Power Allocation for Reliable SIC Detection of Rectangular QAM-based NOMA Systems

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Abstract—This paper considers the power allocation for non-orthogonal multiple access (NOMA) users to enable using the successive interference cancellation (SIC) while providing reliable error performance. The derived closed-form expressions are applicable for arbitrary number of NOMA users each of which has a square or rectangular quadrature amplitude modulation (QAM) constellation with arbitrary order. The obtained numerical results show that power assignment process at the transmitter for the superposition process and at the receiver for the SIC process should be performed meticulously because the power difference between the weakest and strongest users can be tremendous when the number of users or the modulation orders increase. Moreover, the derived expressions can be used to reduce the computational complexity that is required to obtain the optimal power coefficients using brute force methods by significantly reducing the search space.

Index Terms—non-orthogonal multiple access (NOMA), successive interference cancellation (SIC), quadrature amplitude modulation (QAM), error floors, power assignment.

I. INTRODUCTION

DUE to the continuous demand for wireless services, significant efforts are currently devoted to design spectrally efficient multiple access (MAC) protocols. In particular, non-orthogonal multiple access (NOMA) has attracted significant attention due to its potential to serve more users using the same spectrum used by orthogonal MAC protocols. Nevertheless, NOMA generally has some challenges that should be resolved to achieve its full potential. Some of these challenges are the inter-user-interference (IUI), receiver complexity, and the requirement for meticulous power control [1].

In power-domain NOMA, the users' symbols are multiplexed in the power domain by assigning high power to users with poor channel conditions, and low power to users with good channel conditions. Therefore, without loss of generality, a NOMA system that serves N users whose channels' gains are $\alpha_1 < \alpha_2 < \dots < \alpha_N$ allocates the powers such that $P_1 > P_2 > \dots > P_N$. Consequently, the unique power assigned for each user enables the receiver to separate the data for each user using the maximum likelihood detector (MLD) [1], [2]. However, to use the less complex successive interference cancellation (SIC) detector, the power ordering is not sufficient to guarantee that symbols of all users will be detected reliably [3]. It is worth noting that both the MLD and

SIC detectors will have identical bit error rate (BER) given that the power assignment is performed such that the constellation for each user does not overlap with the constellations of other users [1].

Because the data for all users is multiplexed in the power domain, the power assignment for each user is a critical requirement for NOMA systems to offer reliable BER [1], [2], [4]–[8]. Generally speaking, the power assignment in the literature is performed to minimize the average BER of all users [4], or to satisfy the BER requirements for each user [1], [9]. Alternatively, the power can be assigned just to avoid the constellation overlapping for all users [2], [3], [10], [11]. While [2], [10], [11] derived the power ranges for the case of two-user NOMA using quadrature amplitude modulation (QAM) with arbitrary order, Aldababsa *et. al.* [12] derived the power assignment ranges for an arbitrary number of users each of which uses binary phase shift keying (BPSK). More recently, Kara and Kaya [3] derived the ranges for the more general case that considers an arbitrary number of users and QAM with arbitrary modulation order. However, the derived formulas are valid only for square QAM constellations.

Based on the aforementioned literature, and the references listed therein, it can be noted that computing the power ranges for NOMA systems with an arbitrary number of users and a rectangular QAM with arbitrary modulation order remains an open research problem. Therefore, by using a simple representation of constellation points for rectangular QAM, we show that efficient closed-form expressions can be derived. Consequently, the work reported in the existing literature can be considered as special cases of the results presented in this work.

II. SYSTEM MODEL

Consider a NOMA system with N users each of which is using a rectangular QAM with modulation order $M_i = 2^{k_i+1}$, $i \in \{1, 2, \dots, N\}$ and the integer $k_i \geq 1$. The superposition symbol transmitted by the basestation can be written as

$$X = \sum_{i=1}^N \beta_i x^{(i)}, \quad x \in \mathbb{X}^{(i)} \text{ and } X \in \mathbb{X} \quad (1)$$

where $\beta_i^2 \triangleq P_i$ is the power allocation coefficient for the i th user, $x^{(i)}$ is the baseband modulated symbol of the i th user with modulation order M_i and set of constellation points $\mathbb{X}^{(i)}$. The modulation order of symbol X is $\mathcal{M} = M_1 \times M_2 \times \dots \times M_N$ and the set of constellation points is $\mathbb{X}$.

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The power coefficients are typically assigned subject to,

$$\sum_{i=1}^N P_i = 1 \quad (2a)$$

$$\beta_i > \beta_{i+1} > 0, \forall i \in \{1, \dots, \bar{N}\} \quad (2b)$$

where $\bar{N} = N - 1$. Such assignment is necessary to provide fairness among different users by assigning far users more power. However, the constraints in (2a) and (2b) are not sufficient to guarantee that the NOMA system will offer a reliable error performance using SIC detector. The justification is that the SIC detector is designed to use a single user detector successively. Therefore, the constellation for any user should not cross certain limits in the x - y plane [3].

A. Coordinates of Rectangular QAM

For a rectangular QAM with modulation order M , there are $M/4$ constellation points in each quadrant. If we define

$$D_1 = 2^{\lceil \frac{1}{2} \log_2(M/4) \rceil} \quad (3)$$

and

$$D_2 = 2^{\lfloor \frac{1}{2} \log_2(M/4) \rfloor} \quad (4)$$

then the constellation points should be arranged over the rectangle $D_1 \times D_2$ or $D_2 \times D_1$. For the special case of a square QAM, $D_1 = D_2$. The binary information bits are mapped into M two-tuples (x_m, y_n) , each forming the transmitted symbol $s_{m,n}$. When the points are arranged over the rectangle $D_1 \times D_2$, the coordinates of the constellation point $s_{m,n}$ are given by,

$$(x_m, y_n) = \left(\frac{2m - 2D_1 - 1}{\Delta}, \frac{2n - 2D_2 - 1}{\Delta} \right) \quad (5)$$

where $m \in [1, \dots, D_1]$, $n \in [1, \dots, D_2]$, and Δ is a normalization factor chosen such that the average power of all symbols is unity,

$$\Delta = \sqrt{\frac{1}{M} \sum_m \sum_n [2m - 2D_1 - 1]^2 + [2n - 2D_2 - 1]^2}. \quad (6)$$

When the points are arranged over the rectangle $D_2 \times D_1$, the coordinates can be found using (5) and (6) while swapping D_1 and D_2 .

Without loss of generality, let the shape of the rectangle in the first quadrant be $D_x \times D_y$, thus, the coordinates of the constellation points are

$$(x_m, y_n) = \left(\frac{2m - 2D_x - 1}{\Delta}, \frac{2n - 2D_y - 1}{\Delta} \right) \quad (7)$$

and

$$\Delta = \sqrt{\frac{1}{M} \sum_m \sum_n [2m - 2D_x - 1]^2 + [2n - 2D_y - 1]^2}. \quad (8)$$

After some straightforward manipulations,

$$\Delta = \sqrt{\frac{8}{3M} D_x D_y [2D_x^2 + 2D_y^2 - 1]}. \quad (9)$$

Fig. 1: Various constellation shapes for rectangular QAM of a single user.

For a square QAM, $D_x = D_y$, and thus (9) becomes

$$\Delta = \sqrt{\frac{2}{3}(M - 1)} \quad (10)$$

B. The Basic Concept

Fig. 1 shows examples for a possible 8 and 16-QAM constellations for a single user. To prevent constellations overlap, these shapes should not cross the x and y -axes. However, due to the symmetry about the x and y -axis, it is sufficient to prevent overlap by preventing the solid circle in the figure from crossing the axes, i.e., it should remain in the same quadrant. Therefore, for the multiple users case, the power conditions can be based on the bottom-left constellation point of quadrant I for the first user U_1 , and the bottom-left constellation point of quadrant III for the other users as shown in Fig. 2. The quadrant in shades of red is the quadrant of interest for successive users. The figure shows the quadrant for U_2 , and U_3 . For example, if U_1 uses 16-QAM, then we need to consider symbol $S_{3,3}$ in Fig. 3. If U_2 also uses 16-QAM, then we need to consider symbol $S_{1,1}$.

Due to the nature of rectangular constellations, the diagonal constellation points are not necessarily symmetric. Therefore, the power constraints will be involving the real and imaginary parts of the coordinates of the solid circle in Fig. 1. For U_1 , the point of interest is in quadrant I where $m = D_x + 1$ and $n = D_y + 1$. The coordinates according to (7) are

$$(x_{D_x+1}, y_{D_y+1}) = \left(\frac{1}{\Delta}, \frac{1}{\Delta} \right). \quad (11)$$

The point of interest for the other users is the bottom-left constellation point in quadrant III where $m = 1$ and $n = 1$. The coordinates according to (7) are

$$(x_1, y_1) = \left(\frac{1 - 2D_x}{\Delta}, \frac{1 - 2D_y}{\Delta} \right). \quad (12)$$

Finally, the conditions should be applied recursively to enable assigning powers for the remaining users appropriately.

Fig. 2: Constellation diagram showing the regions of interest, $N = 3$.

Fig. 3: Labeled constellation points for a 16-QAM of a single user.

III. POWER ALLOCATION CONDITIONS

Consider a NOMA system with N users, modulation orders $\{M_1, M_2, \dots, M_N\}$, shapes $\{(D_x^{(1)} \times D_y^{(1)}), (D_x^{(2)} \times D_y^{(2)}), \dots, (D_x^{(N)} \times D_y^{(N)})\}$, and normalizing parameters $\{\Delta_1, \Delta_2, \dots, \Delta_N\}$. The condition for U_1 is:

$$\frac{\beta_1}{\Delta_1} > 0. \quad (13)$$

The conditions for U_2 are

$$\frac{\beta_1}{\Delta_1} + \frac{\beta_2}{\Delta_2} [1 - 2D_x^{(2)}] > 0 \quad (14a)$$

$$\frac{\beta_1}{\Delta_1} + \frac{\beta_2}{\Delta_2} [1 - 2D_y^{(2)}] > 0 \quad (14b)$$

and so forth. Therefore, the conditions considering the N users are given by,

$$\frac{\beta_1}{\Delta_1} > 0 \quad (15a)$$

$$\frac{\beta_1}{\Delta_1} + \sum_{i=2}^u \frac{\beta_i}{\Delta_i} [1 - 2D_x^{(i)}] > 0, \forall u \in \{2, 3, \dots, N\} \quad (15b)$$

$$\frac{\beta_1}{\Delta_1} + \sum_{i=2}^u \frac{\beta_i}{\Delta_i} [1 - 2D_y^{(i)}] > 0, \forall u \in \{2, 3, \dots, N\}. \quad (15c)$$

However, since the coordinates of the bottom-left constellation points in quadrant III are always negative, i.e., $\frac{[1-2D]}{\Delta} < 0$, then these conditions can be simplified to,

$$\frac{\beta_1}{\Delta_1} + \sum_{i=2}^N \frac{\beta_i}{\Delta_i} [1 - 2D_x^{(i)}] > 0 \quad (16a)$$

$$\frac{\beta_1}{\Delta_1} + \sum_{i=2}^N \frac{\beta_i}{\Delta_i} [1 - 2D_y^{(i)}] > 0. \quad (16b)$$

These conditions represent the coordinates of the solid circle from the perspective of U_1 that have to be positive, i.e., not crossing the x or y -axes. Furthermore, these conditions have to be satisfied recursively. Hence, the conditions for an M -ary rectangular QAM system with N users can be summarized as:

$$\frac{\beta_s}{\Delta_s} + \sum_{i=s+1}^N \frac{\beta_i}{\Delta_i} [1 - 2D_x^{(i)}] > 0, \forall s \in \{1, \dots, \bar{N}\} \quad (17a)$$

$$\frac{\beta_s}{\Delta_s} + \sum_{i=s+1}^N \frac{\beta_i}{\Delta_i} [1 - 2D_y^{(i)}] > 0, \forall s \in \{1, \dots, \bar{N}\} \quad (17b)$$

$$\frac{\beta_N}{\Delta_N} > 0. \quad (17c)$$

These conditions can be rewritten as (18a) and (18b).

In the case that all the users use square QAM, then $D_1 = D_2 = D$ and the conditions (18a) and (18b) are simplified to

$$\beta_u > \Delta_u \sum_{i=u+1}^N \frac{\beta_i}{\Delta_i} [2D^{(i)} - 1], \quad 1 \leq u \leq N-1 \quad (19a)$$

$$\beta_N > 0. \quad (19b)$$

For this case, the conditions are similar to those in [3, Eq. 7]. Moreover, if $M_i = M \forall i$, then

$$\beta_u > [2D - 1] \sum_{i=u+1}^N \beta_i, \quad 1 \leq u \leq N-1 \quad (20a)$$

$$\beta_N > 0. \quad (20b)$$

As an example, if $M = 4$ then $D = 1$ and the conditions (18a) and (18b) become

$$\beta_u > \sum_{i=u+1}^N \beta_i, \quad 1 \leq u \leq N-1 \quad (21a)$$

$$\beta_N > 0. \quad (21b)$$

IV. NUMERICAL RESULTS

This section presents the power ranges for some special cases of interest.

$$\beta_u > \Delta_u \max \left(\sum_{i=u+1}^N \frac{\beta_i}{\Delta_i} [2D_x^{(i)} - 1], \sum_{i=u+1}^N \frac{\beta_i}{\Delta_i} [2D_y^{(i)} - 1] \right), \quad 1 \leq u \leq N-1 \quad (18a)$$

$$\beta_N > 0 \quad (18b)$$

A. $N = 3$ and $M = 4$

In the case that $N = 3$ and $M_i = 4 \forall i$, then the conditions become

$$\beta_3 > 0 \quad (22a)$$

$$\beta_2 > \beta_3 \quad (22b)$$

$$\beta_1 > \beta_2 + \beta_3. \quad (22c)$$

Solving for β_1 , β_2 , and β_3 , gives

$$\beta_1 = 2\beta_3 + c + d \quad (23a)$$

$$\beta_2 = \beta_3 + c \quad (23b)$$

$$\beta_3 = \frac{\sqrt{24 - 12c^2 - 8d - 6c - 4d}}{12} \quad (23c)$$

with $0 < c < 1/\sqrt{2}$ and $0 < d < \sqrt{1 - c^2} - c$. This result perfectly fits the numerical representation of the space of possible solutions as shown in Fig. 4a, where the x -axis represents the value of P_3 and the y -axis shows the possible values of P_1 and P_2 . The figure also shows the optimal power values, white circles, reported in [4] for different channel conditions. As can be noted from the figure, the optimal power values fall within the derived power ranges. Fig. 4b shows that increasing the modulation order for the users reduces the power range significantly. Therefore, the derived ranges can be used to reduce the complexity of finding the optimal power allocation using brute force techniques. It is also worth noting that increasing P_3 reduces the ranges of P_1 and P_2 . Therefore, in the case that U_3 has low signal-to-noise ratio (SNR) it will reduce the the range of values that other users can choose from to optimize their performance.

B. Other Typical Results

The case of $N = 2$ is widely used in the literature, and hence, we present the power ranges for selected modulation orders where $M_1 = 4$ and $M_2 \in \{4, 8, 16\}$. In this case, the conditions are

$$\beta_1 > \beta_2 \frac{\Delta_1}{\Delta_2} [2D - 1] \quad (24a)$$

$$\beta_2 > 0. \quad (24b)$$

By substituting the values of Δ_1, Δ_2 and D we obtain $0 < P_2 < 0.5$, $0 < P_2 < 0.25$ and $0 < P_2 < 0.37$ for $M_2 = 4, 8, 16$, respectively.

Figs. 5 and 6 depict the ranges of possible power allocations when $N = 4$ for various values of M_i . The x -axis represents the value of P_4 and the y -axis shows the possible values of P_1 , P_2 and P_3 . As can be noted from the figure, the power ranges for P_1 , P_2 and P_3 are generally very narrow for high values of P_4 , particularly when $M_i > 4$ for any i . Consequently, the

knowledge if these ranges will enable the system designer to search for particular power assignment values using extremely small search step-size without significantly increasing the time and computational complexity.

V. CONCLUSION

This work presented a derivation of the power ranges that can be assigned for NOMA system with arbitrary number of users each of which uses a rectangular QAM with arbitrary modulation order. The power ranges obtained are presented in a simple closed-form expressions, and all the work reported in the existing literature can be considered as special cases of the results presented in this work. The obtained expressions are invaluable for power optimizations problems where the search space for the optimum power can be reduced significantly. Particularly for high SNRs because the power ranges become very narrow for all users. Similarly, increasing the number of users or the modulation orders reduce the possible power assignment to a very narrow range of values.

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(a) $M = \{4, 4, 4\}$ (b) $M = \{16, 16, 16\}$ Fig. 4: Space of possible PA for $N = 3$ and different values of M_i . (a) $M = \{4, 4, 4, 4\}$ (b) $M = \{4, 8, 16, 16\}$ Fig. 5: Space of possible PA for $N = 4$ and different values of M . (a) $M = \{8, 8, 16, 16\}$ (b) $M = \{16, 16, 16, 16\}$ Fig. 6: Space of possible PA for $N = 4$ and different values of M .