Optimal feedforward torque control for nonlinear induction machines considering stator & rotor copper losses and current & voltage limits

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Abstract—In order to *analytically* solve the optimal feedforward torque control (OFTC) problem of induction machines (IMs), the unified theory for synchronous machine introduced in [1] is extended by considering relevant IM *nonlinearities* and incorporating stator and rotor copper losses. Instead of the well known *Maximum Torque per (stator) Current* (MTPC) operation strategy, *Maximum Torque per (copper) Losses* (MTPL_{Cu}) is realized and extended by the *Maximum (rotor) Current* (MC_{r,ext}) strategy due to stator and rotor current limitations. Modeling magnetic saturation and cross-coupling effects leads to a constrained *nonlinear* optimization problem which is solved based on the idea of *sequential quadratic programming* (SQP). The second order Taylor approximations are formulated in implicit form as *quadrics*. Applying the Lagrangian formalism to the quadratic problem leads to analytical solution for the optimal rotor currents. For a doubly-fed induction machine (DFIM), a decision tree for optimal operation management is presented and the OFTC is validated in simulations for a real nonlinear IM.

Index Terms—efficiency, copper losses, (doubly-fed) induction machine, Maximum Torque per (copper) Losses (MTPL_{Cu}), Maximum (rotor) Current (MC_{r,ext}), analytical solution, sequential quadratic programming (SQP), quadrics, Lagrangian optimization, operation management

Notation $\mathbb{N}, \mathbb{R}$: natural, real numbers. $\mathbf{x} := (x_1, \dots, x_n)^\top \in \mathbb{R}^n$: column vector, $n \in \mathbb{N}$ where “ $\top$ ” and “ $:=$ ” mean “transposed” and “is defined as”, respectively. $\mathbf{a}^\top \mathbf{b} := a_1 b_1 + \dots + a_n b_n$: scalar product of vectors $\mathbf{a}$ and $\mathbf{b}$. $\|\mathbf{x}\| := \sqrt{\mathbf{x}^\top \mathbf{x}} = \sqrt{x_1^2 + \dots + x_n^2}$: Euclidean norm of $\mathbf{x}$. $\mathbf{X} \in \mathbb{R}^{n \times m}$: matrix with n rows and m columns. $\mathbf{I}_n \in \mathbb{R}^{n \times n} := \text{diag}(1, \dots, 1)$: identity matrix. $\mathbf{O}_{n \times m} \in \mathbb{R}^{n \times m}$: zero matrix. $\mathbf{0}_n \in \mathbb{R}^n$: zero vector. Park transformation angle $\phi_p \in \mathbb{R}$ and $\mathbf{J} := \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$: counter-clock wise rotation matrix by $\frac{\pi}{2}$. “s.t.”: subject to (optimization with constraints). $\mathbb{X} \cap \mathbb{Y}$: intersection of sets $\mathbb{X}, \mathbb{Y} \subset \mathbb{R}^2$.

I. INTRODUCTION

About 50–70 % of all electricity is consumed by electric machines with an overwhelming share of 90 % induction machines (IMs) [2]. Moreover, IMs are also used for converter-based power generation which is the focus of this paper: Doubly-fed induction machines (DFIMs) with grid-connected stator and inverter-fed rotor as widely used in wind energy conversion systems (WECS) [3]. However, the proposed

generic approach can also be applied to other types of inverter-fed IMs, such as squirrel-cage induction machines (SCIMs).

The highest priority of the OFTC is to provide the reference torque while minimizing losses. In a wide operating range, there usually exist (infinitely many) different combinations of reference currents resulting in the same torque. Thus, an optimal reference current computation (ORCC) is desirable which minimizes the current-dependent IM losses while reaching the reference torque and taking into account operating (e.g. current & voltage) limits. To do so, this paper proposes a physics-based nonlinear IM model for analytical ORCC.

In [4–7] the efficiency of DFIM-based WECS is increased based on an improved reactive power sharing between the rotor and the stator circuit considering the dominant copper losses (also iron losses in [4], [6] and converter losses in [6]). Neglecting the impact of the active rotor current, the reactive rotor current calculation in [6] results in a power saving of 15 % of the total losses for low load operation. In [8], a new structure for WECS is proposed which also allows to regulate the DFIM stator voltage without an additional converter for further efficiency improvement based on flux weakening. However, all analytical reference current computations stated above *neglect* nonlinear saturation and cross-coupling effects and operating limits.

In [9], saturation effects are modeled by a current dependent but scalar magnetizing inductance. The numerically and offline calculated reference currents are saved in lookup-tables (LUTs) for OFTC. In [10], an IM identification method based on measurements is used to extract optimal reference current LUTs. As this method requires expensive torque sensors, [10] proposes the MTPC strategy as alternative. However, MTPC only minimizes the copper losses of the primary (inverter-fed) side and not of the secondary (short-circuited or grid-connected) side. Moreover, the step-wise or numerical calculations of the LUT-based approaches result in limited accuracy. Since the copper and iron losses depend on temperatures at different locations, [9] proposes to use multiple LUTs accepting an significantly increased computational effort due to increased dimensions of the LUTs.

To the best knowledge of the authors, all existing approaches

to solve the OFTC problem for IMs are either based on simplified IM models or on LUT-based approaches using numerical calculations or expensive measurements. Thus, this paper provides an advanced nonlinear IM model for OFTC which allows for *analytical* ORCC taking into account (i) *nonlinear magnetic saturation*, (ii) *cross-coupling* effects (i.e. the dependencies between orthogonal current and magnetic flux vectors are *not* neglected), and (iii) stator *and* rotor copper losses. In addition, the proposed OFTC approach is generic as it allows further (online) parameter adaption, such as the temperature dependent stator and rotor resistances or even iron losses [11]. The nonlinear IM model is based on magnetic flux linkage maps obtained by measurements [10] or finite element analysis (FEA) [12]. The focus of this paper is on minimizing the total IM copper losses assuming that the iron losses are negligible or constant for a given torque as in [5].

Contributions of this paper are: The unified theory for synchronous machines presented in [1] & [11] is extended and applied to IMs: Section II introduces the nonlinear IM model and problem statement; Section III-A integrates the measured magnetic flux linkage maps into the quadratic IM model approximation; Section III-B discusses quadratic programming with analytical solution; Section III-C proposes a novel operation management for OFTC in the full operating range (i.e. even at the operating limits); Section IV discusses implementation and simulation results.

II. MODELING AND PROBLEM STATEMENT

A. Nonlinear IM model

For stator angular velocity $\omega_s := -\omega_p$ and rotor relative angular velocity $\omega_r := n_p \omega_m - \omega_p$ with machine angular velocity ω_m and number of pole pairs n_p , the stator and rotor voltages in the dq -reference frame are given by

$$\begin{aligned} \underbrace{\begin{pmatrix} u_s^{dq} \\ u_r^{dq} \end{pmatrix}}_{=: \mathbf{u}^{dq}} &= \underbrace{\begin{bmatrix} \mathbf{R}_s^{dq} & \mathbf{O}_{2 \times 2} \\ \mathbf{O}_{2 \times 2} & \mathbf{R}_r^{dq} \end{bmatrix}}_{=: \mathbf{R}^{dq}} \underbrace{\begin{pmatrix} i_s^{dq} \\ i_r^{dq} \end{pmatrix}}_{=: \mathbf{i}^{dq}} + \frac{d}{dt} \underbrace{\begin{pmatrix} \psi_s^{dq} \\ \psi_r^{dq} \end{pmatrix}}_{=: \boldsymbol{\psi}^{dq}} \\ &\quad - \underbrace{\begin{bmatrix} \omega_s & 0 \\ 0 & \omega_r \end{bmatrix}}_{=: \boldsymbol{\Omega}} \underbrace{\begin{bmatrix} \mathbf{J} & \mathbf{O}_{2 \times 2} \\ \mathbf{O}_{2 \times 2} & \mathbf{J} \end{bmatrix}}_{=: \mathbf{J}_2} \underbrace{\begin{pmatrix} \psi_s^{dq} \\ \psi_r^{dq} \end{pmatrix}}_{=: \boldsymbol{\psi}^{dq}} \\ &= \mathbf{R}^{dq} \mathbf{i}^{dq} - \boldsymbol{\Omega} \mathbf{J}_2 \boldsymbol{\psi}^{dq} + \frac{d}{dt} \boldsymbol{\psi}^{dq} \end{aligned} \quad (1)$$

with voltages $\mathbf{u}_s^{dq} := (u_s^d, u_s^q)^\top$, $\mathbf{u}_r^{dq} := (u_r^d, u_r^q)^\top$, flux linkages $\boldsymbol{\psi}_s^{dq} := (\psi_s^d, \psi_s^q)^\top$, $\boldsymbol{\psi}_r^{dq} := (\psi_r^d, \psi_r^q)^\top$, currents $\mathbf{i}_s^{dq} := (i_s^d, i_s^q)^\top$, $\mathbf{i}_r^{dq} := (i_r^d, i_r^q)^\top$ of stator and rotor, respectively. Moreover, $\mathbf{R}_s^{dq}$, $\mathbf{R}_r^{dq}$ denote the resistance matrix (both $\in \mathbb{R}^{2 \times 2}$) of stator and rotor, respectively, which simplify to $\mathbf{R}_s^{dq} = R_s \mathbf{I}_2$, $\mathbf{R}_r^{dq} = R_r \mathbf{I}_2$ for symmetric, fault-free windings. Using equal sign conventions for ω_m and the torque, latter is given by

$$m_m = - \underbrace{\frac{2}{3\kappa^2} n_p}_{=: \kappa_p} (\mathbf{i}_r^{dq})^\top \mathbf{J} \boldsymbol{\psi}_r^{dq} = -\kappa_p n_p (\mathbf{i}_r^{dq})^\top \mathbf{J} \boldsymbol{\psi}_r^{dq} \quad (2)$$

with Clarke transform scaling factor κ (e.g. $\kappa := \frac{2}{3}$ for amplitude-invariant transformation).

Neglecting iron losses, the IM losses $p_{m,L}$ are copper losses p_{Cu} comprising stator *and* rotor copper losses, i.e.

$$p_{m,L} := p_{Cu} := \underbrace{\kappa_p (\mathbf{i}_s^{dq})^\top \mathbf{R}_s^{dq} \mathbf{i}_s^{dq}}_{=: p_{s,Cu}} + \underbrace{\kappa_p (\mathbf{i}_r^{dq})^\top \mathbf{R}_r^{dq} \mathbf{i}_r^{dq}}_{=: p_{r,Cu}}. \quad (3)$$

B. Problem statement

In the following, the typical DFIM configuration of WECS is considered but the present approach is generic and can also be applied to other IM types (e.g. SCIMs by setting the secondary (short-circuited) terminal voltage to zero).

The goal of OFTC is to determine the rotor current reference $\mathbf{i}_{r,ref}^{dq}$ which reaches the reference torque $m_{m,ref}$ with minimal IM losses taking into account current and voltage limits. Thus, the optimization problem can be stated as

$$\left. \begin{aligned} \mathbf{i}_{r,ref}^{dq} &:= \arg \min_{\mathbf{i}_r^{dq} \in \mathbb{S}} p_{m,L}(\mathbf{i}_r^{dq}) \\ \text{s.t. } m_m &= \overline{m}_{m,ref} := \text{sat}_{m_{m,min}}^{m_{m,max}}(m_{m,ref}) \end{aligned} \right\} \quad (4)$$

with the *feasible set*

$$\mathbb{S} := \left\{ \mathbf{i}_{r,ref}^{dq} \in \mathbb{R}^2 \mid \frac{2}{3\kappa} \|\mathbf{i}_s^{dq}\| \leq \widehat{u}_{s,max} \wedge \frac{2}{3\kappa} \|\mathbf{i}_r^{dq}\| \leq \widehat{u}_{r,max} \wedge \frac{2}{3\kappa} \|\mathbf{u}_s^{dq}\| \leq \widehat{u}_{s,max} \wedge \frac{2}{3\kappa} \|\mathbf{u}_r^{dq}\| \leq \widehat{u}_{r,max} \right\}.$$

The corresponding maximal admissible values are indicated by the subscript ‘max’.

III. PROPOSED SOLUTION

To solve the constrained nonlinear optimization problem (4), the general idea of *sequential quadratic programming* (SQP) [13] and, in particular, the sequence used in [1] & [11] is adopted which consists of three main stages per sequence: (i) quadratic approximation of the loss function and all constraints at the current operating point, (ii) applying the Lagrangian formalism to solve the quadratic (sub-)problems (QPs), (iii) selection of the optimal solution and the operating point for the next iteration. The sequence terminates if the solutions of the QP converged to an operating point which is equal to the local or global minimizer for a convex differentiable objective function [13].

In order to obtain the quadratic approximations, analytical differentiable model functions are required. However, due to magnetic saturation, the magnetic flux linkages are *nonlinear* and usually *unknown* functions of the machine currents, i.e. $\boldsymbol{\psi}^{dq} = \boldsymbol{\psi}^{dq}(\mathbf{i}_s^{dq}, \mathbf{i}_r^{dq})$. Thus, the presented IM model is based on *magnetic flux linkage maps*. Considering the *steady-state* magnetic flux linkages as functions of the rotor current *only*, i.e.

$$\underbrace{\begin{pmatrix} \widetilde{\boldsymbol{\psi}}_r^{dq}(\mathbf{i}_r^{dq}) \\ \widetilde{\boldsymbol{\psi}}_s^{dq}(\mathbf{i}_r^{dq}) \end{pmatrix}}_{=: \widetilde{\boldsymbol{\psi}}^{dq}(\mathbf{i}_r^{dq})} := \underbrace{\begin{pmatrix} \boldsymbol{\psi}_s^{dq}(\mathbf{i}_s^{dq}(\mathbf{i}_r^{dq}), \mathbf{i}_r^{dq}) \\ \boldsymbol{\psi}_r^{dq}(\mathbf{i}_s^{dq}(\mathbf{i}_r^{dq}), \mathbf{i}_r^{dq}) \end{pmatrix}}_{=: \boldsymbol{\psi}^{dq}(\mathbf{i}_s^{dq}(\mathbf{i}_r^{dq}), \mathbf{i}_r^{dq})}, \quad (5)$$

the linearization of the flux linkages can be stated as

$$\psi^{dq} = \tilde{\psi}^{dq}(\mathbf{i}_r^{dq}) \approx \underbrace{\tilde{\mathcal{L}}^{dq}}_{=:\Delta \mathbf{i}_r^{dq}} (\mathbf{i}_r^{dq} - \bar{\mathbf{i}}_r^{dq}) + \underbrace{\tilde{\psi}^{dq}(\bar{\mathbf{i}}_r^{dq})}_{=:\bar{\psi}^{dq}} \quad (6)$$

where

$$\tilde{\mathcal{L}}^{dq} := \begin{bmatrix} \tilde{\mathcal{M}}_{s,r}^{dq} \\ \tilde{\mathcal{L}}_r^{dq} \end{bmatrix} := \left[\begin{array}{c} \frac{\partial \tilde{\psi}_s^{dq}(\mathbf{i}_r^{dq})}{\partial (\mathbf{i}_r^{dq})^\top} \\ \frac{\partial \tilde{\psi}_r^{dq}(\mathbf{i}_r^{dq})}{\partial (\mathbf{i}_r^{dq})^\top} \end{array} \right] \bigg|_{\mathbf{i}_r^{dq} = \bar{\mathbf{i}}_r^{dq}} \in \mathbb{R}^{4 \times 2}. \quad (7)$$

Note that all quantities evaluated at the current operating point are denoted by an *overline* or *underline* for symbols with tilde. The maps of the rotor-side magnetic flux linkages & *steady-state differential inductances* are shown in Fig. 1 & 2 whereas the stator-side maps are not shown due to space restrictions.

A. Definition of machine quadrics

Applying the *second-order Taylor series expansion*¹

$$f(\mathbf{x}) \approx f(\bar{\mathbf{x}}) + \left(\frac{df(\mathbf{x})}{d\mathbf{x}} \right) \bigg|_{\bar{\mathbf{x}}}^\top (\mathbf{x} - \bar{\mathbf{x}}) + \frac{1}{2} (\mathbf{x} - \bar{\mathbf{x}})^\top \left[\frac{d^2 f(\mathbf{x})}{d\mathbf{x} d\mathbf{x}^\top} \right] \bigg|_{\bar{\mathbf{x}}} (\mathbf{x} - \bar{\mathbf{x}}), \quad (8)$$

to all relevant functions of the optimization problem (4) leads to implicit formulations as (approximated) *quadrics*. The corresponding set of an arbitrary quadric with $\mathbf{A} \in \mathbb{R}^{2 \times 2}$, $\mathbf{a} \in \mathbb{R}^{2 \times 1}$, $\alpha \in \mathbb{R}$ and $\mathbf{x} \in \mathbb{R}^{2 \times 1}$ is given by

$$\mathcal{Q}_A := \{ \mathbf{x} \in \mathbb{R}^2 \mid (\mathbf{x})^\top \mathbf{A} \mathbf{x} + 2\mathbf{a}^\top \mathbf{x} + \alpha = 0 \}. \quad (9)$$

In steady state, i.e.

$$\frac{d}{dt} \psi^{dq} = \mathbf{0}_4 \stackrel{(1)}{\Rightarrow} \mathbf{u}^{dq} = \mathbf{R}^{dq} \mathbf{i}_r^{dq} - \Omega \mathbf{J}_2 \psi^{dq} \quad (10)$$

$$\Rightarrow \mathbf{i}_s^{dq} = (\mathbf{R}_s^{dq})^{-1} (\mathbf{u}_s^{dq} + \omega_s \mathbf{J} \psi_s^{dq}), \quad (11)$$

the (approximated) stator copper loss quadric is given by

$$\begin{aligned} \frac{p_{s,Cu}}{\kappa_p} &= (\mathbf{i}_s^{dq})^\top \mathbf{R}_s^{dq} \mathbf{i}_s^{dq} \\ &\stackrel{(8)}{\approx} \underbrace{(\bar{\mathbf{u}}_s^{dq} + \bar{\omega}_s \mathbf{J} \bar{\psi}_s^{dq})^\top (\bar{\mathbf{R}}_s^{dq})^{-1} (\bar{\mathbf{u}}_s^{dq} + \bar{\omega}_s \mathbf{J} \bar{\psi}_s^{dq})}_{=:\bar{p}_{s,Cu}/\kappa_p} \\ &\quad + 2 \underbrace{(\bar{\mathbf{u}}_s^{dq} + \bar{\omega}_s \mathbf{J} \bar{\psi}_s^{dq})^\top (\bar{\mathbf{R}}_s^{dq})^{-1} \bar{\omega}_s \mathbf{J} \tilde{\mathcal{M}}_{s,r}^{dq} \Delta \mathbf{i}_r^{dq}}_{=:\bar{\mathbf{i}}_{s,Cu}^{\star\top}/\kappa_p} \\ &\quad + \underbrace{(\Delta \mathbf{i}_r^{dq})^\top (\bar{\omega}_s)^2 (\tilde{\mathcal{M}}_{s,r}^{dq})^\top \mathbf{J}^\top (\bar{\mathbf{R}}_s^{dq})^{-1} \mathbf{J} \tilde{\mathcal{M}}_{s,r}^{dq} \Delta \mathbf{i}_r^{dq}}_{=:\bar{\mathbf{L}}_{s,Cu}/\kappa_p} \\ &\Rightarrow 0 \approx (\mathbf{i}_r^{dq})^\top \bar{\mathbf{L}}_{s,Cu} \mathbf{i}_r^{dq} + 2 \underbrace{(\bar{\mathbf{L}}_{s,Cu}^\top - (\bar{\mathbf{i}}_r^{dq})^\top \bar{\mathbf{L}}_{s,Cu})}_{=:\bar{\mathbf{i}}_{s,Cu}^{\star\top}} \mathbf{i}_r^{dq} \\ &\quad + \underbrace{\bar{p}_{s,Cu} - 2\bar{\mathbf{i}}_{s,Cu}^{\star\top} \bar{\mathbf{i}}_r^{dq} + (\bar{\mathbf{i}}_r^{dq})^\top \bar{\mathbf{L}}_{s,Cu} \bar{\mathbf{i}}_r^{dq} - p_{s,Cu}}_{=:\rho_{s,Cu}}. \quad (12) \end{aligned}$$

¹For some $\mathbf{M} \in \mathbb{R}^{n \times m}$, $\mathbf{g}(\mathbf{x}) \in \mathbb{R}^n$ and $\mathbf{h}(\mathbf{x}) \in \mathbb{R}^m$, the second-order approximation of the scalar function $f(\mathbf{x}) := \mathbf{g}(\mathbf{x})^\top \mathbf{M} \mathbf{h}(\mathbf{x})$ is given by (8) with *gradient* $\left(\frac{df(\mathbf{x})}{d\mathbf{x}} \right)^\top = \mathbf{h}(\mathbf{x})^\top \mathbf{M}^\top \left[\frac{d\mathbf{g}(\mathbf{x})}{d\mathbf{x}} \right] + \mathbf{g}(\mathbf{x})^\top \mathbf{M} \left[\frac{d\mathbf{h}(\mathbf{x})}{d\mathbf{x}} \right]$ and symmetric *Hessian* matrix $\left[\frac{d^2 f(\mathbf{x})}{d\mathbf{x} d\mathbf{x}^\top} \right] \approx \left[\frac{d\mathbf{h}(\mathbf{x})}{d\mathbf{x}} \right]^\top \mathbf{M}^\top \left[\frac{d\mathbf{g}(\mathbf{x})}{d\mathbf{x}} \right] + \left[\frac{d\mathbf{g}(\mathbf{x})}{d\mathbf{x}} \right]^\top \mathbf{M} \left[\frac{d\mathbf{h}(\mathbf{x})}{d\mathbf{x}} \right]$ where $\frac{d}{d\mathbf{x}} \left[\frac{d\mathbf{g}(\mathbf{x})}{d\mathbf{x}} \right]$ and $\frac{d}{d\mathbf{x}} \left[\frac{d\mathbf{h}(\mathbf{x})}{d\mathbf{x}} \right]$ are neglected (for details see [14]).

Figure 1: Nonlinear magnetic flux linkages ψ_r^{dq} in the stator voltage aligned dq -reference frame, i.e. $u_s^q = 0$, of a real DFIM with tangent plane [■] at some operating point $\bar{\mathbf{i}}_r^{dq}$ [■].

Figure 2: Nonlinear *steady-state differential inductances* $\tilde{\mathcal{L}}_r^{dq}(\mathbf{i}_r^{dq})$ of a real DFIM with some operating point $\bar{\mathbf{i}}_r^{dq}$ [■].

Adding to (12) the rotor copper loss quadric

$$0 = (\mathbf{i}_r^{dq})^\top \underbrace{\kappa_p \mathbf{R}_r^{dq}}_{=:\bar{\mathbf{L}}_{r,Cu}} \mathbf{i}_r^{dq} - \underbrace{p_{r,Cu}}_{=:\rho_{r,Cu}}, \quad (13)$$

the quadric of the total loss objective function (3) is given by

$$\begin{aligned} &\stackrel{(3)}{\Rightarrow} 0 \stackrel{(12)}{\approx} (\mathbf{i}_r^{dq})^\top \underbrace{[\bar{\mathbf{L}}_{r,Cu} + \bar{\mathbf{L}}_{s,Cu}]}_{=:\bar{\mathbf{L}}_{Cu}} \mathbf{i}_r^{dq} + 2 \underbrace{\bar{\mathbf{i}}_{s,Cu}^\top}_{=:\bar{\mathbf{i}}_{Cu}^{\star\top}} \mathbf{i}_r^{dq} \\ &\quad + \underbrace{\rho_{s,Cu} + \rho_{r,Cu}}_{=:\rho_{Cu}}. \quad (14) \end{aligned}$$

The equality constraint in (4) leads to the torque quadric

$$\begin{aligned} \bar{m}_{m,ref} &\stackrel{(2)}{\approx} \underbrace{-\kappa_p n_p (\bar{\mathbf{i}}_r^{dq})^\top \mathbf{J} \bar{\psi}_r^{dq}}_{=:\bar{m}_m} \\ &\quad + 2 \frac{-\kappa_p n_p}{2} \left\{ (\bar{\psi}_r^{dq})^\top \mathbf{J}^\top + (\bar{\mathbf{i}}_r^{dq})^\top \mathbf{J} \tilde{\mathcal{L}}_r^{dq} \right\} \Delta \mathbf{i}_r^{dq} \\ &\quad + (\Delta \mathbf{i}_r^{dq})^\top \frac{-\kappa_p n_p}{2} \left[(\tilde{\mathcal{L}}_r^{dq})^\top \mathbf{J}^\top + \mathbf{J} \tilde{\mathcal{L}}_r^{dq} \right] \Delta \mathbf{i}_r^{dq} \\ &\quad \stackrel{(2)}{\approx} \underbrace{\bar{\mathbf{i}}_r^{\star\top}}_{=:\bar{\mathbf{i}}_r^{\star\top}} \mathbf{i}_r^{dq} \\ &\Rightarrow 0 \approx (\mathbf{i}_r^{dq})^\top \bar{\mathbf{T}} \mathbf{i}_r^{dq} + 2 \underbrace{(\bar{\mathbf{i}}_r^{dq})^\top \bar{\mathbf{T}}}_{=:\bar{\mathbf{i}}_r^{\star\top}} \mathbf{i}_r^{dq} \\ &\quad + \underbrace{\bar{m}_m - 2\bar{\mathbf{i}}_r^{\star\top} \bar{\mathbf{i}}_r^{dq} + (\bar{\mathbf{i}}_r^{dq})^\top \bar{\mathbf{T}} \bar{\mathbf{i}}_r^{dq} - \bar{m}_{m,ref}}_{=:\bar{\tau}}. \quad (15) \end{aligned}$$

The quadrics of the inequality constraints in (4) are given for

the stator current limit by

$$\begin{aligned}
&\stackrel{(4)}{\Rightarrow} \frac{9\kappa_c^2}{4} (\hat{i}_{s,\max})^2 \geq (\mathbf{i}_s^{dq})^\top \mathbf{i}_s^{dq} \\
&\stackrel{(8)}{\approx} \underbrace{\left(\bar{\mathbf{u}}_s^{dq} + \bar{\omega}_s \mathbf{J} \bar{\boldsymbol{\psi}}_s^{dq} \right)^\top (\mathbf{R}_s^{dq})^{-2} \left(\bar{\mathbf{u}}_s^{dq} + \bar{\omega}_s \mathbf{J} \bar{\boldsymbol{\psi}}_s^{dq} \right)}_{=:\frac{4}{9\kappa_c^2} (\hat{i}_s)^2} \\
&\quad + 2 \underbrace{\left(\bar{\mathbf{u}}_s^{dq} + \bar{\omega}_s \mathbf{J} \bar{\boldsymbol{\psi}}_s^{dq} \right)^\top (\mathbf{R}_s^{dq})^{-2} \bar{\omega}_s \mathbf{J} \widetilde{\mathbf{M}}_{s,r}^{dq} \Delta \mathbf{i}_r^{dq}}_{=:\frac{4}{9\kappa_c^2} \bar{\mathbf{c}}_s^{*\top}} \\
&\quad + (\Delta \mathbf{i}_r^{dq})^\top \underbrace{(\bar{\omega}_s)^2 (\widetilde{\mathbf{M}}_{s,r}^{dq})^\top \mathbf{J}^\top (\mathbf{R}_s^{dq})^{-2} \mathbf{J} \widetilde{\mathbf{M}}_{s,r}^{dq}}_{=:\frac{4}{9\kappa_c^2} \bar{\mathbf{C}}_s} \Delta \mathbf{i}_r^{dq} \\
&\Rightarrow 0 \geq (\mathbf{i}_r^{dq})^\top \bar{\mathbf{C}}_s \mathbf{i}_r^{dq} + 2 \underbrace{\left(\bar{\mathbf{c}}_s^{*\top} - (\bar{\mathbf{i}}_r^{dq})^\top \bar{\mathbf{C}}_s \right)}_{=:\bar{\mathbf{c}}_s^\top} \mathbf{i}_r^{dq} \\
&\quad + \underbrace{\left(\hat{i}_s \right)^2 - 2 \bar{\mathbf{c}}_s^{*\top} \bar{\mathbf{i}}_r^{dq} + (\bar{\mathbf{i}}_r^{dq})^\top \bar{\mathbf{C}}_s \bar{\mathbf{i}}_r^{dq} - (\hat{i}_{s,\max})^2}_{=:\bar{\mathbf{i}}_s} \quad (16)
\end{aligned}$$

with $(\mathbf{R}_s^{dq})^{-2} := (\mathbf{R}_s^{dq})^{-\top} (\mathbf{R}_s^{dq})^{-1} = (\mathbf{R}_s^{dq})^{-1} (\mathbf{R}_s^{dq})^{-1}$; for the rotor current limit by

$$\stackrel{(4)}{\Rightarrow} 0 \geq (\mathbf{i}_r^{dq})^\top \underbrace{\frac{4}{9\kappa_c^2} \mathbf{I}_2}_{=:\mathbf{C}_r} \mathbf{i}_r^{dq} - \underbrace{(\hat{i}_{r,\max})^2}_{=:\iota_r} \quad (17)$$

and for the rotor voltage limit by

$$0 \geq (\mathbf{i}_r^{dq})^\top \bar{\mathbf{V}} \mathbf{i}_r^{dq} + 2 \bar{\mathbf{v}}^\top \mathbf{i}_r^{dq} + \bar{\nu} \quad (18)$$

where the derivation of latter quadric is not stated due to space restrictions. Since $\frac{2}{3\kappa} \|\mathbf{u}_s^{dq}(t)\| = \hat{u}_{s,\max}$ for all $t \geq 0$, for a grid-connected stator of the DFIM, the stator voltage constraint in (4) boils down to the usage of $\hat{u}_{s,\max}$ in the stator voltage equation in (10). Moreover, the IM losses $p_{m,L}$ depend on the phase resistances which are assumed to be constant for simplicity. All machine parameters are listed in Tab. I.

Remark 1. Actually, resistances and therefore $p_{m,L}$ in (3) depend on stator & rotor temperature, i. e. $\mathbf{R}_s^{dq}(\vartheta_s)$, $\mathbf{R}_r^{dq}(\vartheta_r)$. In contrast to conventional LUT-based approaches, the analytical ORCC here can easily use these varying parameters online.

Table I: Parameters of the simulated but real DFIM.

Parameters	Symbol	Value
Max. stator current	$\hat{i}_{s,\max}$	20.9 A
Max. rotor current	$\hat{i}_{r,\max}$	17 A
Max. stator voltage	$\hat{u}_{s,\max}$	$\sqrt{2/3} \cdot 400$ V
Max. rotor voltage	$\hat{u}_{r,\max}$	$\sqrt{2/3} \cdot 400$ V
Park angular velocity	$\omega_p = -\omega_s$	$2\pi 50 \frac{\text{rad}}{\text{s}}$
Stator resistance	R_s	0.72Ω
Rotor resistance	R_r	0.55Ω

B. Quadratic programming

Applying the *Lagrangian* formalism to the QP

$$\left. \begin{aligned} \mathbf{i}_{r,\text{ref}}^{dq} &:= \arg \min_{\mathbf{i}_r^{dq} \in \bar{\mathbb{S}}} (\mathbf{i}_r^{dq})^\top \bar{\mathbf{L}}_{\text{Cu}} \mathbf{i}_r^{dq} + 2 \bar{\mathbf{l}}_{\text{Cu}}^\top \mathbf{i}_r^{dq} + \rho_{\text{Cu}} \\ \text{s. t. } &(\mathbf{i}_r^{dq})^\top \bar{\mathbf{T}} \mathbf{i}_r^{dq} + 2 \bar{\mathbf{t}}^\top \mathbf{i}_r^{dq} + \bar{\tau}(m_{m,\text{ref}}) = 0 \end{aligned} \right\} \quad (19)$$

where $\bar{\mathbb{S}}$ denotes the feasible set *approximated* at the current operating point, leads to three equations $\frac{\partial f}{\partial \mathbf{i}_r^{dq}} = \frac{\partial f}{\partial \mathbf{i}_r^{dq}} = \frac{\partial f}{\partial \lambda} = 0$ with Lagrangian function $f(\mathbf{i}_r^{dq}, \lambda)$ and multiplier λ . Eliminating λ , an implicit solution (again a quadric, see [1]) is obtained for the *Maximum Torque per (copper) Losses* (MTPL_{Cu}) strategy. Similarly, the *Maximum Torque per (rotor) Current* (MTPC_r) strategy is obtained by replacing the overall copper loss quadric in (19) by the rotor copper loss quadric (13) only.

All machine quadrics are summarized in Tab. II where ∂ denotes the boundary of an inequality constraint; e. g., the set $\partial \mathbb{I}_r \subset \mathbb{I}_r$ contains all $\mathbf{i}_r^{dq}$ where the rotor current constraint is *active*, i. e. $\frac{2}{3\kappa} \|\mathbf{i}_r^{dq}\| = \hat{i}_{r,\max}$. Note that except $\partial \mathbb{I}_r$, all sets depend on the actual operating point $\bar{\mathbf{i}}_r^{dq}$ and thus, are indicated by symbols with overline.

Table II: Definition of the (approximated) machine quadrics.

Machine quadric	$\mathbb{Q}_A$ or $\mathbb{Q}_B$	$\mathbf{A}$, $\mathbf{a}$, α
Stator copper losses	$\bar{\mathbb{L}}_{s,\text{Cu}}$	$\bar{\mathbf{L}}_{s,\text{Cu}}$, $\bar{\mathbf{l}}_{s,\text{Cu}}$, $\rho_{s,\text{Cu}}$
Rotor copper losses	$\bar{\mathbb{L}}_{r,\text{Cu}}$	$\bar{\mathbf{L}}_{r,\text{Cu}}$, $\mathbf{0}^\top$, $\rho_{r,\text{Cu}}$
Total copper losses	$\bar{\mathbb{L}}_{\text{Cu}}$	$\bar{\mathbf{L}}_{\text{Cu}}$, $\bar{\mathbf{l}}_{\text{Cu}}$, ρ_{Cu}
Torque (equality)	$\bar{\mathbb{T}}(m_{m,\text{ref}})$	$\bar{\mathbf{T}}$, $\bar{\mathbf{t}}$, $\bar{\tau}(m_{m,\text{ref}})$
Stator current limit	$\partial \bar{\mathbb{I}}_s(\hat{i}_{s,\max})$	$\bar{\mathbf{C}}_s$, $\bar{\mathbf{c}}_s$, $\bar{i}_s(\hat{i}_{s,\max})$
Rotor current limit	$\partial \bar{\mathbb{I}}_r(\hat{i}_{r,\max})$	$\mathbf{C}_r$, $\mathbf{0}^\top$, $\iota_r(\hat{i}_{r,\max})$
Rotor voltage limit	$\partial \bar{\mathbb{V}}(\hat{u}_{r,\max})$	$\bar{\mathbf{V}}$, $\bar{\mathbf{v}}$, $\bar{\nu}(\hat{u}_{r,\max})$
MTPL _{Cu} strategy	$\bar{\text{MTPL}}_{\text{Cu}}$	$\bar{\mathbf{M}}_{\text{L}_{\text{Cu}}}$, $\bar{\mathbf{m}}_{\text{L}_{\text{Cu}}}$, $\bar{\mu}_{\text{L}_{\text{Cu}}}$
MTPC _r strategy	$\bar{\text{MTPC}}_r$	$\bar{\mathbf{M}}_{\text{C}_r}$, $\bar{\mathbf{m}}_{\text{C}_r}$, $\bar{\mu}_{\text{C}_r}$

The optimal solution is given by the intersection point of two quadrics, i. e. $\mathbf{i}_{r,\text{ref}}^{dq} = \mathbb{Q}_A \cap \mathbb{Q}_B$, which can be calculated analytically (for details see [1]). The choices of $\mathbb{Q}_A$ and $\mathbb{Q}_B$ depend on which inequality constraints are *active* for the QP to be solved as shown in the next section.

C. Operation Management

Without any active inequality constraint or torque saturation, the optimal rotor current is equal to the intersection point $\bar{\text{MTPL}}_{\text{Cu}} \cap \bar{\mathbb{T}}(m_{m,\text{ref}})$ [•]. As illustrated in Fig. 3(a), the MTPL_{Cu} strategy [—] is the optimal compromise between stator copper losses [---] and rotor copper losses [----] resulting in minimal machine copper losses [—]. The torque mainly depends on i_r^d such that the torque quadric $\bar{\mathbb{T}}$ moves to the left/right for in-/decreasing torque. In contrast to the MTPL_{Cu} strategy, the MTPC_r strategy [—] only minimizes the rotor copper losses. Hence, the intersection point $\bar{\text{MTPC}}_r \cap \bar{\mathbb{T}}$ [•] is only suboptimal.

For high torque reference magnitudes, the current constraints get active whereas the rotor voltage constraint can be neglected for this DFIM. Considering only the rotor current limit, the max./min. torque is given by the intersections of $\bar{\text{MTPC}}_r$ [—] with the rotor current limit set $\partial \bar{\mathbb{I}}_r$ [—] at $\bar{\text{MTPC}}_r \cap \partial \bar{\mathbb{I}}_r$ [•/•]. However, taking into account the stator current limit as well, the max./min feasible torque is given by

Figure 3: Current locus for different torques with quadrics for $\bar{\mathbb{L}}_{s,Cu}$ [---], $\bar{\mathbb{L}}_{r,Cu}$ [---], $\bar{\mathbb{L}}_{Cu}$ [—], $\bar{\mathbb{T}}$ [—], $\partial\bar{\mathbb{L}}_s$ [—], $\partial\bar{\mathbb{L}}_r$ [—], $\overline{\text{MTPL}}_{Cu}$ [—], $\overline{\text{MTPC}}_r$ [—], feasible region $\bar{\mathbb{S}} := \bar{\mathbb{L}}_s \cap \bar{\mathbb{L}}_r$ [■] and reference currents $\bar{i}_{r,\text{ref}}^{dq}$ [★].

the intersections of the stator current limit set $\partial\bar{\mathbb{L}}_s$ [—] and $\partial\bar{\mathbb{L}}_r$ at $\partial\bar{\mathbb{L}}_s \cap \partial\bar{\mathbb{L}}_r$ [●/■].

The max./min. feasible torque of the $\overline{\text{MTPL}}_{Cu}$ strategy is reached at the intersection point $\overline{\text{MTPL}}_{Cu} \cap \partial\bar{\mathbb{L}}_r$ [●/■]. For higher torque magnitudes, the operating point $\overline{\text{MTPL}}_{Cu} \cap \bar{\mathbb{T}}(m_{m,\text{ref}})$ [■] of the $\overline{\text{MTPL}}_{Cu}$ strategy is invalid and the optimal feasible operating point is given by the intersection $\partial\bar{\mathbb{L}}_r \cap \bar{\mathbb{T}}$ [●] as visualized in Fig. 3 (b). During this so-called *Maximum (rotor) Current* ($\text{MC}_{r,\text{ext}}$) strategy, the rotor copper losses are constant due to constant rotor current amplitude. Only the stator current amplitude is increased to reach the max. feasible torque magnitude at $\partial\bar{\mathbb{L}}_s \cap \partial\bar{\mathbb{L}}_r$ [●/■].

Using the gained knowledge about the DFIM characteristics and the minimization problem, an optimal operation management is designed as visualized by the decision tree in Fig. 4. The initial input parameters are the reference torque $\bar{m}_{m,\text{ref}} := m_{m,\text{ref}}$ and the (measured) rotor currents $\bar{i}_r^{dq} := i_r^{dq}$. The magnetic flux linkages $\tilde{\psi}^{dq}(\bar{i}_r^{dq})$, the steady-state differential inductances $\tilde{\mathcal{L}}^{dq}(\bar{i}_r^{dq})$ and, optionally, also other parameters such as the phase resistances $\bar{R}^{dq}$ are updated based on the operating conditions and temperature variations.

In the next step, the torque is calculated at the intersection $\overline{\text{MTPC}}_r \cap \partial\bar{\mathbb{L}}_r$. If the solution is not feasible, i.e. $\bar{i}_r^{dq*} \notin \bar{\mathbb{S}}$, the torque is calculated again at $\partial\bar{\mathbb{L}}_s \cap \partial\bar{\mathbb{L}}_r$. If the reference torque exceeds these limits, the torque reference is saturated to $\bar{m}_{m,\text{ref}} := \text{sat}_{\bar{m}_{m,\text{min}}}^{\bar{m}_{m,\text{max}}}(\bar{m}_{m,\text{ref}})$ and the new operating point is set to the current solution at the active limit, i.e. $\bar{i}_r^{dq} := \bar{i}_r^{dq*}$, before going back to the parameter update step. Otherwise, if the reference torque of the current iteration is within the limits, i.e. $\bar{m}_{m,\text{min}} \leq \bar{m}_{m,\text{ref}} \leq \bar{m}_{m,\text{max}}$, the final condition of the torque saturation $\bar{m}_{m,\text{ref}} = \text{sat}_{\bar{m}_{m,\text{min}}}^{\bar{m}_{m,\text{max}}}(m_{m,\text{ref}})$ is checked. Latter condition ensures that the saturated torque reference has not fallen below the feasible limit at the final operating point. If this happens, the saturated torque reference for the next iteration is reset to the saturated initial torque reference, i.e. $\bar{m}_{m,\text{ref}} := \text{sat}_{\bar{m}_{m,\text{min}}}^{\bar{m}_{m,\text{max}}}(m_{m,\text{ref}})$.

After completion of the torque saturation, the actual loss

minimization starts with the calculation of $\overline{\text{MTPL}}_{Cu} \cap \bar{\mathbb{T}}$. If the solution $\bar{i}_r^{dq*}$ is feasible, the $\overline{\text{MTPL}}_{Cu}$ strategy is chosen, otherwise, overwriting $\bar{i}_r^{dq*}$ with the solution of $\partial\bar{\mathbb{L}}_r \cap \bar{\mathbb{T}}$, leads to the $\text{MC}_{r,\text{ext}}$ strategy. If the current operating point $\bar{i}_r^{dq}$ converged to the solution $\bar{i}_r^{dq*}$, i.e. $\|\bar{i}_r^{dq*} - \bar{i}_r^{dq}\| \leq \epsilon$ with some small threshold ϵ , the optimal rotor current reference $\bar{i}_{r,\text{ref}}^{dq} := \bar{i}_r^{dq*}$ is finally obtained [★].

Figure 4: Decision tree for optimal operation management.

IV. SIMULATION RESULTS

The nonlinear DFIM-model with flux linkage maps shown in Fig. 1, steady-state differential inductance maps shown in Fig. 2 and parameters listed in Tab. I as well as OFTC based on Tab. II and Fig. 4 are implemented and simulated in Matlab & Simulink R2019b. The reference torque is provided by an outer-loop speed controller and the OFTC computes the reference currents for the inner-loop rotor current controller. For the simulated scenario, the DFIM is operated in generator mode and the slip is varied from 30 % super-synchronous to 0 %.

Fig. 5 shows the simulated time series with gradually increasing load torque from zero to its final value $m_1 = 30 \text{ Nm} \approx 53\% \cdot |m_{m,\text{min}}|$ at $t = 0.2 \text{ s}$. At the beginning, the speed reference is constant but is gradually decreased from $\omega_{m,\text{ref}} = 1.3\pi 50 \frac{\text{rad}}{\text{s}}$ at $t = 0.3 \text{ s}$ to its final value $\omega_{m,\text{ref}} = \pi 50 \frac{\text{rad}}{\text{s}}$ at $t = 0.45 \text{ s}$. During the time interval $305 \text{ ms} \leq t \leq 467 \text{ ms}$ the OFTC choses $\text{MC}_{r,\text{ext}}$ strategy and, at the remaining time, the $\overline{\text{MTPL}}_{Cu}$ strategy. With increasing torque magnitude, the d -component of the rotor

Figure 5: Simulation results with transitions between the operation strategies MTPL_{Cu} [] and $\text{MC}_{\text{r,ext}}$ [].

current reference $i_{\text{r,ref}}^d$ increases for both strategies. In contrast, the q -component decreases from $i_{\text{r,ref}}^q \approx -7.55$ A at $t = 0$ s to $i_{\text{r,ref}}^q = -8.04$ A at $t = 305$ ms during the MTPL_{Cu} strategy. However, i_r^q rapidly increases – i. e. the magnitude of i_r^q rapidly decreases – after switching to the $\text{MC}_{\text{r,ext}}$ strategy in order to reach the max. torque magnitude with the max. feasible i_r^d . During the deceleration phase, the reference torque $m_{\text{m,ref}}$ is saturated to $\bar{m}_{\text{m,ref}}$ resulting in a notable deviation between actual and reference speed.

In Fig. 6, the efficiency of the OFTC $\eta_{\text{OFTC}} := \frac{p_s + p_r}{p_m}$ is compared with the efficiency of conventional torque control with $i_s^q = 0$ or $i_r^q = 0$.² For $i_s^q = 0$, a large negative reactive rotor current $i_r^q < -13.8$ A is required such that the torque is limited to $m_{\text{m,min}} \approx -29$ Nm due to rotor current limitation. For $i_r^q = 0$, the torque is limited to $m_{\text{m,min}} \approx -50.5$ Nm at $i_r^d \approx 15$ A due to stator current limitation, cf. Fig. 3 (b). At approximately the same torque, the OFTC switches from MTPL_{Cu} to $\text{MC}_{\text{r,ext}}$. With the $\text{MC}_{\text{r,ext}}$ strategy, the OFTC reaches the max. feasible torque magnitude of $m_{\text{m,min}} \approx -56.7$ Nm. Note the slightly increased curvature at the end of the η_{OFTC} -line near $m_{\text{m,min}}$ in the left plot of Fig. 6 as the $\text{MC}_{\text{r,ext}}$ strategy diverges from the MTPL_{Cu} strategy.

The right plot shows the difference $\Delta\eta$ between the efficiency of OFTC and torque control with $i_s^q = 0$ or $i_r^q = 0$. The efficiency using OFTC is increased by more than 1 % in the whole feasible generator torque range. For torques $m_m \geq -10$ Nm $\approx 18\% \cdot m_{\text{m,min}}$, the efficiency is increased by more than $\Delta\eta > 3\%$.

V. CONCLUSION

A *generic* and *analytical* solution for the optimal feedforward torque control problem (OFTC) of *nonlinear* induction machines (IMs) was presented considering stator *and* rotor copper losses. An operation management was designed for a doubly-fed induction machine (DFIM) which saturates the

²Actually, the reactive power reference at the point of common coupling (PCC) of WECS depends on the grid demand. However, unitary power factor operation with $i_s^q = 0$ is commonly considered, but using the grid-side converter (GSC) for reactive power control is also an option [15]. For OFTC, it is assumed that the GSC compensates the reactive power of the DFIM at the PCC (if required).

Figure 6: At 30 % slip (super-synchronous): (a) Efficiency of the OFTC $\eta_{\text{OFTC}} := \frac{p_s + p_r}{p_m}$ with infeasible area [], MTPL_{Cu} [] and $\text{MC}_{\text{r,ext}}$ [] strategy areas as well as efficiency of conventional torque control with $i_s^q = 0$ or $i_r^q = 0$, and (b) efficiency enhancement $\Delta\eta := \eta_{\text{OFTC}} - \eta_{i_s^q=0/i_r^q=0}$.

reference torque and choses the optimal operating strategy. The OFTC accurately tracks the reference torque and minimizes all copper losses, resulting in significantly increased efficiency compared with conventional unitary power factor control. Moreover, the max. feasible torque is reached taking into account rotor *and* stator current & voltage limits, respectively. Future research will include further types of losses, such as iron and converter losses, and validate the theoretical and simulative results in the laboratory.

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