the transmission reach due to PS which is not of interest. So PS in the COI is absolutely sufficient.

The benefit of the presented evaluation is confirming the fact that PS affects NLI terms differently, i.e., SCI and XCI terms which are dominant NLI terms, are more sensitive to PS, and MCI can be neglected for this kind of analysis. The other advantage is that in a point-to-point fiber-optic system, applying PS to the COI is enough. So one does not need to do it for all of the WDM channels. It simplifies the analysis and allows focusing on system parameters.

Appendix

To calculate SER of a shaped M -ASK or M point pulse amplitude modulation (PAM) constellation, same as uniform scheme, constellation points are divided to two groups: inner and outer points. Probability of error for inner and outer points is computed as [23, Equations 4.3-2, 4.3-3], respectively. The symbol error probability is then given by

$$\begin{aligned} P_{e, M\text{-PAM}} &= \sum_{m=1}^M P_m P[\text{error}|m \text{ sent}] \\ &= \sum_{i=1}^{M-2} P_i P_{ei} + 2P_o P_{eo} \\ &= 2Q\left(\frac{d_{\min}}{\sqrt{2N_0}}\right) \left[\sum_{i=1}^{M-2} P_i + P_o \right] (M \geq 4), \end{aligned} \quad (12)$$

where P_i and P_o is the probability of each interior and exterior symbol, respectively. To find SER of M -QAM system, we have

$$P_{c, M\text{-QAM}} = P_{c, \sqrt{M}\text{-PAM}}^2 = \left(1 - P_{e, \sqrt{M}\text{-PAM}}\right)^2 \quad (13)$$

$$P_{e, M\text{-QAM}} = 1 - P_{c, M\text{-QAM}} = 1 - \left(1 - P_{e, \sqrt{M}\text{-PAM}}\right)^2, \quad (14)$$

where

$$P_{e, \sqrt{M}\text{-PAM}} = P_e = 2Q\left(\frac{d_{\min}}{\sqrt{2N_0}}\right) \left[\sum_{i=1}^{\sqrt{M}-2} P_i + P_o \right] \quad (\sqrt{M} \geq 4) \quad (15)$$

To compute SER for PM M -QAM, we must compute $1 - (1 - (1 - (1 - P_e)^2))^2$ which results in $1 - (1 - P_e)^4$.

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