Quantum Memristor

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Abstract—Research in quantum computing hardware has shown that quantum bits can be implemented with superconducting circuits, these qubits could be used in other applications such as quantum devices for electronic systems. In this paper a nonlinear parameterised two qubits quantum device is presented, the current - voltage characteristic of this element is similar to an adjustable memristor when its parameters are changed.

I. INTRODUCTION

Research in new alternatives of computation, to overcome the limits of current technology, has focused on the development of quantum computing using mainly superconducting circuits for the quantum bits or qubits [1]. The mathematical framework of quantum computation is well known and it has been used to describe quantum hardware and algorithms for different applications [2].

On the other hand, in circuit theory, a nonlinear device known as the memristor was proposed many decades ago, this device behaves as a resistance with memory [3]. The physical implementations of the memristor and its possible applications continue to be an interesting research topic [4,5].

In this paper, a nonlinear parametric Quantum Device (QD) is proposed, the current - voltage characteristic of this element is similar to a memristor when the parameters satisfy a particular condition.

The paper is organized as follows, section two presents the quantum device within the mathematical framework of quantum systems. Section three introduces the quantum device into a resonant circuit with the corresponding differential equations and the simulation diagram. Section four illustrate the QD dynamics for different parameters and input frequencies. Finally conclusions are formulated in section five.

II. QUANTUM DEVICE

A nonlinear device exploiting quantum principles is proposed here, to study this element the basic mathematical framework of quantum systems is applied [2].

A. State and Operator

Consider a quantum device with two qubits,

$$\begin{aligned} |\psi_1(x_1)\rangle &= \begin{bmatrix} \cos \alpha_1(x_1) \\ \sin \alpha_1(x_1) \end{bmatrix} \\ |\psi_2(x_2)\rangle &= \begin{bmatrix} \cos \alpha_2(x_2) \\ \sin \alpha_2(x_2) \end{bmatrix} \end{aligned} \quad (1)$$

and the operator A with real values,

$$A = \begin{bmatrix} a_1 & 0 & 0 & 0 \\ 0 & a_2 & 0 & 0 \\ 0 & 0 & a_3 & 0 \\ 0 & 0 & 0 & a_4 \end{bmatrix} \quad (2)$$

The arguments $\{\alpha_1, \alpha_2\}$ are function of classical variables $\{x_1, x_2\}$ with gains $\{k_1, k_2\}$,

$$\alpha_1(x_1) = \frac{\pi/2}{1 + e^{-k_1 x_1}}; \quad \alpha_2(x_2) = \frac{\pi/2}{1 + e^{-k_2 x_2}} \quad (3)$$

The state $|\psi(x_1, x_2)\rangle$ of this quantum device is the tensor product of the individual states,

$$\begin{aligned} |\psi(x_1, x_2)\rangle &= |\psi_1(x_1)\rangle |\psi_2(x_2)\rangle \\ |\psi(x_1, x_2)\rangle &= \begin{bmatrix} \cos \alpha_1(x_1) \cos \alpha_2(x_2) \\ \cos \alpha_1(x_1) \sin \alpha_2(x_2) \\ \sin \alpha_1(x_1) \cos \alpha_2(x_2) \\ \sin \alpha_1(x_1) \sin \alpha_2(x_2) \end{bmatrix} \end{aligned} \quad (4)$$

After measuring the state, the expectation value of operator A is a function of the classical variables,

$$f(x_1, x_2) = \langle \psi(x_1, x_2) | A | \psi(x_1, x_2) \rangle \quad (5)$$

where,

$$\begin{aligned} f(x_1, x_2) &= a_1 [\cos \alpha_1(x_1) \cos \alpha_2(x_2)]^2 \\ &+ a_2 [\cos \alpha_1(x_1) \sin \alpha_2(x_2)]^2 \\ &+ a_3 [\sin \alpha_1(x_1) \cos \alpha_2(x_2)]^2 \\ &+ a_4 [\sin \alpha_1(x_1) \sin \alpha_2(x_2)]^2 \end{aligned} \quad (6)$$

It is important to mention that (6) is a continuous map from $\{x_1, x_2\}$ to $f(x_1, x_2)$, passing through the discrete state space of the quantum system (4).

B. Expectation Value Zero

For this particular quantum device the expectation value $f(x_1, x_2)$ is zero in two situations:

(a) $f(x_1, x_2) = 0, x_2 = 0$:

$$\begin{aligned} x_2 = 0, \quad \alpha_2 &= \frac{\pi}{4} \\ f(x_1, x_2) &= \frac{1}{2}(a_1 + a_2)(\cos \alpha_1(x_1))^2 \\ &+ \frac{1}{2}(a_3 + a_4)(\sin \alpha_1(x_1))^2 \\ f(x_1, x_2) &= 0, \quad a_1 = -a_2, \quad a_3 = -a_4 \end{aligned} \quad (7)$$

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(b) $f(x_1, x_2) = 0, x_1 = 0$:

$$\begin{aligned} x_1 = 0, \quad \alpha_1 &= \frac{\pi}{4} \\ f(x_1, x_2) &= \frac{1}{2}(a_1 + a_3)(\cos \alpha_2(x_2))^2 \\ &\quad + \frac{1}{2}(a_2 + a_4)(\sin \alpha_2(x_2))^2 \\ f(x_1, x_2) &= 0, \quad a_1 = -a_3, \quad a_2 = -a_4 \end{aligned} \quad (8)$$

In the following the quantum device (6) is used as a nonlinear circuit element, some simulations illustrate its behavior.

III. LCQD CIRCUIT

In this section, the quantum device is part of a classical circuit, the device inputs are $\{x_1 = q, x_2 = i\}$ and the output $v_0 = f(x_1, x_2)$, see figure 1.

Fig. 1. LC circuit with quantum device, $L = 1$ H, $C = 1$ F, notice the Josephson junction symbol.

Current implementations of qubits mostly use superconducting circuits, the nonlinear QD includes the symbol of the Josephson junction.

A. Equations

The differential equations that relate the three variables, charge : $x_1 = q$, current : $x_2 = i$, capacitor voltage : x_3 are,

$$\begin{aligned} \frac{dx_1}{dt} &= x_2 \\ \frac{dx_2}{dt} &= \frac{1}{L}v_i - \frac{1}{L}x_3 - \frac{1}{L}f(x_1, x_2) = \frac{1}{L}[v_i - x_3 - f(x_1, x_2)] \\ \frac{dx_3}{dt} &= \frac{1}{C}x_2 \end{aligned} \quad (9)$$

The circuit input is,

$$v_i(t) = 10. \sin \omega t \text{ V}$$

B. Simulation Diagram

The differential equations (9) can be represented with a simulation diagram using integration blocks, see figure 2. The voltage of the quantum device is a nonlinear function of the current and charge, $v_0 = f(x_1, x_2)$.

Fig. 2. Simulation diagram corresponding to equations (9), circuit LCQD.

IV. SIMULATIONS

To illustrate the dynamics of the LCQD circuit, in particular the two situations (7, 8) for $v_0 = f(x_1, x_2)$, some numerical simulations are presented.

A. Current - Voltage

The frequency ω and the parameters of the quantum device are,

$$\begin{aligned} \omega &= 5.0 \text{ rad/s} \\ a_1 &= -0.2, \quad a_2 = 0.2, \quad a_3 = -0.5, \quad a_4 = 0.5 \\ k_1 &= 2.0, \quad k_2 = 2.0 \end{aligned}$$

Figure 3 shows the nonlinear map (6) for the selected parameters, the function is constant at higher values of charge x_1 and current x_2 .

Fig. 3. Nonlinear function $f(x_1, x_2)$ for the quantum device (6).

Figure 4 presents the quantum device voltage $v_0(t)$ for the circuit periodic input $v_i(t)$. Figure 5 is the current - voltage characteristic, notice the pinched hysteresis loop that reveals a memristive behavior [3-5].

Fig. 4. Quantum device voltage for the periodic circuit input, $\omega = 5\text{rad/s}$.

Fig. 6. Quantum device voltage for the periodic circuit input, the memristor value is increased by changing parameters $\{a_i\}$, same frequency $\omega = 5\text{rad/s}$.

Fig. 5. Quantum device pinched hysteresis loop that shows a memristor like behavior, $\omega = 5\text{rad/s}$.

B. Current - Voltage

In this simulation the quantum device parameters are modified but following (7), the frequency is the same,

$$\begin{aligned}\omega &= 5.0 \text{ rad/s} \\ a_1 &= -1.0, \quad a_2 = 1.0, \quad a_3 = -2.0, \quad a_4 = 2.0 \\ k_1 &= 2.0, \quad k_2 = 2.0\end{aligned}$$

Figure 6 is the quantum device voltage for the circuit periodic input. Figure 7 presents the pinched hysteresis loop, the current - voltage slope has changed meaning a different value of memristance.

Comparing figures 4 and 6, an increase in the output voltage v_0 can be seen due to an increase in the memristor parameters $\{a_i\}$.

Fig. 7. Quantum device pinched hysteresis loop when the parameters are increased, $\omega = 5\text{rad/s}$, the slope changes meaning an increase in memristance.

C. Current - Voltage

In this simulation the frequency ω has been doubled, the parameters of the quantum device are,

$$\begin{aligned}\omega &= 10.0 \text{ rad/s} \\ a_1 &= -0.2, \quad a_2 = 0.2, \quad a_3 = -0.5, \quad a_4 = 0.5 \\ k_1 &= 2.0, \quad k_2 = 2.0\end{aligned}$$

Figure 8 is the quantum device voltage for the circuit periodic input. Figure 9 presents the pinched hysteresis loop, notice that increasing the frequency changes the loop, approaching a straight line, which is consistent with a memristive behavior [3-5].

Fig. 8. Quantum device voltage for the periodic circuit input, the frequency is doubled $\omega = 10\text{rad/s}$.

Fig. 9. Quantum device pinched hysteresis loop when the input frequency is increased, $\omega = 10\text{rad/s}$, notice that the loop changes approaching a straight line similar to memristors.

D. Charge - Voltage

The previous simulations illustrate the behavior of the quantum device (6) as a memristor with its characteristic pinched hysteresis loop when the mathematical conditions (7) are satisfied.

The proposed quantum device (6) is capable of more behaviors besides the memristor. In this simulation it is shown a pinched hysteresis loop between charge and voltage (8), the frequency and parameters are,

$$\begin{aligned}\omega &= 5.0 \text{ rad/s} \\ a_1 &= 0.2, \quad a_2 = 0.5, \quad a_3 = -0.2, \quad a_4 = -0.5 \\ k_1 &= 2.0, \quad k_2 = 2.0\end{aligned}$$

Figure 10 is the quantum device voltage $v_0(t)$ for the circuit input $v_i(t)$ and conditions (8). Figure 11 is the pinched hysteresis loop between charge and voltage.

Fig. 10. Quantum device $v_0(t)$ when the circuit input is $v_i(t)$ and the parameters satisfy (8), $\omega = 5\text{rad/s}$.

Fig. 11. Quantum device pinched hysteresis loop between charge and voltage (8), $\omega = 5\text{rad/s}$.

V. CONCLUSIONS

Many quantum computing applications implement qubits with superconducting circuits, but there is a possibility to take this technology further and develop analogue (digital) quantum devices for electric and electronic circuits.

In this paper a two qubits device has been studied, this nonlinear element behaves as an adjustable memristor when the parameters are changed under some conditions. Other dynamics are possible such as the pinched hysteresis loop in the voltage - charge plot.

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REFERENCES

- [1] National Academies of Sciences, Engineering, and Medicine. Quantum Computing: Progress and Prospects. The National Academies Press, Washington, DC, 2019.
- [2] M.A. Nielsen, and I.L. Chuang, Quantum Computation and Quantum Information. Cambridge: Cambridge University Press, 2010.
- [3] L.O. Chua, Memristor - The Missing Circuit Element, IEEE Trans. on Circuit Theory, vol. 18, no. 5, pp. 507-519, Sept. 1971.
- [4] D.B. Strukov, G.S. Snider, D.R. Stewart, and R.S. Williams, The Missing Memristor Found, Nature, vol. 453, pp. 80-83, May 2008.
- [5] H.H.-C. Iu, and A.L. Fitch, Development of Memristor Based Circuits. Singapore: World Scientific, 2013.