Product of κ - μ and α - μ Distributions and Their Composite Fading Distributions

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Abstract—In this study, product of two independent and non-identically distributed (i.n.i.d.) random variables (RVs) for κ - μ fading distribution and α - μ fading distribution is considered. The novel exact series formulas for the product of two i.n.i.d. fading distributions κ - μ and α - μ are derived instead of Fox H-function to solve the problem that Fox H-function with multiple RVs cannot be implemented in professional mathematical software packages as MATHEMATICA and MAPLE. Exact close-form expressions of probability density function (PDF) and cumulative distribution function (CDF) are deduced to represent provided product expressions and generalized composite multipath shadowing models. At last, these analytical results are validated with Monte Carlo simulations, it shows that for provided κ - μ / α - μ model non-linear parameter has more important influence than multipath component in PDF and CDF when the ratio between the total power of the dominant components and the total power of the scattered waves is same.

Keywords—Composite fading channels, κ - μ distribution, α - μ distribution, cognitive radios networks (CRs)

I. INTRODUCTION

A. Related works

In cognitive radios networks (CRs) wireless communication systems, there have been a number of important studies with regard to the product of random variables (RVs), which is applied to optimize spectrum efficiency and evaluate performance of communication systems [1]-[4]. Different models of the product with multiple RVs have been universally investigated to model high-resolution synthetic radar clutter [5], obtain the channel gain and keyhole effects over cascaded fading channels in multiple-input multiple-output (MIMO) [6], [7], and combine long term fading and short term fading in vehicle-to-vehicle communications and body area networks [8]-[12].

The composite fading distributions describe the common fading phenomena for long term fading and short term fading in non-stationary stochastic environments [8], [13], it is more practical to jointly represent the large-scale and small-scale fading characteristics in realistic wireless fading channels. The short term fading can be observed within short distance with few wavelengths, such as κ - μ , η - μ , α - μ , Rayleigh, Nakagami- m and Weibull [14], [15], and in general the long term fading is

described by lognormal distribution which can be replaced by α - μ distribution (consists of special cases as gamma, negative exponential, Nakagami- m , Weibull, one-sided Gaussian and Rayleigh) [8], [9]. The κ - μ distribution is a generalized small-scale line-of-sight (LOS) multipath fading distribution. With the aid of circularly symmetric random variables, the κ - μ distribution can model the scattering cluster in homogeneous communication environments and it includes Rice ($\kappa=k$, $\mu=1$), Rayleigh ($\kappa=0$, $\mu=1$), Nakagami- m ($\kappa=0$, $\mu=m$) and one-sided Gaussian distributions ($\kappa=0$, $\mu=0.5$) for two fading parameters κ and μ [3], [4], [15]. Moreover, the α - μ distribution is general fading model which describes the signal is made up of multipath waves clusters in non-linear fading condition, and it can better accommodate statistical characteristic variations for non-linear signal because of its flexibility. Besides, a large number of field testing results show that gamma distribution can accurately approximate the lognormal distribution to solve the problem that the close-form expressions of lognormal process is hard to obtain [8], [9], [16].

B. Main contributions

Although there are many papers about the product method for fading models, such as, the PDF and CDF for the product model of multiple α - μ variates have been investigated in [8], [9] by calculating finite sum of hypergeometric functions. The MATHEMATICA routine for product of Nakagami- m distributions has been proposed to deal the problem that Fox H-function could not overall be implemented in MATHEMATICA and MAPLE [18]. The CDF and PDF for product of three α - μ variates are derived by using Meijer G-function [19]. Furthermore, in [12] the general structure for PDF and CDF with Fox H-function has been derived as [12, Eq. (1), Eq. (15) and Eq. (30)] with residues calculus, and outage capacity [12, Eq. (27)] and detection probability in a UHF (Ultra High Frequency) RFID (Radio Frequency Identification) system [12, Eq. (28)] have been derived by the form of integrals, however, the exact close-form series expressions for κ - μ / α - μ and η - μ / α - μ with generalized hypergeometric function have not been derived in [12, Table III and Table IV] and [12, Table III and Table IV] are series cases for product of fading models with Gauss hypergeometric function [17, Eq. (15.1.1)]. At the same time, in [12] the conclusion states that ‘there are maybe other more exact series expressions but with more algebraic manipulations’ for the unknown product models, besides, the single-variable and multi-variable Fox H-functions are not yet available in MATHEMATICA and MAPLE.

Motivated by above, as series formulas with generalized hypergeometric functions provide more precise evaluations algebraically, and single-variable and multi-variable Fox-H functions are not executable in MATHEMATICA, besides there are no studies related to close-form series expressions for the product of two i.n.i.d. RVs with κ - μ and α - μ distributions in the open literature. Hence, in this paper, the generalized product models κ - μ/α - μ have been proposed to approximate generalized composite multipath shadowing channels. The main contributions of this paper are summarized as follows:

- This paper provides exact series product model κ - μ/α - μ for two i.n.i.d. RVs with fading distributions by employing generalized hypergeometric function instead of Fox H-function [12], which describes the intertwined effects of line-of-sight (LOS) short-term fading κ - μ distribution and long-term fading distribution that is substituted by α - μ distribution. The provided novel model can consider special cases, for example, Rice/ α - μ , Rayleigh/ α - μ , Nakagami- m/α - μ and one-sided Gaussian/ α - μ (where α - μ distribution consists of gamma, Nakagami- m , negative exponential, Weibull, one-sided Gaussian and Rayleigh distribution).

- The representations for PDF and CDF of κ - μ/α - μ are derived with the help of fading envelopes to represent the combined product model with short term fading and long term fading.

II. FADING DISTRIBUTIONS

The κ - μ fading distribution model small-scale LOS signals propagated in homogeneous scattering environments [15]. For fading signal with envelope R , normalized envelope $P(P = R/\hat{r})$, the root-mean-square (rms) of R is $\hat{r}$ ($\hat{r} = \sqrt{E(R^2)}$, $E(\cdot)$ denotes the expectation), the envelope PDF $f_P(\rho)$ can be written as

$$f_P(\rho) = \frac{2\mu(1+\kappa)^{\frac{\mu+1}{2}}}{\kappa^{\frac{\mu-1}{2}} e^{\kappa\mu}} \rho^\mu e^{-\mu(1+\kappa)\rho^2} I_{\mu-1}(2\mu\sqrt{\kappa(1+\kappa)}\rho) \quad (1)$$

where $\kappa(\kappa>0)$ is the ratio between the total power of the dominant components and the total power of the scattered waves, $\mu(\mu>0)$ is the number of multipath clusters, $I_\nu(\cdot)$ is the modified Bessel function of the first kind with the order ν . For fading signal with power $W=R^2$ and normalized power $\Omega=W/\bar{w}$ ($\bar{w}=E(W)$), the power PDF $f_\Omega(w)$ is given by

$$f_\Omega(w) = \frac{\mu(1+\kappa)^{\frac{\mu+1}{2}}}{\kappa^{\frac{\mu-1}{2}} e^{\kappa\mu}} w^{\frac{\mu-1}{2}} e^{-\mu(1+\kappa)w} I_{\mu-1}[2\mu\sqrt{\kappa(1+\kappa)}w] \quad (2)$$

The rms of κ - μ distribution is given by

$$\hat{r}^{\kappa-\mu} = \sqrt{E(R^2)} = \frac{\bar{r}\Gamma(\mu)(1+\kappa)^{1/2}\mu^{1/2}e^{\kappa\mu}}{\Gamma(\mu+\frac{1}{2})_1F_1(\mu+1/2, \mu, \kappa\mu)} \quad (3)$$

where $\bar{r} = E(R)$, $\Gamma(\cdot)$ is the gamma function.

The α - μ fading distribution is a non-linear physical fading model with fading parameters α and μ [8], [9], for the fading signal envelope R , the envelope PDF can be expressed as

$$f_R(r) = \frac{\alpha\mu^\mu}{\Gamma(\mu)} \frac{r^{\alpha\mu-1}}{\hat{r}^{\alpha\mu}} e^{-\frac{r^\alpha}{\hat{r}^\alpha}} \quad (4)$$

where the parameter $\alpha(\alpha>0)$ represents the nonlinearity of the propagation medium and the parameter $\mu(\mu>0)$ is the number of multipath clusters, the α -rms of α - μ distribution is given by

$$\hat{r}^{\alpha-\mu} = \sqrt[\alpha]{E(R^\alpha)} = \frac{\bar{r}\mu^\alpha\Gamma(\mu)}{\Gamma(\mu+\frac{1}{\alpha})} \quad (5)$$

III. PRODUCT MODEL OF I.N.I.D FADING DISTRIBUTIONS

A. Product model of two i.n.i.d.RVs

Assuming two i.n.i.d. positive RVs X for κ - μ fading distribution and Y for α - μ fading distribution respectively, let $Z = X \times Y$, with the help of the principles of probability theory and statistics, the PDF for RV Z is given by [8]

$$f_Z(z) = \int_0^\infty f_Y(y) f_{Z|Y}(z|y) dy \quad (6)$$

where $f_Z(\cdot)$ and $f_Y(\cdot)$ are the PDFs of Z and Y respectively, $f_{Z|Y}(\cdot)$ is the conditional PDF of Z/Y and it can be given by

$$F_{Z|Y}\left(\frac{z}{y}\right) = \frac{1}{y} f_X\left(\frac{z}{y}\right) \quad (7)$$

where $F_{Z|Y}(z/y)$ denotes the CDF of Z/Y , $X=Z/Y$, the PDF of Z with κ - μ distribution for RV X and α - μ distribution for RV Y can be expressed as

$$f_Z(z; \bar{x}, \bar{y}) = \int_0^\infty \frac{1}{y} f_{Z|Y}^{\kappa-\mu}(z|y; \bar{x}) f_Y^{\alpha-\mu}(y; \bar{y}) dy \quad (8)$$

where $\bar{x}=E(X)$ and $\bar{y}=E(Y)$. For κ - μ distribution of RV X in (8), with the aid of (1)-(3), the conditional PDF in (8) can be expressed as

$$f_{R|X}^{\kappa-\mu}(r_x) = A_1 \frac{r_x^{\mu_x}}{\hat{r}_x^{\mu_x+1}} e^{-\mu_x(1+\kappa_x)\frac{r_x^2}{\hat{r}_x^2}} I_{\mu_x-1}(2\mu_x\sqrt{\kappa_x(1+\kappa_x)}\frac{r_x}{\hat{r}_x}) \quad (9)$$

where

$$A_1 = \frac{2\mu_x(1+\kappa_x)^{\frac{\mu_x+1}{2}}}{\kappa_x^{\frac{\mu_x-1}{2}} e^{\kappa_x\mu_x}}, \quad \hat{r}_x = \frac{\bar{r}_x\Gamma(\mu_x)(1+\kappa_x)^{1/2}\mu_x^{1/2}e^{\kappa_x\mu_x}}{\Gamma(\mu_x+\frac{1}{2})_1F_1(\mu_x+1/2, \mu_x, \kappa_x\mu_x)} \quad (10)$$

where $\bar{r}_x=E(R_x)$. With the help of (4) and (5), the envelope PDF for α - μ distribution of RV Y in (8) is given by

$$f_{R|Y}^{\alpha-\mu}(r) = \frac{\alpha_y\mu_y^{\mu_y}}{\Gamma(\mu_y)} \frac{r_y^{\alpha_y\mu_y-1}}{\hat{r}_y^{\alpha_y\mu_y}} e^{-\frac{r_y^{\alpha_y}}{\hat{r}_y^{\alpha_y}}} \quad (11)$$

where

$$\hat{r}_y = \sqrt[\alpha_y]{E(R_y^{\alpha_y})} = \frac{\bar{r}_y\mu_y^{\frac{1}{\alpha_y}}\Gamma(\mu_y)}{\Gamma(\mu_y+\frac{1}{\alpha_y})} \quad (12)$$

in (12) $\bar{r}_y=E(R_y)$. By substituting (9)-(12) into (8), the product PDF for Z can be obtained as

$$f_Z(z) = A_1 A_2 \int_0^\infty y^{-\mu_x+\alpha_y\mu_y-2} e^{-\frac{\mu_x(1+\kappa_x)z^2}{\hat{r}_x^2 y^2} - \frac{\mu_y}{\hat{r}_y^{\alpha_y}} y^{\alpha_y}} I_{\mu_x-1}\left(\frac{2\mu_x\sqrt{\kappa_x(1+\kappa_x)}}{\hat{r}_x y} z\right) dy \quad (13)$$

where

$$A_2 = \frac{\alpha_y \mu_y^{\mu_y} z^{\mu_x}}{\Gamma(\mu_y) \hat{r}_y^{\alpha_y \mu_y} \hat{r}_x^{\mu_x+1}} \quad (14)$$

B. Product PDF

Having established the PDF of Z for κ - μ distribution of RV X with parameters $\kappa_x, \mu_x, \hat{r}_x$, and α - μ distribution of RV Y with $\alpha_y, \mu_y, \hat{r}_y$, the PDF can be derived as Theorem 1.

Theorem 1: The PDF of the product of i.n.i.d. RV X and Y ($Z = X \times Y$) can be expressed as (15), where RV X is κ - μ distribution and RV Y is α - μ distribution, in (15) where

$$\begin{aligned} \varphi(k, \kappa_x, \mu_x, \alpha_y, \mu_y) &= \frac{A_1 A_2 A_3}{2\Gamma(k+1)\Gamma(k+\mu_x)} \left(\frac{\mu_x^2 \kappa_x (1+\kappa_x)}{\hat{r}_x^2} \right)^k \\ &\quad z^{2k} \frac{2}{\alpha_y B_1^{k+\mu_y} B_2^{\frac{\alpha_y}{2}}}, \\ A_3 &= \left(\frac{\mu_x \kappa_x^{\frac{1}{2}} (1+\kappa_x)^{\frac{1}{2}} z}{\hat{r}_x} \right)^{\mu_x-1}, B_1 = \frac{\mu_x (1+\kappa_x) z^2}{\hat{r}_x^2}, B_2 = \left(\frac{\mu_y}{\hat{r}_y^{\alpha_y}} \right)^{\frac{2}{\alpha_y}}, \\ \Xi(a, b) &= \frac{b}{a}, \frac{b+1}{a}, \dots, \frac{b+a-1}{a}, \frac{p}{q} = \frac{1}{\frac{\alpha_y}{2}} \end{aligned} \quad (16)$$

in which ${}_mF_n()$ is the generalized hypergeometric function [20, Eq. (9.14.1)].

Proof: For integral C_1 in (13), suppose $t=y^2$, the C_1 is expressed as

$$C_1 = \frac{1}{2} \int_0^\infty t^{\frac{-\mu_x + \alpha_y \mu_y - 2}{2}} e^{-\frac{\mu_x (1+\kappa_x) z^2}{\hat{r}_x^2 t} - \frac{\mu_y}{\hat{r}_y^{\alpha_y} t^{\frac{\alpha_y}{2}}}} I_{\mu_x-1} \left(\frac{2\mu_x \sqrt{\kappa_x (1+\kappa_x)} z}{\hat{r}_x t^{\frac{1}{2}}} \right) dt \quad (17)$$

where $I_\nu(z)$ can be obtained as [20, Eq. (8.445)].

$$I_\nu(z) = \sum_{k=0}^{\infty} \frac{1}{\Gamma(k+1)\Gamma(\nu+k+1)} \left(\frac{z}{2} \right)^{2k+\nu} \quad (18)$$

Substituting (18) into (17), the C_2 can be simplified as

$$C_2 = \sum_{k=0}^{\infty} \frac{1}{\Gamma(k+1)\Gamma(k+\mu_x)} \left(\frac{\mu_x \sqrt{\kappa_x (1+\kappa_x)} z}{\hat{r}_x t^{\frac{1}{2}}} \right)^{2k+\mu_x-1} \quad (19)$$

Then substituting (17) and (19) into (13), the PDF of Z can be given by

$$f_Z(z) = \frac{A_1 A_2 A_3}{2} \sum_{k=0}^{\infty} \frac{1}{\Gamma(k+1)\Gamma(k+\mu_x)} \left(\frac{\mu_x^2 \kappa_x (1+\kappa_x)}{\hat{r}_x^2} \right)^k z^{2k} \underbrace{\int_0^\infty t^{-(k+\mu_x) + \frac{\alpha_y \mu_y}{2} - 1} e^{-\frac{\mu_x (1+\kappa_x) z^2}{\hat{r}_x^2 t} - \frac{\mu_y}{\hat{r}_y^{\alpha_y} t^{\frac{\alpha_y}{2}}}} dt}_{I_1} \quad (20)$$

To the best of authors' knowledge, the integral I_1 in (20) could not be solved with any formula for the uncertainty coefficients of t in exponential function, because the integral diverges as an infinite series expansion when it is applied to the exponential function. However, with the aid of [8, Eq. (12)-(15)] and [21, Eq. (2.3.2.14)], after algebraic manipulations we construct the integral (21) and let $\alpha_y/2:1=p:q$ (p and q are the

positive coprime integers). It is noteworthy that when p or q are positive even numbers, similarly, like [8], [9] and [12], we get $p-1$ for p or $q-1$ for q . Then, the formula (20) can be derived as (15).

C. Product CDF

Theorem 2: The CDF of the product of κ - μ distribution for RV X and α - μ distribution for RV Y can be expressed as (22), when X and Y are i.n.i.d RVs.

In (22) where

$$M_1 = \frac{\alpha_y \mu_y^{\mu_y}}{\Gamma(\mu_y) \hat{r}_y^{\alpha_y \mu_y} \hat{r}_x^{\mu_x+1}}, M_2 = \left(\frac{\mu_x \kappa_x^{\frac{1}{2}} (1+\kappa_x)^{\frac{1}{2}}}{\hat{r}_x} \right)^{\mu_x-1}, M_3 = \frac{\mu_x (1+\kappa_x)}{\hat{r}_x^2} \quad (23)$$

Proof: the CDF of Z can be obtain as

$$F_Z(z) = \int f_Z(z) dz \quad (24)$$

With the help of (15), the CDF of Z can be derived by substituting (15) into (24), after some algebraic simplification, the CDF of Z could be derived as (22) with the aid of (16), (21) and [22, Eq. (1.16.1.1)].

D. Composite fading channels

The composite fading characteristics can be represented as short-term distribution/long-term distribution and it will be considered as the special expressions when $\bar{x}=1$ for the product model κ - μ/α - μ . Then the special composite fading expressions for Rice/ α - μ , Rayleigh/ α - μ , Nakagami- m/α - μ and one-sided Gaussian/ α - μ will be obtained from (15) and (22).

IV. NUMERICAL RESULTS AND APPLICATION

The PDF, and CDF over the composite fading distribution model κ - μ/α - μ have been simulated in MATHEMATICA [25] with different values of fading parameters $\kappa_1, \mu_1, \alpha_2$ and μ_2 (where we define 1 for x and 2 for y). Unlike α - μ/α - μ in [8] and κ - μ/κ - μ in [12], this paper investigates the product series representation of κ - μ/α - μ , so the α_y is the most important research variable for simulation in (15), and (22).

Fig.1 and Fig.2 show the PDF of the composite distribution κ - μ/α - μ , in Fig.1 when $\alpha_2=2$, with different value of κ_1 , as μ_1 and μ_2 increase, more concentrated around $\bar{y}=1$ have been produced, on the other hand, if μ_1 and μ_2 are very low, the PDF curves are closed to the axis of ordinates. Likewise, when $\alpha_2=2, 6, 10$, we can obtain the same simulation performance for μ_1 and μ_2 . Fig.3 presents the CDF of composite fading distribution κ - μ/α - μ when $\alpha_2=2, 6, 10$ for varied μ_2 , as α_2 and μ_2 increase at the same time, the CDF curves become more abruptly and converges to 1 more fast, besides α_2 is more important than μ_2 on convergence property. Furthermore, it is noteworthy that although the phenomena of singularities for series representation have been produced by the product of the fading models like [8], [9] and [12], etc., it does not affect the accuracy and convergence for PDF and CDF.

V. CONCLUSIONS

In this paper, the product model of two i.n.i.d. RVs for κ - μ and α - μ fading distributions have been derived, this model can

$$\begin{aligned}
f_Z(z) &= \sum_{k=0}^{\infty} \varphi(k, \kappa_x, \mu_x, \alpha_y, \mu_y) (B_1 B_2)^{k+\mu_x} \sum_{i=0}^{q-1} \frac{(-1)^i}{i!} \Gamma(-\frac{p}{q}(i+k+\mu_x)+\mu_y) (B_1 B_2)^i \\
& {}_1F_{p+q}(1; \Xi(p, \frac{p}{q}(i+k+\mu_x)-\mu_y+1), \Xi(q, i+1); \frac{(B_1 B_2)^q}{(-p)^p(-q)^q}) + \frac{\alpha_y}{2} (B_1 B_2)^{\frac{\alpha_y \mu_y}{2}} \sum_{i=0}^{p-1} \frac{(-1)^i}{i!} \Gamma(k+\mu_x-\frac{q}{p}(i+\mu_y)) (B_1 B_2)^{\frac{\alpha_y i}{2}} \\
& {}_1F_{p+q}(1; \Xi(q, \frac{q}{p}(i+\mu_y)-\mu_x+1), \Xi(p, i+1); \frac{(B_1 B_2)^{\frac{\alpha_y p}{2}}}{(-p)^p(-q)^q})
\end{aligned} \tag{15}$$

$$\begin{aligned}
& (B_1)^{\mu_x} (B_2)^{\frac{\alpha_y \mu_y}{2}} \int_0^{\infty} t^{\frac{\alpha_y \mu_y}{2}-\mu_x-1} \exp(-B_1 t^{-1} - (B_2)^{\frac{\alpha_y}{2}} t^{\frac{\alpha_y}{2}}) dt = \frac{2}{\alpha_y} ((B_1 B_2)^{\mu_x} \sum_{i=0}^{q-1} \frac{(-1)^i}{i!} \Gamma(-\frac{p}{q}(i+\mu_x)+\mu_y) (B_1 B_2)^i \\
& {}_1F_{p+q}(1; \Xi(p, \frac{p}{q}(i+\mu_x)-\mu_y+1), \Xi(q, i+1); \frac{(B_1 B_2)^q}{(-p)^p(-q)^q}) + \frac{\alpha_y}{2} (B_1 B_2)^{\frac{\alpha_y \mu_y}{2}} \sum_{i=0}^{p-1} \frac{(-1)^i}{i!} \Gamma(\mu_x-\frac{q}{p}(i+\mu_y)) (B_1 B_2)^{\frac{\alpha_y i}{2}} \\
& {}_1F_{p+q}(1; \Xi(q, \frac{q}{p}(i+\mu_y)-\mu_x+1), \Xi(p, i+1); \frac{(B_1 B_2)^{\frac{\alpha_y p}{2}}}{(-p)^p(-q)^q})
\end{aligned} \tag{21}$$

$$\begin{aligned}
F_Z(z) &= \sum_{k=0}^{\infty} \frac{A_1 M_1 M_2}{2\Gamma(k+1)\Gamma(k+\mu_x)} (\frac{\mu_x^2 \kappa_x (1+\kappa_x)}{\hat{\kappa}_x^2})^k \frac{2}{\alpha_y M_3^{k+\mu_x} B_2^{\frac{\alpha_y \mu_y}{2}}} ((M_3 B_2)^{k+\mu_x} \\
& \sum_{i=0}^{q-1} \frac{(-1)^i}{i!} \Gamma(-\frac{p}{q}(i+k+\mu_x)+\mu_y) (M_3 B_2)^i \frac{z^{2i+2(k+\mu_x)}}{2i+2(k+\mu_x)} \\
& {}_2F_{p+q+1}(1, \frac{2i+2(k+\mu_x)}{2q}; \Xi(p, \frac{p}{q}(i+k+\mu_x)-\mu_y+1), \Xi(q, i+1), \frac{2i+2(k+\mu_x)}{2q}+1; \frac{(M_3 B_2 z^2)^q}{(-p)^p(-q)^q}) + \frac{\alpha_y}{2} (M_3 B_2)^{\frac{\alpha_y \mu_y}{2}} \\
& \sum_{i=0}^{p-1} \frac{(-1)^i}{i!} \Gamma(k+\mu_x-\frac{q}{p}(i+\mu_y)) (M_3 B_2)^{\frac{\alpha_y i}{2}} \frac{z^{\alpha_y i + \alpha_y \mu_y}}{\alpha_y i + \alpha_y \mu_y} \\
& {}_2F_{p+q+1}(1, \frac{\alpha_y i + \alpha_y \mu_y}{\alpha_y q}; \Xi(q, \frac{q}{p}(i+\mu_y)-k+\mu_x+1), \Xi(p, i+1), \frac{\alpha_y i + \alpha_y \mu_y}{\alpha_y q}+1; \frac{(M_3 B_2 z^2)^{\frac{\alpha_y p}{2}}}{(-p)^p(-q)^q})
\end{aligned} \tag{22}$$

Fig.1. PDF for the composite fading distribution of $\kappa\text{-}\mu/\alpha\text{-}\mu$, with $\alpha_2=2$, $\hat{\kappa}_x = \hat{\kappa}_y = 1$ and fading values for κ_1 , μ_1 and μ_2

improve channel capacity in CRs and evaluate the performance of communication systems as vehicle-to-vehicle communications, cascaded fading channels and body area networks. The novel exact series representations for PDF and

CDF of $\kappa\text{-}\mu/\alpha\text{-}\mu$ are derived to investigate the performance metrics of fading parameters, and novel composite fading channels such as Rice/ $\alpha\text{-}\mu$, Rayleigh/ $\alpha\text{-}\mu$, Nakagami- $m/\alpha\text{-}\mu$ and One-sided Gaussian/ $\alpha\text{-}\mu$ can be obtained from $\kappa\text{-}\mu/\alpha\text{-}\mu$ model as special cases, the exact series representation with generalized hypergeometric function have been derived

Fig.2. PDF for the composite fading distribution of $\kappa\text{-}\mu/\alpha\text{-}\mu$, with $\hat{r}_x = \hat{r}_y = 1$ and fading values for $\kappa_1, \mu_1, \alpha_2$ and μ_2

instead of Fox H-function. The analytical results and simulations indicate that for $\kappa\text{-}\mu/\alpha\text{-}\mu$ model, the nonlinear parameter plays more important role than multipath variables and higher α will evidently improve channel capacity in CRs. In general, the proposed $\kappa\text{-}\mu/\alpha\text{-}\mu$ model can be widely used as generalized composite multipath-shadowing fading scenarios in wireless communication fields.

Fig.3. CDF for the composite fading distribution of $\kappa\text{-}\mu/\alpha\text{-}\mu$, with $\kappa_1=1.1$, $\mu_1=1.2$, $\hat{r}_x = \hat{r}_y = 1$ and fading values for α_2 and μ_2

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