Power Allocation for Reliable SIC Detection of Rectangular QAM-based NOMA Systems

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Abstract—This paper considers the power allocation for non-orthogonal multiple access (NOMA) users to enable using the successive interference cancellation (SIC) while providing reliable error performance. The derived closed-form expressions are applicable for arbitrary number of NOMA users each of which has a square or rectangular quadrature amplitude modulation (QAM) constellation with arbitrary order. The obtained numerical results show that power assignment process at the transmitter for the superposition process and at the receiver for the SIC process should be performed meticulously because the power difference between the weakest and strongest users can be tremendous when the number of users or the modulation orders increase. Moreover, the derived expressions can be used to reduce the computational complexity that is required to obtain the optimal power coefficients using brute force methods by significantly reducing the search space.

Index Terms—non-orthogonal multiple access (NOMA), successive interference cancellation (SIC), quadrature amplitude modulation (QAM), error floors, power assignment.

I. INTRODUCTION

DUE to the continuous demand for wireless services, significant efforts are currently devoted to design spectrally efficient multiple access (MAC) protocols. In particular, non-orthogonal multiple access (NOMA) has attracted significant attention due to its potential to serve more users using the same spectrum used by orthogonal MAC protocols. Nevertheless, NOMA generally has some challenges that should be resolved to achieve its full potential. Some of these challenges are the inter-user-interference (IUI), receiver complexity, and the requirement for meticulous power control [1].

In [R3,1] downlink power-domain NOMA, the users' symbols are multiplexed at the base station in the power domain by assigning high power to users with poor channel conditions, and low power to users with good channel conditions. Therefore, without loss of generality, a NOMA system that serves N users whose channels' gains are $\alpha_1 < \alpha_2 < \dots < \alpha_N$ allocates the powers such that $P_1 > P_2 > \dots > P_N$. Consequently, the unique power assigned for each user enables the receiver to separate the data for each user using the maximum likelihood detector (MLD) [1], [2]. However, to use the less complex successive interference cancellation (SIC) detector, the power ordering is not sufficient to guarantee that symbols of all users will be detected reliably [3]. It is

worth noting that both the MLD and SIC detectors will have identical bit error rate (BER) given that the power assignment is performed such that the constellation for each user does not overlap with the constellations of other users [1].

Because the data for all users is multiplexed in the power domain, the power assignment for each user is a critical requirement for NOMA systems to offer reliable BER [1], [2], [4]–[8]. Generally speaking, the power assignment in the literature is performed to minimize the average BER of all users [4], or to satisfy the BER requirements for each user [1], [9]. Alternatively, the power can be assigned just to avoid the constellation overlapping for all users [2], [3], [10]–[12]. While [2], [11], [12] derived the power ranges for the case of two-user NOMA using quadrature amplitude modulation (QAM) with arbitrary order, Aldababsa *et. al.* [13] derived the power assignment ranges for an arbitrary number of users each of which uses binary phase shift keying (BPSK). More recently, Kara and Kaya [3] derived the ranges for the more general case that considers an arbitrary number of users and QAM with arbitrary modulation order. However, the derived formulas are valid only for square QAM constellations.

[R2,2] Although the square QAM is widely used in practical applications, the rectangular QAM is commonly preferred for transmitting odd number of bits per symbol, because its power efficiency is significantly higher than phase shift keying (PSK), particularly for high modulation orders [14]. Therefore, the rectangular QAM has been also adopted for various applications such as digital video broadcasting (DVB) and digital multi-program television networks [14], [15]. Moreover, the low complexity modulation and demodulation made it attractive for high rate transmissions [16].

Based on the aforementioned literature, and the references listed therein, it can be noted that computing the power ranges for NOMA systems with an arbitrary number of users and a rectangular QAM with arbitrary modulation order remains an open research problem. Therefore, [R3,2] [R1,3] the contribution of this correspondence can be summarized as:

- Derived the power ranges for downlink NOMA systems to enable the SIC receiver to work without constellation points overlap, and thus, avoid severe performance degradation.
- Using a generalized representation of constellation points for rectangular QAM, efficient closed-form expressions are recursively derived for NOMA systems with arbitrary number of users where each of which uses QAM with an arbitrary modulation order. Consequently, the work reported in the existing literature can be considered as special cases of the results presented in this work.

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- The derived expressions are further simplified for several cases of interest.
- The relation between power ranges and optimum power allocation is demonstrated graphically for the three users scenario.
- The obtained results are used to provide interesting insights about the power allocation for NOMA when the number of users or the modulation are large.

II. SYSTEM MODEL

Consider $[R_{3,1}]$ a downlink NOMA system with N users each of which is using a rectangular QAM with modulation order $M_i = 2^{k_i+1}$, $i \in \{1, 2, \dots, N\}$ and the integer $k_i \geq 1$. The superposition symbol transmitted by the base station can be written as

$$X = \sum_{i=1}^N \beta_i x^{(i)}, \quad x \in \mathbb{X}^{(i)} \text{ and } X \in \mathbb{X} \quad (1)$$

where $\beta_i^2 \triangleq P_i$ is the power allocation coefficient for the i th user, $x^{(i)}$ is the baseband modulated symbol of the i th user with modulation order M_i and set of constellation points $\mathbb{X}^{(i)}$. The modulation order of symbol X is $\mathcal{M} = M_1 \times M_2 \times \dots \times M_N$ and the set of constellation points is $\mathbb{X}$.

The power coefficients are typically assigned subject to,

$$\sum_{i=1}^N P_i = 1 \quad (2a)$$

$$\beta_i > \beta_{i+1} > 0, \quad \forall i \in \{1, \dots, \bar{N}\} \quad (2b)$$

where $\bar{N} = N - 1$. Such assignment is necessary to provide fairness among different users by assigning far users more power. Therefore, (2a) and (2b) should be always satisfied throughout this article, even if they are not stated explicitly. However, these constraints are not sufficient to guarantee that the NOMA system will offer a reliable error performance using SIC detector. The justification is that the SIC detector is designed to use a single user detector successively. Therefore, the constellation for any user should not cross certain limits in the x - y plane [3]. $[R_{1,s}]$ Fig. 1 shows two examples for constellation points overlap even when (2a) and (2b) are satisfied. The solid circles in the figure represent the cases where $P_1 \gg P_2$, and thus, there is no constellation points overlap. The other constellation points are for the cases where $P_1 > P_2$, but there is constellation points overlap. The constellation points that are supposed to be in the same quadrant are given the same shape. The overlap regions in both figures are shown as the shaded rectangles.

A. Coordinates of Rectangular QAM

For a rectangular QAM with modulation order M , there are $M/4$ constellation points in each quadrant of the x - y plane. If we define

$$D_1 = 2^{\lceil \frac{1}{2} \log_2(M/4) \rceil} \quad (3)$$

and

$$D_2 = 2^{\lfloor \frac{1}{2} \log_2(M/4) \rfloor} \quad (4)$$

$[R_{3,3}][R_{1,5}]$ then the constellation points should be arranged over a rectangular $D_1 \times D_2$ or $D_2 \times D_1$ grid. In other words, if the constellation points in a quadrant are seen as points in a rectangular grid, D_1 will be the number of points on the side of the grid along the x -axis, and D_2 will be the number of points on the side of the grid along the y -axis. For the special case of a square QAM, we have $D_1 = D_2$. As an example, if we consider 16-QAM, in each quadrant there are 2×2 constellation points. In this case, $D_1 = D_2 = 2$. In the case of 32-QAM, the constellation points in each quadrant can be arranged as a grid of 4×2 points. In this case, $D_1 = 4$ and $D_2 = 2$. Note that 32-QAM can also be arranged as a 2×4 grid in each quadrant. In this case $D_1 = 2$ and $D_2 = 4$.

The binary information bits are mapped into M two-tuples (x_m, y_n) , each forming the transmitted symbol $s_{m,n}$. When the points are arranged over the rectangle $D_1 \times D_2$, the coordinates of the constellation point $s_{m,n}$ are given by,

$$(x_m, y_n) = \left(\frac{2m - 2D_1 - 1}{\Delta}, \frac{2n - 2D_2 - 1}{\Delta} \right) \quad (5)$$

where $m \in [1, \dots, 2D_1]$, $n \in [1, \dots, 2D_2]$, and Δ is a normalization factor chosen such that the average power of all symbols is unity,

$$\Delta = \sqrt{\frac{1}{M} \sum_m \sum_n [2m - 2D_1 - 1]^2 + [2n - 2D_2 - 1]^2}. \quad (6)$$

When the points are arranged over the rectangle $D_2 \times D_1$, the coordinates can be found using (5) and (6) while swapping D_1 and D_2 .

Without loss of generality, let the shape of the rectangle in Quadrant I be $D_x \times D_y$, thus, the coordinates of the constellation points are

$$(x_m, y_n) = \left(\frac{2m - 2D_x - 1}{\Delta}, \frac{2n - 2D_y - 1}{\Delta} \right) \quad (7)$$

and

$$\Delta = \sqrt{\frac{1}{M} \sum_m \sum_n [2m - 2D_x - 1]^2 + [2n - 2D_y - 1]^2}. \quad (8)$$

After some straightforward manipulations,

$$\Delta = \sqrt{\frac{8}{3M} D_x D_y [2D_x^2 + 2D_y^2 - 1]}. \quad (9)$$

For a square QAM, $D_x = D_y$, and thus (9) becomes

$$\Delta = \sqrt{\frac{2}{3}} (M - 1). \quad (10)$$

B. NOMA Power Allocation Basic Concept

$[R_{2,3}][R_{1,s}]$ Fig. 1 shows examples for two NOMA configurations where $N = 2$. In both cases $M_1 = 4$, while $M_2 = 8$ in a) and 16 in b). To prevent constellations overlap, the closest points to the x and/or y -axis should not cross the x and/or y -axes. This point will be termed as the critical point, and it is represented by the solid circle which is marked by the arrow in Quadrant I. Although there are four critical points

Figure 1: Example of constellation points overlap for $N = 2$ and $M_1 = 4$. a) $M_2 = 8$, $P_1 = 0.9, 0.51$. b) $M_2 = 16$, $P_1 = 0.9, 0.6$.

for this configuration, it is sufficient to consider only the point in Quadrant I due to the constellation symmetry about the x and y -axis. Therefore, the power conditions can be calculated based on the critical point. Another example for $N = 3$ is depicted in Fig. 2 where $M_1 = M_2 = M_3 = 4$. As can be noted from this figure, because $N = 3$, there are two critical points in Quadrant I, marked by white circles in the solid squares. Therefore, to prevent overlap, the critical point that is close to the x - y axis should not cross the x - y axis, and the second point should not cross the virtual x - y axis marked by the dashed lines.

$[R_{2,3}][R_{3,3}][R_{1,7}]$ The power allocation constraints are derived from the fact that the critical point should not cross the x - y axis. For this we need to compute the coordinates of the critical point. These coordinates can be computed from the coordinates of specific constellation points of the involved users. For user 1, the point of interest is the bottom-left constellation point in Quadrant I. For all the other users, the point of interest is the bottom-left constellation point in Quadrant III. For example, for a three-user NOMA using $M_1 = M_2 = M_3 = 4$, the coordinates of the critical point are $\beta_1 s_{2,2}^{(1)} + \beta_2 s_{1,1}^{(2)} + \beta_3 s_{1,1}^{(3)}$, where $s_{m,n}^{(k)}$ are the coordinates of the constellation point $s_{m,n}$ of user k as shown in Fig. 3 for the QPSK case. And for a three-user NOMA using $M_1 = M_2 = M_3 = 16$, the coordinates of the critical point are $\beta_1 s_{3,3}^{(1)} + \beta_2 s_{1,1}^{(2)} + \beta_3 s_{1,1}^{(3)}$, where $s_{m,n}^{(k)}$ are the coordinates of the constellation point $s_{m,n}$ of user k for the 16-QAM case.

Due to the nature of rectangular QAM, the constellation points are not necessarily symmetric about the diagonals in the x - y plane. Therefore, the power constraints will be involving the x - and y -coordinates of the critical point. More specifically, for user 1, the point of interest is in Quadrant I where $m = D_x + 1$ and $n = D_y + 1$. The coordinates of the point of

Figure 2: Constellation diagram of a three-user NOMA using $M_1 = M_2 = M_3 = 4$ showing various constellation points boundaries, $P_1 = 0.73$, $P_2 = 0.25$, and $P_3 = 0.02$.

Figure 3: Labeled constellation points for QPSK and 16-QAM.

interest according to (7) are,

$$(x_{D_x+1}, y_{D_y+1}) = \left(\frac{1}{\Delta}, \frac{1}{\Delta} \right). \quad (11)$$

The point of interest for the other users is the bottom-left constellation point in Quadrant III where $m = 1$ and $n = 1$. The coordinates of the point of interest according to (7) are

$$(x_1, y_1) = \left(\frac{1 - 2D_x}{\Delta}, \frac{1 - 2D_y}{\Delta} \right). \quad (12)$$

Finally, the conditions should be applied recursively to enable assigning powers for the remaining users appropriately.

III. POWER ALLOCATION CONDITIONS

Consider a NOMA system with N users $\{U_1, U_2, \dots, U_N\}$, modulation orders $\{M_1, M_2, \dots, M_N\}$, shapes $\{(D_x^{(1)} \times D_y^{(1)}), (D_x^{(2)} \times D_y^{(2)}), \dots, (D_x^{(N)} \times D_y^{(N)})\}$, and normalizing parameters $\{\Delta_1, \Delta_2, \dots, \Delta_N\}$. Given that all users adopt a rectangular QAM constellation, then the power assignment for all users should satisfy the conditions given in the following proposition:

(a) $M = \{4, 4, 4\}$ (b) $M = \{16, 16, 16\}$ Figure 4: Space of possible power allocation for $N = 3$ and different values of M_i .

Proposition 1. ^[R2,3] *The power assignment for a NOMA system with N users each of which is using a rectangular QAM with order M_i , $i = 1, 2, \dots, N$ should satisfy the following conditions:*

$$\beta_u > \Delta_u \max \left(\sum_{i=u+1}^N \frac{\beta_i}{\Delta_i} [2D_x^{(i)} - 1], \sum_{i=u+1}^N \frac{\beta_i}{\Delta_i} [2D_y^{(i)} - 1] \right), \quad u = 1, 2, \dots, \bar{N} \quad (13a)$$

$$\beta_N > 0 \quad (13b)$$

$$\sum_{i=1}^N \beta_i^2 = 1. \quad (13c)$$

Proof. The condition for U_1 is:

$$\frac{\beta_1}{\Delta_1} > 0. \quad (14)$$

The conditions for U_2 are

$$\frac{\beta_1}{\Delta_1} + \frac{\beta_2}{\Delta_2} [1 - 2D_x^{(2)}] > 0 \quad (15a)$$

$$\frac{\beta_1}{\Delta_1} + \frac{\beta_2}{\Delta_2} [1 - 2D_y^{(2)}] > 0 \quad (15b)$$

and so forth. Therefore, the conditions considering the N users are given by,

$$\frac{\beta_1}{\Delta_1} > 0 \quad (16a)$$

$$\frac{\beta_1}{\Delta_1} + \sum_{i=2}^u \frac{\beta_i}{\Delta_i} [1 - 2D_x^{(i)}] > 0, \forall u \in \{2, 3, \dots, N\} \quad (16b)$$

$$\frac{\beta_1}{\Delta_1} + \sum_{i=2}^u \frac{\beta_i}{\Delta_i} [1 - 2D_y^{(i)}] > 0, \forall u \in \{2, 3, \dots, N\}. \quad (16c)$$

However, since the coordinates of the bottom-left constellation point in Quadrant III are negative, i.e., $(1 - 2D)/\Delta < 0$, then these conditions can be simplified to,

$$\frac{\beta_1}{\Delta_1} + \sum_{i=2}^N \frac{\beta_i}{\Delta_i} [1 - 2D_x^{(i)}] > 0 \quad (17a)$$

$$\frac{\beta_1}{\Delta_1} + \sum_{i=2}^N \frac{\beta_i}{\Delta_i} [1 - 2D_y^{(i)}] > 0. \quad (17b)$$

^[R1,7] These are the conditions on the coordinates of the critical point from the perspective of U_1 that have to be positive, i.e., the critical point not crossing the x or y -axes. Furthermore, these conditions have to be satisfied recursively for a system without user 1 and so on. Hence, the conditions for an M -ary rectangular QAM system with N users can be summarized as:

$$\frac{\beta_s}{\Delta_s} + \sum_{i=s+1}^N \frac{\beta_i}{\Delta_i} [1 - 2D_x^{(i)}] > 0, \forall s \in \{1, \dots, \bar{N}\} \quad (18a)$$

$$\frac{\beta_s}{\Delta_s} + \sum_{i=s+1}^N \frac{\beta_i}{\Delta_i} [1 - 2D_y^{(i)}] > 0, \forall s \in \{1, \dots, \bar{N}\} \quad (18b)$$

$$\frac{\beta_N}{\Delta_N} > 0. \quad (18c)$$

These conditions can be rewritten as (13a) and (13b). $\square$

For the case that all users use square QAM, $D_1 = D_2 = D$ and the conditions (13a) and (13b) are simplified to

$$\beta_u > \Delta_u \sum_{i=u+1}^N \frac{\beta_i}{\Delta_i} [2D^{(i)} - 1], \quad 1 \leq u \leq \bar{N} \quad (19a)$$

$$\beta_N > 0. \quad (19b)$$

For this case, the conditions are similar to those in [3, Eq. 7]. Moreover, if $M_i = M \forall i$, then the conditions become

$$\beta_u > [2D - 1] \sum_{i=u+1}^N \beta_i, \quad 1 \leq u \leq \bar{N} \quad (20a)$$

$$\beta_N > 0. \quad (20b)$$

As an example, if $M = 4$ then $D = 1$ and the conditions (13a) and (13b) become

$$\beta_u > \sum_{i=u+1}^N \beta_i, \quad 1 \leq u \leq \bar{N} \quad (21a)$$

$$\beta_N > 0. \quad (21b)$$

Figure 5: $[R_{2,4}][R_{3,5}][R_{3,6}]$ Comparison between the proposed system and [3].

IV. NUMERICAL RESULTS

This section presents the power ranges for some special cases of interest.

A. $N = 3$ and $M = 4$

In the case that $N = 3$ and $M_i = 4 \forall i$, the conditions become

$$\beta_3 > 0 \quad (22a)$$

$$\beta_2 > \beta_3 \quad (22b)$$

$$\beta_1 > \beta_2 + \beta_3. \quad (22c)$$

Solving for β_1 , β_2 , and β_3 , gives

$$\beta_1 = 2\beta_3 + c + d \quad (23a)$$

$$\beta_2 = \beta_3 + c \quad (23b)$$

$$\beta_3 = (\sqrt{6 - 3c^2 - 2d - 3c - 2d})/6 \quad (23c)$$

with $0 < c < 1/\sqrt{2}$ and $0 < d < \sqrt{1 - c^2} - c$. This result perfectly fits the numerical representation of the space of possible solutions as shown in Fig. 4a, where the x -axis represents the value of P_3 and the y -axis shows the possible values of P_1 and P_2 . The figure also shows the optimal power values, white circles, reported in [4] for different channel conditions. As can be noted from the figure, the optimal power values fall within the derived power ranges. Fig. 4b shows that increasing the modulation order for the users reduces the power range significantly. Therefore, the derived ranges can be used to reduce the complexity of finding the optimal power allocation using brute force techniques. It is also worth noting that increasing P_3 reduces the ranges of P_1 and P_2 . Therefore, in the case that U_3 has low signal-to-noise ratio (SNR) it will reduce the range of values that other users can choose from to optimize their performance.

B. Other Typical Results

The case of $N = 2$ is widely used in the literature, and hence, we present the power ranges for selected modulation

orders where $M_1 = 4$ and $M_2 \in \{4, 8, 16, 32\}$. In this case, the conditions are

$$\beta_1 > \beta_2 \frac{\Delta_1}{\Delta_2} \max \left\{ [2D_x^{(2)} - 1], [2D_y^{(2)} - 1] \right\} \quad (24a)$$

$$\beta_2 > 0. \quad (24b)$$

$[R_{2,4}][R_{3,6}]$ Fig. 5 shows a comparison of the power allocation ranges produced by our proposed system to those produced by [3] for different square and rectangular QAM scenarios. Specifically for the cases where $M_1 = 4$ and $M_2 \in \{4, 8, 16, 32\}$. As expected, for square QAM, $M_2 \in \{4, 16\}$, the power ranges are exactly the same. For rectangular QAM, $M_2 \in \{8, 32\}$, the ranges produced by [3] are larger as shown in the figure where the green line exceeds the red line. In those cases, even if the conditions proposed in [3] are satisfied, there will be overlap where some constellation points cross some decision boundaries.

Figs. 6 and 7 depict the ranges of possible power allocations when $N = 4$ for various values of M_i . The x -axis represents the value of P_4 and the y -axis shows the possible values of P_1 , P_2 and P_3 . As can be noted from the figure, the power ranges for P_1 , P_2 and P_3 are generally very narrow for high values of P_4 , particularly when $M_i > 4$ for any i . Consequently, the knowledge of these ranges will enable the system designer to search for particular power assignment values using extremely small search step-size without significantly increasing the time and computational complexity.

V. CONCLUSION

This work presented a derivation of the power ranges that can be assigned for NOMA systems with arbitrary number of users each of which uses a rectangular QAM with arbitrary modulation order. The power ranges obtained are presented in simple closed-form expressions, and all the work reported in the existing literature can be considered as special cases of the results presented in this work. The obtained expressions are invaluable for power optimizations problems where the search space for the optimum power can be reduced significantly, particularly when assigning the nearest user a high power. Similarly, increasing the number of users or the modulation

Figure 6: Space of possible power allocation for $N = 4$ and different values of M . Figure 7: Space of possible power allocation for $N = 4$ and different values of M .

orders reduces the possible power assignment to a very narrow range of values.

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Response to Reviewers' Comments

Power Allocation for Reliable SIC Detection of Rectangular QAM-based NOMA Systems (VT-2021-00242)

Dear Prof Zhou.

We would like to thank you for the professional handling of the review process of our paper, and for giving us the chance to address the reviewers' comments and resubmit the paper. We also thank the reviewers for the time, effort and valuable comments.

We found the comments reasonable and constructive, and thus, all comments have been addressed diligently. Therefore, we hereby submit the revised version of the paper along with a point-for-point response to the reviewers' comments. Moreover, all changes have been clearly marked and linked to the reviewers' comments. For the convenience of the editor and reviewers, we have clearly highlighted all the amendments made to the paper using color coding, where the response to each reviewer is given a particular color. Moreover, for each response, we added the comment and reviewer numbers, for example, [\[R_{1,2}\]](#) corresponds to changes made in response to Reviewer 1 comment 2.

Sincerely yours,

On behalf of the authors,
Dr A. Al-Dweik, SM, IEEE, P. Eng.
Department of Electrical and Computer Engineering
Western University
London, ON, Canada

Reviewer 1:

We thank the reviewer for the time, effort and valuable comments. We believe that all comments are valuable and contribute to improving the paper presentation and enriching its contents. Therefore, we have addressed all comments diligently. Please see the detailed response below.

- 1) Is this submission a corresponding article or not? It seems to the reviewer that the length is too short for a regular submission.

Indeed, it is a correspondence.

- 2) There are some other studies on QAM-based NOMA systems, the authors may want to discuss the difference between this study and the existing ones, for instance, [1] "Sparse Vector Coding-Based Multi-Carrier NOMA for In-Home Health Networks," in IEEE Journal on Selected Areas in Communications, vol. 39, no. 2, pp. 325-337, Feb. 2021.

We thank the reviewer for referring this article. This article has been cited as suggested, please see [10] in the updated manuscript. Please note that the power is selected in [10] such that the constellations for both users do not overlap without specifying the possible power ranges. Also, the system considers only the two-user scenario.

- 3) The contribution can be summarized with some items.

We thank the reviewer for this suggestion. In response, we have listed the contribution in an itemized manner. Please see [R_{1,3}] on page 1.

- 4) We generally allocate the power values with an inversed order as the channel gains to maintain the fairness among different users. But in other purpose, for instance, to maximum the system throughput of NOMA-based systems, we do not allocate the power in such a manner.

We agree with the reviewer interesting remark. However, please note that the relation between the powers allocated for the users should be maintained to enable the SIC to operate correctly. But the order, as the reviewer hinted, does not have to be the inverse of the channel gain.

- 5) What do you mean by D_1, D_2 in equations (3,4)?

We thank the reviewer for this comment. We have elaborated on the definition to increase the clarity with examples. Please see the modifications labeled [R_{1,5}] on page 2.

- 6) There are no other constellation points in Quadrant III in Fig. 2 as claimed. Additionally, the authors need to explain what different colors are used for in Quadrant I. Please also note the first letter of quadrant are respectively with uppercase and lower case in Fig. 2 and the main text.

We appreciate the reviewer comment and agree that the figure and the corresponding discussions should be improved. Therefore, we have updated Fig. 2 and all the corresponding discussions.

- 7) What do you mean by $S_{3,3}, S_{1,1}$ and other symbols? There are many symbols used without definition, which makes this submission hard to follow. Please double check.

We thank the reviewer for this comment. We have corrected a typo where we referred to $s_{1,1}$ as $S_{1,1}$. We have also added additional explanations to clarify the used symbols. Please see the modifications labeled [R_{1,7}] on pages 3 and 4.

- 8) The reviewer cannot follow what the authors want to explain in subsection II. B.

We agree that discussion in section II-B should be improved for more clarity. In response to this comment, we have updated Fig. 1, Fig. 2, and the corresponding discussions. Please see the updated subsection on page 2.

- 9) What a specific problem have you solved from this study?

We are deriving a generalized approach for power allocation of NOMA with any number of users using rectangular QAM with any modulation order.

10) High-definitional figures are required in IEEE publications.

We thank the reviewer for this comment. We have reproduced all the figures with high resolution and more clarity.

Reviewer 2:

We thank the reviewer for the time, effort and valuable comments. We believe that all comments are valuable and contribute to improving the paper presentation and enriching its contents. Therefore, we have addressed all comments diligently. Please see the detailed response below.

This paper considers the power allocation for NOMA systems, and derive closed-form solutions specifically for an arbitrary number of users and a rectangular QAM with arbitrary modulation order. Numerical results have demonstrated promising error rate performance. My comments are below:

- 1) What is the major design challenges regarding the power allocation scheme from the case of two users to a general case of K users? This is critical to justify the novelty/contribution of this work.

We thank the reviewer for this comment. For the two-user scenario, there are only two conditions that should be satisfied: 1) $P_1 + P_2 = 1$ and 2) The coordinates of the critical point should be greater than zero. For $N > 2$, there are $N - 1$ critical points that should not cross $N - 1$ particular regions in the x - y plane. Finding the coordinates for all critical points and their corresponding regions in the x - y plane for arbitrary N and M of a rectangular QAM is a challenging problem, which we solved in this correspondence. To the best of our knowledge, this is the first work that solves this problem for such a generalized conditions.

- 2) I'm a bit confused by the description regarding the 'Rectangular QAM'. Based on my opinion, I think square QAM is the mostly considered QAM constellation that is of interest. What are the benefits of other Rectangular QAM types shown in Fig. 1 over square QAM? To further motivate Rectangular QAM, the authors should perhaps discuss its potential application scenarios.

We appreciate the reviewer comment, and in response, we have added a new paragraph that discusses the benefit of rectangular QAM. Please [R_{2,2}] on page 1, and the newly added references [14]–[16].

- 3) The descriptions in Section II-B and Section III can be improved. Since the paper considers a general power allocation scheme for M-QAM constellation, I suggest that the authors present the result as a 'Proposition', in which the closed-form power values are clearly shown as functions of the user number and modulation order.

We thank the reviewer for the interesting comment. The proposition has been added as suggested. Please see Proposition 1 on page 3. Moreover, we have thoroughly revised and updated several parts of the paper including Section II-B and Section III.

- 4) The proposed scheme lacks a numerical comparison to existing works. Based on the introduction, there indeed exist several very relevant works, which should be compared.

We thank the reviewer for this comment. We have compared the power allocation ranges produced by our proposed system to those produced by [3] for different square and rectangular QAM scenarios. The results are shown in Fig. 5. The associated explanation can be found in the paragraph labeled [R_{2,4}] on page 5.

Reviewer 3:

We thank the reviewer for the time, effort and valuable comments. We believe that all comments are valuable and contribute to improving the paper presentation and enriching its contents. Therefore, we have addressed all comments diligently. Please see the detailed response below.

- 1) According to the application scenario, NOMA can be divided into uplink NOMA and downlink NOMA. There are differences in power allocation principles in different application scenarios. Application scenario of this paper is downlink NOMA. In order to increase the readability of the article and avoid misunderstanding, it is necessary to explain the application scene clearly. These questions arise in detail in line 41 on page 1.

We thank the reviewer for the careful reading. We agree with the reviewer comment, and in response, we have explicitly stated that our work considers the downlink NOMA. Please see $[R_{3,1}]$ on pages 1 and 2.

- 2) In the introduction, the author draws a solution according to the research status of the existing related technologies. Noting that computing the power ranges for NOMA systems with an arbitrary number of users and a rectangular QAM with arbitrary modulation order remains an open research problem. And the work reported in the existing literature can be considered as special cases of the results presented in this work. These only highlight the novelty of the paper and do not indicate whether the research content of the paper is valuable. The reviewer think the contribution of the paper can be briefly described at the end of the introduction.

We thank the reviewer for the valuable comment. In response, we have clearly listed the main contributions of the paper. Please see $[R_{3,2}]$ on page 1.

- 3) In order to increase the readability of the paper, it is necessary to explain each physical quantity that appears in the paper, such as D1 and D2 .

We thank the reviewer for this comment. We have elaborated on the definition to increase the readability with examples. We have also added additional explanations to clarify other used symbols. Please see the modifications labeled $[R_{3,3}]$ on pages 2 and 3.

- 4) How to prove the equivalence of inequality group (22) and equality group (23).

We thank the reviewer for this comment. The inequality group (22) can be solved using any analytical package like Matlab.

- 5) The B part of the NUMERICAL RESULTS begins with only the numerical analysis of a variety of cases where the $N = 2$, $M1 = 4$ and $M2 \in \{4, 8, 16\}$. If the corresponding result diagram is drawn, will it be more vivid ?

We thank the reviewer for this comment. We have represented the results in diagrams added in Fig. 5.

- 6) The contribution and uniqueness of this paper can be highlighted by comparing the performance of rectangular QAM and square QAM horizontally in the NUMERICAL

We thank the reviewer for this comment. We have compared the power allocation ranges produced by our proposed system to those produced by [3] for different square and rectangular QAM scenarios. The results are shown in Fig. 5. The associated explanation can be found in the paragraph labeled $[R_{3,6}]$ on page 5.