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Orthogonal Time Sequence Multiplexing Modulation

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Abstract—This paper proposes *orthogonal time sequence multiplexing (OTSM)*, a novel single carrier modulation scheme based on the well known Walsh-Hadamard transform (WHT) combined with row-column interleaving, and zero padding (ZP) between blocks in the time-domain. The information symbols in OTSM are multiplexed in the delay and sequency domain using a cascade of time-division and Walsh-Hadamard (sequency) multiplexing. By using the WHT for transmission and reception, the modulation and demodulation steps do not require any complex multiplications. We then propose two low-complexity detectors: (i) a simpler non-iterative detector based on a single tap minimum mean square time-frequency domain equalizer and (ii) an iterative time-domain detector. We demonstrate, via numerical simulations, that the proposed modulation scheme offers high performance gains over orthogonal frequency division multiplexing (OFDM) and exhibits the same performance of orthogonal time frequency space (OTFS) modulation, but with lower complexity. In proposing OTSM, along with simple detection schemes, we offer the lowest complexity solution to achieving reliable communication in high mobility wireless channels, as compared to the available schemes published so far in the literature.

Index Terms—OTSM, Walsh Hadamard Transform, Orthogonal Time Sequence Multiplexing, Delay-Sequency, OTFS, Detector, Matched Filtering, Delay-Doppler Channel, Gauss Seidel.

I. INTRODUCTION

Orthogonal time sequence multiplexing (OTSM) modulation is a novel single-carrier modulation scheme where the information symbols are multiplexed on the *delay-sequency* domain, where *sequency* is defined as the number of zero crossing per unit time interval, [1], [2]. It can also be viewed as a time-domain precoding scheme where the information symbols are split into blocks and each block is precoded using the Walsh-Hadamard transform (WHT) followed by row-column interleaving of the samples in time. In other words, OTSM is a combination of sequency multiplexing and time division multiplexing. Each column of the WHT matrix represents a Walsh function characterized by its unique sequency i.e., the number of sign changes along that column. This is similar to the ordering of Fourier components in increasing harmonic number [1], [2].

Sequency, introduced by Harmuth in [1], similar to frequency, can be equally used to represent the different frequency components of a signal. But, different from frequency

(which represents the period of a signal), sequency can be used to measure non-periodical signals as well as periodic ones, [2], [3]. Walsh functions can be viewed as a discrete, digital counterpart of the Fourier transform basis functions given by the continuous set of complex exponential sinusoidal functions on the unit circle. Naturally, the discrete Fourier response gives the most compact representation when the signal component frequencies are assumed to be multiples of the discrete Fourier transform (DFT) frequency resolution. The most appealing feature of the Fourier transform is that it is invariant to the standard shape transformations such as scaling and translation. Hence, delay and Doppler representation is commonly used to best characterize the physical wireless channel models. However, in practice, the channel Doppler shifts are not necessarily integer multiples of the DFT resolution (fractional Doppler shifts), [4]. Then the sequency domain may offer an equivalent representation to the Doppler domain in terms of its sparsity [1], [2]. The bottom chain in Fig. 1 shows the relation between the Doppler and sequency¹ domains. It can be noted that Doppler and sequency domains are related by unitary transformation operations (cascade of IFFT/FFT and WHT/IWHT). This is useful for high mobility channel representation and estimation in the delay-sequency domain, which is unexplored yet in wireless communications.

The success of a candidate waveform for high mobility wireless communications is tied to the transceiver complexity as well as the availability of efficient detection methods to solve a $\mathbf{y} = \mathbf{H} \cdot \mathbf{x} + \text{noise}$; detection problem at the receiver. Some properties of the channel matrix $\mathbf{H}$, such as sparsity and eigenvalue distribution, determines the complexity of the detection process. This leads us to the motivation behind proposing OTSM: (i) low complexity transceiver design due to using only WHT, (ii) low complexity detection comparable to a traditional baseline OFDM scheme and (iii) good performance in high mobility channels similar to the most advanced modulations schemes in the literature such as orthogonal time frequency space modulation (OTFS), [5]. In this paper, we show how OTSM achieves the above demands and outperforms OTFS (in terms of complexity) by comparing the OTSM uncoded BER performance with OTFS using similar low complexity

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¹To be precise, the delay-sequency domain should be called *delay-sequency-shift* domain. For brevity we will just refer to sequency since, unlike a *frequency-shift* (which is well known as Doppler), no term is defined for a *sequency-shift* in the literature.

Fig. 1. Relation between the different discrete information symbol domains and the corresponding modulation schemes

detectors among the ones proposed for OTFS in the literature [6]–[13].

Related Work – Recently, OTFS modulation was proposed based on delay-Doppler representation of doubly-selective wireless channels, [5]. The proposed modulation scheme replaces the traditional Doppler spread representation of the channel with the concept of sequency spread. In OTFS, the channel delay and Doppler spread introduces interference along orthogonal dimensions (delay and Doppler), [5]. Similarly, in OTSM, we allow delay spread and Doppler spread to cause inter-symbol interference along the delay and sequency dimensions, respectively. This allows the interference caused by the channel delay and Doppler spread to remain separable at the receiver. This is something that the well known OFDM scheme fails to achieve, since OFDM only converts a static multipath channel without any Doppler spread into a multiplicative channel. However, the simple detection comes at the cost of unequal sub-carrier gains, which results in the error performance to be dominated by the worst sub-carrier SNR. Moreover, the OFDM single-tap equalization fails in the presence of significant Doppler, due to the loss of orthogonality.

OFDM variants such as universal filtered multi carrier (UFMC) and filter bank multicarrier (FBMC) included the spreading of information symbols over a number of independent sub-carriers. Further schemes like, generalized frequency division multiplexing (GFDM), were developed where the information symbols are spread along sub-carriers as well as time slots, using non-orthogonal transformations (filtering). In contrast, OTFS, [5], and the very similar orthogonal signal-division multiplexing (OSDM) modulation, [14], spread all information symbols in all sub-carriers as well as all time slots using a 2-D orthogonal transformation (symplectic Fourier transform), thereby making sure that each symbol gets the average power of all the time and frequency (sub-carriers) resources available. In [6], the OTFS scheme was generalized as a 2-D unitary transformation on the time-frequency domain, i.e., OFDM with 2-D precoding. It was then proved that any orthogonal 2-D transformation with *constant modulus* basis

functions operating on the entire time-frequency domain allows the receiver to exploit maximum time-frequency diversity in doubly-selective channels. Even though such precoding was shown to offer good error performance, 2-D precoding in the time-frequency domain may increase the transceiver complexity, since an OFDM modulator on top of 2-D precoding is still required for transmission, [6].

One solution to mitigating the 2-D time-frequency precoding complexity in [6] is to restrict the unitary transformation precoding along the frequency dimension to the Fourier transform, specifically FFT. As shown in Fig. 1, such precoding along the frequency dimension transforms the frequency-time information samples directly in to the delay-time domain, thereby bypassing the need for an OFDM transmitter. By using FFT as above, the 2-D precoding along the frequency-time domain is reduced to just a 1-D precoding along the time dimension of the delay-time domain (see Fig. 1). OTSM can then be imagined as a precoded single-carrier modulation scheme. For maximum time-frequency diversity in doubly-selective channels, any *constant modulus* unitary transformation can be chosen for precoding along the time-dimension of the delay-time domain. The best possible solution, in terms of transceiver complexity, is then the WHT (as highlighted by green box in Fig. 1), which has the advantage of requiring only additions and subtractions for modulation and demodulation.

Remark: Note that OTFS modulation can also be interpreted as a precoded single-carrier modulation scheme (see Fig. 1), with FFT as the unitary precoding along the time domain. Then, OTFS is nothing but the well known *Zak* transform, [15]. Using the same concept, OTSM modulation can then be considered as a *Walsh-Zak* transform, where the FFT in the traditional discrete *Zak* transform is replaced by the significantly lower complexity WHT.

Advantages – OTSM offers the following attractive advantages:

- The low complexity modulation and demodulation steps makes it quite appealing for VLSI chip implementation and is suitable for very low power wireless devices, such as wireless sensors, IOT devices, and deep space

communication transmitters.

- The OTSM transceiver complexity is comparable to a single-carrier scheme, but achieves the advantages of a multi-carrier system without sacrificing performance.
- Due to only using WHT operation, the time domain samples are all integers if the information symbols are also integers. Hence, a smaller number of bits may be used to represent the transmit signal without any quantization error.
- OTSM is suitable for integer forcing linear receivers with significantly reduced complexity, [16].
- OTSM performance improves in the presence of multiple Doppler paths, which typically degrades single carrier and OFDM modulation performances. This makes it suitable for high mobility and under-water acoustic wireless communications.
- Offers similar performance to OTFS and OSDM modulation schemes at lower complexity, both in static and highly mobility wireless channels.
- It can also be easily overlaid on top of existing OFDM based systems: the time-frequency samples can be generated using the relations given in Fig. 1 (FFT along delay and IWHT along sequency) and then transmitted as an OFDM signal.

The rest of the paper is organised as follows. In Section II, we present the OTSM system model followed by *two* low complexity detection algorithms in Section III. Section IV presents the simulation results and discussions followed by our concluding remarks in Section V.

Notations: The following notations will be followed in this paper: a , $\mathbf{a}$, $\mathbf{A}$ represent a scalar, vector, and matrix, respectively; $\mathbf{a}(n)$ and $\mathbf{A}(m, n)$ represent the n -th and (m, n) -th element of $\mathbf{a}$ and $\mathbf{A}$, respectively; $\mathbf{A}^T$, $\mathbf{A}^\dagger$, $\mathbf{A}^*$ and $\mathbf{A}^n$ represent the transpose, Hermitian transpose, complex conjugate and n -th power, respectively, of $\mathbf{A}$. The set of $M \times N$ dimensional matrices with complex entries are denoted by $\mathbb{C}^{N \times M}$. Let $\circledast$ represent circular convolution, $\otimes$, the Kronecker product, $\circ$, the Hadamard product (i.e., the element wise multiplication) and, $\oslash$, the Hadamard division (i.e., the element wise division). Let $|\mathcal{S}|$ denote the cardinality of the set $\mathcal{S}$, $\text{tr}(\mathbf{A})$, the trace of the square matrix $\mathbf{A}$, $\text{vec}(\mathbf{A})$, the column-wise vectorization of the matrix $\mathbf{A}$ and $\text{vec}_{N,M}^{-1}(\mathbf{a})$ is the matrix formed by folding a vector $\mathbf{a}$ into a $N \times M$ matrix by filling it column wise. Let $\mathbf{F}_N$ and $\mathbf{W}_N$ be the normalized N point discrete Fourier transform (DFT) matrix with elements $\mathbf{F}_N(i, k) = (1/\sqrt{N})e^{-j2\pi ik/N}$ and the normalized N -point WHT matrix, [2], respectively.

II. OTSM SYSTEM MODEL

A. Transmitter and Receiver Operation

We will be using the following matrix/vector representation throughout the paper. Let $\mathbf{x}, \mathbf{y} \in \mathbb{C}^{NM \times 1}$ be the transmitted and received information symbols. The total frame duration and bandwidth of the transmitted OTSM signal frame are $T_f = NT$ and $B = M\Delta f$, respectively, where N is chosen² to be

a power of 2. We consider the case where $T\Delta f = 1$, i.e., the OTSM signal is critically sampled for any pulse shaping waveform. Fig. 2 shows the OTSM transceiver operation.

1) *Transmitter:* At the transmitter, the information symbols $\mathbf{x} = [\mathbf{x}_0^T, \dots, \mathbf{x}_{M-1}^T]^T$ are split into vectors $\mathbf{x}_m \in \mathbb{C}^{N \times 1}$. The symbol vectors are arranged into a matrix $\mathbf{X} \in \mathbb{C}^{M \times N}$ by placing each symbol vector $\mathbf{x}_m$ in the m -th row. The column and row indices represent the delay and sequency indices, respectively, of the delay-sequency grid.

$$\mathbf{X} = [\mathbf{x}_0, \mathbf{x}_1, \dots, \mathbf{x}_{M-1}]^T \quad (1)$$

The last $l_{\max}$ symbol vectors, i.e., $\mathbf{x}_m$ for $m \geq M - l_{\max}$, are then made zero vectors, where $l_{\max}$ is the discrete channel delay spread index. As shown in Fig. 2, this zero padding³ (ZP) along the delay domain helps to avoid the inter-block interference due to channel delay spread. A N -point Walsh-Hadamard transform is applied on each of these symbol vectors to transform it to the delay-time domain,

$$\tilde{\mathbf{X}} = \mathbf{X} \cdot \mathbf{W}_N \quad (2)$$

The matrix $\tilde{\mathbf{X}}$ contains the delay-time samples which are vectorized to obtain the time-domain samples $\mathbf{s} \in \mathbb{C}^{NM \times 1}$ to be transmitted into the physical channel

$$\mathbf{s} = \text{vec}(\tilde{\mathbf{X}}) \quad (3)$$

The transmitter operation above can be expressed in the simple matrix form as

$$\mathbf{s} = \mathbf{P} \cdot (\mathbf{I}_M \otimes \mathbf{W}_N) \cdot \mathbf{x} \quad (4)$$

where $\mathbf{P}$ is the row-column interleaver matrix. Such permutation is known in the literature as a *perfect shuffle*, and has the following property [17]: given square matrices $\mathbf{A}$ and $\mathbf{B}$

$$\mathbf{A} \otimes \mathbf{B} = \mathbf{P} \cdot (\mathbf{B} \otimes \mathbf{A}) \cdot \mathbf{P}^T. \quad (5)$$

Moreover, since $\mathbf{P}$ is a permutation matrix, $\mathbf{P}^{-1} = \mathbf{P}^T$. Using the perfect shuffle property in (5), the transmitter operation in (4) can be simplified as

$$\mathbf{s} = (\mathbf{W}_N \otimes \mathbf{I}_M) \cdot (\mathbf{P} \cdot \mathbf{x}) \quad (6)$$

The time-domain samples are pulse shaped, and digital to analog converted, and transmitted into the wireless channel as $s(t)$.

2) *Receiver:* At the receiver, $r(t)$ after analog to digital conversion and sampling, the transmitter steps are reversed to demodulate the received time-domain samples. Let $\mathbf{r} \in \mathbb{C}^{NM \times 1}$ be the time-domain vector after the RF front-end operations and ADC sampling. The received information symbols $\mathbf{y}$ are folded into the matrix $\tilde{\mathbf{Y}}$ column-wise as

$$\tilde{\mathbf{Y}} = \text{vec}_{M,N}^{-1}(\mathbf{r}) \quad (7)$$

The received information symbols are obtained by taking a N -point WHT of the received delay-time samples $\tilde{\mathbf{Y}}$ as

$$\mathbf{Y} = [\mathbf{y}_0, \mathbf{y}_1, \dots, \mathbf{y}_{M-1}]^T = \tilde{\mathbf{Y}} \cdot \mathbf{W}_N \quad (8)$$

³Alternatively, a cyclic prefix (CP) can also be added in time domain between blocks, similar to CP-OFDM, to prevent inter-block interference.

²The value of N is set to 6 in Fig. 2 just for illustrative purposes.

Fig. 2. OTSM transceiver operation with ZP for $N = 6$, $M = 8$ and the maximum discrete delay spread index $l_{\max} = 2$.

The receiver operation can be simplified in the matrix form as

$$\mathbf{y} = (\mathbf{I}_M \otimes \mathbf{W}_N) \cdot (\mathbf{P}^T \cdot \mathbf{r}) \quad (9)$$

where $\mathbf{y} = [\mathbf{y}_0^T, \dots, \mathbf{y}_{M-1}^T]^T$.

B. Channel

Consider a baseband equivalent channel model with P propagation paths, where h_i is the complex path gain, τ_i and ν_i are the delay shift and Doppler shift, respectively, associated with the i -th path. Let $\tau_{\max}$ and $\nu_{\max}$ denote the maximum delay spread and Doppler spread, respectively of the channel. We assume that the channel is *under-spread*, i.e., $\tau_{\max}\nu_{\max} \ll 1$. Under the under-spread assumption, $l_{\max} < M$, where $l_{\max} = \frac{\tau_{\max}}{M\Delta f}$ is the maximum delay spread normalized to the receiver delay resolution. Since the number of channel coefficients P in the delay-Doppler

domain is typically limited, the channel response has a sparse representation [5], [18]:

$$h(\tau, \nu) = \sum_{i=1}^P h_i \delta(\tau - \tau_i) \delta(\nu - \nu_i). \quad (10)$$

The corresponding continuous time-varying channel impulse response function can be written as

$$g(\tau, t) = \int_{\nu} h(\tau, \nu) e^{j2\pi\nu(t-\tau)} d\nu. \quad (11)$$

C. Input-Output Relation

Starting from the received time-domain signal $r(t)$, the continuous time domain input-output relation can be written as

$$r(t) = \int_0^{\tau_{\max}} g(\tau, t) s(t - \tau) d\tau + w(t). \quad (12)$$

At the receiver, the channel impaired signal is down-converted to baseband and sampled, thereby limiting the received waveform to finite complex samples. The discrete time model is obtained by sampling the received waveform $r(t)$ at sampling intervals $t = qT/M$, where $0 \leq q \leq NM-1$. Following, [18], the time-domain input-output relation in the simple matrix form can then be expressed as

$$\mathbf{r} = \mathbf{G} \cdot \mathbf{s} + \mathbf{w}. \quad (13)$$

where $\mathbf{G} \in \mathbb{C}^{NM \times NM}$ is the time-domain discrete baseband channel matrix. Substituting (6) and (9) in (13) (and omitting the noise term for brevity) we get

$$(\mathbf{W}_N \otimes \mathbf{I}_M) \cdot (\mathbf{P} \cdot \mathbf{y}) = \mathbf{G} \cdot (\mathbf{W}_N \otimes \mathbf{I}_M) \cdot (\mathbf{P} \cdot \mathbf{x}) \quad (14)$$

Reversing the transmitter operations, the input-output relation in (6) can be expressed in terms of the information symbols as

$$\mathbf{y} = \mathbf{H} \cdot \mathbf{x} + \bar{\mathbf{w}} \quad (15)$$

where the delay-sequency channel and AWGN noise experienced by the information symbols

$$\begin{aligned} \mathbf{H} &= (\mathbf{I}_M \otimes \mathbf{W}_N) \cdot (\mathbf{P}^T \cdot \mathbf{G} \cdot \mathbf{P}) \cdot (\mathbf{I}_M \otimes \mathbf{W}_N), \\ \bar{\mathbf{w}} &= (\mathbf{I}_M \otimes \mathbf{W}_N) \cdot (\mathbf{P}^T \cdot \mathbf{w}). \end{aligned} \quad (16)$$

Note that, the band-width of the matrix $\mathbf{G}$ is $l_{\max} + 1$ and there are L non-zero elements in each row of $\mathbf{G}$, where $L \leq l_{\max} + 1$ in the number of distinct delay taps as seen by the discrete receiver. It can be seen from Fig. 2 that, the null symbols in the delay-sequency domain acts as interleaved guard bands in the time-domain hence preventing inter-block interference. This means that the elements in the strictly upper triangular part of the time-domain channel matrix can be ignored.

III. LOW COMPLEXITY DETECTION

As shown in Fig. 2, due to the time-domain ZP, the interference between the time domain blocks is prevented. This allows the time-domain input-output relation in (6) to be split and independently processed as

$$\mathbf{r}_n = \mathbf{G}_n \cdot \mathbf{s}_n + \mathbf{w}_n, \quad n = 0, \dots, N-1 \quad (17)$$

where $\mathbf{s} = [\mathbf{s}_0^T, \dots, \mathbf{s}_{N-1}^T]^T$ and $\mathbf{r} = [\mathbf{r}_0^T, \dots, \mathbf{r}_{N-1}^T]^T$ and $\mathbf{G}_n$ is the time-domain channel at the n -th time-domain block. Note that all the N time-domain blocks have an equal component of each delay-sequency domain information symbol due to the Walsh-Hadamard spreading.

A. Block-wise Frequency Domain Single Tap Equalizer

In the case of static (or very low Doppler spread) wireless channels, the time-domain channel matrix for each block can be assumed to be a circulant matrix, which can be diagonalized in the frequency domain. However, in a mobile channel Doppler spread introduces interference between the frequency domain samples of each block as the time-domain channel matrix is no more circulant due to the time-varying channel. However, since the duration of each time-domain

Fig. 3. OTSM BER performance using single tap equalizer at different UE speeds.

block is small compared to the whole frame, the channel can be assumed to be time-invariant in each block, but varying from block to block. The advantage is that, this allows the use of a single tap MMSE equalizer for detection in each block followed by the WHT to combine the block estimates.

The received time-frequency samples can be obtained by the M -point FFT operation on the received time domain blocks

$$\bar{\mathbf{r}}_n = \mathbf{F}_M \cdot \mathbf{r}_n. \quad (18)$$

as in a standard OFDM receiver. We can then equalize each block in parallel as

$$\bar{\mathbf{s}}_n(m) = \frac{\bar{\mathbf{h}}_n^*(m) \cdot \bar{\mathbf{r}}_n(m)}{|\bar{\mathbf{h}}_n(m)|^2 + \sigma_w^2} \quad (19)$$

for $m = 0, \dots, M-1$ and $n = 0, \dots, N-1$, where σ_w^2 , the AWGN noise variance and the frequency domain channel coefficients for each time-domain block are given by

$$\bar{\mathbf{h}}_n = \text{diag}[\mathbf{F}_M \cdot \mathbf{G}_n \cdot \mathbf{F}_M^\dagger] \quad (20)$$

where $\text{diag}[\mathbf{A}]$ denotes the column vector containing the diagonal elements of the square matrix $\mathbf{A}$. As shown in Fig. 1, the information symbol estimates in the delay-sequency domain can then be obtained by the M -point IFFT operation on the time-frequency domain estimates $\bar{\mathbf{s}}_n$ followed by the N -point WHT as

$$\hat{\mathbf{X}} = \mathbf{F}_M^\dagger \cdot [\bar{\mathbf{s}}_0, \bar{\mathbf{s}}_1, \dots, \bar{\mathbf{s}}_{N-1}] \cdot \mathbf{W}_N \quad (21)$$

Fig. 3 shows the OTSM BER performance using a single tap MMSE equalizer for QPSK modulation at different user equipment (UE) speeds. It can be seen that the single tap equalizer can offer very good performance at moderate speeds (or moderate Doppler spread channels). Similar to a traditional OFDM receiver, the single tap equalizer performance can be improved by increasing the sub-carrier spacing Δf , as the time-frequency domain samples remain roughly orthogonal, when the channel Doppler spread is a small fraction of the sub-carrier spacing. This is particularly useful for dealing with low-latency and delay-critical wireless applications.

B. Block-wise Time Domain Iterative detector

As discussed in the previous subsection, splitting the whole frame into short blocks allows the channel to be assumed as invariant for the block duration T . However, in the case of high Doppler spread (see Fig. 3), such assumptions may cause performance degradation, especially for higher order QAM symbols. In this section, we use the well studied Gauss Seidel (GS) method available in the literature [20], [21], for low complexity detection.

In the proposed detector, GS iteration is done on the matched filtered channel matrix blocks $\mathbf{R}_n = \mathbf{G}_n^\dagger \cdot \mathbf{G}_n$. The matrix input-output relation in (17) after the matched filtering operation can be written as

$$\mathbf{z}_n = \mathbf{R}_n \cdot \mathbf{s}_n + \bar{\mathbf{w}}_n \quad (22)$$

where $\mathbf{R}_n = \mathbf{G}_n^\dagger \cdot \mathbf{G}_n$, $\mathbf{z}_n = \mathbf{G}_n^\dagger \cdot \mathbf{r}_n$ and $\bar{\mathbf{w}}_n = \mathbf{G}_n^\dagger \cdot \mathbf{w}_n$. The GS method is used to iteratively find the least squares solution

$$\hat{\mathbf{s}}_n = \min_{\mathbf{s}_n} \|\mathbf{z}_n - \mathbf{R}_n \mathbf{s}_n\|^2 \quad (23)$$

of the M -dimensional linear system of equations in (22).

Let $\mathbf{D}_n$ and $\mathbf{L}_n$ be the matrix containing the diagonal elements and the strictly lower triangular elements of the matched filter matrix $\mathbf{R}_n$. From [20], [21], the GS iterative method for finding the estimate of $\mathbf{s}_n$ in each iteration is then given as

$$\hat{\mathbf{s}}_n^{(i)} = -\mathbf{T}_n \cdot \hat{\mathbf{s}}_n^{(i-1)} + \mathbf{b}_n \quad (24)$$

$$\mathbf{T}_n = (\mathbf{D}_n + \mathbf{L}_n)^{-1} \cdot \mathbf{L}_n^\dagger, \quad \mathbf{b}_n = (\mathbf{D}_n + \mathbf{L}_n)^{-1} \cdot \mathbf{z}_n. \quad (25)$$

where $\mathbf{T}_n \in \mathbb{C}^{M \times M}$ is the GS iteration matrix. The vector $\hat{\mathbf{s}}_n^{(i)} \in \mathbb{C}^{M \times 1}$ represents the estimate of the transmitted time-domain samples of the n -th block in the i -th iteration. The delay-sequency domain information symbols in the i -th iteration is then given as

$$\hat{\mathbf{X}}^{(i)} = \mathcal{D}(\mathbf{C}^{(i)}), \quad \text{where } \mathbf{C}^{(i)} = [\hat{\mathbf{s}}_0^{(i)}, \hat{\mathbf{s}}_1^{(i)}, \dots, \hat{\mathbf{s}}_{N-1}^{(i)}] \cdot \mathbf{W}_N \quad (26)$$

where $\mathcal{D}(\cdot)$ denotes the decision making function replacing all the elements of the input with the nearest QAM symbol (in terms of the Euclidean distance). The hard decision estimates are transformed back to the time domain to update the time domain estimate to be used in the next iteration.

$$\hat{\mathbf{s}}^{(i)} \leftarrow (1 - \delta) \hat{\mathbf{s}}^{(i)} + \delta \text{vec}(\hat{\mathbf{X}}^{(i)} \cdot \mathbf{W}_N) \quad (27)$$

where δ is the relaxation parameter to improve the detector convergence for higher modulation schemes like 64-QAM, [13], [20], [21]. As initial estimate to the iterative detection, we can chose either $\mathbf{X}^{(0)} = \mathbf{0}$ or the MMSE solution in (21) yielding faster convergence. Fig. 4 shows the OTSM BER performance using the iterative equalizer for QPSK modulation at different UE speeds. It can be noted that, in contrast to the single tap equalizer performance in Fig. 3, the iterative equalizer offers significantly better performance in high mobility channels. The availability of low complexity detection methods makes OTSM an attractive candidate for future wireless communication networks.

Fig. 4. OTSM BER performance using iterative time-domain detector at different UE speeds.

Fig. 5. BER performance using iterative time-domain detector compared with OTFS and single carrier (SC) transmission schemes for different QAM alphabet sizes.

IV. SIMULATION RESULTS AND DISCUSSION

For simulations we generate OTFS frames for $N = 64$ and $M = 64$. The sub-carrier spacing Δf is taken as 15 kHz. The maximum delay spread (in terms of integer taps) is taken to be 4 ($l_{\max} = 3$) which is approximately 4 μs , i.e., maximum number of delay taps as seen by the discrete receiver $L = 4$. The channel delay model is generated according to the standard EVA model⁴ (with a speed of 120 km/h) with the Doppler shift for the i -th path generated from a uniform distribution $U(0, \nu_{\max})$, where $\nu_{\max}$ is the maximum Doppler shift [22]. For our simulations, we assume perfect knowledge of the channel state information at the receiver. For BER plots, 10^5 frames are sent for every point in the BER curve.

Fig. 5 shows the BER performance of OTSM compared with a single carrier (SC) scheme and OTFS employing the iterative time-domain algorithm in Section III-B. The relaxation

⁴The EVA channel power-delay profile is given by [0, -1.5, -1.4, -3.6, -0.6, -9.1, -7.0, -12.0, -16.9] dB with excess tap delays [0, 30, 150, 310, 370, 710, 1090, 1730, 2510] ns.

Fig. 6. BER performance compared with OFDM and OTFS transmission schemes for 4-QAM modulation using the proposed iterative and non-iterative detectors.

parameter is set to $\delta = 1$ for both 4-QAM and 16-QAM, and $\delta = 0.25$ for 64-QAM. Fig. 6 shows the performance with a both a single tap frequency domain equalizer and an iterative equalizer for 4-QAM compared with OTFS and OFDM. The BER plots show the superior performance of OTSM over OFDM and SC transmission. OTSM also offers similar performance to OTFS at significantly lower transmitter and receiver complexity.

In [13], it was shown that the proposed matched filtered GS algorithm in Section III-B can be implemented in the time domain in $NM(2L+1)$ complex multiplications per iteration. Moreover, in OTFS employing similar detector, taking the hard estimates between every iteration requires converting the time-domain samples to the delay-Doppler domain and replacing them with the nearest QAM symbol and then converting them back to get the improved time domain estimates. This demodulation and modulation operation requires $2NM \log_2(N)$ complex multiplications per iteration, which increases the detector complexity to $NM(2L+1+2\log_2(N))$ complex multiplications. Fortunately, the OTSM scheme gains from the fact that the corresponding modulation (27) and demodulation (26) steps require only complex additions, which makes OTSM detection completely linear in the number of transmitted symbols NM .

V. CONCLUSION

We proposed a novel single carrier modulation scheme based on a combination of Walsh-Hadamard transform and block interleaving. This is equivalent to multiplexing the information symbols on the time-shifted and sequency-shifted versions of Walsh functions. The error performance was shown to be significantly better than OFDM and SC transmission, and similar to the recently proposed OTFS scheme, although with much lower complexity. The proposed scheme is very suitable for low complexity transceivers as no complex multiplications are required. Combined with other attractive advantages of the proposed modulation scheme as listed in the introduction,

OTSM can be considered a candidate for wireless communications for 6G. Finally, OTSM analysis including delay-sequency domain channel representation and practical channel estimation will be explored in future work.

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