Robust CPHD Fusion for Distributed Multitarget Tracking Using Asynchronous Sensors

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Abstract—This paper concentrates on tracking multiple targets using an asynchronous network of sensors with different sampling rates. First, a timely fusion approach is proposed for handling measurements from asynchronous sensors. In the proposed approach, the arithmetic average fusion of the estimates provided by local cardinalized probability hypothesis density filters is recursively carried out according to the network-wide sampling time sequence. The corresponding intersensor communication is conducted by a partial flooding protocol, in which either cardinality distributions or intensity functions pertinent to local posteriors are disseminated among sensors. Moreover, both feedback and non-feedback fusion-filtering modes are provided to meet the performance and real-time requirements, respectively. Second, an extension of the timely fusion approach referred to as robust bootstrap approach is presented, which can deal with unknown clutter and detection parameters by exploiting a local bootstrap filtering scheme. Finally, numerical simulations are performed to test the proposed approaches.

Index Terms—Multitarget tracking, arithmetic average fusion, asynchronous sensors, flooding protocol, different sampling rates, bootstrap filtering, random finite sets.

I. INTRODUCTION

Nowadays, distributed sensor networks are widespread in military and civilian fields [1], [2] owing to the advantages of high reliability, strong robustness and excellent scalability over their centralized counterparts. A significant topic in the distributed sensor network is distributed multitarget tracking (DMT) [3]–[6], which involves tracking stochastic appearing and disappearing targets using decentralized but interconnected sensors in a collaborative manner. The main concerns of DMT are twofold: 1) how to obtain local estimates at each sensor from sensors observations contaminated by clutter and noise; 2) how to fuse estimates provided by different sensors in the presence of unknown and ubiquitous intersensor cross-correlation.

The random finite set (RFS) approach [7], [8] provides a rigorous and promising framework for the first concern of which the centerpiece lies in the multitarget Bayes (MTB) filter. As the first tractable approximation of the MTB filter, the probability density hypothesis (PHD) filter [9] recursively propagates the intensity function of the multitarget RFS. To reduce the covariance of target number estimates, the cardinalized PHD (CPHD) [10] filter was subsequently derived

by Mahler, which is a generalization of the PHD filter that additionally propagates the cardinality distribution. Practical implementations of the two filters have been reported in [11]–[13]. These results render the PHD and CPHD filters accessible to a wider audience.

Aiming at the second concern, two average fusion methods, i.e., arithmetic average (AA) [14]–[16] and geometric average (GA) [5], [17], [18], have been proposed, which are immune to the correlations among sensors. Compared with the GA fusion, the AA fusion has much lower computational complexity [19]. This merit makes the AA fusion suitable for realistic sensor networks with limited computing resources. Furthermore, the AA fusion of RFS densities leads to a mixture distribution and it admits the closed-form Markov-Bayesian recursion [20]. With these facts in mind, we focus on the AA fusion in this work. Recently, the AA fusion has been applied in conjunction with the PHD and CPHD filters in various scenarios, demonstrating outstanding performance not only in the context of target tracking [14], [21]–[27] but also vehicular network positioning [28] and multi-robot collaboration [29].

Most of the existing distributed fusion approaches depend on the ideal assumption that the sampling rates of sensors are identical. In fact, even in the simplest homogeneous network, communication delay is unavoidable, which poses the need for asynchronous fusion. Several multisensor PHD filters [30]–[32] have been proposed for asynchronous radars with different sampling rates. Unfortunately, these filters merely suit scenarios with few sensors. Extensions to an arbitrary number of sensors have been studied in [33], [34], where the fusion of local estimates are performed in accordance with

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a fixed period via the central node. A robust PHD fusion algorithm that is capable of joint estimating target states and time-offsets among sensors has been investigated in [35]. However, estimating time-offsets at sensors using a sampling-based optimization strategy requires extra consumption of the communication and computing resources and means limited applicability to the energy-constrained sensor network. The works mentioned above are all on the foundation of the PHD filter. Furthermore, none of these works takes unknown clutter and detection parameters into account.

When unknown model parameters are involved, robust PHD and CPHD filters have been presented in [36]–[38], which are able to estimate the clutter and detection parameters while filtering. Further improvements have been reported in [39]–[41]. Adaptive PHD and CPHD filters relying on the inverse gamma Gaussian mixture (GM) model have been given in [42], where the non-Gaussian feature parameters are utilized in addition to the position information in order to adapt the time-varying detection probability. Robust estimators [43]–[45] have also been presented to simultaneously estimate the multitarget state and clutter rate under the PHD filtering framework. However, they are designed for the single sensor case.

Based on the analyses mentioned above, we present the first ever distributed CPHD filter that is able to deal with different sampling rates among sensors and unknown clutter and detection parameters in this paper. The main contributions are summarized below.

- 1) We propose a timely fusion approach to DMT based on asynchronous sensors, where potential communication delay among sensors is allowable. In the proposed approach, the AA fusion of local CPHD estimates is performed as long as new measurements have been received by any sensor.
- 2) We adopt a partial flooding protocol for intersensor communication, in which merely local posteriors are disseminated among sensors. At each flooding iteration, the information exchanged between neighbors can either be the individual cardinality distribution or the pairwise cardinality distribution and intensity function.
- 3) We provide both feedback and non-feedback fusion-filtering modes with the intention of achieving preferable performance and reducing/avoiding sensor data delay, respectively. In the feedback mode, the local estimates are improved through fusion, whereas the fusion outcome does nothing help to the local estimates in the non-feedback mode.
- 4) We extend the timely fusion approach to handle unknown model parameters by exploiting a local bootstrap filter consisting of a PHD estimator and a CPHD filter. In particular, the PHD estimator provides the clutter and detection parameters for the CPHD filter, while the CPHD filter generates local estimates.

The paper is outlined as follows. Section II gives the relevant preliminaries. Sections III and IV detail the proposed timely fusion and robust bootstrap approaches, respectively. Section V presents numerical studies. Section VI concludes

this paper.

II. PRELIMINARIES

A. Background on RFS

An RFS is essentially a set-valued random variable of which the randomness can be captured by its probability density. Let $\mathcal{X}$ be an RFS and $f(\mathcal{X})$ be its probability density. The cardinality distribution of $\mathcal{X}$ denoted by $\rho(n)$ stands for the probability of the event that $\mathcal{X}$ comprises n elements. Another crucial statistic of $\mathcal{X}$ is the intensity function $\nu(x)$, whose integral, i.e., $\int_{x \in \mathcal{X}} \nu(x) dx$, stands for the expected number of elements in $\mathcal{X}$. Definitions and properties of the cardinality distribution and intensity function have been explicitly given in [7], [8].

We specify two kinds of RFSs used throughout this paper. An identical independently distributed cluster (IIDC) RFS $\mathcal{Y}$ has the probability density

$$f(\mathcal{Y}) = n! \rho(n) \prod_{y \in \mathcal{Y}} s(y). \quad (1)$$

where $s(y)$ represents the spatial distribution. The corresponding intensity function is given by

$$\nu(y) = \sum_{n=0}^{\infty} n \rho(n) s(y). \quad (2)$$

An IIDC RFS reduces to a Poisson RFS when the cardinality is Poisson distributed. Hence, the probability density and intensity function of a Poisson RFS $\mathcal{Z}$ can be respectively expressed as

$$f(\mathcal{Z}) = e^{-\lambda} \prod_{z \in \mathcal{Z}} \lambda s(z), \quad (3)$$

$$\nu(z) = \lambda s(z), \quad (4)$$

where λ denotes the mean of the Poisson distribution.

The Poisson and IIDC RFSs are closely related to the PHD and CPHD filters. To be specific, the intensity function of a Poisson RFS describing the multitarget state is recursively propagated in the PHD filter [9]. Whereas the CPHD filter [10] models the multitarget state as an IIDC RFS and jointly propagates its cardinality distribution and intensity function as time progresses.

B. Multisensor Multitarget System Model

Assume that there are N_k targets with states $x_k^i, i = 1, \dots, N_k$ in the scene at time k . The current multitarget state can then be modeled by an RFS $\mathcal{X}_k = \{x_k^1, \dots, x_k^{N_k}\}$. In this paper, $\mathcal{X}_k$ is formed by

$$\mathcal{X}_k = \left[\bigcup_{x_{k-1} \in \mathcal{X}_{k-1}} \mathcal{S}_k(x_{k-1}) \right] \cup \mathcal{B}_k, \quad (5)$$

where

$$\mathcal{S}_k(x_{k-1}) = \begin{cases} \{x_k\}, & \text{if } x_{k-1} \text{ continues to exist} \\ \emptyset, & \text{if } x_{k-1} \text{ vanishes} \end{cases} \quad (6)$$

and $\mathcal{B}_k$ is a Poisson RFS modeling newborn targets. It is supposed that newborn targets and existing targets are mutually independent and that the transition of single target state from x_{k-1} to x_k is determined by the Markov density $\varphi_{k|k-1}(x_k|x_{k-1})$.

A sensor network consisting of S sensors with different sampling rates $T_1, \dots, T_S$ is considered. Each sensor is capable of acquiring measurement data, processing local information and communicating with neighbors. Mathematically, the sensor network can be represented by $(\mathcal{E}, \mathcal{L})$, where $\mathcal{E} = \{1, \dots, S\}$ and $\mathcal{L} \subseteq \mathcal{E} \times \mathcal{E}$ denote the set of sensors and the set of communication links, respectively. The diameter of the graph is denoted by D_m . The sampling time instants (STIs) of all sensors are available and collected by $\mathcal{T} = \bigcup_{i=1}^S \mathcal{T}_i$, where $\mathcal{T}_i = \{\tau_i + mT_i, m \in \mathbb{N}^+\}$ and τ_i denotes the initial STI of sensor i . If sensor i can acquire measurement data at time k , i.e., $k \in \mathcal{T}_i$, the multitarget measurement at sensor i and time k can then be formulated by

$$\mathcal{Z}_{i,k} = \left[\bigcup_{x_k \in \mathcal{X}_k} \mathcal{D}_{i,k}(x_k) \right] \cup \mathcal{C}_{i,k}, \quad (7)$$

where

$$\mathcal{D}_{i,k}(x_k) = \begin{cases} \{z_{i,k}\}, & \text{if } x_k \text{ detected} \\ \emptyset, & \text{if } x_k \text{ misdetected} \end{cases} \quad (8)$$

and the detection probability is denoted by $p_{i,k}^d(x_k)$. If detected, a target-originated measurement $z_{i,k}$ is generated in accordance with the likelihood function $\psi_{i,k}(z_{i,k}|x_k)$. $\mathcal{C}_{i,k}$ is a Poisson RFS modeling clutter measurements (not originating from any target). The intensity function of $\mathcal{C}_{i,k}$ is given by $\kappa_{i,k} = \lambda_{i,c}/V_i$, where $\lambda_{i,c}$ and V_i denote the clutter rate and volume of the field of view (FOV), respectively. The clutter measurements and target-originated measurements are not related to each other.

C. AA Fusion

The AA fusion of S multitarget probability distributions¹ (MPDs) $f_1(\mathcal{X}), \dots, f_S(\mathcal{X})$ is given by

$$\bar{f}(\mathcal{X}) = \sum_{i=1}^S w_i f_i(\mathcal{X}), \quad (9)$$

where w_i denotes the positive fusing weight associated with sensor i and $\sum_{i=1}^S w_i = 1$.

As indicated in [46], [47], the AA fusion can be interpreted in the sense of Kullback-Leibler divergence (KLD). That is,

$$\bar{f}(\mathcal{X}) = \arg \min_g \sum_{i=1}^S w_i D_{\text{KL}}(f_i||g), \quad (10)$$

where $D_{\text{KL}}(f||g)$ stands for the KLD between MPDs $f(\mathcal{X})$ and $g(\mathcal{X})$, which can be computed as

$$D_{\text{KL}}(f||g) = \int f(\mathcal{X}) \log \frac{f(\mathcal{X})}{g(\mathcal{X})} \delta \mathcal{X}. \quad (11)$$

¹When an RFS is employed to model the multitarget state, its probability density is known as multitarget probability distribution.

It has also been proved in [6], [47] that the AA fusion is a Fréchet mean as follows:

$$\bar{f}(\mathcal{X}) = \arg \min_g \sum_{i=1}^S w_i \|f_i - g\|^2, \quad (12)$$

where $\|f\|^2 \triangleq \int (f(\mathcal{X}))^2 \delta \mathcal{X}$.

Recently, Li [20] pointed out that for MPDs $f_i(\mathcal{X}), i = 1, \dots, S$, the KLD of any MPD $g(\mathcal{X})$ relative to their AA fusion $\bar{f}(\mathcal{X})$ satisfies

$$\begin{aligned} D_{\text{KL}}(\bar{f}||g) &= \sum_{i=1}^S w_i (D_{\text{KL}}(f_i||g) - D_{\text{KL}}(f_i||\bar{f})), \\ &\leq \sum_{i=1}^S w_i D_{\text{KL}}(f_i||g), \end{aligned} \quad (13)$$

where the equation holds if and only if $f_1(\mathcal{X}), \dots, f_S(\mathcal{X})$ are identical. Let $g(\mathcal{X})$ be the target distribution, then we can observe from (13) that the AA fusion of a number of MPDs matches the target distribution better than all MPDs on average. This offers an information-theoretic justification for the AA fusion.

The following Lemma presents theoretical results of the AA fusion of IIDC MPDs.

Lemma 1: Given fusing IIDC MPDs $f_i(\mathcal{X}), i = 1, \dots, S$ and positive fusing weights $w_1, \dots, w_S$ subject to $\sum_{i=1}^S w_i = 1$. Their AA fusion $\bar{f}(\mathcal{X})$ has the cardinality distribution $\bar{\rho}(n) = \sum_{i=1}^S w_i \rho_i(n)$ and intensity function $\bar{\nu}(x) = \sum_{i=1}^S w_i \nu_i(x)$. Unfortunately, $\bar{f}(\mathcal{X})$ is not an IIDC MPD. To maintain the closure property, an IIDC MPD approximation of $\bar{f}(\mathcal{X})$ is given by

$$\tilde{f}(\mathcal{X}) = n! \bar{\rho}(n) \prod_{x=1}^n \bar{s}(x), \quad (14)$$

where $\bar{s}(x) = \bar{\nu}(x) / \sum_{n=0}^{\infty} n \bar{\rho}(n)$ and the intensity function as well as the cardinality distribution of $\tilde{f}(\mathcal{X})$ and $\bar{f}(\mathcal{X})$ are equivalent. Mathematical proofs of Lemma 1 have been given in [14], [46] and are therefore omitted here.

III. PROPOSED TIMELY FUSION APPROACH

In the timely fusion approach, the AA fusion of local CPHD estimates is carried out at each sensor $i \in \mathcal{E}$ and fusion time instant (FTI) $k \in \mathcal{T}$. A simple example is given in Fig. 1 to illustrate the timely fusion mechanism. It can be clearly seen that sensor S5 performs fusion at not only its own but also other sensors' STIs. In fact, the FTIs of sensors S1-S4 are consistent with those of sensor S5. The timely fusion approach comprises filtering, communication and fusion three steps. In what follows, we detail these steps in order.

A. Local Multitarget Filtering

Due to different sampling rates among sensors, measurements from distinct sensors are often time-misaligned. Hence, merely part of the sensors can receive measurement data at a generic FTI k . We name these sensors as measurement available sensors (MASSs) and the remaining sensors as measurement unavailable sensors (MUSs). Correspondingly, the

Fig. 1. Illustration of the timely fusion mechanism.

sensor set $\mathcal{E}$ can be decomposed into two subsets $\mathcal{E}_{a,k}$ and $\mathcal{E}_{u,k}$ with constraints $\mathcal{E} = \mathcal{E}_{a,k} \cup \mathcal{E}_{u,k}$ and $\mathcal{E}_{a,k} \cap \mathcal{E}_{u,k} = \emptyset$. Here, $\mathcal{E}_{a,k}$ and $\mathcal{E}_{u,k}$ stand for the set of MASs and the set of MUSs, respectively. At time k , each sensor i carries out a single sensor CPHD filter to get its local cardinality distribution $\rho_{i,k}(n)$ and intensity function $\nu_{i,k}(x)$. If $i \in \mathcal{E}_{a,k}$, $\rho_{i,k}(n)$ and $\nu_{i,k}(x)$ are obtained by sequential performing the single sensor CPHD prediction and update stages, otherwise, $\nu_{i,k}(x)$ and $\rho_{i,k}(n)$ are obtained by performing the single sensor CPHD prediction stage only. For details on the single sensor CPHD filter, readers are referred to [10], [12].

We employ the GM method [11], [12] to implement the local CPHD filter. As a result, the intensity function $\nu_{i,k}(x)$ is represented by $J_{i,k}$ Gaussian terms (GTs) with positive weights $\alpha_{i,k}^j$, $j = 1, \dots, J_{i,k}$, i.e.,

$$\nu_{i,k}(x) = \sum_{j=1}^{J_{i,k}} \alpha_{i,k}^j \mathcal{N}(x; \mu_{i,k}^j, \Sigma_{i,k}^j), \quad (15)$$

where $\mathcal{N}(x; \mu, \Sigma)$ denotes the Gaussian probability density with mean μ and covariance Σ . Equivalently, $\nu_{i,k}(x)$ is characterized by the parameter set $\mathcal{G}_{i,k} \triangleq \left\{ \left(\alpha_{i,k}^j, \mu_{i,k}^j, \Sigma_{i,k}^j \right) \right\}_{j=1, \dots, J_{i,k}}$. In principle, the cardinality distribution $\rho_{i,k}(n)$ is computationally infeasible since it involves an infinite number of terms. Fortunately, it is sufficient to truncate $\rho_{i,k}(n)$ at $n = N_{max}$ and approximate it with $\{\rho_{i,k}(n)\}_{n=0}^{N_{max}}$ [12]. The estimated target number $\hat{N}_{i,k}$ can be calculated by the mode $\hat{N}_{i,k} = \arg \max_n \rho_{i,k}(n)$ or the mean $\hat{N}_{i,k} = \sum_{j=1}^{J_{i,k}} \alpha_{i,k}^j$. Since the mode is superior to the mean in general, it can be used to modify the weights of GTs as done in (19).

Abiding by [27], merely target-like GTs (T-GTs) are disseminated while the remaining false-alarm-like GTs are discarded. This has been proved to be able to not only reduce the computational burden and communication cost but also reduce the adverse effect of the AA fusion. We adopt a threshold rule to identify the T-GTs, where the GTs with weights greater than a preset threshold r_{gate} are regarded as the T-GTs.

B. Partial Flooding Communication

We begin with a brief review of the distributed flooding protocol [48], in which each sensor plays a role analogous to a fusion center and collects information from all other sensors through iterative peer to peer (P2P) communication. Under the premise of strongly connected sensor network, complete consensus, namely the information set of each sensor is identical, can be achieved through communication iterations. Determination of the total number of communication iterations has been explicitly investigated in [19]. In spite of its appealing results in theory, the distributed flooding protocol poses severe challenges to the communication and storage capabilities in practice.

In this work, we adopt an efficient flooding protocol for intersensor communication, where only local posteriors are disseminated among sensors. The resulting partial flooding is conditioned on the relevant information from a part of the sensors (more precisely stated, MASs), which differs from the existing flooding protocol conditioned on the information from all sensors. The motivation for the partial flooding is the fact that the local posterior is more accurate than the local prediction from the Bayesian inference perspective.

To gain insights of the consensus of the partial flooding, we encapsulate the information that needs to be transmitted into a set and define the following dynamics:

$$\mathcal{I}_i(\ell + 1) = \mathcal{I}_i(\ell) \cup \mathcal{U}_i(\ell), \quad (16)$$

where $\mathcal{I}_i(\ell)$ and $\mathcal{U}_i(\ell)$ stand for the existing and newly received information sets of sensor i at iteration ℓ , respectively, $\mathcal{I}_i(0)$ stands for the initial information set of sensor i satisfying $\forall i \in \mathcal{E}, |\mathcal{I}_i(0)| = 1$ and $\mathcal{U}_i(0) = \emptyset$.

As a result of partial flooding, at any time $k \in \mathcal{F}$, we have the following results:

- 1) $\forall i \in \mathcal{E}_{a,k}, \ell \geq D_m, \mathcal{I}_i(\ell) = \bigcup_{j \in \mathcal{E}_{a,k}} \mathcal{I}_j(0)$.
- 2) $\forall i \in \mathcal{E}_{u,k}, \ell \geq D_m, \mathcal{I}_i(\ell) = \bigcup_{j \in \mathcal{E}_{a,k} \cup \{i\}} \mathcal{I}_j(0)$.

Clearly, complete consensus is achieved in MASs and MUSs, respectively.

C. AA-Based CPHD Fusion

After the final flooding iteration L , the collection of cardinality distributions and intensity functions at sensor i and time

k is given by $\{\rho_{r,k}(n), \nu_{r,k}(x)\}_{r \in \mathcal{M}_i(L) \cup \{i\}}$, where $\mathcal{M}_i(L)$ stands for the collection of sensors that are no more than L hops away from sensor i . Note that $i \notin \mathcal{M}_i(L)$. Under the premise of uniform fusion weights, Lemma 1 is realized in the following Proposition.

Proposition 1: The mean cardinality distribution and mean intensity function are respectively computed as follows:

$$\bar{\rho}_{i,k}^{[L]}(n) = \frac{1}{|\mathcal{M}_{i,k}(L)|} \sum_{r \in \mathcal{M}_{i,k}(L)} \rho_{r,k}(n), \quad (17)$$

$$\bar{\nu}_{i,k}^{[L]}(x) = \frac{1}{|\mathcal{M}_{i,k}(L)|} \sum_{r \in \mathcal{M}_{i,k}(L)} \nu_{r,k}(x), \quad (18)$$

where $\mathcal{M}_{i,k}(L) = \mathcal{M}_i(L) \cup \{i\} \cap \mathcal{E}_{a,k}$ and the initial conditions are specified by $\bar{\rho}_{i,k}^{[0]}(n) = \rho_{i,k}(n)$ and $\bar{\nu}_{i,k}^{[0]}(n) = \nu_{i,k}(n)$, respectively. After pruning and merging steps [11], [12], $\bar{\nu}_{i,k}^{[L]}(x)$ can be represented by the fused GM parameters $\bar{\mathcal{G}}_{i,k} \triangleq \left\{ \left(\bar{\alpha}_{i,k}^j, \bar{\mu}_{i,k}^j, \bar{\Sigma}_{i,k}^j \right) \right\}_{j=1, \dots, \bar{J}_{i,k}}$. The target number estimate $\hat{N}_{i,k} = \arg \max_n \bar{\rho}_{i,k}^{[L]}(n)$ is then exploited to modify the weights $\bar{\alpha}_{i,k}^j, j = 1, \dots, \bar{J}_{i,k}$ as follows:

$$\bar{\alpha}_{i,k}^j = \beta_i \bar{\alpha}_{i,k}^j, \quad (19)$$

where $\beta_i = \hat{N}_{i,k} / \sum_{j=1}^{\bar{J}_{i,k}} \bar{\alpha}_{i,k}^j$.

The procedures of the AA-CPHD fusion are summarized in Algorithms 1.

Algorithm 1: AA-CPHD Fusion Based on Partial Flooding.

Input: $\{\rho_{i,k}(n), \mathcal{G}_{i,k}\}_{i=1, \dots, S}$

Output: $\{\bar{\rho}_{i,k}(n), \bar{\mathcal{G}}_{i,k}\}_{i=1, \dots, S}$

- 1: **for** each sensor $i = 1, \dots, S$ **do**
 - 2: Identify T-GTs using the threshold rule stated in Section III-A.
 - 3: **end for**
 - 4: **for** L communication iterations **do**
 - 5: Each sensor exchanges the cardinality distribution and intensity function with its neighbors according to the partial flooding protocol presented in Section III-B.
 - 6: **end for**
 - 7: **for** each sensor $i = 1, \dots, S$ **do**
 - 8: Calculate $\bar{\rho}_{i,k}(n)$ and $\bar{\nu}_{i,k}(n)$ via (17) and (18), respectively.
 - 9: Re-weight GTs via (19).
 - 10: Prune and merge the GTs leading to $\bar{\mathcal{G}}_{i,k}$.
 - 11: **end for**
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Remark 1: As indicated in Lemma 1, the fusion of $\rho_{i,k}(n)$ is irrelative to $\nu_{i,k}(x)$. It is therefore feasible to disseminate and fuse $\rho_{i,k}(n)$ only. That means merely the cardinality distribution average in (17) is required. Afterward, the target number estimate $\hat{N}_{i,k}$ is exploited to scale the weights of unfused T-GCs in a way similar to (19). The resulting approach is named as AA-based cardinality consensus (hereafter abbreviated as AA-CC).

Remark 2: We provide feedback and non-feedback fusion-filtering modes to the AA-CPHD fusion and the AA-CC fusion. In the feedback mode, the fusion outcome takes part

Fig. 2. Schematic diagram of the feedback and non-feedback modes.

in the update of local estimates. When the communication and fusion operations are time-costly due to plenty of reasons such as high sampling rate and large network size, the feedback mode suffers from sensor data delay since the measurement data is unused until the end of the fusion operation. To mitigate this drawback, we adopt a non-feedback alternative [49], in which the fusion outcome does nothing help to the local estimates. A schematic diagram of the two schemes with regard to sensor i and its three successive FTIs r, s, t , is presented in Fig. 2, where $r < s < t$, $r = mT_i + \tau_i$, and $t = (m+1)T_i + \tau_i$. As shown, it is the feedback mode rather than the non-feedback mode that improves the local estimates via fusion.

IV. EXTENSION TO UNKNOWN MODEL PARAMETERS

In this section, we focus on the proposed robust bootstrap approach, which extends the timely fusion approach discussed in the previous section to accommodate unknown clutter and detection parameters using the bootstrap filtering scheme analogous to [39], [40]. The basic idea of the scheme is to recursively estimate, at each sensor $i \in \mathcal{E}$ and time $k \in \mathcal{T}_i$, the required model parameters by exploiting a local PHD estimator first and then feed its estimates to a local CPHD filter, as demonstrated in Fig. 3. Such a scheme inherits the benefit of the PHD filter regarding linear complexity and the benefit of the CPHD filter in that it provides high quality estimates. What distinguishes the timely fusion approach from the robust bootstrap approach is the local filtering step. Specifically, the former adopts the local CPHD filtering scheme, whereas the bootstrap filtering scheme is employed in the latter. In the following, we present the local bootstrap filtering step for the cases of unknown parameters in clutter and detection, respectively.

A. Unknown Clutter Rate Case

The centerpiece of dealing with unknown clutter rate is to model clutter as a set of false targets (distinct from actual targets). Both false and actual targets follow standard dynamic and measurement models. The multitarget state is therefore

Fig. 3. Schematic diagram of the local bootstrap filtering scheme at sensor i and time k .

formulated by a hybrid RFS, which consists of a disjoint union of the RFSs associated with actual and false targets. Estimating the hybrid RFS yields the number and the state information on actual targets together with the clutter rate.

The PHD filter with unknown clutter rate (hereafter referred to as λ -PHD filter) recursively propagates the intensity function $\check{\nu}_k(\check{x})$ of the hybrid RFS $\check{\mathcal{X}}_k = \mathcal{X}_k^f \uplus \mathcal{X}_k^a$, which can be decomposed into

$$\check{\nu}_k(\check{x}) = \begin{cases} \nu_k^f(c), & \check{x} = c \\ \nu_k^a(x), & \check{x} = x \end{cases}, \quad (20)$$

where $\nu_k^f(\cdot)$ and $\nu_k^a(\cdot)$ denote the intensity functions for the false target RFS $\mathcal{X}_k^f$ and actual target RFS $\mathcal{X}_k^a$, respectively, $\uplus$ denotes the disjoint union, $\check{x} \in \check{\mathcal{X}}_k$, $c \in \mathcal{X}_k^f$ and $x \in \mathcal{X}_k^a$. Since the measurements generated by false targets are independent of the clutter state c , $\nu_k^{(0)}(\cdot)$ is completely determined by the mean number of false targets $N_k^{(0)}$. Hence, the λ -PHD filter jointly propagates $\nu_k^{(1)}(\cdot)$ and $N_k^{(0)}$ over time. More details about the λ -PHD filter can be found in [36], [38].

In this work, at each sensor $i \in \mathcal{E}$ and every STI $k \in \mathcal{T}_i$, the λ -PHD filter is first updated using the received measurement $\mathcal{Z}_{i,k}$ based on the GM implementation. Then, the estimated clutter rate is computed by $\hat{r}_{i,c} = N_{i,k}^{(0)} p_{D,k}^{(0)}$, where $p_{D,k}^{(0)}$ stands for the detection probability of false targets. Finally, $\hat{r}_{i,c}$ is exploited in the CPHD filter to obtain the posterior cardinality distribution $\rho_{i,k}(n)$ and intensity function $\nu_{i,k}(x)$.

B. Unknown Detection Probability Case

Based on the augmented state model [37], the standard PHD filter can be easily extended to deal with unknown detection probability. The augmented PHD filter recursively propagates the intensity function $\underline{\nu}_k(x, a)$ of the multitarget RFS $\underline{\mathcal{X}}_k$ containing information on the kinematic state x and detection probability a . A closed-form solution to the augmented PHD recursion named the Beta GM PHD (BGM-PHD) filter was derived in [36], where the Gaussian and Beta distributions are employed to model the kinematic state and detection probability, respectively.

In the present paper, the BGM-PHD filter is first performed at each sensor $i \in \mathcal{E}$ and every STI $k \in \mathcal{T}_i$. As a result, the intensity function $\underline{\nu}_{i,k}(x, a)$ is represented by $J_{i,k}$ Beta-Gaussian terms with positive weights $\alpha_{i,k}^j$, $j = 1, \dots, J_{i,k}$, i.e.,

$$\underline{\nu}_{i,k}(x, a) = \sum_{j=1}^{J_{i,k}} \alpha_{i,k}^j \mathcal{N}(x; \mu_{i,k}^j, \Sigma_{i,k}^j) \beta(a; s_{i,k}^j, t_{i,k}^j), \quad (21)$$

where $\beta(\cdot; s, t)$ denotes the Beta distribution with shape parameters s and t . The equivalent detection probability at sensor i is then calculated by

$$p_{i,k}^d = \frac{\sum_{j=1}^{J_{i,k}} \alpha_{i,k}^j \xi_{i,k}^j}{\sum_{j=1}^{J_{i,k}} \alpha_{i,k}^j}, \quad (22)$$

where $\xi_{i,k}^j = \frac{s_{i,k}^j}{s_{i,k}^j + t_{i,k}^j}$ denotes the mean of the j -th Beta term $\beta(a; s_{i,k}^j, t_{i,k}^j)$. With the help of $p_{i,k}^d$, the posterior intensity function $\nu_{i,k}(x)$ and cardinality distribution $\rho_{i,k}(n)$ can be obtained by updating the local CPHD filter.

V. SIMULATION RESULTS

This section provides two simulations examples to test the approaches proposed in Sections III and IV. In both examples, a sensor network consisting of 20 sensors S1-S20 with diameter $D_m = 5$ is deployed in the surveillance region $[-2\text{km}, 2\text{km}] \times [-2\text{km}, 2\text{km}]$. The sampling rates of sensors S1-S10 and sensors S11-S20 are 2s and 3s, respectively. All sensors start sampling at the initial time $\tau = 1\text{s}$. At each example, 100 trials are carried out. The duration time of each trial is 120s. The network optimal subpattern assignment (N-OSPA) and time-averaged N-OSPA (TN-OSPA) errors [6] are used throughout the examples to evaluate the tracking performance. The corresponding cut-off parameter and order parameter (see [50] for the meaning of these parameters) are set as $c = 200\text{m}$ and $p = 2$, respectively.

A. Linear Example With Known Model Parameters

In this example, each target follows a constant velocity dynamic model and each sensor generates linear measurements. The target state is a vector $x_k = [p_k^x, p_k^y, v_k^x, v_k^y]^T$ of position $[p_k^x, p_k^y]^T$ and velocity $[v_k^x, v_k^y]^T$. The single target Markov density is specified by $\varphi_{k|k-1}(x_k | x_{k-1}) = \mathcal{N}(x_k; F_k x_{k-1}, Q_k)$ with

$$F_k = \begin{bmatrix} \mathbf{I}_{2 \times 2} & \Delta \mathbf{I}_{2 \times 2} \\ \mathbf{0}_{2 \times 2} & \mathbf{I}_{2 \times 2} \end{bmatrix}, Q_k = \sigma_v^2 \begin{bmatrix} \frac{\Delta^4}{4} \mathbf{I}_{2 \times 2} & \frac{\Delta^3}{2} \mathbf{I}_{2 \times 2} \\ \frac{\Delta^3}{2} \mathbf{I}_{2 \times 2} & \Delta^2 \mathbf{I}_{2 \times 2} \end{bmatrix},$$

where $\mathbf{I}_{n \times m}$ and $\mathbf{0}_{n \times m}$ stand for the $n \times m$ identity and zero matrices, respectively, $\Delta = 1\text{s}$ and $\sigma_v = 1\text{m/s}^2$. The birth intensity function γ_k is given by $\gamma_k = \sum_{j=1}^4 \alpha_{\gamma}^j \mathcal{N}(x; \mu_{\gamma}^j, \Sigma_{\gamma})$ where $\alpha_{\gamma} = 0.03$, $\mu_{\gamma}^1 = [0, 0, 500\text{m}, 0]^T$, $\mu_{\gamma}^2 = [400\text{m}, 0, -600\text{m}, 0]^T$, $\mu_{\gamma}^3 = [-800\text{m}, 0, -200\text{m}, 0]^T$, $\mu_{\gamma}^4 = [-200\text{m}, 0, 800\text{m}, 0]^T$, $\Sigma_{\gamma} = \text{diag}([10\text{m}, 10\text{m/s}, 10\text{m}, 10\text{m/s}]^T)^2$. The survival probability is a constant 0.98.

If target x_k is detected by the sensor i at time k , then a noisy position measurement $z_{i,k}$ is generated according to the likelihood function $\psi_{i,k}(z_{i,k} | x_k) = \mathcal{N}(z_{i,k}; H_{i,k} x_k, R_{i,k})$ with

$$H_{i,k} = [\mathbf{I}_{2 \times 2} \quad \mathbf{0}_{2 \times 2}], R_{i,k} = \sigma_{\varepsilon}^2 \mathbf{I}_{2 \times 2},$$

where $\sigma_{\varepsilon} = 10\text{m}$. The FOV of each sensor is the surveillance region. The clutter rate and detection probability of all sensors

Fig. 4. Linear simulation scenario. Sensors are positioned at magenta squares and cyan pentagrams, respectively. Target trajectories are plotted by red lines with the start and end positions given by circles and triangles, respectively. Communication links are indicated by black dashed lines. The estimates of the non-cooperative and AA-CPHD-F filters are marked by blue and green points, respectively.

Fig. 5. Averaged target number estimates and N-OSPA errors versus time.

are known a priori and set as $\lambda_{i,c} = 10$ and $p_{i,k}^d = 0.95, i = 1, \dots, 20$, respectively.

Filters resulting from the timely fusion approach including the AA-CPHD with feedback (in short AA-CPHD-F), AA-CPHD with non-feedback (in short AA-CPHD-NF), AA-CC with feedback (in short AA-CC-F) and AA-CC with non-feedback (in short AA-CC-NF) are compared with the non-cooperative filter without communication and fusion operations. Each local CPHD filter truncates the cardinality distribution at $N_{max} = 50$, makes use of at most 100 GCs to represent the intensity function, selects T-GCs using a weight threshold $r_{gate} = 0.001$ and merges T-GCs with Mahalanobis distance below 4 m.

The true trajectories and the sensor topology together with position estimates of the AA-CPHD-F filter (when $L = 5$) and non-cooperative filter are plotted in Fig. 4. Fig. 5 presents the averaged target number estimates and N-OSPA errors for the filters under study. The results indicate the following findings:

- The proposed filters perform better than the non-

Fig. 6. Averaged TN-OSPA errors versus communication iteration.

Fig. 7. Averaged TN-OSPA errors versus varying detection probability.

Fig. 8. Averaged TN-OSPA errors versus varying clutter rate.

cooperative filter thanks to the AA fusion.

- The feedback mode is superior to its non-feedback counterpart since the fusion outcome is helpful to improve the local estimates.
- The AA-CPHD fusion prefers over the AA-CC fusion, which can be attributed to the fact that more target-related information is used in the fusion operation.

Fig. 6 shows the TN-OSPA errors versus communication iteration for different filters. As expected, the TN-OSPA errors for different filters decrease with the increase of the number of communication iterations. We also test the TN-OSPA errors versus varying clutter rate and varying detection probability for the AA-CPHD-F, AA-CPHD-NF, AA-CC-F and AA-CC-NF filters, respectively. The corresponding results are given in Figs. 7 and 8, which further consolidate the results in Fig. 5.

B. Nonlinear Example With Unknown Model Parameters

In this example, the single target state is given by $x_k = [\tilde{x}_k^T, \omega_k]^T$ with position and velocity $\tilde{x}_k^T = [p_k^x, v_k^x, p_k^y, v_k^y]^T$ together with turn rate ω_k . The state Markov density is spec-

Fig. 9. Nonlinear simulation scenario. The estimates of the R-AA-CPHD-F filter for unknown clutter and detection parameters are plotted by green and cyan points, respectively. The other landmarks are consistent with Fig. 4.

ified by $\varphi_{k|k-1}(x_k|x_{k-1}) = \mathcal{N}(x_k; F(\omega_k)x_{k-1}, Q_k)$, where

$$F(\omega) = \begin{bmatrix} 1 & \frac{\sin \omega \Delta}{\omega} & 0 & \frac{1 - \cos \omega \Delta}{\omega} & 0 \\ 0 & \cos \omega \Delta & 0 & -\sin \omega \Delta & 0 \\ 0 & \frac{1 - \cos \omega \Delta}{\omega} & 1 & \frac{\sin \omega \Delta}{\omega} & 0 \\ 0 & \sin \omega \Delta & 0 & \cos \omega \Delta & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix},$$

and $Q_k = \text{diag}([I_2 \otimes G], \sigma_u^2)$ with

$$G = \begin{bmatrix} \frac{\sigma_\omega^2}{4} & \frac{\sigma_\omega^2}{2} \\ \frac{\sigma_\omega^2}{2} & \sigma_\omega^2 \end{bmatrix},$$

where $\otimes$ denotes the Kronecker product, $\Delta = 1\text{s}$, $\sigma_\omega = 1\text{m/s}^2$ and $\sigma_u = (\pi/360)\text{rad/s}$. The birth intensity function γ_k is given by $\gamma_k = \sum_{j=1}^4 \alpha_j^i \mathcal{N}(x; \mu_j^i, \Sigma_\gamma)$ where $\alpha_\gamma^1 = \alpha_\gamma^2 = 0.02$, $\alpha_\gamma^3 = \alpha_\gamma^4 = 0.03$, $\mu_\gamma^1 = [900\text{m}, 0, 400\text{m}, 0, 0]^T$, $\mu_\gamma^2 = [-300\text{m}, 0, 700\text{m}, 0, 0]^T$, $\mu_\gamma^3 = [-400\text{m}, 0, 0, 0, 0]^T$, $\mu_\gamma^4 = [250\text{m}, 0, 750\text{m}, 0, 0]^T$, $\Sigma_\gamma = \text{diag}([20\text{m}, 20\text{m/s}, 20\text{m}, 20\text{m/s}, 4(\pi/180)\text{rad/s}]^T)^2$. If detected by the sensor i at time k , the single target measurement model is determined by

$$z_{i,k} = \left[\frac{\arctan\left(\frac{p_k^x - p_i^x}{p_k^y - p_i^y}\right)}{\sqrt{(p_k^x - p_i^x)^2 + (p_k^y - p_i^y)^2}} \right] + \varepsilon_{i,k}$$

where $[p_i^x, p_i^y]^T$ denotes the location of sensor i , $\varepsilon_{i,k} \sim \mathcal{N}(\cdot; 0_{2 \times 1}, R_{i,k})$, $R_{i,k} = \text{diag}([\sigma_\theta^2, \sigma_r^2]^T)$, $\sigma_\theta = (\pi/180)\text{rad/s}$ and $\sigma_r = 10\text{m}$. The FoV of each sensor is a circle with the sensor's location as the center and 5000 m as the radius, which can cover the surveillance region.

We demonstrate the performance of the robust bootstrap approach in two separate cases. In particular, the AA-CPHD-F and AA-CPHD-NF filters are considered to accommodate unknown model parameters on account of their preferable performance, yielding the robust AA-CPHD-F (in short R-AA-CPHD-F) filter and the robust AA-CPHD-NF (in short R-AA-CPHD-NF) filter. Each case also contains a comparison with the AA-CPHD-F, AA-CPHD-NF and non-cooperative filters.

Fig. 10. Averaged TN-OSPA errors versus communication iterations for the unknown clutter rate case.

Fig. 11. Average TN-OSPA errors versus communication iterations for the unknown detection probability case.

For the case of unknown clutter rate, the mean clutter rate for each sensor is 20. For the case of unknown detection probability, the detection probability for each sensor is a constant 0.9. These information is not known to the R-AA-CPHD-F and R-AA-CPHD-NF filters. The other parameters used in the local PHD estimators follow the choice of [36]. The local CPHD filters adopt the unscented transform to handle the system nonlinearity.

Fig. 9. plots the position estimates of the R-AA-CPHD-F (when $L = 5$) filter and non-cooperative filters for two cases in addition to the true trajectories as well as the sensor topology. The TN-OSPA errors versus communication iteration for the two cases are shown in Figs. 10 and 11. As shown, the R-AA-CPHD-F filter performs almost as well as its benchmark filter, i.e., the AA-CPHD-F filter and so does the R-AA-CPHD-NF filter. These results suggest that the proposed robust bootstrap approach can deal with unknown clutter and detection parameters without difficulty.

VI. CONCLUSION

In this paper, two DMT approaches are proposed to handle different sampling rates among sensors. In the first timely fusion approach, the CPHD filter is carried out at each sensor for generating local estimates, while the AA fusion is employed to combine local estimates from different sensors. The information exchange among sensors follows a partial flooding protocol, in which either intensity functions or cardinality distributions with respect to local posteriors are broadcast throughout the network. In addition to the feedback fusion-

filtering mode, where the fusion outcome is utilized to update local estimates, a non-feedback alternative is also investigated for the purpose of avoiding/reducing sensor data delay. The second robust bootstrap approach is an extension of the timely fusion approach, which takes unknown clutter and detection parameters into consideration. The centerpiece of the robust bootstrap approach is the local bootstrap filter consisting of a PHD estimator and a CPHD filter. Specifically, the PHD estimator is run first and its estimates of the model parameters are then provided to the CPHD filter to generate the local posterior estimates. Numerical simulation are performed and their results demonstrate the validity of the proposed timely fusion and robust bootstrap approaches. Our future work will consider extensions of the proposed approaches to labeled RFS filters [51], [52] and resource-limited sensor networks [53]–[55].

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