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Resolving Range and Velocity Ambiguity Effectively and Efficiently with GPU

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Abstract—Resolving the ambiguity in both range and velocity domain is one of the essential issues for the pulse Doppler (PD) radars using medium pulse repetition frequency (PRF) waveforms. The existing solutions are mainly based on the Chinese Remainder Theorem (CRT) and its variations and extensions, but these algorithms typically have an unsatisfactory accuracy and are highly complex in computation, especially for the case of a large false-alarm rate. In this work, we proposed a new approach to tackle with ambiguity based on the partial distance matrices (PDM) and implemented it on the NVIDIA graphics processing unit (GPU) platform. The evaluations based on both Monte-Carlo simulations and real measured data present the advantages of the proposed approach in accuracy and efficiency. The results also show the proposed approach can be utilized as a real-time algorithm even for a low-threshold detection scheme.

Index Terms—radar detection, ambiguity resolution, range and velocity ambiguity, medium PRF, GPU

I. INTRODUCTION

Medium pulse repetition frequency (PRF) waveforms are typically selected to balance the maximum unambiguous velocity and maximum unambiguous range of pulse Doppler (PD) radars. However, both range and velocity ambiguity will exist in this case. In order to resolve the ambiguity, multiple PRFs are employed to produce different ambiguous target range and velocity measurements.

Currently, the prevailing algorithm are mostly based on the Chinese Remainder Theorem (CRT) [1]–[3]. It is an analytic procedure for calculating the unambiguous range from the measured range on different PRFs. A graphical implementation of the CRT called coincidence algorithm and another variation of CRT referred to as residual look-up table (RLUT) have also been proposed in [4]–[7]. In these algorithms, the error in measurements is likely to degrade resolution results. In addition, a high computational complexity and a lot of memory space are required especially for the case where the interested range and velocity scopes are considerably large.

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To gain better anti-error ability, clustering algorithm has been proposed in [8], [9]. It rearranges all the possible ambiguous range corresponding to different PRFs in ascending order and calculate the cluster variation in order. The best cluster occurs at the value when the variation is a minimum. Although the clustering algorithm outperforms CRT, but it requires substantial computation resources, preventing it being applied in the practical real-time engineering.

Therefore, this work proposes a new ambiguity resolution approach based on partial distance matrices (PDM). We also implement the proposed algorithm on the NVIDIA graphics processing unit (GPU) platform with compute unified device architecture (CUDA). The evaluations show that the ambiguity in both range and velocity can be resolved effectively and efficiently. To our best knowledge, this is also the first time that an ambiguity resolution algorithm has been implemented on GPU and has exhibited the significant potential of GPU for the real-time practical engineering applications.

This paper is organized as follows. Section II states the ambiguity resolution problem, followed by the elaborations of the proposed algorithm with its implementation on GPU in Section III. IV evaluates and analyzes the proposed method with Monte-Carlo simulations and real measurements. Finally, V concludes the work.

II. PROBLEM STATEMENT

Assume a target is located at range R and moves with radial velocity v . For PRF i , the measured range gate number g_i by radar can be formulated as

$$R = m_i R_{0,i} + (g_{f,i} + g_i) \Delta R + \delta R_i, \quad (1)$$

where $g_{0,i}$ and $R_{f,i}$, δR_i are the maximum unambiguous range, the front gate number and range error for PRF i , respectively, ΔR is the range gate width, and m_i is a non-negative integer. The measured doppler gate d_i satisfies

$$v = \begin{cases} n_i v_{0,i} + d_i \Delta v + \delta v_i, & d_i = 1, 2, \dots, \frac{N}{2} \\ n_i v_{0,i} + (d_i - N) \Delta v + \delta v_i, & d_i = \frac{N}{2} + 1, \dots, N \end{cases} \quad (2)$$

where $v_{0,i} = \lambda f_i / 2$ is the maximum unambiguous velocity for PRF i , $\Delta v = \lambda f_i / (2N)$ is the doppler gate width, f_i is the value of the PRF, δv_i is the velocity error, N is the total doppler gate number of pulse processing, and n_i is a positive integer.

Typically, the maximum unambiguous range of PRF i is designed as certain multiple of range gate width, *i.e.* $R_{0,i} = m_{0,i} \Delta R$, where $n_{0,i}$ is a positive integer. Consider a simple case where only one target is presented and no false-alarm exists, three PRFs are utilized to resolve the range ambiguity, and the velocity is unambiguous. Also assume there is no measurement error, *i.e.* $\delta R_i = 0$ and $\delta v_i = 0$. Then Eq. (1) can be reduced to

$$R = (m_i m_{0,i} + (g_{f,i} + g_i)) \Delta R, \quad i = 1, 2, 3. \quad (3)$$

The traditional CRT method gives the resolved range R as

$$R = \text{mod} (C_1(g_{f,1} + g_1) + C_2(g_{f,2} + g_2) + C_3(g_{f,3} + g_3), m_{0,1}m_{0,2}m_{0,3}\Delta R, \quad (4)$$

where the variables C_i are constants related to $m_{0,i}$, and the details can be found in [1].

It should be noted that the calculation in CRT based on discrete range gates, causing possible errors if measurement errors exist or the target is located at the margin of a range gate. Other errors in the system, such as the errors in PRFs, front range gate number *etc.*, will lead to performance degradation of CRT. Furthermore, if there exist more than one target and some false alarms, the computational complexity grows explosively.

The graphic version of CRT can mitigate the complexity to some extent, but a large memory space to store the range-velocity information of all target candidates will be searched frequently, slowing down the processing speed. The computational complexity reaches $O(N_r N_v)$, where N_r and N_v are the number of total range gates and doppler gates of the maximum range and velocity of interest, respectively. For example, if maximum range of interest is 200 km and velocity of interest ranges from -300 m/s to 300 m/s, and the range resolution is 30 m and the velocity resolution is 1 m/s, then it requires at least $2 \times 10^5 / 30 \cdot 600 \approx 3.8$ MB memory space for one PRF. If there are 20 candidates in the unambiguous range, and the maximum range of interest is 30 times of the unambiguous range, then at least 1200 indexing and assignment followed by 4×10^6 addition operations are required for the ambiguity resolution with two PRFs and at least 4×10^6 sequential search operations are demanded. This prevents an efficient ambiguity resolution implementation.

III. SOLVING AMBIGUITY BASED ON PARTIAL DISTANCE MATRICES

The problem of resolving ambiguity can be treated as the problem of matching. The ambiguous measurements in range and velocity firstly are extended to different ambiguous regions according to Eq. (1) and (2), respectively. The generated possible targets with the extended range and velocity together with the original ambiguous measurements of targets are

Fig. 1: (a) is the concept of resolving range and velocity ambiguity with the distance matrix, and (b) example of a distance matrix and partial distance matrices.

called target candidates. Then the similarity of the target candidates of each PRF can be calculated. If any similarity is over a threshold, the corresponding candidates are from the measurements of one target, and the range and velocity of the real target can be determined.

In this work, we utilize the distance of target candidates of different PRFs to evaluate their similarity. The difference in range and velocity are utilized to evaluate the distance. Information in other dimensions can also be leveraged to calculate distance and evaluate similarity. In order to use multiple PRFs to resolve ambiguity, the beamwidth should be sufficient large to cover the space in the sequential several frames. Therefore, we also use the measured angle to differentiate the real target from false alarms. Mathematically, the test of a target can be formulated as

$$D(R_i, v_i, \theta_i; R_j, v_j, \theta_j) \begin{matrix} > \\ < \end{matrix} \begin{matrix} H_0 \\ H_1 \end{matrix} Th, \quad (5)$$

where $D(\cdot)$ is the distance function of two candidates, and any target candidate p_i is described by its range, velocity and azimuth, *i.e.* R_i, v_i, θ_i , Th is the threshold, and H_1 and H_0 represents the hypotheses that these two candidates are matched and mismatched, respectively.

To reduce the computational complexity, we use the distance function as below.

$$D(R_i, v_i, \theta_i; R_j, v_j, \theta_j) = \begin{cases} \infty, & |v_i - v_j| \geq v_{th} \text{ or } |\theta_i - \theta_j| \geq \theta_{th}, \\ |R_i - R_j|, & |v_i - v_j| < v_{th} \text{ and } |\theta_i - \theta_j| < \theta_{th}, \end{cases} \quad (6)$$

where v_{th} and θ_{th} are the threshold of velocity and azimuth, respectively.

Fig. 1(a) shows a simple example. For simplicity, ambiguous measurements of two different PRFs are considered, and only one target is detected for each PRF. In the figure, blue circles and red stars represent the target candidates extended from the ambiguous measurements of two targets of two different PRFs, respectively. The extension is conducted in both range and velocity domains. The distance of two target candidates marked inside the red circles is smaller than the distance

Fig. 2: Ambiguity resolution results of (a) the conventional CRT and (b) the proposed PDM based on Monte-Carlo simulations.

threshold. Thus, they are determined to be matched, *i.e.* the unambiguous measurements of one target under two PRFs. The range and velocity of the target are the weighted average of the range and velocity of these two candidates.

Directly using distance matrix to resolve ambiguity also demands for considerable resources. The space and computational complexity both are as large as $O(N^2)$, where N is the average number of ambiguous measurements per PRF. Consider the aforementioned example in Section II. Resolving ambiguity requires 4.32 MB memory space for single-float precision, and at least 1.08×10^6 subtraction and 3×10^5 search operations are demanded.

To further reduce the computation and memory complexity, we divided all the target candidates into several blocks according to the range of the target candidates. Instead of calculating the distance among all the candidates, calculating the distance between any two candidates in the same block is sufficient. In other words, the calculation of the whole distance matrix will be reduced to the calculation of parts of the original distance matrix, referred as the partial distance matrices in this work. Fig. 1(b) illustrates an example of the value of a distance matrix, and the regions with red box are partial distance matrices. For simplicity, the velocity and azimuth of all the candidates are assumed the same.

Assume the target candidates are located uniformly in the whole range scope in the aforementioned example, and the range scope of a partial region is 10 km. Then, there will be 20 blocks, and only 30 candidates exist in each block. In consequence, the required memory space will be reduced to 216 KB, the subtraction operations will be reduced to 5.4×10^4 , and the search operations will be reduced to 1.8×10^4 . In general, both of the space and computational complexity are reduced to $O(N^2/M)$, where M is the number of partial distance matrices.

It is worth noting that generating partial distance matrices can be fully parallelized, and extending ambiguous targets to all possible range and velocity according to Eq. 1 and 2 and

extracting matched targets can also be partially parallelized. This makes GPU extremely suitable for ambiguity resolution with PDM. GPUs excel processing of a large amount of data by using the same set of instructions, referred to as single instruction multiple data (SIMD) technology. For example, the 5.4×10^4 subtraction can be performed simultaneously, thus greatly speeding up calculation.

Due to page limit, we briefly introduce the implementation of our proposed algorithm on GPU platform with CUDA as follows. When extending ambiguous targets, the thread dimension is set to the number of measured targets, and the grained dimension is set to the number of extending times. Meanwhile, all the target candidates are divided into different blocks according to their range, preparing for the calculation of partial distance matrices. Here, a vital technology called warp aggregated atomics using cooperative groups (CG) will be utilized to realize a lock when arrange all candidates in parallel. Then, another kernel generates partial distance matrices between two different PRFs with the grained dimension of the number of the target candidates of the current frame and the block dimension of the number of the candidates of the last frame. Next, the minimum of each column is selected in parallel. Two candidates are determined to be matched if the minimum is smaller than the threshold. Finally, the unambiguous target information will be extracted.

IV. EVALUATIONS AND ANALYSES

We first evaluate the performance of the proposed PDM based on Monte-Carlo simulations. The comparison with a graphic version of CRT, the coincidence algorithm discussed in [10], is conducted. To overcome measurement errors, a tolerance of one range gate is established. This means that a target is declared if a detection occurs in all PRFs at some range gate $g \pm 1$. At each iteration of the simulations, the range and velocity of targets are generated in the first step. The number of targets is a Gaussian random variable ranging from 1 to 5. Given the values of PRFs, range gate width and doppler

Fig. 3: Ambiguity resolution results of the conventional CRT and the proposed approach based on simulations.

TABLE I: Monte-Carlo simulation results of the conventional CRT and the proposed PDM.

Method	Average detection rate	Average false-alarm rate
CRT	0.9267	1.93×10^{-6}
Proposed	0.9642	6.92×10^{-7}

gate width, the ambiguous range gates and doppler gates of the targets can be obtained. Here, random errors in range and velocity in Gaussian distribution will be added. Meanwhile, some false alarms will be generated randomly as well. Here, we use two different PRFs to resolve the range and velocity ambiguity. Fig. 2 shows the results of the two methods at one iteration in the Monte-Carlo simulations. Table I shows the detection rate and false alarm rate after ambiguity resolution of 5000-iteration Monte-Carlo simulations.

As shown in Fig. 2, both PDM and CRT can resolve the range and velocity ambiguity correctly in this case. However, the false alarms of PDM are noticeably fewer than those

of CRT, which is supported by the statistics of the Monte-Carlo simulations as shown in Table I. The false alarm rate of ambiguous measurements is around 5.38×10^{-4} before resolution. Both PDM and CRT can reduce the false alarm rate after ambiguity resolution, since the resolution process can be treated as binary integration. However, in addition to the true target, a series of false targets also appear periodically at the same velocity of the true target in the results of CRT, while no such phenomenon exists in the results of PDM. This results from a looser matching condition in CRT due to the existence of the one tolerance gate. As shown in Table I, compared to CRT, PDM achieves over 4% improvement in detection rate and over 64% reduction of false alarm rate.

Apart from Monte-Carlo simulations, we also evaluate the proposed PDM and compared it with CRT in real applications. Fig. 3(a) shows the ambiguous measurements of a ground-based radar. Four sequential frames with different PRFs are utilized to resolve the range and velocity ambiguity of an area. The range is normalized to the maximum unambiguous range. As we can note, identifying targets is almost impossible in this case, due to the small maximum unambiguous range and high speed of the targets.

Fig. 4: Processing time of PDM algorithm on CPU and GPU platform. (a) is the average processing time per frame, and (b) is the average processing time per detected target.

Fig. 3(b) and 3(c) show the ambiguity resolution results of CRT and our proposed PDM algorithm, respectively. Both algorithms can accomplish to obtain unambiguous targets. However, more false alarms appear when using CRT. PDM algorithm is capable of resolving ambiguity remarkably better than CRT. This agrees well to the results of Monte-Carlo simulations. The improvement stems from using range and velocity information in the continuous spaces in PDM rather than the discrete ones in CRT. In other words, a finer-grained information is exploited in PDM. Another important reason is the utilization of additional information in azimuth domain.

We also assess the processing time of PDM algorithm on CPU platform and GPU platform. The results are shown in Fig. 4. The number of detected ambiguous targets per frame is changed by adjusting detection threshold. It should be noted that the processing time of PDM on CPU platform grows rapidly with the average number of ambiguous targets per frame, while that of PDM on GPU platform only slightly increases. Moreover, the average processing time per detected target even decreases with the number of targets per frame, mainly because the overhead of launching CUDA kernels and sequential operations are amortized. This also proves that PDM has been effectively accelerated by the parallelism of GPU platform.

The average frame time is in the order of 10s microseconds. This indicates that CPU-based PDM can be used as a real-time ambiguity resolution algorithm if the detection threshold is relatively high. Even if low-threshold detection is adopted, the average processing time of GPU-based PDM can be well limited to within 2 ms, making it highly efficient in real applications.

V. CONCLUSION

This work has proposed a new ambiguity resolution algorithm based on partial distance matrices (PDM). The proposed algorithm exploits fine-grained range and velocity information

and additional information in azimuth domain, facilitating its capability of high ambiguity resolution accuracy, strict false alarm suppression and insensitive to measurement errors. Extensive analyses present the small complexity of PDM. The parallel implementation of PDM on GPU platform accelerates the ambiguity resolution process, ensuring the average execution time per frame below 2 ms even for a low-threshold detection and paving a way for the practical real-time engineering applications.

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