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Physical Basis of Lognormals in Reliability for Microelectronics With Deposited Metal Layers

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Abstract

Historically, many electronic device lifetimes were empirically modeled with Lognormals and Weibulls, but a physical basis was elusive until now. We show Lognormals naturally arise for long thin conducting *metal layers* deposited on substrates. Key elements are: (1) an applied voltage V_M across the *metal layer* ends; (2) good thermal connectivity through the substrate to a constant temperature T_B baseplate; and (3) a *metal layer* failure mechanism that has an *activation energy* U_M . Since these (1)-(3) conditions are common to many microelectronic devices, Lognormals should also be applicable to a wide variety of similar situations.

Ohm's Law, $V_M = I R_M$, with power dissipation $P = I^2 R_M$, produces extra heat, giving a *metal layer* temperature $T_L > T_B$, which sets a baseline $\mathcal{F}(T_L)$ failure rate. What we derive here is that having a Gaussian variation in *metal layer geometry* produces Lognormal distributions for the *metal layer* lifetimes, which provides a new physical basis for using Lognormals in microelectronic device reliability.

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1 Introduction

Probability Density Functions (pdfs) for electronic device lifetimes have been modeled using Lognormals, Exponentials (*random failures*), Gaussians (*wearout*), and Weibulls, but a physical basis for a

using Lognormals or Weibulls has remained elusive until now. Here, we show why long thin conducting *metal layers*, deposited as *wires* on a non-conducting bulk substrate, should exhibit Lognormal *pdfs*.

This derivation relies on these *metal layers* having: (1) An Arrhenius-type failure mechanism with *activation energy* U_M ; (2) An applied voltage V_M across the *metal layer*, and (3) Good thermal connectivity through the substrate down to a underlying heat-sink held at temperature of T_B .

Many modern-day devices and components obey the above (1)-(3) conditions, and Lognormal *pdfs* have often been empirically observed, such as in *LED Numeric Display Failures*¹⁻² and in semiconductor device accelerated aging data³⁻⁵. However, this Lognormal *pdf* connection was apparently not well known as late as 1957⁶, as that was prior to transistor miniaturization technology.

When *failure rates* $\mathcal{F}$ obey an Arrhenius plot, that process is characterized by activation energy U_M , giving a nearly constant negative slope on a $\ln(\mathcal{F})$ vs $\beta \equiv (1/k_B T)$ plot, with k_B being *Boltzmann's Constant*. This signature indicates a diffusion-driven failure mechanism due to unwanted atomic migration. A nominal *wire temperature* T_L sets a baseline $\mathcal{F}(T_L)$ *metal layer* lifetime. We then show how a Gaussian variation in *metal layer geometry* results in a Lognormal distribution for *metal layer* lifetimes, providing a Lognormal physical basis.

2 Lognormals for *Metal Layers*

When deposited *metal layers* atop a substrate are used for *wire* interconnects, their failure rate $\mathcal{F}$ is often set by an activation energy U_M , and a nominal operating temperature T_L :

$$\mathcal{F}_M(T_L) = \mathcal{F}_\infty \exp[-U_M / k_B T_L], \quad [2.0.1]$$

where $\mathcal{F}_\infty$ is an asymptotic material constant. If an anomalous *wire* section has a different local temperature $T \neq T_L$, its failure rate will be:

$$\mathcal{F}(T) = \mathcal{F}_M \exp[-U_M (\frac{1}{k_B T} - \frac{1}{k_B T_L})], \quad [2.0.2a]$$

$$\ln[\mathcal{F}(T) / \mathcal{F}_M] = -U_M (\frac{1}{k_B T} - \frac{1}{k_B T_L}). \quad [2.0.2b]$$

Our key point is any device **geometry** change that alters the local temperature from T_L to T , also changes the *wire* lifetime, making $\mathcal{F}(T)$ a function of **geometry**. If those **geometry** changes are due to manufacturing variations, as characterized by a Gaussian random variable X with zero mean and a fixed standard deviation, once it is shown that:

$$\ln[\mathcal{F}(T) / \mathcal{F}_M] = +\mathbf{K} X [1 + \vartheta(X^2)], \quad [2.0.3]$$

for small X and some constant $\mathbf{K}$, then it immediately follows that $\mathcal{F}(T)$ is a Lognormal distribution.

2.1 Geometrical and Thermal Factors

Figure 1 shows two different deposited *metal layers* atop a non-conducting substrate, as used in *wire* interconnects between devices. In (1a), the normal *metal layer* section has length L_a , depth d_o , a normal width w_o , and a cross-sectional area of $A_o = w_o d_o$. In (1b), the anomalous *metal layer* section has the same $\{L_a; d_o\}$ length and depth, but with improper width w_a , and a different cross-sectional area of $A_a = w_a d_o$. Each L_a section is also part of a longer thin deposited *wire* of total length:

$$L_{TOT} = (2 L_n + L_a), \quad [2.1.1]$$

with the other L_n lengths having a normal *metal layer* geometry.

Figure 2 shows a side view, with a *metal layer* of thickness d_o at $Z = 0$, sitting atop a substrate of thickness Z_B , which is underlain by a heat-sink at $Z = Z_B$ held at temperature T_B . When the *metal layer* is at temperature $T \neq T_B$, a linear temperature profile forms across the substrate between $Z = 0$ and $Z = Z_B$.

Figure 1: Nominal (1a) vs Anomalous (1b) Metal Layer Geometries.

The anomalous portion is assumed to have an improper width w_a , spanning a length L_a , as compared to a normal width w_o . To the left and right of this length L_a are two long normal w_o sections, each of length L_n , giving a total length of $(2 L_n + L_a)$.

Figure 2: Side View of Deposited Metal Layer atop a Substrate.

Central portion of length L_a , may have a normal (w_o) or anomalous (w_a) width. It abuts two long normal w_o sections of length L_n , for a total length $(2 L_n + L_a)$. The Metal Layer depth is d_o , and it sits atop a substrate of thickness Z_B , with a heat-sink at the bottom that is held at a constant temperature T_B .

The *Wiedemann-Franz Law* applies⁷ to the *metal layer* electrical resistance:

$$[K_M / (\sigma_M T)] \equiv \Lambda_o = \frac{\pi^2}{3} \left(\frac{k_B}{e} \right)^2 \approx 2.445 \times 10^{-8} (\text{Watt} - \text{ohms}) / K^2, \quad [2.1.2]$$

due to electrons near the metal Fermi Level contributing both to the *metal layer* K_M thermal conductivity [$\text{Watts} / (\text{meter} - K)$] and its σ_M electrical conductivity [$1 / (\text{ohm} - \text{meter})$], where Λ_o is the *Lorentz Number*. For metals, K_M is fairly temperature independent, making $\sigma_M = \sigma_M(T)$, with T in Kelvin.

When the *metal layer* carries a current I , its power dissipation P [Watts] needs to be conducted out of the *metal layer*, through the substrate, down to the $Z = Z_B$ heat-sink. The heat flux J_S [Watts / m^2] out of the *metal layer*, through the substrate, is controlled by the K_S *substrate* thermal conductivity [$\text{Watts} / (\text{meter} - K)$].

For the anomalous *wire* in (1b), its L_n normal geometry sections are assumed to have a boundary condition of $T(Z = 0) = T_n$ and $T(Z = Z_B) = T_B$, giving an in-substrate temperature profile of:

$$T(Z) = (T_n) - (T_n - T_B) Z / Z_B, \quad [2.1.3]$$

with a temperature gradient and J_n^S heat flux of:

$$\nabla T(Z) = -(T_n - T_B) / Z_B, \quad [2.1.4a]$$

$$J_n^S = -\frac{K_S}{Z_B} (T_n - T_B). \quad [2.1.4b]$$

In contrast, the anomalous section of length L_a in (1b) will have:

$$T(Z) = (T_a) - (T_a - T_B) Z / Z_B, \quad [2.1.5a]$$

$$J_a^S = -\frac{K_S}{Z_B} (T_a - T_B). \quad [2.1.5b]$$

Under constant V_M conditions, the presence of an anomalous *wire* section changes the equilibrium temperatures for both the normal w_o and the anomalous w_a width sections, compared to the totally uniform *wire* case of (1a).

2.2 Electrical Factors

Applying a voltage V_M across a *metal layer* with resistance R_M gives:

$$V_M = I R_M, \quad [2.2.1a]$$

$$P = I^2 R_M. \quad [2.2.1b]$$

The power dissipation P [Watts] in the *metal layer* generates heat, causing it to reach a new equilibrium temperature $T_L > T_B$. The deposited *metal layer*

has an electrical resistivity ρ , and electrical conductivity $\sigma = 1 / \rho$. Each length L of the *wire*, with cross-sectional area A has a resistance R set by:

$$R = \rho L / A = (1 / \sigma) (L / A). \quad [2.2.2]$$

Two L_n length long *wires* each with resistance R_n , plus one L_a length with resistance R_a , as in (1b) of *Figure 1*, gives a net resistance R_M :

$$R_M = 2 R_n + R_a, \quad [2.2.3]$$

The Wiedemann-Franz Law of Eq. [2.1.2] is used next. Let L_n be at T_n Kelvin, with the L_a length at T_a , then Eq. [2.1.2] and Eq. [2.2.2] give:

$$R_a = \rho_a L_a / A_a = \frac{L_a}{\sigma_a w_a d_o} = \frac{L_a}{\sigma_a w_a d_o} \left[\frac{\Lambda_o}{(d_o K_M)} \right] (T_a / w_a), \quad [2.2.4a]$$

$$R_n = \rho_n L_n / A_o = \frac{L_n}{\sigma_n w_o d_o} = \frac{L_n}{\sigma_n w_o d_o} \left[\frac{\Lambda_o}{(d_o K_M)} \right] (T_n / w_o), \quad [2.2.4b]$$

$$\gamma_o \equiv [L_a / L_n], L_a = \gamma_o L_n, \quad [2.2.4c]$$

where $\rho_a \neq \rho_n$ if $T_a \neq T_n$. Applying a constant voltage V_M across the whole wire length gives a total current I_a , and a P_M power dissipation of:

$$P_M \equiv 2 P_n + P_a = I_a V_M = \quad [2.2.5]$$

$$(I_a)^2 R_M = 2 (I_a)^2 R_n + (I_a)^2 R_a.$$

The power dissipated per unit area for each *metal layer* section is:

$$P_n / (L_n w_o) = \frac{K_S}{Z_B} (T_n - T_B) \quad [2.2.6a]$$

$$= (I_a)^2 R_n / (L_n w_o) \\ = (I_a / w_o)^2 [\Lambda_o / (d_o K_M)] (T_n),$$

$$P_a / (L_a w_a) = \frac{K_S}{Z_B} (T_a - T_B) \quad [2.2.6b]$$

$$= (I_a)^2 R_a / (L_a w_a) \\ = (I_a / w_a)^2 [\Lambda_o / (d_o K_M)] (T_a).$$

These 5 equations result:

$$V_M = \{2 R_n + R_a\} I_a, \quad [2.2.7a]$$

$$R_n = \left[\frac{\Lambda_o L_n}{d_o K_M} \right] \left(\frac{T_n}{w_o} \right)_{(1)}, \quad [2.2.7b]$$

$$R_a = \left[\frac{\Lambda_o L_a}{d_o K_M} \right] \left(\frac{T_a}{w_a} \right)_{(2)} = \left[\frac{\Lambda_o L_n}{d_o K_M} \right] \left(\frac{\gamma_o T_a}{w_a} \right)_{(2)}, \quad [2.2.7c]$$

$$[(I_a)^2 R_n]_{(1)} = \frac{K_S}{Z_B} (L_n w_o) (T_n - T_B)_{(1)}, \quad [2.2.7d]$$

$$[(I_a)^2 R_a]_{(2)} = \gamma_o \frac{K_S}{Z_B} (L_n w_a) (T_a - T_B)_{(2)}, \quad [2.2.7e]$$

with 5 unknowns $\{R_a; R_n; T_a; T_n; I_a\}$. The subscript $[_]_{(1)}$ indicates the T_n -equation, while $[_]_{(2)}$ indicates the T_a -equation. First replacing $\{R_a; R_n\}$:

$$V_M = I_a \left[\frac{\Lambda_o L_n}{d_o K_M} \right] [2 (T_n / w_o) + (\gamma_o T_a / w_a)], \quad [2.2.8a]$$

$$(I_a)^2 (T_n / w_o) \left[\frac{\Lambda_o L_n}{d_o K_M} \right]_{(1)} = \quad [2.2.8b]$$

$$\frac{K_S}{Z_B} (L_n w_o) [T_n - T_B]_{(1)},$$

$$(I_a)^2 (\gamma_o T_a / w_a) \left[\frac{\Lambda_o L_n}{d_o K_M} \right]_{(2)} = \quad [2.2.8c]$$

$$\gamma_o \frac{K_S}{Z_B} (L_n w_a) [T_a - T_B]_{(2)},$$

$$(I_a)^2 (T_n) \left[\frac{\Lambda_o L_n}{d_o K_M} \right]_{(1)} = \quad [2.2.8d]$$

$$\frac{K_S}{Z_B}(L_n w_o^2)[T_n - T_B]_{(1)}, \quad [2.2.8e]$$

$$(I_a)^2 (T_a) \left[\frac{\Lambda_o L_n}{d_o K_M} \right]_{(2)} =$$

When $w_a \rightarrow w_o$, it is the normal wire configuration of **(1a)** in Figure 1. Using $T_a \rightarrow T_L$, $T_n \rightarrow T_L$, and $I_a \rightarrow I_o$ in Eq. [2.2.8d] then gives:

$$V_M = I_o \left[\frac{\Lambda_o L_n}{d_o K_M} \right] [2 + \gamma_o] (T_L / w_o), \quad [2.2.9a]$$

$$(I_o)^2 (T_L) \left[\frac{\Lambda_o L_n}{d_o K_M} \right] = \frac{K_S}{Z_B} (L_n w_o^2) (T_L - T_B). \quad [2.2.9b]$$

Replacing I_o in Eq. [2.2.9b] with V_M from Eq. [2.2.9a] gives:

$$(V_M)^2 (T_L) \left[\frac{\Lambda_o L_n}{d_o K_M} \right] = \quad [2.2.10a]$$

$$\left(\frac{K_S}{Z_B} \right) (L_n w_o^2) (T_L - T_B) \times \left[\frac{\Lambda_o L_n}{d_o K_M} \right]^2 [2 + \gamma_o]^2 (T_L / w_o)^2,$$

$$(V_M)^2 = \quad [2.2.10b]$$

$$\Lambda_o (K_S / K_M) [(L_n)^2 / (d_o Z_B)] \times [2 + \gamma_o]^2 (T_L) (T_L - T_B).$$

For this **(1a)** wire case, the single parameter Γ_o in Eq. [2.2.11a] sets T_L and $\mathcal{F}_M$:

$$\Gamma_o \equiv (V_M)^2 / \left\{ \Lambda_o \left(\frac{K_S}{K_M} \right) \left[\frac{(L_n)^2}{d_o Z_B} \right] \right\}, \quad [2.2.11a]$$

$$\Gamma_o = [2 + \gamma_o]^2 (T_L) (T_L - T_B), \quad [2.2.11b]$$

$$\mathcal{F}_M(T_L) = \mathcal{F}_\infty \exp[-U_M / k_B T_L]. \quad [2.2.11c]$$

2.3 Deviations from Nominal

Assuming the same V_M constant voltage conditions for both the **(1a)** and **(1b)** wire configurations in Figure 1, the w_a anomalous width section is allowed to modify the equilibrium temperature for both the normal wire section [$T_n \neq T_L$] and the anomalous wire section [$T_a \neq T_L$]. Using Γ_o from Eq. [2.2.11a], Eqs. [2.2.8a]-[2.2.8c] become:

$$V_M = I_a \left[\frac{\Lambda_o L_n}{d_o K_M} \right] [(2 T_n / w_o) + (\gamma_o T_a / w_a)], \quad [2.3.1a]$$

$$[T_n \Gamma_o]_{(1)} = \quad [2.3.1b]$$

$$[(2 T_n) + \gamma_o T_a (w_o / w_a)]^2 [T_n - T_B]_{(1)},$$

$$[T_a \Gamma_o]_{(2)} = \quad [2.3.1c]$$

$$[2 T_n (w_a / w_o) + (\gamma_o T_a)]^2 [T_a - T_B]_{(2)}.$$

Let X be small deviation of w_a from the normal w_o width, which can induce a temperature change to T_a for the w_a geometry, as compared to T_n for w_o :

$$w_a \equiv w_o / (1 + X), \quad [2.3.2a]$$

$$T_a \equiv T_n (1 + \alpha X), \quad [2.3.2b]$$

then Eqs. [2.3.1b]-[2.3.1c] become:

$$[\Gamma_o]_{(1)} = \quad [2.3.3a]$$

$$[2 + \gamma_o (1 + \alpha X) (1 + X)]^2 T_n [T_n - T_B]_{(1)},$$

$$[\Gamma_o (1 + \alpha X)]_{(2)} = \quad [2.3.3b]$$

$$[(\frac{2}{1+X}) + \gamma_o (1 + \alpha X)]^2 T_n [T_n (1 + \alpha X) - T_B]_{(2)}.$$

When $X \rightarrow 0$, then $T_n \rightarrow T_L$, and both Eqs. [2.3.3a]-[2.3.3b] give Eq. [2.2.11b]. Normally X is given, and $\{\alpha; T_n\}$ are solved for. Let:

$$\Delta \equiv (T_n - T_L), \quad [2.3.4]$$

which converts Eqs. [2.3.3a]-[2.3.3b] into:

$$[\Gamma_o]_{(1)} = [2 + \gamma_o (1 + \alpha X) (1 + X)]^2 \times \quad [2.3.5a]$$

$$(T_L + \Delta - T_B) (T_L + \Delta)_{(1)},$$

$$[\Gamma_o]_{(2)} = [(\frac{2}{1+X}) + \gamma_o (1 + \alpha X)]^2 \times \quad [2.3.5b]$$

$$[(T_L + \Delta) - (\frac{T_B}{1+\alpha X})] (T_L + \Delta).$$

Since Eqs. [2.3.5a]-[2.3.5b] only has (αX) terms, α may not be small, but both $\{X; \Delta\}$ can be small, so that only linear terms in either variable are needed:

$$(T_L + \Delta) (T_L - T_B + \Delta) \approx \quad [2.3.6a]$$

$$\{(T_L) (T_L - T_B) + \Delta (2 T_L - T_B)\},$$

$$(T_L + \Delta) [(T_L + \Delta) - T_B / (1 + \alpha X)] \approx \quad [2.3.6b]$$

$$\{(T_L) (T_L - T_B) + [\Delta (2 T_L - T_B) + \alpha X T_B T_L]\}.$$

Similarly:

$$\{[2 + \gamma_o (1 + \alpha X) (1 + X)]^2\} \approx \quad [2.3.7a]$$

$$\{[2 + \gamma_o]^2 + [2(2 + \gamma_o)(1 + \alpha) \gamma_o X]\},$$

$$\{[2 / (1 + X) + \gamma_o (1 + \alpha X)]^2\} \approx \quad [2.3.7b]$$

$$\{[2 + \gamma_o]^2 - [2(2 + \gamma_o)(2 - \gamma_o \alpha) X]\},$$

which converts Eqs. [2.3.5a]-[2.3.5b] into:

$$[\Gamma_o]_{(1)} \approx \quad [2.3.8a]$$

$$\{[2 + \gamma_o]^2 + [2(2 + \gamma_o)(1 + \alpha) \gamma_o X]\}_{[1]} \times$$

$$\{(T_L) (T_L - T_B) + \Delta (2 T_L - T_B)\}$$

$$\approx [2 + \gamma_o]^2 (T_L) (T_L - T_B) +$$

$$(T_L) (T_L - T_B) [2(2 + \gamma_o)(1 + \alpha) \gamma_o X] +$$

$$[2 + \gamma_o]^2 \Delta (2 T_L - T_B),$$

$$[\Gamma_o]_{(2)} \approx \quad [2.3.8b]$$

$$\{[2 + \gamma_o]^2 - [2(2 + \gamma_o)(2 - \gamma_o \alpha) X]\}_{[2]} \times$$

$$\{(T_L) (T_L - T_B) + [\Delta (2 T_L - T_B) + \alpha X T_B T_L]\}_{[2]}$$

$$\approx [2 + \gamma_o]^2 (T_L) (T_L - T_B) +$$

$$[2 + \gamma_o]^2 [\Delta (2 T_L - T_B) + \alpha X T_B T_L]$$

$$- (T_L) (T_L - T_B) [2(2 + \gamma_o)(2 - \gamma_o \alpha) X],$$

Using Eq. [2.2.11b] to subtract out Γ_o gives:

$$0 \approx (2 + \gamma_o) [\Delta (2 T_L - T_B)]_{(1)} \quad [2.3.9a]$$

$$+ X (T_L) (T_L - T_B) [2(1 + \alpha) \gamma_o]_{(1)},$$

$$0 \approx (2 + \gamma_o) [\Delta (2 T_L - T_B) + \alpha X T_B T_L]_{(2)} \quad [2.3.9b]$$

$$- X (T_L) (T_L - T_B) [2(2 - \gamma_o \alpha)]_{(2)}.$$

As the Δ -terms match, matching X -terms sets α :

$$(T_L) (T_L - T_B) [2(1 + \alpha) \gamma_o]_{[1]} = \quad [2.3.10a]$$

$$\{(2 + \gamma_o) (\alpha T_B T_L) - (T_L) (T_L - T_B) [2(2 - \gamma_o \alpha)]\}_{[2]},$$

$$(2 + \gamma_o) (\alpha T_B T_L) = \quad [2.3.10b]$$

$$\begin{aligned}
& 2(T_L)(T_L - T_B)[(1 + \alpha)\gamma_o + (2 - \gamma_o)\alpha] \\
& = 2(T_L)(T_L - T_B)[2 + \gamma_o], \\
(\alpha) & = 2(T_L - T_B)/T_B. \quad [2.3.10c] \\
(1 + \alpha) & = 1 + 2\frac{(T_L - T_B)}{T_B} = \frac{1}{T_B}(2T_L - T_B). \quad [2.3.10d] \\
\text{Then Eq. [2.3.9a], becomes:} \\
-\{\Delta/X\}(2 + \gamma_o)(2T_L - T_B) & \approx \quad [2.3.11a] \\
& (T_L)(T_L - T_B)[2(1 + \alpha)\gamma_o] \\
& = 2\gamma_o(T_L/T_B)(T_L - T_B)(2T_L - T_B), \\
-\{\Delta/X\}(2 + \gamma_o) & \approx \quad [2.3.11b] \\
& 2\gamma_o(T_L/T_B)(T_L - T_B), \\
-(\Delta) & \approx 2\left[\frac{\gamma_o}{2 + \gamma_o}\right](T_L - T_B)(T_L/T_B)X, \quad [2.3.11c] \\
\text{showing } \Delta < 0. \text{ Using Eq. [2.3.4] then sets } T_n: \\
T_n \equiv T_L + \Delta & = \quad [2.3.12a] \\
& T_L \left[1 - 2\left(\frac{\gamma_o}{2 + \gamma_o}\right)\left[\frac{(T_L - T_B)}{T_B}\right]X\right], \\
\frac{(T_n - T_L)}{T_L} & = -\left(\frac{\gamma_o}{2 + \gamma_o}\right)\left[\frac{2(T_L - T_B)}{T_B}\right]X. \quad [2.3.12b] \\
\text{Using Eq. [2.3.2b] for } T_a, \text{ and } \{\alpha; T_n\} \text{ from Eq.} \\
\text{[2.3.10b] and [2.3.12a]:} \\
T_a \equiv T_n(1 + \alpha X) & = T_n \left[1 + \frac{2(T_L - T_B)}{T_B}X\right] \quad [2.3.13a] \\
& = T_L \left[1 - 2\left(\frac{\gamma_o}{2 + \gamma_o}\right)\left(\frac{T_L - T_B}{T_B}\right)X\right] \left[1 + \frac{2(T_L - T_B)}{T_B}X\right] \\
& \approx T_L \left\{1 + \left[\frac{2(T_L - T_B)}{T_B}\right]\left[\frac{2}{2 + \gamma_o}\right]X\right\}, \\
\frac{(T_a - T_L)}{T_L} & = +\left(\frac{2}{2 + \gamma_o}\right)\left[\frac{2(T_L - T_B)}{T_B}\right]X. \quad [2.3.13b] \\
\text{Combining Eq. [2.3.12b] and Eq. [2.3.13b] gives:} \\
\frac{(T_a - T_n)}{T_L} & = \quad [2.3.14a] \\
& \left[\frac{2(T_L - T_B)}{T_B}\right]\left[\frac{2}{2 + \gamma_o}\right]X + \left(\frac{\gamma_o}{2 + \gamma_o}\right)\left[\frac{2(T_L - T_B)}{T_B}\right]X, \\
(T_a - T_n) & = 2\left(\frac{T_L}{T_B}\right)(T_L - T_B)X. \quad [2.3.14b] \\
\text{When } X \rightarrow 0, \text{ then } w_a \rightarrow w_o, \text{ and } T_a \rightarrow T_n \rightarrow T_L. \\
\text{Since the temperatures } \{T_L; T_B\} \text{ for the normal (1a) } \\
\text{wire and heat-sink are assumed known, Eq. [2.3.14b]} \\
\text{shows the temperature difference between the two} \\
\text{sections of the (1b) anomalous wire configuration is} \\
\text{a direct measure of the fractional deviation from nor-} \\
\text{mal for the anomalous wire geometry.}
\end{aligned}$$

2.4 Lognormal Completion

Given a *wire* of length $(2L_n + L_a)$ atop a substrate, as in *Figure 1*, with a heat-sink at depth Z_B and temperature T_B , applying a V_M operating voltage across the normal *wire* configuration **(1a)** gives a nominal T_L *wire* temperature and failure rate $\mathcal{F}_M$:

$$\mathcal{F}_M(T_L) = \mathcal{F}_\infty \exp[-U_M / k_B T_L]. \quad [2.4.1]$$

Let the anomalous *wire* **(1b)** have a section anomalous w_a width of:

$$w_a \equiv w_o / (1 + X), \quad [2.4.2a]$$

$$\gamma_o \equiv [L_a / L_n], \quad [2.4.2b]$$

extending over a length L_a . Assuming that X is small, corrections of $\vartheta(X^2)$ can be ignored. For $X > 0$, the w_a section will reach its own new equilibrium temperature $T_a > T_L$, giving a new failure rate $\mathcal{F}_M(T_a)$:

$$T_a = T_L \left\{1 + \left(\frac{2}{2 + \gamma_o}\right)\left[\frac{2(T_L - T_B)}{T_B}\right]X\right\}, \quad [2.4.3a]$$

$$\mathcal{F}(T_a) = \mathcal{F}_M \exp[-U_M \left(\frac{1}{k_B T_a} - \frac{1}{k_B T_L}\right)]. \quad [2.4.3b]$$

$$\ln[\mathcal{F}(T_a) / \mathcal{F}_M] = \quad [2.4.3c]$$

$$\begin{aligned}
& -U_M \left(\frac{1}{k_B T_a} - \frac{1}{k_B T_L}\right) = +\frac{U_M}{k_B T_a} \left(\frac{T_a - T_L}{T_L}\right) \\
& = +\frac{U_M}{k_B T_a} \left[\frac{2(T_L - T_B)}{T_B}\right]\left[\frac{2}{2 + \gamma_o}\right]X - \vartheta(X^2).
\end{aligned}$$

In contrast, the two normal geometry *wire* lengths in the anomalous *wire* **(1b)**, each of length L_n and width w_o , will have a cooler temperature $T_n < T_L$:

$$\frac{(T_n - T_L)}{T_L} = -\left[\frac{2(T_L - T_B)}{T_B}\right]\left[\frac{\gamma_o}{2 + \gamma_o}\right]X, \quad [2.4.4a]$$

$$T_n = T_L \left\{1 - \left[\frac{2(T_L - T_B)}{T_B}\right]\left[\frac{\gamma_o}{2 + \gamma_o}\right]X\right\}, \quad [2.4.4a]$$

$$\mathcal{F}(T_n) = \mathcal{F}_M \exp[-U_M \left(\frac{1}{k_B T_n} - \frac{1}{k_B T_L}\right)]. \quad [2.4.4b]$$

$$\ln[\mathcal{F}(T_n) / \mathcal{F}_M] = \quad [2.4.4c]$$

$$\begin{aligned}
& -U_M \left(\frac{1}{k_B T_n} - \frac{1}{k_B T_L}\right) = +\frac{U_M}{k_B T_n} \left(\frac{T_n - T_L}{T_L}\right) \\
& = -\frac{U_M}{k_B T_L} \left[\frac{2(T_L - T_B)}{T_B}\right]\left[\frac{\gamma_o}{2 + \gamma_o}\right]X - \vartheta(X^2),
\end{aligned}$$

showing both Eq. [2.4.3c] and Eq. [2.4.4c] depend on the $\{L_a; L_n\}$ lengths due to their γ_o -term. However, the companion to Eq. [2.3.14b]:

$$\ln[\mathcal{F}(T_a) / \mathcal{F}(T_n)] = \quad [2.4.5]$$

$$+\frac{U_M}{k_B T_L} \left[\frac{2(T_L - T_B)}{T_B}\right]X - \vartheta(X^2),$$

is independent of those lengths, and is primarily a function of the **geometry** changes, as set by X in Eq. [2.4.2a]. When those **geometry** changes are due to random manufacturing deviations about a mean, X is then a random variable, with zero mean and a given standard deviation. When X is a Gaussian, then the failure rates $\{\mathcal{F}(T_a); \mathcal{F}(T_n)\}$ and the Eq. [2.4.5] ratio, all become Lognormal distributions.

3 Summary

When a *wire* connection is established by depositing a *metal layer* atop a substrate, the dominant *wire* failure mechanism is likely due to an unwanted gradual interaction between the *wire* and its substrate, which is likely diffusion dominated. The *wire* failure rate $\mathcal{F}$

then depends on its nominal operating temperature T_L , and an activation energy U_M :

$$\mathcal{F}_M(T_L) = \mathcal{F}_\infty \exp[-U_M / k_B T_L]. \quad [3.0.1]$$

The normal *wire* configuration top view is shown as **(1a)** in *Figure 1*, which has a uniform *metal layer* width w_o . A side view is shown in *Figure 2*, with the *metal layer* of thickness d_o at $Z = 0$ atop a substrate of thickness Z_B , while a heat-sink at $Z = Z_B$ maintains a constant temperature T_B plane.

The Wiedemann-Franz Law⁷ sets $\mathcal{F}_M$ and T_L for the normal *wire* configuration of **(1a)** in *Figure 1*, along with this Eq. [3.0.2a] Γ_o lumped parameter:

$$\Gamma_o \equiv \frac{(V_M)^2}{\{\Lambda_o(K_S / K_M) [(L_n)^2 / (d_o Z_B)]\}}, \quad [3.0.2a]$$

$$\Gamma_o = [2 + \gamma_o]^2 (T_L) (T_L - T_B), \quad [3.0.2b]$$

where K_M and K_S , respectively, are the *metal layer* and *substrate* thermal conductivities [$Watts / (meter-K)$], and Λ_o is the *Lorentz Number*:

$$\Lambda_o = \frac{\pi^2}{3} \left(\frac{k_B}{e} \right)^2 \approx 2.445 \times 10^{-8} \frac{Watt-ohms}{(K)^2}. \quad [3.0.3]$$

Under constant V_M voltage conditions, let a wire section of length L_a have a different deposited *metal layer* width w_a [**(1b)** in *Figure 1*], abutted by two normal geometry regions of $\{L_n; w_o\}$ length and width. Both the anomalous $\{L_a; w_a\}$ region and the adjacent normal $\{L_n; w_o\}$ regions will reach new equilibrium temperatures $\{T_a; T_o\}$ respectively, compared to **(1a)** in *Figure 1*. The following modified local failure rates then result:

$$\mathcal{F}(T_a) = \mathcal{F}_M \exp[-U_M (\frac{1}{k_B T_a} - \frac{1}{k_B T_L})], \quad [3.0.4a]$$

$$\mathcal{F}(T_n) = \mathcal{F}_M \exp[-U_M (\frac{1}{k_B T_n} - \frac{1}{k_B T_L})]. \quad [3.0.4a]$$

Using X as a measure of the w_a deviation from its normal w_o value:

$$w_a \equiv w_o / (1 + X), \quad [3.0.5a]$$

$$\gamma_o \equiv [L_a / L_n], \quad [3.0.5b]$$

for small X , ignoring terms of $\vartheta(X^2)$, we showed:

$$(T_a - T_n) = 2 \left(\frac{T_L}{T_B} \right) (T_L - T_B) X, \quad [3.0.6a]$$

$$\ln[\mathcal{F}(T_a) / \mathcal{F}_M] = \quad [3.0.6b]$$

$$+ \frac{U_M}{k_B T_L} \left[\frac{2(T_L - T_B)}{T_B} \right] \left[\frac{2}{2 + \gamma_o} \right] X,$$

$$\ln[\mathcal{F}(T_n) / \mathcal{F}_M] = \quad [3.0.6c]$$

$$- \frac{U_M}{k_B T_L} \left[\frac{2(T_L - T_B)}{T_B} \right] \left[\frac{\gamma_o}{2 + \gamma_o} \right] X,$$

$$\ln[\mathcal{F}(T_a) / \mathcal{F}(T_n)] = + \frac{U_M}{k_B T_L} \left[\frac{2(T_L - T_B)}{T_B} \right] X, \quad [3.0.6d]$$

If the Eq. [3.0.5a] **geometry** changes are due to random manufacturing deviations about a mean, X becomes a random variable with a standard deviation

and zero mean. When X is a Gaussian, then the failure rates $\mathcal{F}(T_a)$ and $\mathcal{F}(T_n)$, and the Eq. [3.0.6d] ratio, all become Lognormal distributions.

4 References

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