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SUBMISSION DATE / POSTED DATE

25-01-2022 / 01-02-2022

CITATION

Al-Dweik, Arafat; Al-Rimawi, Ashraf (2022): On the Performance of RIS-Assisted Communications with Direct Link in κ - μ Shadowed Fading Channels. TechRxiv. Preprint. <https://doi.org/10.36227/techrxiv.19060085.v1>

DOI

[10.36227/techrxiv.19060085.v1](https://doi.org/10.36227/techrxiv.19060085.v1)

On the Performance of RIS-Assisted Communications with Direct Link in κ - μ Shadowed Fading Channels

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Abstract—Reconfigurable intelligent surfaces (RISs) is a new technology that can be used to create a virtual line-of-sight (LOS) link when the direct link is blocked. The signal-to-noise ratio (SNR) can be significantly improved by optimizing the reflecting elements' phases to make the reflected signals add coherently at the receiver. Nevertheless, the RIS cannot control the phase of the direct link if it exists. In such scenarios, the RIS phase can still be controlled such that the direct and reflected signals add coherently, however, the gain obtained by using the RIS might be hindered. Therefore, this paper considers the performance analysis of such scenarios where a novel analytical framework is developed to evaluate the SNR, outage probability and bit error rate (BER). To capture a broad range of fading conditions, the channels are modeled as independent but not identically distributed generalized κ - μ shadowed fading channels. The Laguerre expansion is used to derive the probability density function (PDF) and cumulative distribution function (CDF) of the instantaneous channel fading, which are used to derive the PDF and CDF of the instantaneous SNR. The paper also considers deriving the asymptotic PDF, CDF, moment generating function (MGF) of the SNR, as well as the outage probability and BER. The derived expressions are used to evaluate the system performance in various fading environments such as Rayleigh, One-Sided Gaussian, Nakagami- m , Rician, and Rician-shadowed distributions since they are special cases of the κ - μ shadowed distribution. The obtained analytical results corroborated by Monte Carlo simulation show that a strong direct link can generally eliminate the gain obtained using the RIS. Therefore, the overhead required for the RIS operation becomes a burden on the network and may cause severe throughput degradation. In such scenarios, the RIS involvement should dynamically controlled to avoid unnecessary complexity and throughput reduction.

Index Terms— κ - μ shadowed fading, reconfigurable intelligent surface (RIS), intelligent surfaces, diversity order, outage probability, moment generating function (MGF), bit error rate (BER), sixth generation (6G).

I. INTRODUCTION

The demand for mobile data access has been growing rapidly in the past few years, driven by the large number of new subscribers, high data rate applications, and emerging Internet of Things (IoT) applications, which require massive connectivity. Therefore, the limited capacity problem became

prominent [1], [2]. According to Ericsson mobility report [3], mobile network data traffic grew 44% between 2020 and 2021, and reached 72 Exabyte per month generated by about 8 billion subscribers. As per Cisco report [4], 50% of all IoT networked devices in 2023 will be connected to cellular networks. Therefore, the wireless network is expected to support about 15 billion IoT devices in addition to 8 billion mobile users. Although the advancements achieved by fifth generation (5G) wireless networks are considered substantial as compared to the fourth generation (4G), 5G networks capacity improvement is still way below the $1000\times$ capacity increase specified by the International Mobile Telecommunications-2020 (IMT-2020) vision.

To resolve the limited capacity problem, extensive research efforts are being dedicated to design efficient solutions. For example, spectral capacity solutions aim at introducing more spectrum by utilizing new frequency bands, particularly high frequencies such as millimeter waves (mmW). However, mmW poses several challenges [5] such as the poor propagation characteristics of mmW as compared to classical sub-6 GHz spectrum. Moreover, in highly-dense urban areas, the high-rise buildings cause shadowing and mostly block line-of-sight (LOS) connectivity, which makes it more challenging for high frequency transmission. Consequently, reconfigurable intelligent surfaces (RISs) were introduced to control the wireless environment [6], [7]. An RIS panel consists of two-dimensional meta-surfaces arrays in which each meta-surface element can independently change the phase-shift of the incident electromagnetic signal to create a preferable wireless environment [8]. RIS panels do not require high hardware complexity or cost overhead during their integration, while they can provide substantial gain in terms of reliability and network capacity [9].

A. Related Work

RIS has recently attracted extensive research activities, particularly performance analysis. The performance analysis was obtained for various system and channel models using several performance metrics such as signal-to-noise ratio (SNR), bit error rate (BER), symbol error rate (SER), outage probability and capacity. The system model can be generally classified into two main categories, RIS without direct link [6], [8], [10]–[12], [12]–[16], which is more common, and with direct link [17]–[21]. Table I shows the classification of selected recent articles based on their consideration of a direct link,

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This work was supported by Khalifa University Competitive Internal Research Award CIRA-2020-056.

system model, channel model, and performance metric. More specifically, In [6], [8], [10]–[16], the authors provide closed-form approximations for several performance metrics through modeling an end-to-end RIS channel amplitude by Gaussian [6], K_G [12], Gamma [8], and Nakagami- $\bar{m}$ [10] distribution. Moreover, the channel in [11] are based on Central Limit Theorem (CLT) which does not accurate for small number of reflecting elements whereas, in [15] a unified analytical frameworks based on Fox- H function are developed which are shown to be highly accurate even for small number of reflecting elements, particularly for continuous phase shifts.

B. Motivation and Contribution

Although the above mentioned works provide good results about the performance of the wireless system in existences of RIS, they are neglecting the direct link between base stations (BS) and user equipment (UE). In practice, it is not necessary that the direct link is completely blocked since the system suffers from an intrinsic product path loss [27]. i.e; the direct link cannot be ignored as generally it has relatively a smaller path loss [16]. Motivated by this, the direct link is considered in References [18]–[21], [24]–[26], as summarized in Table I. In these works (except [24] that considered K -fading model where the approximation of the performance of the system based on the tight approximation of end-to-end SNR was derived), the authors considered Nakagami- $\bar{m}$ [25] and Rayleigh [18]–[20], [26] models for the direct link. It is worth mentioned that these fading models do not accurately fit since there is a high probability that all links experience composite fading/shadowing. To this end, $\kappa - \mu$ shadowed distribution is widely used in literature for modeling composite fading/shadowing. It has a clear physical interpretation, good analytical properties and unifies popular fading models such as One-Side Gaussian, Rayleigh, Nakagami- $\bar{m}$, Rician, $\bar{\kappa}$ - $\bar{\mu}$, and η - $\bar{\mu}$ fading channels as special cases [28], and [29, Table. I].

To the best of our knowledge, no prior work in literature has derived an analytical framework for studying RIS-assisted wireless communication systems in the more generalized $\kappa - \mu$ shadowed fading channel. All scenarios introduced in literature considered Rice [24], Nakagami- $\bar{m}$ [25] and Rayleigh [26], which are derived as special cases in our framework. To be more specific, the main contribution of this study can be summarized as follows:

- A new and generalized analytical framework model to characterize the performance of the RIS-assisted wireless communication systems has been developed.
- Generalized and exact closed-form expressions for the probability density function (PDF), moment generating function (MGF), and cumulative distribution function (CDF) of the end-to-end SNR have been derived.
- Generalized and closed-form expression for the Outage Probability (Pout), and BER have also been derived.
- To get more insights, at high SNR, new and accurate closed-form expression for the asymptotic of Pout and BER have been obtained, and the system diversity order and coding gain are also provided.

Fig. 1: System model.

- The derived results are valid for arbitrary number of reflecting elements, and various values of $\kappa - \mu$ shadowed parameters.
- Simulations are also provided to validate our analysis.

C. Paper Organization and Notations

The remainder of this work is organized as follows: Section II System Model. Next, Section III Channel Model. Performance Evaluation presented in Section IV while, Section V reports respective numerical results and discussions. Finally, conclusions are provided in Section VI. The notations used throughout the paper are listed in Table II.

II. SYSTEM AND CHANNEL MODELS

A. System and Channel Models

The simplified system model is given in Fig.1, where an RIS-assisted wireless communications network that includes a BS, RIS panel equipped with N reflecting elements, and K UEs. Each UE can access the RIS through an orthogonal multiple access protocol such as time division multiple access (TDMA). In this work we consider that the UE can receive the transmitted signal through two different links, an indirect link through the RIS and a direct link from the BS. Because the distance between the RIS panel, BS and UE is much larger than the distance between the reflecting elements within the RIS panel, then all RIS elements are assumed to be at the same distance from the BS and UE. Therefore, the distances from BS to UE, BS to RIS, and RIS to UE are respectively denoted as d_0 , d_h , and d_g . Unlike the distances, the channel gains between the BS and each RIS element are typically assumed to be independent and identically distributed (i.i.d.) because the elements are spaced no less than half the wavelength of the carrier frequency. Therefore, the channel gains between the BS and RIS, RIS and UE are denoted as h_i , and g_i , respectively, $i = 1, 2, \dots, N$. The channel gain between the BS and UE is denoted as h_0 . The channel state information (CSI) of each UE is communicated to the BS by means of channel reciprocity algorithms, or via a feedback channel. The BS controls the phase of each RIS element through programmable controller

TABLE I: References which considered direct and in-direct links. The abbreviations are defined as: outage probability (OP), coverage probability (CP), average error probability (AEP), ergodic capacity (EC).

Ref	Direct Link	Performance Metric	No. of Antennas	Approach for Analysis	Channel	Phase knowledge	Modulation Schemes
[8]	—	OP, AEP and EC	SISO	Gamma distribution	Rayleigh	Perfect	M-QAM
[10]	—	OP, and AEP	SISO	K -distribution	Rayleigh/Rician	Perfect	BPSK/QAM
[11]	—	OP, AEP and EC	SISO	CLT	Rayleigh	Perfect	BPSK, and M-QAM
[12]	—	OP, AEP and EC	MIMO	K_G distribution	Rayleigh	Perfect	DPSK
[12]	—	OP, and EC	SISO	Gamma distribution	Rayleigh	Perfect	—
[13]	—	AEP	SISO	Nakagami- $\bar{m}$	Arbitrary	Imperfect	BPSK
[14]	—	OP and EC	SISO	Gamma distribution	Arbitrary	Perfect	—
[15]	—	OP and EC	SISO	Fox H function	Arbitrary	Imperfect	—
[16]	—	EC	MIMO	CLT	Arbitrary	Imperfect	—
[17]	—	EC	MIMO	CLT	Rician	Perfect	—
[22]	—	AEP	SISO	CLT	Rayleigh	Perfect	MPSK/QAM
[23]	—	OP	SIMO	CLT	Rician	Imperfect	—
[18]	✓	EC	SISO	CLT	Rayleigh	Imperfect	—
[19]	✓	OP, and EC	SISO	CLT	Rayleigh	Imperfect	—
[20]	✓	OP, and EC	SISO	leveraging Theorem	Rayleigh, and Rician	perfect	—
[21]	✓	OP	SISO	Gamma Distribution	Nakagami- $\bar{m}$	Perfect	—
[24]	✓	OP, AEP and EC	SISO	Tight Approximation	Rician and K -distribution	Perfect	BPSK
[25]	✓	CP	SISO	CLT	Nakagami- $\bar{m}$	Perfect	—
[26]	✓	EC	SISO	CLT	Rayleigh	Perfect	—

TABLE II: List of functions, $\mathbf{a}_p = a_1, \dots, a_p$; $\mathbf{b}_q = b_1, \dots, b_q$.

Head	Head
$\mathbb{E}[\cdot]$	Statistical expectation
$\text{Var}(\cdot)$	Statistical variance
$\Pr[\cdot]$	Probability operation
$\mathcal{L}[\cdot]$	Laplace transform
$\mathcal{L}^{-1}[\cdot]$	Inverse Laplace transform
$M_v(\cdot)$	Moment generating function
$f_X(\cdot)$	Probability density function
$F_X(\cdot)$	Cumulative distribution function
$\Gamma(\cdot)$	gamma function [30, eq. (8.310)]
$\gamma(\cdot, \cdot)$	Lower incomplete gamma function [30, eq. (8.350)]
$K_v(\cdot)$	Modified Bessel function of order v [31, eq. (9.6.2)]
Φ_2	bivariate confluent hypergeometric function (HF) [32]
${}_1F_1(\cdot)$	Regularized HF
${}_1F_1(\cdot; \cdot; \cdot)$	Confluent HF [30, eq. (8.310)]
${}_pF_q(\mathbf{a}_p; \mathbf{b}_q; x)$	generalized HF [30, eq. (9.14)]
$B(\cdot, \cdot)$	Beta function [30, eq. (8.38)]

attached to the RIS panel through a feedback channel [6], [33], [34].

For mathematical tractability, we consider that the BS and UE are equipped with single antenna [35]–[37]. Therefore, the received signal at any UE can be expressed as

$$Y = Y_D + Y_R + n \quad (1)$$

where Y_D is the signal received through the direct link, Y_R is the signal received through the RIS, and $n \sim \mathcal{CN}(0, N_0)$ is the additive white Gaussian noise (AWGN). The direct signal can be written as [25]

$$h_D = \sqrt{P} d_0^{-\frac{\alpha_D}{2}} \zeta^{\frac{1}{2}} |h_0| e^{-j\theta_0} s \quad (2)$$

where P is the transmitted power, $\zeta = \left(\frac{c}{4\pi f_c}\right)^2$ denotes the average channel gain at a reference distance of 1 m based on the free space path-loss model, c is the speed of light, f_c is the carrier frequency, s is the transmitted information symbol, $|h_0|$ and θ_0 are the magnitude and phase of the channel from the BS to UE, and α_D is the pathloss exponent at direct link.

Similarity, Y_R denotes the received signal at UE through the RIS link, which can be written as,

$$Y_R = \sqrt{P} \zeta d_h^{-\frac{\alpha_R}{2}} d_g^{-\frac{\alpha_R}{2}} s \sum_{i=1}^N |h_i| e^{-j\theta_i} v_i e^{j\Psi_i} |g_i| e^{-j\phi_i} \quad (3)$$

where $|h_i|$ and θ_i denote the magnitude and phase of h_i , respectively, while, $|g_i|$ and ϕ_i are the magnitude and phase of g_i the UE, respectively. The i th RIS element gain and phase are v_i and Ψ_i . Without loss of generality, we consider that $v_i = 1$ [8], [35]–[38]. Therefore, we can write

$$Y_R + Y_D = \left(\zeta d_h^{-\frac{\alpha_R}{2}} d_g^{-\frac{\alpha_R}{2}} \sum_{i=1}^N |h_i| |g_i| e^{-j(\theta_i - \phi_i + \Psi_i - \theta_0)} + d_0^{-\frac{\alpha_D}{2}} \zeta^{\frac{1}{2}} |h_0| \right) s \sqrt{P} e^{-j\theta_0}. \quad (4)$$

If we choose $\theta_i - \phi_i + \Psi_i - \theta_0 = 0$ [38]–[41], then

$$Y_R + Y_D = \sqrt{P} e^{-j\theta_0} \left(\frac{\zeta}{d_h^{\frac{\alpha_R}{2}} d_g^{\frac{\alpha_R}{2}}} \sum_{i=1}^N |h_i| |g_i| + d_0^{-\frac{\alpha_D}{2}} \zeta^{\frac{1}{2}} |h_0| \right) s = (H_R + H_D) e^{-j\theta_0} s \quad (5)$$

Consequently, the overall channel gain can be expressed as

$$V = H_D + H_R. \quad (6)$$

B. Channel Model

Due to the wide range of use cases and applications of 5G and sixth generation (6G) wireless networks, conventional channel models that consider large scale shadowing will be mostly insufficient to capture the complex propagation effects of such applications [42]. In the literature, several composite channel models such as the $\bar{\kappa}\text{-}\bar{\mu}$ /inverse and $\eta\text{-}\bar{\mu}$ /inverse gamma fading are used to capture the statistical behavior of the combined small-scale and large-scale fading in various propagation environments [43]. However, such models assume that the shadowing affects the scattered and dominant waves

equally, and thus, such models are referred to as multiplicative shadow fading models [44], [45]. However, in practice the shadowing often affects only the dominant components. Therefore, such model is referred to as LOS shadow fading model [46], [47].

The κ - μ shadowed model, denoted as κ - μ , is a generalized distribution that plays an essential role in modeling fading for LOS scenarios. The κ - μ shadowed model has an additional parameter ω , as compared to $\bar{\kappa}$ - $\bar{\mu}$ model, which is related to shadowing. Consequently, it has a clear physical interpretation, flexible analytical properties and can represent common fading models such as One-Side Gaussian, Rayleigh, Nakagami- $\bar{m}$, Rician, $\bar{\kappa}$ - $\bar{\mu}$, and η - $\bar{\mu}$ fading channels as special cases [28], [29, Table. I]. In addition, it can be used in several real-world applications [46], [48]–[50]. By limiting the fading parameters μ and ω to integer values, minor differences are obtained in practice when fitting field measurements to the κ - μ , while it will make the mathematical analysis more tractable and may lead to efficient closed-form expressions [51]. Consequently, in this work we consider that all channel gains follow the κ - μ with integer μ and ω parameters. Therefore, the PDF and CDF are respectively given by [51]

$$f_X(x) = \sum_{i=0}^M \frac{C_i x^{m_i-1}}{\Gamma(m_i) m_i^{m_i}} e^{-\frac{x}{m_i}} \quad (7)$$

and

$$F_X(x) = 1 - \sum_{i=0}^M C_i e^{-\frac{x}{m_i}} \sum_{k=0}^{m_i-1} \frac{1}{k!} \left(\frac{x}{m_i} \right)^k \quad (8)$$

where M , C_i , m_i , and Ω_i are given in Table III.

C. PDF of the Received Signal Power

As can be noted from (5) and (6), the received signal power depends on V , and thus, its PDF should be derived. Towards this goal, define $|h_i||g_i| \triangleq Z_i$, and for notational simplicity we drop the reflector index i unless it is necessary to include. Therefore, the PDF of Z can be evaluated as [52],

$$f_Z(z) = \int_0^\infty \frac{1}{x} f_{|h|}(x) f_{|g|}\left(\frac{z}{x}\right) dx. \quad (9)$$

By substituting the PDFs $h \sim \kappa\mu(M_1, C_i, m_i, \Omega_i)$, and $g \sim \kappa\mu(M_2, C_j, m_j, \Omega_j)$ expressed in (7), into (9), and using the identity [30, eq. (3.471.9)]

$$\int_0^\infty x^{v-1} e^{\frac{\beta}{x} - \gamma x} dx = 2 \left(\frac{\beta}{\gamma} \right)^{\frac{v}{2}} K_v \left(2\sqrt{\beta\gamma} \right) \quad (10)$$

we obtain,

$$f_Z(z) = 2 \sum_{j=0}^{M_2} \sum_{i=0}^{M_1} C_j C_i \frac{z^{-1+\frac{m_j+m_i}{2}}}{\Gamma(m_j) \Gamma(m_i) (\Omega_j \Omega_i)^{\frac{m_j+m_i}{2}}} \times K_{m_j-m_i} \left(2\sqrt{\frac{z}{\Omega_j \Omega_i}} \right). \quad (11)$$

Moreover, the mean and variance of Z are respectively given by,

$$\mathbb{E}[Z] \triangleq \mu_Z = \sum_{j=0}^{M_2} \sum_{i=0}^{M_1} C_j C_i \Omega_j^{m_j+1} \Omega_i^{m_i+1} m_j m_i \quad (12)$$

and

$$\begin{aligned} \text{Var}[Z] &\triangleq \sigma_Z^2 = \mathbb{E}[Z^2] - (\mathbb{E}[Z])^2 \\ &= \sum_{j=0}^{M_2} \sum_{i=0}^{M_1} C_j C_i (\Omega_j \Omega_i)^2 m_j^2 m_i^2 \\ &\quad \times [(m_j+1)(m_i+1) - C_j C_i m_j m_i]. \end{aligned} \quad (13)$$

In the special case where all the links are identical, i.e., $M_1 = M_2 = M$, $\Omega_j = \Omega_i = \Omega_l$, and $C_j = C_i = C_l$, then

$$f_Z(z) = \sum_{l=0}^M C_l^2 \frac{2z^{-1+m_l}}{\Gamma^2(m_l) (\Omega_l)^{m_l}} K_0 \left(\frac{2\sqrt{z}}{\Omega_l} \right) \quad (14)$$

$$\mu_Z = \sum_{l=0}^M (C_l \Omega_l^{m_l+1} m_l)^2 \quad (15)$$

$$\sigma_Z^2 = \sum_{l=0}^M C_l^2 (\Omega_l m_l)^4 \left[(m_l+1)^2 - C_l^2 m_l^2 \right]. \quad (16)$$

According to [53], [54], the PDF and CDF of random variable (RV) $Z = \sum_{i=1}^N Z_i$ can be tightly approximated by the first term of a Laguerre expansion as follows,

$$f_Z(z) = \frac{z^a}{b^{a+1} \Gamma(a+1)} \exp\left(-\frac{z}{b}\right) \quad (17)$$

and

$$F_Z(z) = \frac{\gamma(1+a, \frac{z}{b})}{\Gamma(a+1)} \quad (18)$$

where the parameters a and b are related to the mean and variance of Z as

$$a = \frac{\mu_Z^2}{\sigma_Z^2} - 1 \quad (19)$$

and

$$b = \frac{\sigma_Z^2}{\mu_Z} \quad (20)$$

where $\mu_Z = \sum_{i=1}^N \mu_{Z_i}$ and $\sigma_Z^2 = \sum_{i=1}^N \sigma_{Z_i}^2$. In case of i.i.d. channel gains, $\mu_Z = N\mu_{Z_i}$ and $\sigma_Z^2 = N\sigma_{Z_i}^2$.

By applying the transformation method between two random variables in (3), the PDF and CDF of H_R can be given by

$$f_{H_R}(h_R) = \frac{(h_R)^a}{(L_R b)^{a+1} \Gamma(a+1)} \exp\left(-\frac{h_R}{L_R b}\right) \quad (21)$$

and

$$F_{H_R}(h_R) = \frac{\gamma\left(1+a, \frac{h_R}{L_R b}\right)}{\Gamma(a+1)} \quad (22)$$

where $L_R = \sqrt{P} \zeta d_h^{-\frac{\alpha_R}{2}} d_g^{-\frac{\alpha_R}{2}}$.

Similarly, the PDF and CDF of H_D can be expressed as,

$$f_{H_D}(h_D) = \sum_{q=0}^{M_3} \frac{C_q}{\Gamma(m_q)} \frac{h_D^{m_q-1}}{(L_D \Omega_q)^{m_q}} e^{-\frac{h_D}{L_D \Omega_q}} \quad (23)$$

and

$$F_{H_D}(h_D) = 1 - \sum_{q=0}^{M_3} C_q e^{-\frac{h_D}{L_D \Omega_q}} \sum_{k=0}^{m_q-1} \frac{1}{k!} \left(\frac{h_D}{L_D \Omega_q} \right)^k \quad (24)$$

TABLE III: Parameter values for the κ - μ shadowed distribution [51].

Case $\mu > \omega$		Case $\mu \leq \omega$
$M = \mu$		$M = \mu - \omega$
$C_i = \begin{cases} 0 & i = 0 \\ (-1)^\omega \binom{\omega+i-2}{i-1} \left[\frac{\omega}{\mu k + \omega} \right]^\omega \left[\frac{\mu k}{\mu k + \omega} \right]^{-\omega-i+1} & 0 < i \leq \mu - \omega \\ (-1)^{i-\mu+\omega-1} \binom{i-2}{i-\mu+\omega-1} \left[\frac{\omega}{\mu k + \omega} \right]^{i-\mu+\omega-1} \left[\frac{\mu k}{\mu k + \omega} \right]^{1-i} & \mu - \omega < i \leq \mu \end{cases}$		$(\omega - \mu) \left[\frac{\omega}{\mu k + \omega} \right]^i \left[\frac{\mu k}{\mu k + \omega} \right]^{\omega - \mu - i}$
$m_i = \begin{cases} \mu - \omega - i + 1 & 0 \leq i \leq \mu - \omega \\ \mu - i + 1 & \mu - \omega \leq i \leq \mu \end{cases}$		$m_i = \omega - i$
$\Omega_i = \begin{cases} \frac{\gamma}{\mu(1+k)} & 0 \leq i \leq \mu - \omega \\ \frac{\mu k + \omega}{\omega} \frac{\gamma}{\mu(1+k)} & \mu - \omega \leq i \leq \mu \end{cases}$		$\Omega_i = \frac{\mu k + \omega}{\omega} \frac{\gamma}{\mu(1+k)}$

respectively, where $L_D = \sqrt{P}d_0^{-\frac{\alpha_D}{2}} \zeta^{\frac{1}{2}}$, M_3 , C_q , m_q , and Ω_q are given in Table III.

By noting the superposition of H_D and H_R in (6), then the PDF of V can be obtained by applying the convolution theorem to (21) and (23). Thus,

$$\begin{aligned}
 f_V(v) &= \int_{-\infty}^{\infty} f_{H_D}(u) f_{H_R}(v-u) du \\
 &= \sum_{q=0}^{M_3} \frac{C_q}{\Gamma(m_q)} \frac{1}{(L_D \Omega_q)^{m_q}} \frac{1}{(L_R b)^{a+1} \Gamma(a+1)} \\
 &\quad \times \exp\left(-\frac{v}{L_R b}\right) \int_0^v (v-u)^a u^{m_q-1} \\
 &\quad \times \exp\left(-\left(\frac{1}{L_D \Omega_q} - \frac{1}{L_R b}\right)u\right) du \\
 &= \sum_{q=0}^{M_3} \frac{C_q}{(L_D \Omega_q)^{m_q} (L_R b)^{a+1}} v^{a+m_q} \\
 &\quad \times \exp\left(-\frac{v}{L_R b}\right) \frac{1}{\Gamma(a+m_q+1)} \\
 &\quad \times {}_1F_1\left(m_q, a+m_q+1; \left(\frac{1}{L_R b} - \frac{1}{L_D \Omega_q}\right)v\right). \tag{25}
 \end{aligned}$$

D. Moment Generating Function of V

By applying the Laplace transform to (25), we obtain the MGF of V ,

$$\begin{aligned}
 M_V(s) &= \mathcal{L}\left[f_V(v); -\left(s + \frac{1}{L_R b}\right)\right] \\
 &= \sum_{q=0}^{M_3} \frac{C_q}{(L_D \Omega_q)^{m_q}} \frac{1}{(L_R b)^{a+1} \Gamma(a+m_q+1)} \\
 &\quad \times \mathcal{L}\left[v^{a+m_q} {}_1F_1(m_q; a+m_q+1; U_4 v)\right] \tag{26}
 \end{aligned}$$

where $U_4 = \left(\frac{1}{L_R b} - \frac{1}{L_D \Omega_q}\right)$. Using [55, eq.4.23.1],

$$\begin{aligned}
 &\mathcal{L}\left[v^{a+m_q} {}_1F_1\left(m_q; a+m_q+1; \left(\frac{1}{L_R b} - \frac{1}{L_D \Omega_q}\right)v\right)\right] \\
 &= \Gamma(a+m_q+1) \left(s + \frac{1}{L_R b}\right)^{-(a+1)} \left(s + \frac{1}{L_D \Omega_q}\right)^{-m_q}. \tag{27}
 \end{aligned}$$

Therefore, (26) becomes

$$\begin{aligned}
 M_V(s) &= \sum_{q=0}^{M_3} \frac{C_q}{(L_D \Omega_q)^{m_q}} \frac{1}{(L_R b)^{a+1}} \left(s + \frac{1}{L_R b}\right)^{-(a+1)} \\
 &\quad \times \left(s + \frac{1}{L_D \Omega_q}\right)^{-m_q}. \tag{28}
 \end{aligned}$$

E. The CDF of V

By noting that the MGF in (28) and $f_V(v)$ has the relation

$$f_V(v) = \mathcal{L}^{-1}[M_V(s); v] \tag{29}$$

then $F_V(v)$ we can be obtained as

$$F_V(v) = \mathcal{L}^{-1}\left[\frac{M_V(s)}{s}; v\right]. \tag{30}$$

By substituting (28) in (30), we obtain

$$\begin{aligned}
 F_V(v) &= \mathcal{L}^{-1}\left[\frac{M_V(s)}{s}\right] \\
 &= \sum_{q=0}^{M_3} \frac{C_q}{(L_D \Omega_q)^{m_q}} \frac{1}{(L_R b)^{a+1}} \\
 &\quad \times \mathcal{L}^{-1}\left[\frac{1}{s} \left(s + \frac{1}{L_R b}\right)^{-(a+1)} \left(s + \frac{1}{L_D \Omega_q}\right)^{-m_q}\right]. \tag{31}
 \end{aligned}$$

Using [55, eq. (4.24.3)], the Laplace inverse can be evaluated as,

$$\begin{aligned}
 &\mathcal{L}^{-1}\left[\frac{1}{s} \left(s + \frac{1}{L_R b}\right)^{-(a+1)} \left(s + \frac{1}{L_D \Omega_q}\right)^{-m_q}\right] = v^{a+m_q+1} \\
 &\quad \times \frac{\Phi_2\left(a+1, m_q, a+m_q+2; -\frac{1}{L_R b}v, -\frac{1}{L_D \Omega_q}v\right)}{\Gamma(a+m_q+2)}. \tag{32}
 \end{aligned}$$

Substituting (32) into (31) gives,

$$\begin{aligned}
 F_V(v) &= \sum_{q=0}^{M_3} \frac{C_q}{(L_D \Omega_q)^{m_q}} \frac{1}{(L_R b)^{a+1}} \frac{v^{a+m_q+1}}{\Gamma(a+m_q+2)} \\
 &\quad \times \Phi_2\left(a+1, m_q, a+m_q+2; -\frac{v}{L_R b}, -\frac{v}{L_D \Omega_q}\right) \tag{33}
 \end{aligned}$$

where the function Φ_2 can be efficiently computed by means of an inverse Laplace transform [56].

F. End-to-End SNR

The instantaneous end-to-end SNR at the UE can be expressed as

$$\gamma_Y = \frac{P_s}{N_0} v^2 = v^2 \bar{\gamma} \quad (34)$$

where P_s is the transmitted symbol power, N_0 is thermal noise power, and $\bar{\gamma}$ is the average SNR. Therefore, the PDF, CDF and MGF of γ_Y are given by,

$$f_{\gamma_Y}(\gamma) = \frac{1}{2\sqrt{\bar{\gamma}}} \sum_{q=0}^{M_3} \frac{C_q}{(L_D \Omega_q)^{m_q} (L_R b)^{a+1}} \frac{1}{\Gamma(a + m_q + 1)} \\ \times {}_1F_1\left(m_q, a + m_q + 1; \left(\frac{1}{L_R b} - \frac{1}{L_D \Omega_q}\right) \sqrt{\frac{\gamma}{\bar{\gamma}}}\right) \\ \times \left(\frac{\gamma}{\bar{\gamma}}\right)^{\frac{a+m_q}{2}} \exp\left(-\frac{1}{L_R b} \sqrt{\frac{\gamma}{\bar{\gamma}}}\right) \quad (35)$$

$$F_{\gamma_Y}(\gamma) = \sum_{q=0}^{M_3} \frac{C_q}{(L_D \Omega_q)^{m_q} (L_R b)^{a+1}} \frac{1}{\Gamma(a + m_q + 2)} \\ \times \left(\frac{\gamma}{\bar{\gamma}}\right)^{\frac{a+m_q+1}{2}} \Phi_2(a + 1, m_q, a + m_q + 2; U_1, U_2) \quad (36)$$

$$M_{\gamma_Y}(s) = \sum_{q=0}^{M_3} \frac{C_q}{(L_D \Omega_q)^{m_q} (L_R b)^{a+1}} \left(\frac{1}{\bar{\gamma}}\right)^{\frac{a+m_q+1}{2}} \\ \times \left(s + \frac{1}{\sqrt{\bar{\gamma}} L_R b}\right)^{-(a+1)} \left(s + \frac{1}{\sqrt{\bar{\gamma}} L_D \Omega_q}\right)^{-m_q} \quad (37)$$

respectively, where $U_1 = -\frac{1}{L_R b} \sqrt{\frac{\gamma}{\bar{\gamma}}}$, and $U_2 = -\frac{1}{L_D \Omega_q} \sqrt{\frac{\gamma}{\bar{\gamma}}}$.

To obtain more insights about the system performance, we derive the asymptotic PDF, CDF and MGF in the high SNR region, i.e., $\bar{\gamma} \rightarrow \infty$. For the PDF,

$$f_{\gamma_Y}(\gamma)|_{\bar{\gamma} \rightarrow \infty} \triangleq f_{\gamma_Y}^{\infty}(\gamma) \quad (38)$$

by noting that [57]

$$\lim_{\bar{\gamma} \rightarrow \infty} {}_1F_1\left(m_q, a + m_q + 1; \left(\frac{1}{L_R b} - \frac{1}{L_D \Omega_q}\right) \sqrt{\frac{\gamma}{\bar{\gamma}}}\right) \\ \times \exp\left(-\frac{1}{L_R b} \sqrt{\frac{\gamma}{\bar{\gamma}}}\right) = 1 \quad (39)$$

then we obtain from (35),

$$f_{\gamma_Y}^{\infty}(\gamma) = \frac{1}{2\sqrt{\bar{\gamma}}} \sum_{q=0}^{M_3} \frac{C_q}{(L_D \Omega_q)^{m_q} (L_R b)^{a+1}} \\ \times \frac{1}{\Gamma(a + m_q + 1)} \left(\frac{\gamma}{\bar{\gamma}}\right)^{\frac{a+m_q}{2}}. \quad (40)$$

Similarly,

$$F_{\gamma_Y}^{\infty}(\gamma) = \sum_{q=0}^{M_3} \frac{C_q}{(L_D \Omega_q)^{m_q} (L_R b)^{a+1}} \\ \times \frac{1}{\Gamma(a + m_q + 2)} \left(\frac{\gamma}{\bar{\gamma}}\right)^{\frac{a+m_q+1}{2}}, \quad (41)$$

and

$$M_{\gamma_Y}^{\infty}(s) = \sum_{q=0}^{M_3} \frac{C_q}{(L_D \Omega_q)^{m_q} (L_R b)^{a+1}} \left(\frac{1}{\bar{\gamma} s^2}\right)^{\frac{a+m_q+1}{2}}. \quad (42)$$

III. PERFORMANCE EVALUATION

A. Outage Probability

The derived distributions can be used to compute the outage probability, which is defined as

$$P_{out} = \Pr[\gamma_Y \leq \gamma_{out}] \\ = F_{\gamma_Y}(\gamma_{out}) \quad (43)$$

where γ_{out} is a predetermined SNR threshold. From (41), it can be concluded that P_{out} can be approximated as

$$P_{out} \approx (G_o)^{-G_d} \quad (44)$$

where G_o and G_d denote the coding gain and diversity order which are respectively given by

$$G_o = \left(\sum_{q=0}^{M_3} \frac{\Gamma(a + m_q + 2) (L_D \Omega_q)^{m_q} (L_R b)^{a+1}}{C_q} \right)^w \frac{\bar{\gamma}}{\gamma} \quad (45)$$

where $w = \frac{2}{a+m_q+1}$, and

$$G_d = \frac{a + m_q + 1}{2}. \quad (46)$$

B. Bit Error Rate (BER)

Based on the MGF expressed in (37), the BER can be expressed as,

$$P_B = \frac{a_m}{\pi} \int_0^{\frac{\pi}{2}} M_{\gamma_Y} \left(\frac{b_m^2}{2 \sin^2(\vartheta)} \right) d\vartheta \quad (47)$$

where a_m and b_m are parameters that depend on the modulation scheme. By substituting (37) in (47), the BER becomes

$$P_B = \sum_{q=0}^{M_3} \frac{C_q}{(L_D \Omega_q)^{m_q} (L_R b)^{a+1}} \left(\frac{1}{\bar{\gamma}}\right)^{\frac{a+m_q+1}{2}} \frac{a_m}{\pi} \\ \times \int_0^{\frac{\pi}{2}} \left(\frac{b_m^2}{2 \sin^2(\vartheta)} + \frac{1}{\sqrt{\bar{\gamma}} L_R b} \right)^{-(a+1)} \\ \times \left(\frac{b_m^2}{2 \sin^2(\vartheta)} + \frac{1}{\sqrt{\bar{\gamma}} L_D \Omega_q} \right)^{-m_q} d\vartheta. \quad (48)$$

By performing the change of variable $t = \cos^2(\vartheta)$, and after some algebraic manipulations, we obtain

$$P_B = \sum_{q=0}^{M_3} \frac{C_q}{(L_D \Omega_q)^{m_q} (L_R b)^{a+1}} \left(\frac{1}{\bar{\gamma}}\right)^{\frac{a+m_q+1}{2}} \\ \times \left(\frac{b_m^2}{2} + \frac{1}{\sqrt{\bar{\gamma}} L_R b} \right)^{-(a+m_q)} \\ \times \left(\frac{b_m^2}{2} + \frac{1}{\sqrt{\bar{\gamma}} L_D \Omega_q} \right)^{-m_q} \frac{a_m}{\pi} \int_0^1 t^{-\frac{1}{2}} \\ \times (1-t)^{a+m_i+\frac{1}{2}} (1-u_1 t)^{-(a+1)} (1-u_2 t)^{-m_i} dt \quad (49)$$

TABLE IV: Simulation Parameters.

Parameters	Value
System Bandwidth (BW)	1 MHz
Power of AWGN (N_0)	-114 dBm
Distance between BS and UE (d_0)	500 m
Distance between BS and RIS (d_h)	500 m
The distance between BS and RIS (d_g)	100 m
Power attenuation at reference (1 m)	-30 dBm
Transmitted Power (P)	1 W
Path loss exponent at all links	3.5
Target Threshold SNR γ_{out}	20 dB
Monte Carlo runs	10^4

with the help of [30, eq. (3.211)]

$$\int_0^1 t^{\lambda_1-1} (1-t)^{\mu_1-1} (1-u_1 t)^{-\varrho} (1-u_2 t)^{-\varsigma} dt = B(\mu_1, \lambda_1) F_1(\lambda_1, \varrho, \varsigma, \lambda_1 + \mu_1; u_1, u_2) \quad (50)$$

(49) becomes

$$P_B = \sum_{q=0}^{M_3} \frac{a_m}{4\pi} \frac{C_q}{(L_D \Omega_q)^{m_q}} \frac{1}{(L_R b)^{a+1}} B\left(a + m_q + \frac{3}{2}, \frac{1}{2}\right) \times \left(1 + \frac{b_m^2 \sqrt{\bar{\gamma}} L_R b}{2}\right)^{-w_2} \left(1 + \frac{b_m^2 \sqrt{\bar{\gamma}} L_D \Omega_q}{2}\right)^{-m_q} \times F_1\left(\frac{1}{2}, a+1, m_q, a+m_q+2; u_1, u_2\right) \quad (51)$$

where the values of w_2 , u_1 , and u_2 are respectively given by

$$w_2 = a + 1 \quad (52)$$

$$u_1 = \frac{2}{1 + 2b_m^2 L_R b \sqrt{\bar{\gamma}}} \quad (53)$$

and

$$u_2 = \frac{2}{1 + 2b_m^2 L_D \Omega_q \sqrt{\bar{\gamma}}}. \quad (54)$$

For the asymptotic case where $\bar{\gamma} \rightarrow \infty$, we obtain

$$P_B^\infty = \sum_{q=0}^{M_3} \frac{a_m}{4\pi} \frac{C_q}{(L_D \Omega_q^2)^{m_i} (L_R b^2)^{a+1}} B\left(a + m_q + \frac{3}{2}, \frac{1}{2}\right) \times \left(\frac{b_m^2 \sqrt{\bar{\gamma}}}{2}\right)^{-(a+m_q+1)} \quad (55)$$

where the asymptotic BER can also be expressed in terms of coding G_p and diversity gain G_d , i.e., $P_B \approx (G_p \bar{\gamma})^{-G_d}$ [58, eq. (1)]

$$G_p = \left(\sum_{q=0}^{M_3} \frac{a_m}{4\pi} \frac{C_q}{(L_D \Omega_q^2)^{m_q} (L_R b^2)^{a+1}} \right)^{-\frac{2}{a+m_q+1}} \times \left(B\left(a + m_q + \frac{3}{2}, \frac{1}{2}\right) \frac{b_m^2}{2} \right)^{-\frac{2}{a+m_q+1}} \quad (56)$$

and

$$G_d = \frac{a + m_q + 1}{2} \quad (57)$$

Fig. 2: The PDF $f_Z(z)$ for different fading models as special cases of κ - μ shadowed distribution for $N = 10$.

IV. NUMERICAL RESULTS

This section presents the theoretical and Monte Carlo simulation results for various cases of interest. The main simulation parameters are given in Table IV, unless is specified otherwise.

Fig. 2 demonstrates the effectiveness and validity of the derived PDFs by comparing the generalized theoretical expression of the PDF in (17) to Monte Carlo simulation results. The values of μ , κ and ω of the individual PDFs are selected to correspond to some of the widely used PDFs, while N is fixed at 10. As the figure shows, the theoretical expression perfectly matches the simulation results for all considered scenarios. Moreover, the generalized distribution model can be used to represent the widely known fading distributions such as the Rayleigh, Nakagami- m , Rician, and Rician-Shadowed. In Fig. 3, PDF $f_Z(z)$ is evaluated for fixed and equal fading parameters where $N \in \{1, 3, 5, 7, 10\}$. As the figure shows, the analytical and simulation results match perfectly, which confirms the accuracy of the derived expressions. Moreover, it can be noted that increasing N can significantly improve the fading conditions.

Fig. 4 compares the analytical, asymptotic and simulated outage probability versus SNR using various number of reflecting elements N . As can be noted from the figure, the analytical and simulation results match very well for all the considered scenarios. It can be also observed that at high SNRs, the asymptotic and exact results converge. The improvement gained by increasing the number of reflectors increases non-linearly versus N . For example, increasing the number of reflectors from 5 to 10 reduced the SNR required to achieve P_{out} of 10^{-5} from 8.25 dB to -1.45 dB, i.e., the gain is about 9.7 dB. However, increasing N from 10 to 20 reflectors offers only ~ 6 dB gain.

Fig. 3: The PDF $f_Z(z)$ for different values of N where $\mu_R = 2$ and $\omega_R = 30$.

Fig. 4: Outage probability versus $\bar{\gamma}$ for $N = 5, 10, 20$, $\omega_R = \omega_D = 6$, $\mu_R = \mu_D = 3$, $\kappa_R = \kappa_D = 1$.

Fig. 5, Fig. 6 and Table. V evaluate the impact of the various system and channel conditions on the outage and BER performance. As can be noted from Fig. 5, outage probability (OP) is significantly dominated by the strength and quality of the direct path that is indicated by μ_D . For $\mu_D = 1$, the fading of the direct path is severe, and thus $P_{out} > 0.1$ for $\bar{\gamma} > 0$ dB. Increasing the value of N can reduce P_{out} , but the impact

Fig. 5: The outage probability for various values of μ_D and N where $\omega_R = \omega_D = 30$, $\mu_R = 5$, $\kappa_R = 5$, $\kappa_D = 3$, and $\gamma_{out} = 5$ dB.

TABLE V: P_{out} for various κ_D and ω_D at BS-UE link, $\omega_R = 15$, $\mu_R = \mu_D = 1$, and $\kappa_R = 5$, $\bar{\gamma} = 0$ dB, $\gamma_{out} = -20$ dB.

N	κ_D	ω_D			
		1	5	10	15
Without RIS	1	0.096	0.134	0.074	0.073
	3	0.096	0.067	0.029	0.073
	5	0.096	0.035	0.011	0.008
	7	0.096	0.020	0.0042	0.0026
1	1	0.0676	0.0547	0.0523	0.0515
	3	0.0676	0.0268	0.0201	0.0178
	5	0.0676	0.0137	0.0074	0.0056
	7	0.0676	0.0077	0.0029	0.0019
3	1	0.0330	0.0266	0.0255	0.0251
	3	0.0330	0.0130	0.0098	0.0087
	5	0.0330	0.0067	0.0036	0.0027
	7	0.0330	0.0037	0.0014	9.0×10^{-4}
5	1	0.0157	0.0125	0.0122	0.0120
	3	0.0157	0.0062	0.0047	0.0041
	5	0.0157	0.0032	0.0017	0.0013
	7	0.0157	0.0018	6.8×10^{-4}	4.3×10^{-4}
10	1	0.0074	0.0059	0.0057	0.0056
	3	0.0074	0.0029	0.0022	0.0019
	5	0.0074	0.0015	8.0×10^{-4}	6.1×10^{-4}
	7	0.0074	8.2×10^{-4}	3.2×10^{-4}	2.0×10^{-4}

appears generally for $\bar{\gamma} > 0$ dB. At low SNRs, all P_{out} values approach that of the direct path regardless the value of N .

Table V shows the variation of the outage probability with respect to the shadowing parameter of the direct path ω_D between BS and UE for different values of κ_D and N . The results are obtained for $\mu_R = \mu_D = 1$. The shadowing parameters between BS-RIS and RIS-UE links is $\omega_R = 30$. As can be noted from the table, P_{out} is generally dominated by ω_D . For example, when $\omega_D = 1$, the value of P_{out} remains fixed regardless the value of κ_D . However, when ω_D improves, then P_{out} improves by improving κ_D . Moreover, it can be noted that ω_D also limits the impact of N , where increasing

Fig. 6: The impact of the number of reflecting elements N on the BER, $\omega_R = \omega_D = 3$, and $\mu_R = \mu_D = 2$.

Fig. 8: The impact of the distance between RIS and UE on average bit error rate (ABER): $\bar{\gamma} = 0$ dB, $\gamma_{out} = 20$ dB, $\omega_R = \omega_D = 2$, and $\mu_R = \mu_D = 1$ and $\kappa_R = \kappa_D = 1$.

Fig. 7: The impact of the path loss exponent (α_R) on the bit error rate (BER) where $\bar{\gamma} = -30$ dB, $\alpha_D = 3.5$, $\omega_R = \omega_D = 2$, $\mu_R = \mu_D = 1$, and $\kappa_R = \kappa_D = 1$.

Fig. 9: Variation in diversity order with respect to number of reflecting elements N where $\omega_R = \omega_D$, and $\mu_R = \mu_D = 1$ and $\kappa_R = \kappa_D = 1$.

the value of N from 1 to 10 reduced P_{out} only by a factor of 10, while the improvement for the same scenario is much more significant for $\omega_D = 15$. For high values of ω_D , it can be observed that the impact of κ_D becomes more tangible where P_{out} decreases by increasing κ_D .

Fig. 6 shows the effect of changing the number of reflecting elements N on the BER. The exact (51) and asymptotic

(55) BERs are included and compared to the Monte Carlo simulation results. The figure shows that the analytical and simulation results match very well, even for the case of very small number of reflectors such as $N = 1$ and 3. The BER improvement gained by using RIS seems to be significant, even when for small values of N . The asymptotic BER approaches the exact when the exact BER becomes roughly linear. For the

case of no RIS and $N = 1$, the curves converge for $\bar{\gamma} \gtrsim 30$ dB, while for the case of $N = 3$ they converge for $\bar{\gamma} \gtrsim 28$ dB.

Fig. 7 evaluates the impact of the path loss exponent at RIS link (α_R) on the BER using (51). As can be noted from the figure, the BER achieved by the RIS increases by increasing α_R , and ultimately approaches the BER of the direct link regardless the value of N . Such performance is obtained because increasing α_R causes severe signal attenuation, and hence, the signal reflected by the RIS becomes extremely weak. However, when α_R is small, a significant performance gain can be achieved by increasing N .

Fig. 8 demonstrates the advantage of using RIS-aided transmission over the direct link. Therefore, the figure illustrates the effect of increasing the distance between the RIS and UE (d_g) on P_{out} . In particular, the graph provides a comparative analysis of three transmission schemes, which are the direct transmission only, RIS without direct transmission, and RIS transmission with a direct link. It can be noted that if the direct link is available, it is beneficial to combine both signals, notably when N increases. Moreover, it is generally difficult for the direct path alone to achieve reliable P_{out} when the outage threshold P_{out} is high. Therefore, the location of the RIS should be optimized by taking into consideration the possibility of exploiting the direct and reflected links to minimize the outage and BER.

Fig. 9 presents the diversity order of the system versus the number of reflecting elements N for various values of ω_D . Results indicate that ω_D has a significant impact on the diversity order for high values of N . When $\omega_D \gg 1$, all the reflected links have generally strong signals, and thus the diversity gained by using the RIS is limited. For low values of ω_D , the channel variations increase significantly, which increases the diversity order achieved by using the RIS.

V. CONCLUSIONS

In this paper, we developed a new analytical framework model to investigate the end-to-end SNR, outage probability and BER performance of RIS assisted wireless communication systems over more generalized κ - μ shadowed distribution channels. Novel accurate closed-form expressions for the PDF, CDF of end-to-end SNR have been derived. Moreover, closed-form expressions have been derived for the outage and BER using the Laguerre expansion, which provided near-exact results. The asymptotic outage and BER are derived to provide more insights on the achievable diversity order. The derived expressions were used to holistically study the performance of RIS systems in the presence of a direct link between the BS and UE. The obtained results show that the direct path can significantly affect the RIS system performance, and hence, should be taken into consideration during the planning and design stages of the system. Monte Carlo simulation was used to validate all the derived expressions.

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