Stabilization with Prescribed Instant for High-Order Integrator Systems

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Abstract—This paper develops a new controller design approach to stabilize system states x onto the equilibrium at an arbitrarily selected time instant t_f irrespective of initial system states x_0 and parameters. Therefore, the actual convergence time is independent with x_0 , and $\dot{x} \neq 0$ until $t \geq t_f$, which is different with traditional fixed, predefined, and prescribed time stability. In this work, a series of potential practicable time-varying feedback functions are explored, by which a novel controller is designed for high-order integrator systems. The proposed control is bounded and can gradually go to zero at an arbitrarily selected time instant, at which the system states reach zero simultaneously. This special stability of the controlled system is analyzed by reduction to absurdity rather than the Lyapunov method. The effectiveness of the proposed results is illustrated by numerical simulations.

Index Terms—Finite-time control, Prescribed-time control, Time-varying feedback, Reduction to absurdity

I. INTRODUCTION

AS is well-known, the central philosophy of finite-time stabilization (FNTS) is the guarantee of the state convergence over a finite time interval while providing an estimation of the convergence time (i.e., the settling time) depending on initial states and control parameters [1], [2]. For autonomous systems, the finite-time stability analysis was formally presented in [3] in which a Lyapunov-based inequality $\dot{V}(x) \leq -aV^b(x)$ with $b \in (0, 1)$ and $a > 0$ was developed. The real convergence time $T(x_0)$ satisfies $T(x_0) \leq \frac{V(x_0)^{1-b}}{a(1-b)}$, and there is no uniform bounds for $T(x_0), \forall x_0 \in \mathbb{R}^n$. Many variants around this Lyapunov inequality were further proposed, for example, the homogeneous approach [4], the implicit Lyapunov function approach [5], the adaptive control [6]. These years, the FNTS method was further developed as three branches: the fixed-time stabilization (FTS), the predefined-time stabilization (PDTs), and the prescribed-time stabilization (PSTS). These control methods are classified into two categories, one is the FTS and the PDTs, the other is the PSTS.

The FTS and the PDTs get finite-time stability by utilizing the power of some special functions of system states. the

FTS ensures the convergence time $T(x_0)$ to be bounded by a constant $T_{\max}$, which is independent of the initial conditions x_0 , in another word, $T(x_0) \leq T_{\max}, \forall x_0 \in \mathbb{R}^n$ [7], [8]. The constant $T_{\max}$ is determined by some controller parameters, so one should calculate the parameters according to an assigned $T_{\max}$. As two important applications, fixed-time differentiator [9] and fixed-time observer [10] were then proposed. Moreover, the robust FTS of MIMO systems under matched and unmatched disturbances were also studied [11]. However, the actual convergence time may be far smaller than the designed $T_{\max}$ [12]. Hence, it is logical to pursue less conservativeness in the FTS. To this end, the notion of the PDTs was proposed as a result of $\sup_{x_0 \in \mathbb{R}^n} T(x_0) = T_f$ [13], [14]. Another improvement of the PDTs among the FTS is that the assigned time T_f is a constant parameter explicit in the controller. This feature is quite similar to the PSTS. One defect of the traditional predefined-time controller is that the initial control grows exponentially with x_0 . This weakness was overcome by the construction of a series of new controllers with polynomial terms instead of exponential ones [15].

The idea of the PSTS is inspired by some time-varying feedback functions that seem to go to infinity towards the terminal time T_f [16]. This method ensures $T(x_0) \leq T_f, \forall x_0 \in \mathbb{R}^n$, where T_f is a constant parameter can be arbitrarily assigned and is explicit in the controller [17], [18]. The PSTS was combined with parametric Lyapunov equations and then got a uniformly bound of the control variable [19]. The PSTS was also appropriately recast within the framework of time-varying homogeneity [20]. Moreover, the PSTS with uncertainties was studied for nonlinear systems [21], [22]. Compared with the PDTs, the assigned time T_f of the PSTS is more conservative. It is noted that the singularity or discontinuity of prescribed-time control may appear when t tends to $t_0 + T_f$, as there is usually a term $\frac{1}{t_0 + T_f - t}$ in time-varying feedback functions. Hence, such controllers on PSTS should be carefully designed to ensure their boundedness [23].

Currently, the methods to achieve the FNTS introduced above have been combined and learned from each other. Some interesting achievements and applications are listed below. Notably, by designing autonomous stabilizing controllers with some added time-varying gains, the work in [24] derived a predefined upper bound of the settling time and provided sufficient conditions such that the time-varying gains remain bounded. To mention a few, Michiels et al. generalized the fixed-time control to linear systems with input delays by using the so-called act-and-wait control strategy [25]. Hu et al. improved the FTS to deal with the problem of synchronization for

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discontinuous dynamic networks [26]. Zhang et al. proposed a novel bipartite fixed-time observer to address the bipartite fixed-time output consensus problem of heterogeneous linear multiagent systems [27]. Kunal et al. proposed a fixed-time stable gradient-flow scheme to yield convergence to an optimal point of a convex optimization problem within a fixed time [28]. Mechali et al. proposed an observer-based fixed-time continuous nonsingular terminal sliding mode control method to solve the problem of robust trajectory tracking for a quadrotor aircraft under disturbances [29].

To sum up, the abovementioned methods more or less have some conservativeness on the real convergence time $T(x_0)$, i.e., $T(x_0) \leq T_f, \forall x_0 \in \mathbb{R}^n$ and $T(x_0) \leq T_{\max}, \forall x_0 \in \mathbb{R}^n$. This paper aims to design a controller based on the n -order chain of integrators to achieve $T(x_0) = T_f, \forall x_0 \in \mathbb{R}^n$. Firstly, the n -order chain of integrators is transformed to a set of differential equations containing special time-varying functions. Secondly, the transformed system is proved to converge to zero at a selected time instant. Then, the convergence of the original system and the transformed system are proved to be equivalent. This solution is also inspired by time-varying feedback functions, and somewhat belongs to the PSTS. Moreover, the convergence to zero happening at a selected time instant is guaranteed. For these reasons, the proposed stabilization method of this paper is called the prescribed-instant stabilization (PSIS¹) in the following context. Our main contribution can be summarized as follows:

- a) Several new time-varying feedback functions are proposed.
- b) A recursive controller based on the n -order chain of integrators is proposed by applying the time-varying feedback functions.
- c) The prescribed-instant stability is proved by reduction to absurdity, which is rarely utilized in the stability analysis.

The rest of the paper is structured as follows: Section II provides some basic knowledge and followed the problem statement. Section III gives some new time-varying feedback functions, based on which the general form of the controller is presented. Then reduction to absurdity is utilized in the analysis of the PSIS. In Section IV, simulations are provided to validate the proposed results. Section V concludes the paper.

II. PRELIMINARIES AND PROBLEM STATEMENT

A. Preliminaries

Consider the following autonomous system:

$$\dot{x} = f(x, \eta), \quad x(t_0) = x_0, \quad (1)$$

where $x \in \mathbb{R}^n$ is the system state, and the vector $\eta \in \mathbb{R}^l$ stands for the tunable parameters of (1). The function $f : \mathbb{R}_{\geq 0} \times \mathbb{R}^n \rightarrow \mathbb{R}^n$ may be discontinuous, and such solutions of (1) exist and are unique in the sense of Filippov [30]. Thus, $\Phi(t, x_0)$ denotes the solution of (1) starting from $x_0 \in \mathbb{R}^n$ at $t = 0$. Moreover, the origin $x = 0$ is the unique equilibrium point of (1).

¹The PSIS will be the abbreviation of “prescribed-instant stable”, “prescribed-instant stability” and “prescribed-instant stabilization” without distinction in the context.

As stated in [31], the parameter-dependent system (1) is equivalent to the controlled system

$$\dot{x} = g(x, u), \quad (2)$$

where $g : \mathbb{R}^n \times \mathbb{R}^m \rightarrow \mathbb{R}^n$, the control $u \in \mathbb{R}^m$ is a feedback function of x with tunable parameters η , i.e., $u = w(x, \eta)$, with $w : \mathbb{R}^n \rightarrow \mathbb{R}^m$. Substituting $u = w(x, \eta)$ in (2) eliminates u and yields $f(x, \eta) := g(x, w(x, \eta))$. All the notions defined and treated hereafter are global, so we will omit to indicate it.

Definition 1: The origin of the system (1) is said to be

(1) finite-time stable (FNTS) [1], if it is Lyapunov stable and for any $x_0 \in \mathbb{R}^n$ there exists $\mathcal{T} > 0$ such that $\Phi(t, x_0) = 0, \forall t \geq t_0 + \mathcal{T}$. The settling-time function of (1) is defined as, $T(x_0) = \inf \{ \mathcal{T} \geq 0 : \Phi(t, x_0) = 0, \forall t \geq t_0 + \mathcal{T} \}$.

(2) fixed-time stable (FTS) [8], if it is finite-time stable and the settling-time function is bounded, i.e., $\exists 0 < T_{\max} < \infty$ such that $T(x_0) \leq T_{\max}, \forall x_0 \in \mathbb{R}^n$.

Remark 1: It is important to highlight that $T_{\max}$ is usually a function of η , and $T_{\max}$ is not explicit in the control u . On the other hand, $T_{\max}$ can be chosen larger than the calculated result from η . For instance, if the settling time function is bounded by T_m , it is also bounded by $\lambda T_m, \forall \lambda \geq 1$. How to find the smallest $T_{\max}$ motivates the following definition.

Definition 2: [12] The origin of the system (1) is said to be predefined-time stable (PDTS) if it is fixed-time stable, and for any positive number T_f , there exists some $\eta \in \mathbb{R}^l$ such that either of the following can be established,

- (a) $T(x_0) \leq T_f, \forall x_0 \in \mathbb{R}^n$ (weak-predefined time)
- (b) $\sup_{x_0 \in \mathbb{R}^n} T(x_0) = T_f$ (strong-predefined time)

Remark 2: In a strict sense, the time T_f can be considered as the true fixed-time in which the system (1) is stabilized. Moreover, T_f used in PDTS is usually a single constant parameter explicit in the controller, i.e., $u = w(x, \eta, T_f)$.

The FNTS, the FTS, and the PDTS are realized by the control $u = w(x, \eta)$ independent with time t . Now, let us consider a time-varying control $u = w(x, t, \eta)$. Then, substituting it into (2) yields a nonautonomous system:

$$\dot{x} = f(x, t, \eta) := g(x, w(x, t, \eta)), \quad x(t_0) = x_0. \quad (3)$$

Definition 3: [18], [23] The origin of the system (3) is said to be prescribed-time stable (PSTS) if it is finite-time stable, and for any positive number T_f , there exists some $\eta \in \mathbb{R}^l$ such that $T(x_0) \leq T_f, \forall x_0 \in \mathbb{R}^n$.

Remark 3: T_f in PSTS is usually a constant parameter (independent with other parameters) explicit in the control, i.e., $u = w(x, t, \eta, T_f)$. In a strict sense, existing proof of PSTS only demonstrated that $T(x_0) \leq T_f$, so there may exist a positive number $T_c < T_f$ such that $x = 0, \dot{x} = 0, \forall t \geq t_0 + T_c$. To eliminate this conservativeness, we propose the following definition.

Definition 4: Consider that the origin of the system (3) is prescribed-time stable with $T_f > 0$. Then, the origin of the system (1) is prescribed-instant stable (PSIS) if $T(x_0) = T_f$.

Remark 4: $T_f = t_f - t_0$ is the assigned convergence time, and t_f is the assigned instant when the system states convergent to zero. $T(x_0) = t_a - t_0$ is the actual convergence time or settling time, and t_a is the actual instant when the system states

convergent to zero. It is noted that the proposed prescribed-instant stability in this paper means the extraordinary accuracy on time, i.e., the true convergence time exactly equal to the assigned time. Therefore, $T(x_0)$ is actually independent with x_0 , and $\dot{x} \neq 0$ until $t \geq t_f$. It is obviously different with the FTS, the PDTs, and the PSTs.

B. Comparison of Control Structures

Consider the following single-integrator system:

$$\dot{x} = u, x(t_0) = x_0 > 0, t_0 = 0. \quad (4)$$

1) Traditional finite-time control [3]:

$$u_{\text{FNTS}} = -\eta_1 x^{1-\frac{1}{\eta_2}}, \eta_1 > 0, \eta_2 \in (1, \infty),$$

$$T(x_0) = \frac{\eta_2 x_0^{\frac{1}{\eta_2}}}{\eta_1}.$$

2) Fixed-time control [8]:

$$u_{\text{FTS}} = -x^{\eta_3} - x^{\eta_4}, \eta_3 \in (0, 1), \eta_4 \in (1, \infty),$$

$$T(x_0) \leq T_{\max} = \frac{1}{(1-\eta_3)} + \frac{1}{(\eta_4-1)}.$$

3) Predefined-time control [12]:

$$u_{\text{PDTs}} = -\frac{\eta_5}{T_f} e^{x^{\frac{1}{\eta_5}}} x^{1-\frac{1}{\eta_5}}, \eta_5 \in (1, \infty),$$

$$\sup_{x_0 \in \mathbb{R}} T(x_0) = T_f.$$

4) Prescribed-time control [18]:

$$u_{\text{PSTs}} = \begin{cases} -\frac{\eta_6 x}{T_f - t}, \eta_6 \in (1, \infty), & 0 \leq t < T_f, \\ 0, & T_f \leq t. \end{cases}$$

$$T(x_0) \leq T_f.$$

Remark 5: It is interesting to notice that the solution of system in (4) under the control u_{PSTs} is $x = (T_f - t)^{\eta_6}$, which is also the solution of

$$\dot{x} = u = -\frac{\eta_6}{T_f} x_0^{\frac{1}{\eta_6}} x^{1-\frac{1}{\eta_6}}.$$

One can see that the prescribed-time control is very similar to predefined-time control. Moreover, taking $\frac{\eta_6}{T_f} x_0^{\frac{1}{\eta_6}}$ as η_1 shows the unification of the PSTs and the FNTs.

C. Problem Statement

It is known for all that a SISO nonlinear system with a relative degree n can usually be transformed to the n -order chain of integrators by utilizing input-output linearization method (Section 2, Chapter 13 of [32]). A controllable MIMO linear system can also be transformed to the controller canonical form, which can be seen as the combination of several chains of integrators. Hence, let us consider the following n -order chain of integrators:

$$\begin{cases} \dot{x}_i = x_{i+1}, i = 1, \dots, n-1, \\ \dot{x}_n = u. \end{cases} \quad (5)$$

For a designed time varying control $u(x, t, \eta)$, the system (5) belongs to the form in (1). Our goal is to

- 1) design a continuous control $u(x, t, \eta)$ goes to zero at a selected instant t_f , and
- 2) ensure that the equilibrium of the system in (5) is PSIS by exploring new techniques of stability analysis.

III. MAIN RESULTS

In this section, the method to design the PSIS controller for the system (5) is presented. Before that, some new time-varying feedback functions, the raw material of the presented controller, are introduced.

A. Time-varying Feedback Functions

Definition 5: Consider a differentiable function $\phi(s) \in \kappa, \kappa_\infty$ (the class κ or κ_∞ function [32]), and $s = \text{arc}(\phi(s))$ (the existence of inverse function is guaranteed by the monotonicity of ϕ). Suppose that ϕ satisfy

- $\phi(s) \sim s$ as $s \rightarrow 0$, i.e. $\lim_{s \rightarrow 0} \frac{\phi(s)}{s} = 1$,
- $\phi'(s) = \beta(\phi(s))$.

Let $s(t) = (t_f - t)^\eta, x(t) = \phi(s(t)) = \phi((t_f - t)^\eta), t \in [0, t_f], \eta > 1$. We have

$$\frac{dx}{dt} = \frac{d\phi}{ds} \frac{ds}{dt} = -\frac{\eta \beta(\phi(s)) \text{arc}(\phi(s))}{t_f - t} = -\frac{\zeta(x)}{t_f - t} = -\psi(x, t).$$

Then, $\phi((t_f - t)^\eta)$ is defined as a reference convergence function (RCF), while $\psi(x, t)$ is defined as a reference convergence differential function (RCDF).

Remark 6: The defined RCDFs above are exactly our desired time-varying feedback functions. According to Definition 5, ϕ monotonously goes to zero as s goes to zero, thus $\psi(\phi, t)$ and $\zeta(\phi)$ have the same sign with ϕ and will not be zero unless ϕ is zero. These functions are also expected to have some important merits. For example, $\psi(\phi, t)$ should be designed to be a symmetrical function (same properties for positive and negative ϕ). It is noted that $\lim_{t \rightarrow t_f} \phi'((t_f - t)^\eta) =$

$\lim_{t \rightarrow t_f} \frac{\phi((t_f - t)^\eta) - 0}{t - t_f} \sim (t_f - t)^{\eta-1}$, for $\eta > 1$. For the first-order system $\dot{x} = u$, if u is designed as $-\psi(x, t)$, then both $u(t)$ and $x(t)$ will converge to zero at $t = t_f$. Let us see some examples on Definition 5.

Example 1: Notice that $x(t) = \tan((t_f - t)^\eta)$ (an RCF) will converge to zero at t_f , we obtain the following RCDF:

$$\dot{x} = \frac{-\eta}{\cos^2((t_f - t)^\eta)} \frac{(t_f - t)^\eta}{t_f - t} = \frac{-\eta(x^2 + 1) \arctan(x)}{t_f - t}. \quad (6)$$

One can also design analogous RCDFs according to the thought obtaining the RCDF in (6), e.g.

$$\begin{cases} x = \text{sh}((t_f - t)^\eta) = \frac{e^{(t_f - t)^\eta} - e^{-(t_f - t)^\eta}}{2}, \\ \dot{x} = -\frac{\eta \sqrt{1+x^2} \ln(x + \sqrt{1+x^2})}{t_f - t}. \end{cases}$$

In this paper, the following three typical RCFs (ϕ) are discussed in detail,

$$\begin{aligned} x_1 &= \tan(t_f - t)^\eta, \\ x_2 &= (t_f - t)^\eta, \\ x_3 &= \ln(1 + (t_f - t)^\eta). \end{aligned}$$

Fig. 1. States and control variables of the three typical RCDFs of (7)–(9).

The corresponding RCDFs (ψ) are:

$$\dot{x}_1 = u_{x1} = -\frac{\eta(x_1^2 + 1)\arctan(x_1)}{t_f - t}, \quad (7)$$

$$\dot{x}_2 = u_{x2} = -\frac{\eta x_2}{t_f - t}, \quad (8)$$

$$\dot{x}_3 = u_{x3} = -\frac{\eta(1 - e^{-|x_3|})}{t_f - t} \text{sign}(x_3). \quad (9)$$

It is interesting to notice that the control in (8) is exactly that of PSTS with single-integrator system. However, the control for high-order integrator systems will be different. Fig. 1 shows the convergence of the states to the origin and the control variables corresponding to the RCDFs in (7)–(9), where $t_f = 5$, $x(0) = 3.5$, and $\eta = 2$.

B. Controller Design

Consider the system (5). The proposed RCDFs such as in (7)–(9) are utilized as time-varying feedback functions. The controller is demonstrated by a recursive form. The initial condition is chosen as $x_{1,d} = 0$ and $x_{2,d} = -\psi_1$. The recurrence relation ($i \geq 2$) is

$$x_{i+1,d} = \dot{x}_{i,d} - \sigma_i - \psi_i, \quad (10)$$

where $\sigma_i = x_i - x_{i,d}$, and ψ_i belongs to the RCDFs. To avoid divergence to infinity or discontinuity of the control law at $t = t_f$, we select each parameter η_i of function ψ_i as $\eta_i > n+1-i$, $i = 1, 2, \dots, n$. The proposed controller for the system (5) is:

$$u_n = \begin{cases} x_{n+1,d}, & t_0 \leq t < t_f, \\ 0, & t_f \leq t, \end{cases} \quad (11)$$

where $x_{n+1,d}$ is obtained from (10). On the other hand, one can obtain that

$$\dot{\sigma}_i = \dot{x}_i - \dot{x}_{i,d} = x_{i+1} - x_{i+1,d} - \sigma_i - \psi_i$$

Fig. 2. The sketch map of Claim 1.

$$\begin{aligned} &= \sigma_{i+1} - \sigma_i - \psi_i, \\ \dot{\sigma}_n &= \dot{x}_n - \dot{x}_{n,d} = u - \dot{x}_{n,d} \\ &= -\sigma_n - \psi_n. \end{aligned}$$

Hence, the system (5) is transformed to:

$$\begin{cases} \dot{\sigma}_1 = \sigma_2 - \psi_1, \\ \dot{\sigma}_i = \sigma_{i+1} - \sigma_i - \psi_i, i = 2, \dots, n-1, \\ \dot{\sigma}_n = -\sigma_n - \psi_n. \end{cases} \quad (12)$$

It is noted that the dynamics of σ_i is only affected by ψ_i , σ_{i+1} and(or) itself. This is very important for our further proof of the PSIS.

C. Prescribed-instant Stability (PSIS)

In this subsection, we will directly study the character of the differential equations in the system (12), and illustrate the PSIS of the system (5) under the control in (11). Several claims and lemmas are introduced for developing our main results.

Claim 1: Consider the following differential equation,

$$\dot{\sigma} = \sigma - \psi(\sigma, t), \quad (13)$$

where ψ is an RCDF. Then, σ reaches zero at $t = t_f$.

Proof: We consider a positive initial value $\sigma(t_0)$ for the symmetry of the equation in (13). According to the comparison Lemma [32], σ is lower bounded by $\phi(t)$ corresponding to the selected ψ and $t_a \geq t_f$, as the illustration in Fig. 2. Reduction to absurdity is utilized as follows. We assume $t_a > t_f$, i.e., $\sigma(t_f) = \delta > 0$.

Utilizing the mean value theorem of integrals, we have,

$$\begin{aligned} \sigma(t_f) &= \delta + \int_{t_0}^{t_f} \sigma(t)dt - \int_{t_0}^{t_f} \frac{\zeta(\sigma)}{t_f - t} dt \\ &= k - \int_{t_0}^{t_f} \frac{\zeta(\sigma)}{t_f - t} dt, \end{aligned}$$

where k is a constant calculated from the first two terms and $\frac{\zeta(\sigma)}{t_f - t}$ is ψ . According to Definition 5 and Remark 6, $\delta \leq \sigma(t) \leq \sigma_0$ leads to $m \leq \zeta(\sigma) \leq M$, where m and M are some positive constants. We have

$$\int_{t_0}^{t_f} \frac{m}{t_f - t} dt \leq \int_{t_0}^{t_f} \frac{\zeta(\sigma)}{t_f - t} dt \leq \int_{t_0}^{t_f} \frac{M}{t_f - t} dt.$$

Let $\tau = t_f - t$. Then, we obtain that

$$\int_{t_0}^{t_f} \frac{m}{t_f - t} dt = \int_0^{t_f - t_0} \frac{m}{\tau} d\tau = +\infty,$$

$$\int_{t_0}^{t_f} \frac{M}{t_f - t} dt = \int_0^{t_f - t_0} \frac{M}{\tau} d\tau = +\infty.$$

Therefore, by the Squeeze Theorem, we have $\int_{t_0}^{t_f} \frac{\zeta(\sigma)}{t_f - t} dt = +\infty$, which leads to $\sigma(t_f) = -\infty$. This is in conflict with $\sigma(t_f) = \delta > 0$. Hence, we can draw the conclusion that $\sigma(t_f) = 0$, i.e., σ reaches zero at $t = t_f$. ■

Claim 2: Consider the following differential equation,

$$\dot{\sigma} = -\sigma - \psi(\sigma, t), \quad (14)$$

where ψ is an RCDF. Then, σ reaches zero at $t = t_f$.

Proof: We consider a positive initial value $\sigma(t_0)$ for the symmetry of the equation in (14). According to the Comparison Lemma, σ is limited by $\phi(t)$ that corresponding to the selected ψ and $t_a \leq t_f$, as the illustration in Fig. 3(a). Reduction to absurdity is utilized as follows. Let us suppose $t_a < t_f$.

By overturning the coordinate axis in Fig. 3(a), we have the coordinate axis in Fig. 3(b). The corresponding differential equation in (14) is transformed to:

$$\frac{dt}{d\sigma} = -\frac{1}{\sigma + \psi}. \quad (15)$$

We have,

$$t_a = -\int_{\sigma_0}^0 \frac{1}{\sigma + \psi} d\sigma = \int_0^{\sigma_0} \frac{1}{\sigma + \psi} d\sigma. \quad (16)$$

When σ tends to zero, t tends to t_a , so $t_f - t$ will not tend to zero. According to Definition 5, we have $\psi = \lim_{t \rightarrow t_a} \frac{\sigma(t) - 0}{t_f - t} \sim \sigma$. Therefore, $t_a = +\infty$, which is conflict with $t_a < t_f$. The conclusion is drawn that σ reaches zero at $t = t_f$. ■

Fig. 3. Abridged general view of the case for Claim 2.

In analogy with Claim 1 and Claim 2, Lemma 1 and Lemma 2 are introduced with the proof in Appendix.

Lemma 1: Consider the following differential equation,

$$\dot{\sigma} = b(t) - \psi(\sigma, t), \quad (17)$$

where ψ is an RCDF. If $b(t)$ is bounded but never stabilized at zero until $t = t_f$, then σ reaches zero at $t = t_f$.

Lemma 2: Consider the following differential equation,

$$\dot{\sigma} = b(t) - \sigma - \psi(\sigma, t),$$

where ψ is an RCDF. If $b(t)$ is bounded but never stabilized at zero until $t = t_f$, then σ reaches zero at $t = t_f$.

Theorem 1: Consider the system (5) starting from any initial condition of x . The controller (11) drives the trajectory of the system (5) to the equilibrium $x = 0$ at the selected instant $t = t_f$. Moreover, the continuous control $u(t)$ also gradually goes to zero at the instant $t = t_f$ and ensures $x(t) = u(t) = 0, \forall t \geq t_f$. In another word, the equilibrium $x = 0$ is PSIS.

Proof: As shown in the last subsection, the controller (11) transforms the system (5) to:

$$\begin{cases} \dot{\sigma}_1 = \sigma_2 - \psi_1, \\ \dot{\sigma}_i = \sigma_{i+1} - \sigma_i - \psi_i, i = 2, \dots, n-1, \\ \dot{\sigma}_n = -\sigma_n - \psi_n. \end{cases} \quad (18)$$

Consider the last differential equation in (18). According to Claim 2, σ_n monotonously goes to zero at t_f . Then, σ_n is taken as $b(t)$, according to Lemma 2, σ_{n-1} will go to zero at t_f . Therefore, $\sigma_{n-2}, \dots, \sigma_2$ can be recursively proved to go to zero at the instant t_f . Finally, the convergence of σ_1 is a direct result of Lemma 1. Therefore, σ is PSIS at $t = t_f$ and leads to:

$$x_i(t_f) = \sigma_i(t_f) + x_{i,d}(t_f) = x_{i,d}(t_f).$$

For the system (5), combining (10) and (11), the relation between the states and the detailed control variables u_i is presented as:

$$\begin{cases} x_1 = x_{1,d} = 0, \\ x_2 = x_{2,d} = -\psi_1 = u_1, \\ x_3 = x_{3,d} = -\dot{\psi}_1 - \sigma_2 - \psi_2 = u_2, \\ x_4 = x_{4,d} = -\dot{\psi}_1 - \dot{\sigma}_2 - \psi_2 - \sigma_3 - \psi_3 = u_3, \\ \vdots \\ x_{n+1} = x_{n+1,d} = \dot{x}_{n,d} - \sigma_n - \psi_n = u_n, \end{cases} \quad (19)$$

where each equation is formed by ψ_i, σ_i , and their derivatives. It is noted that the derivative order of ψ_1 is the highest among the equations. For u_n , the derivative order of ψ_1 is $n-1$ and the derivative order of ψ_i is $n-i$. According to Definition 5, when t tends to t_f , we have $\psi \sim (t_f - t)^{\eta-1}$ and $\psi^{(n-1)} \sim (t_f - t)^{\eta-n}$. As mentioned in Subsection 3.2, for each ψ_i , the corresponding parameter is selected as $\eta_i > n+1-i$, which ensures that the derivative of ψ_i tends to zero. Therefore, the proposed control (11) is bounded as t tends to t_f . Combining the PSIS of σ , we obtain that the trajectory of the system (5) converges to the equilibrium $x = 0$ at the instant $t = t_f$. At the same time, $u(t)$ gradually goes to zero and then $u(t) = 0, \forall t \geq t_f$, which leads to $x(t) = 0, \forall t \geq t_f$. Hence, the system (5) is PSIS. This completes the proof. ■

IV. SIMULATIONS

A. Simulations on a Double Integrator System

Consider a double integrator system,

$$\begin{cases} \dot{x}_1 = x_2, \\ \dot{x}_2 = u. \end{cases}$$

According to (10) and (11), the detailed controller is:

$$u_2 = \begin{cases} -\frac{\partial\psi_1}{\partial x_1}(\sigma_2 - \psi_1) - \frac{\partial\psi_1}{\partial t} - \sigma_2 - \psi_2, & t_0 \leq t < t_f, \\ 0, & t_f \leq t. \end{cases} \quad (20)$$

The detailed RCDFs are selected as:

$$\begin{cases} \psi_1 = \frac{\eta_1 \sigma_1}{t_f - t}, \eta_1 = 3, \\ \psi_2 = \frac{\eta_2 \sigma_2}{t_f - t}, \eta_2 = 2. \end{cases}$$

In simulations, we choose $t_f = 1$ s and three different sets of initial values: set 1: $x_1(0) = 1, x_2(0) = -3$; set 2: $x_1(0) = 2, x_2(0) = -1$; set 3: $x_1(0) = 3, x_2(0) = 1$. The simulation results are depicted as Fig. 4. Fig. 4(a) shows the trajectories of x_1 , Fig. 4(b) shows the trajectories of x_2 , Fig. 4(c) shows the trajectories of control variables under the three different initial conditions.

Fig. 4. States and control variables of the double integrator system.

B. Simulations on a Simple Pendulum System

Consider a simple pendulum system [32]. $l = 0.5$ m is the length of the rod and $m = 0.1$ kg denotes the mass of the bob. The rod is assumed to be rigid and has zero mass. The acceleration due to gravity is $g = 9.81$ m/s² and the friction coefficient is denoted as $k = 0.01$. Let θ (rad) denote the angle subtended by the rod and the vertical axis through the pivot point. Applying a torque T to the pendulum, we have,

$$ml\ddot{\theta} = -mg\sin\theta - kl\dot{\theta} + \frac{1}{l}T.$$

Taking the states as $x_1 = \theta = y$ and $x_2 = \dot{\theta}$, we have,

$$\begin{cases} \dot{x}_1 = x_2, \\ \dot{x}_2 = -\frac{g}{l}\sin x_1 - \frac{k}{m}x_2 + \frac{1}{ml^2}T. \end{cases}$$

Let the torque T be

$$T = ml^2\left(\frac{g}{l}\sin x_1 + \frac{k}{m}x_2 + u\right).$$

We have,

$$\begin{cases} \dot{x}_1 = x_2, \\ \dot{x}_2 = u. \end{cases}$$

Assume the initial value $x_1(0) = 0.09$ rad and $x_2(0) = 0.1$ rad/s. Our goal is to make $x_1 = 0.15$ rad at $t_f = 0.5$ s, i.e., $x_{1,d} = 0.15$ rad. We replace the variable x_1 as $\sigma_1 = x_1 - x_{1,d}$ in the controller presented in (20) and then obtain

$$u_2 = \begin{cases} -\frac{\partial\psi_1}{\partial x_1}(\sigma_2 - \psi_1) - \frac{\partial\psi_1}{\partial t} - \sigma_2 - \psi_2, & t_0 \leq t < t_f, \\ 0, & t_f \leq t. \end{cases}$$

The detailed RCDFs are also selected as:

$$\begin{cases} \psi_1 = \frac{\eta_1 \sigma_1}{t_f - t}, \eta_1 = 3, \\ \psi_2 = \frac{\eta_2(\sigma_2^2 + 1)\arctan(\sigma_2)}{t_f - t}, \eta_2 = 2. \end{cases}$$

The simulation result is depicted as Fig. 5. Fig. 5(a) shows the states of the pendulum system and Fig. 5(b) shows the control variables.

Fig. 5. States and control variables of the pendulum system.

C. Simulations on a Magnetic Suspension

Consider the magnetic suspension [32]. The vertical (downward) position of the ball measured from a reference point is denoted as z_1 , which is also the output y . z_2 denotes the speed of the ball and z_3 denotes the current of the electromagnet. The state equations are:

$$\begin{cases} \dot{z}_1 = z_2, \\ \dot{z}_2 = g - \frac{k}{m}z_2 - \frac{L_0 a z_3^2}{2m(a+z_1)^2}, \\ \dot{z}_3 = g - \frac{1}{L_1 + \frac{aL_0}{a+z_1}} \left[-Rz_3 + \frac{L_0 a z_2 z_3}{(a+z_1)^2} + v \right], \end{cases}$$

where the voltage v is the control input. We assume $m = 0.1$ kg, $k = 0.001$ N/m/s, $g = 9.81$ m/s², $a = 0.05$ m, $L_0 = 0.01$ H, $L_1 = 0.02$ H, and $R = 1$ Ω . By choosing

$$\begin{cases} x_1 = y = z_1, \\ x_2 = \dot{y} = z_2, \\ x_3 = \ddot{y} = g - \frac{k}{m}z_2 - \frac{L_0 a z_3^2}{2m(a+z_1)^2}, \end{cases}$$

one can obtain:

$$\begin{cases} \dot{x}_1 = x_2, \dot{x}_2 = x_3, \\ \dot{x}_3 = -\frac{k}{m}x_3 - \frac{aL_0 z_3}{m(a+z_1)^2(L_1 + \frac{aL_0}{a+z_1})}(-Rz_3 - \frac{L_1 z_2 z_3}{a+z_1} + v). \end{cases}$$

As shown in Subsection 2.2, let the control be:

$$v = Rz_3 + \frac{L_1 z_2 z_3}{a + z_1} - \frac{m(a + z_1)^2(L_1 + \frac{aL_0}{a+z_1})}{aL_0 z_3}(\frac{k}{m}x_3 + u).$$

Then, we obtain that

$$\begin{cases} \dot{x}_1 = x_2, \\ \dot{x}_2 = x_3, \\ \dot{x}_3 = u. \end{cases}$$

Assume the initial value $x_1(0) = z_1(0) = 0.09$ m, $x_2(0) = z_2(0) = -0.03$ m/s and $z_3(0) = -6.6$ A. Our goal is to stabilize $x_1 = z_1 = 0.05$ m at $t_f = 0.5$ s, i.e., $x_{1,d} = 0.05$ m. We replace the variable x_1 by $\sigma_1 = x_1 - x_{1,d}$ in the controller presented in (10) and (11) and then obtain

$$u_3 = \begin{cases} -\sigma_3 - \psi_3 - \frac{\partial x_{3,d}}{\partial x_1}(\sigma_2 - \psi_1) - \frac{\partial x_{3,d}}{\partial t}, \\ -\frac{\partial x_{3,d}}{\partial \sigma_2}(\sigma_3 - \sigma_2 - \psi_2), & t_0 \leq t < t_f, \\ 0, & t_f \leq t. \end{cases}$$

Specifically, ψ_i is chosen as:

$$\begin{cases} \psi_1 = -\frac{\eta_1 \sigma_1}{t_f - t}, \eta_1 = 5, \\ \psi_2 = -\frac{\eta_2(1 - e^{-|\sigma_2|})}{t_f - t} \text{sign}(\sigma_2), \eta_2 = 4, \\ \psi_3 = -\frac{\eta_3(\sigma_3 + 1) \arctan(\sigma_3)}{t_f - t}, \eta_3 = 3. \end{cases}$$

Fig. 6(a) shows the states of the magnetic suspension system and Fig. 6(b) shows the control variables. The initial value of the real controller v is about -7 V. Moreover, u is bounded and goes to zero exactly at $t_f = 0.5$.

Remark 7: Although the convergence time can be arbitrarily chosen, the magnitude of the control will inevitably be very large when the values of the initial condition are very large or the regulation time is set to be very small. This is unexpected in practice [19]. Hence, the bound of the control variables will be considered qualitatively in our future study.

V. CONCLUSIONS

This paper has dealt with the problem to stabilize system states to the origin at a selected time instant irrespective of initial system states and parameters. This can be achieved by the feature of a series of well-designed time-varying feedback functions, which are easy to be obtained according to the presented guidance and are defined as RCDFs. We directly considered the feature of differential equations instead of the Lyapunov method, and then reduction to absurdity has been applied and the proof is unexpectedly elegant. In addition to

Fig. 6. States and control of the magnetic suspension system.

Fig. 7. The sketch map of Case 1 for Lemma 1.

mathematical proof, the control scheme has also been verified by numerical simulations. This control strategy would be of great importance in industrial and engineering applications with the requirement of the precise time.

APPENDIX

Proof of Lemma 1: Set t_b the last time when $b(t)$ passed through zero ($t_b = t_0$ if $b(t)$ never passes through zero). $b(t)$ may have same or opposite sign with $\sigma(t_b)$. We consider a positive initial value $\sigma(t_b)$. Then, $b(t)$ with $t \in (t_b, t_f)$ could be caused by positive or negative. It is obvious that σ will converge to zero at t_f if $\dot{\sigma} = -\psi(\sigma, t)$. Now, let us consider:

$$\dot{\sigma} = b(t) - \psi(\sigma, t). \quad (21)$$

Case 1. $b(t) > 0, t \in (t_b, t_f)$, of which an illustration is shown in Fig. 7. According to the Comparison Lemma, the real convergence time of the equation in (21) would be $t_a \geq t_f$. Reduction to absurdity is utilized as follows.

Fig. 8. The sketch map of Case 2 for Lemma 2.

Assume that $t_a > t_f$, i.e., $\sigma(t_f) = \delta > 0$. Based on the form of ψ and the fact that $b(t)$ is bounded, $\exists t_1$, s.t. $\sigma'(t) < 0$, $t \in (t_1, t_f)$. Taking $t_2 \geq \max(t_1, t_b)$, $\sigma(t_2) = \sigma_2$, we have,

$$\sigma(t_f) = \sigma_2 + \int_{t_2}^{t_f} b(t)dt - \int_{t_2}^{t_f} \frac{\zeta(\sigma)}{t_f - t} dt, \quad (22)$$

where $\frac{\zeta(\sigma)}{t_f - t}$ is ψ . Utilizing the mean value theorem of integrals, we have,

$$\sigma(t_f) = k - \int_{t_2}^{t_f} \frac{\zeta(\sigma)}{t_f - t} dt,$$

where k is a constant. As shown in the proof of Claim 1, one can obtain $\sigma(t_f) = -\infty$, which is conflicted with the assumption that $\sigma(t_f) = \delta > 0$. Therefore, $\sigma(t_f) = 0$, $t_a = t_f$.

Case 2. $b(t) < 0$, $t \in (t_b, t_f)$, of which an illustration is shown in Fig. 8. There exists three subcases:

(i) The value of $b(t)$ cannot drive σ to be negative or zero before t_f . According to the sign of $b(t)$, σ will tend to zero at or before the instant t_f (due to the Comparison Lemma). So, σ reaches zero at the instant t_f .

(ii) The value of $|b(t)|$ is large enough and drive σ to be zero at $t_m \in (\max(t_1, t_b), t_f)$. Because $b(t) < 0$, $t \in (t_b, t_f)$, σ will go across zero and it turns to the subcase (iii).

(iii) The value of $|b(t)|$ is large enough and drive σ to be negative at $t_m \in (\max(t_1, t_b), t_f)$. Then $b(t)$ and σ have the same sign and the proof on (t_m, t_f) is similar to Case 1.

Combining Case 1 and Case 2, we know that σ will reach zero at the instant t_f . ■

Proof of Lemma 2: The proof imitates that of Lemma 1 and the differential equation (14) is taken as a reference differential equation to utilize the Comparison Lemma. The only difference is that (22) is replaced by

$$\sigma(t_f) = \sigma_2 + \int_{t_2}^{t_f} b(t)dt - \int_{t_2}^{t_f} \sigma dt - \int_{t_2}^{t_f} \frac{\zeta(\sigma)}{t_f - t} dt.$$

Then, the detailed proof can be easily completed by following that of Lemma 1 and is thus omitted. ■

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