Total variation as a metric for complementarity in energy resources time series

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Abstract

Complementarity has become an essential concept in energy supply systems. Although there are some other metrics, most studies use correlation coefficients (ρ) to quantify complementarity. The standard interpretation is that a high negative correlation indicates a high degree of complementarity. However, we show that the correlation is not an entirely satisfactory measure of complementarity. As an alternative, we propose a new index based on the mathematical concept of the total variation. For two time series, the new index ϕ is one minus the ratio of the total variation of the sum to the sum of the two series' total variation. We apply the index first to an auto-regressive (AR) process and then to various Colombian electric system series. The AR case clearly illustrates the limitations of the correlation coefficient as a measure of complementarity. We then evaluate complementarity across various space-time scales in the Colombian power sectors, considering hydro and wind projects. The complementarity assessment on a broad temporal and geographical scale helps analyze large power systems with different energy sources. The case study of the Colombian hydropower systems suggests that ϕ is better than ρ because (i) it considers scale, whereas ρ , being non-dimensional, is insensitive to the scale and even to the physical dimensions of the variables; (ii) one can apply ϕ to more than two resources; and (iii) ρ tends to overestimate complementarity.

Keywords: electricity supply, natural resources, climatology, hydrology, electricity markets, reliability, flexibility

1 Introduction

The proliferation of energy sources caused by technological advances and their consequent cost reduction brought the concept of complementarity to the energy jargon. This concept is of particular interest when the resource's availability or their costs are variable in time. Although, up to now, there is no formal definition of resource complementarity in energy supply systems, intuition suggests that complementarity refers to the possibility of alternatively using different resources according to their availability.

Correlation coefficients are by far the most widely used metric for assessing complementarity between energy sources time series. This approach has been used, for example, in Poland (Jurasz, 2017), Portugal (Moura and de Almeida, 2010; Castro and Crispim, 2018), Spain (Ren et al., 2019),

Italy (Monforti et al., 2014; François et al., 2014), USA (Slusarewicz and Cohan, 2018), Colombia (Parra et al., 2020; Peña Gallardo et al., 2020b; Henao et al., 2020; Cantor et al., 2019), Germany (Schindler et al., 2020), Australia (Kay, 2015), Brazil (Silva et al., 2016; de Oliveira Costa Souza Rosa et al., 2017; dos Anjos et al., 2015; Cantão et al., 2017), China (Xu et al., 2017; Li et al., 2019; Cao et al., 2019), Sweden (Widen, 2011), Mexico (Peña Gallardo et al., 2020a), Canada (Denault et al., 2009), Lesotho (D'Isidoro et al., 2020), Finland (Solomon et al., 2020), Argentina (Genchi et al., 2018), Britain (Bett and Thornton, 2016), Saudi Arabia (Sahin, 2000) and Chile (Odeh and Watts, 2019). It has also been used at the international level (e.g. Latin America (Viviescas et al., 2019), Iberian Peninsula (Gutiérrez et al., 2017)), and even at intercontinental level (e.g. Canada vs. Noruega (Denault et al., 2009) and

Europe vs. Africa (Kougias et al., 2016)). Under this approach, there is a high complementarity between two resources when their time series have a high negative correlation coefficient, sometimes called *anti-correlation* (e.g. Bett and Thornton, 2016; Cao et al., 2019; Kougias et al., 2016; Gutiérrez et al., 2017; Slusarewicz and Cohan, 2018). In our opinion, a complementarity metric should measure the regularity of the sum of all available resources, and correlation coefficients fail to do that. The use of correlation coefficients also has the following issues:

1. Negative correlation is not complementarity. Although complementarity entails correlation, correlation does not entail complementarity. For example, the correlation between speed and height in a pendulum is negative, but they are not complementary.
2. Dimensions matter. In the above example, speed and height have different units. Obviously, for two or more variables to be complementary, they must have the same dimension. If they have different dimensions, one can not sum them, and their complementarity has no sense. In the pendulum example above, the complementary variables are kinetic energy and potential energy.
3. The scale of the variables does matter. For example, streamflows from a large river and a small creek could exhibit a sizeable negative correlation, but the large one dominates their sum, and hence the complementarity value should be low.
4. Linearity of the relation is also an issue. Even if correlation coefficients were suitable for assessing complementarity, the selection of Pearson's, Kendall's, Spearman's or any other coefficient should be done on the basis of some physical or mathematical considerations.

Besides correlation, among the other methods used to evaluate complementarity, one can mention the one proposed by Beluco et al. (2008) as the product of three partial indices: (i) partial time complementarity index, which evaluates the time interval between the minimum values of two sources; (ii) partial energy complementarity

index, which evaluates the relationship between the average values of two sources; (iii) partial amplitude-complementarity index, which assesses the differences between maximum and minimum values of the two energy sources. This idea works well for simple annual sinusoidal series but fails for more complicated series. Borba and Brito (2017) proposed the dimensionless ratio between the actual generation discarding excess power and the average generation. This index is more a measure of volatility than complementarity. Han et al. (2019) presented two indices of the complementary rate of fluctuation (CROF) and complementary rate of the ramp (CROR) to analyze the complementary degree of sequences with different fluctuation characteristics. Their rate of fluctuation seems to follow the concept of total variation that we present. Neto et al. (2020) used the concept of Daily Physical Guarantee (DPG), similar to firm power, to evaluate different renewable energy source's complementarity. The general idea is interesting but depends on the system's particular characteristics and does not provide a direct complementarity metric. Canales et al. (2020a); Arias et al. (2015); Canales et al. (2020b) introduced a method that assesses temporal complementarity between three variable energy sources, using a combination of correlation coefficients, Euclidean vectors, compromise programming, and normalization. The method's basis is correlation analysis, and therefore their proposal shares the criticism we made above. For a complete review of different methods to evaluate complementarity see Jurasz et al. (2020). We think that none of those proposals is a satisfactory way of measuring complementarity.

Looking to solve these issues, we propose two novel indices based on the Total Variation theory, described in section 2. To illustrate the use and properties of the new indices, we apply them to four cases of study (section 3): 1) two autoregressive processes; 2) wind vs. hydro in the Colombian electricity market; 3) two hydropower sources in the Colombian electricity market; and 4) fifteen hydropower sources in the Colombian electricity market.

2 Method

130 In ordinary language, a complement of something is a thing that completes or brings perfection. In math, two numbers complement each other when their sum is constant, usually 1 or 100. A 175 measure of complementarity or complementariness between two functions should assess the perfection of their sum. For perfection here, we will take the regularity of the sum. Therefore, the measure of complementarity is related to the measure of regularity. A satisfactory way of measuring the 140 regularity of a function is through its total variation, which allows for simple discontinuities and has more satisfactory properties than the variance (Kolmogorov and Fomin, 1970, p. 328) (Lasota and Mackey, 1994, p. 140).

145 In section 2.1 we review the concept and main mathematical properties of Total Variation. Then, we propose two indices, ϕ and ϕ' , as novel metrics of complementarity (sections 2.2 and 2.3). Afterwards, we present one theoretical and two 150 real case studies to evaluate the suitability of ϕ and ϕ' to quantify complementarity: 1) two AR(1) processes (section 3.1), 2) daily wind and hydropower generation time series in Colombia (section 3.2), and 3) the daily inflows to two 155 hydropower reservoirs in Colombia (section 3.3). Results are compared between ϕ and ϕ' . In section 3.4 we present the application of ϕ index for the complementarity evaluation of multiple time series.

2.1 Total variation

160 According to Kolmogorov and Fomin (1970); Lasota and Mackey (1994), the total variation of a function f defined on an interval $[a, b]$ is

$$\bigvee_a^b f = \sup \sum_{i=1}^n |f(t_i) - f(t_{i-1})|, \quad (1)$$

where the supremum is taken over all possible finite partitions $a = t_1 < \dots < t_n = b$ of $[a, b]$.

165 If the sum is bonded, f is said to be of *bounded variation* on $[a, b]$. Some important properties of the total variation are worth recalling:

1. The total variation of a constant function is zero, and conversely, if $\bigvee_a^b f = 0$ then f is constant 170 on $[a, b]$

2. The total variation of a monotonic function f on $[a, b]$ is $|f(b) - f(a)|$
3. $\bigvee_a^b(\alpha f) = |\alpha| \bigvee_a^b(f)$ for any constant α .
4. If f_1 and f_2 are functions of bounded variation then so is $f_1 + f_2$, and

$$\bigvee_a^b(f_1 + f_2) \leq \bigvee_a^b(f_1) + \bigvee_a^b(f_2) \quad (2)$$

5. If $a < b < c$ then

$$\bigvee_a^c(f) = \bigvee_a^b(f) + \bigvee_b^c(f). \quad (3)$$

From the above properties it is clear that the total variation of a function f with values $f(t_k)$ at the points $a = t_0 < t_1 < \dots < t_n = b$, and linear interpolation in between the given points is

$$\bigvee_a^b(f) = \sum_{k=0}^{n-2} |f(t_{k+1}) - f(t_k)|. \quad (4)$$

This property is useful to compute the total variation of a time series.

To illustrate, consider the following sequence of functions $g_n(t) = \sin(2\pi nt)$, $n \geq 1$. Clearly, in 185 $[0, 1]$ all the functions have 0 mean and variance $1/2$, but $\bigvee_0^1(g_n) = 4n$ Lasota and Mackey (1994).

Taking into account the above, it is possible to define a complementarity index.

2.2 Total Variation Complementarity Index

Given two functions $f_1(t)$ and $f_2(t)$, not both constant, their complementarity index $\phi(f_1, f_2)$ over the time interval $[a, b]$ is defined as

$$\phi(f_1, f_2) = 1 - \frac{\bigvee_a^b(f_1 + f_2)}{\bigvee_a^b(f_1) + \bigvee_a^b(f_2)}. \quad (5)$$

Clearly, $0 \leq \phi(f_1, f_2) \leq 1$, and when $\phi(f_1, f_2) = 1$ there is perfect complementary between f_1 and f_2 , for in that case $f_1 + f_2$ is a constant. The generalization to any number of functions is direct.

As an especial case, consider the Total Variation Complementarity index between the series of anomalies. Clearly, the total variation

complementarity ϕ between the series $f_1, f_2, \dots, f_n$, each function $\phi(f_1, \dots, f_n) = \phi(f_1 - \mu_1, \dots, f_n - \mu_n)$, because of anomalies with respect to a constant mean for

$$\frac{V_a^b(f_1 + f_2 + \dots + f_n)}{V_a^b(f_1) + V_a^b(f_2) + \dots + V_a^b(f_n)} = \frac{V_a^b[(f_1 - \mu_1) + (f_2 - \mu_2) + \dots + (f_n - \mu_n)]}{V_a^b(f_1 - \mu_1) + V_a^b(f_2 - \mu_2) + \dots + V_a^b(f_n - \mu_n)}. \quad (6)$$

Summary of the ϕ index characteristics

by means of

1. If f_1 is the vertical reflection of f_2 , $(f_1 + f_2)$ is a constant, then $V_{t_a}^b(f_1 + f_2) = 0$ and $\phi = 1$.
2. $0 \leq \phi \leq 1$. For the extreme cases $\phi = 1$ means that there is perfect complementarity and for $\phi = 0$ there is no complementarity.
3. ϕ is symmetric.
4. The ϕ index presents the same result when is evaluated on anomaly series compared to their corresponding pure series.
5. The ϕ index could be applied to two or more series.

$$\phi'(f_1, f_2) = (\phi_v + 1)/2. \quad (9)$$

Summary of the ϕ' index characteristics

1. If f_1 is the vertical reflection of f_2 , $\text{Var}[f_1] = \text{Var}[f_2]$, then $\rho_{1,2} = -1$, $\phi_v = 1$ and $\phi' = 1$.
2. $0 \leq \phi' \leq 1$. For the extreme cases $\phi' = 1$ means that there is perfect complementarity and for $\phi' = 0$ there is no complementarity.
3. ϕ' is symmetric.
4. The ϕ' index presents the same result when is evaluated on anomaly series compared to their corresponding pure series.
5. The ϕ' index could be applied just to two series.

2.3 Variance Complementarity Index

If one wants to stay within the correlation concept, it is possible to correct the use of ρ as a metric for complementarity to consider the variables' scale. For that, considering Eq. 5, one can replace total variation by variance. This is, given two functions $f_1(t)$ and $f_2(t)$, not both constant, their variance complementarity index $\phi_v(f_1, f_2)$ is

$$\phi_v(f_1, f_2) = 1 - \frac{\text{Var}[f_1 + f_2]}{\text{Var}[f_1] + \text{Var}[f_2]}, \quad (7)$$

which simplifies to

$$\begin{aligned} \phi_v(f_1, f_2) &= -\frac{2 \text{Cov}[f_1, f_2]}{\text{Var}[f_1] + \text{Var}[f_2]} \\ &= -\rho_{1,2} \frac{2\sqrt{\text{Var}[f_1] \text{Var}[f_2]}}{\text{Var}[f_1] + \text{Var}[f_2]}. \end{aligned} \quad (8)$$

Notice that ϕ_v so defined is the negative of the correlation coefficient times the ratio of the geometric and arithmetic means of the variances. This last factor is always less or equal to one, with equality when the two variances are equal. Therefore, $\phi_v(f_1, f_2)$ varies between -1 and 1 . It is convenient to re-scale it to the range zero to one

3 Cases of study

3.1 Auto-regressive process

Auto-regressive processes are a well-known time series method widely used for all sorts of applications (Box et al., 2011). This section shows an expression for the total variance complementarity index between two cross-correlated auto-regressive processes of order one. The purpose is to clarify the two ways of measuring complementarity, namely the total variation and the variance complementarity index. Let $\mathbf{z}_t$ be a vector time series with two components, x_t and y_t , both with zero mean and unit variance. The multivariate auto-regressive process of order one is

$$\mathbf{z}_{t+1} = \mathbf{A}\mathbf{z}_t + \mathbf{B}\mathbf{v}_{t+1}, \quad (10)$$

where the matrices $\mathbf{A}$ and $\mathbf{B}$ are constant parameters, and $\mathbf{v}_t$ is a vector time series of uncorrelated random variables with zero mean and

unit variance. This is:

$$E[\mathbf{v}_t] = \mathbf{0}, \quad (11)$$

$$\text{Var}[\mathbf{v}_t] = E[\mathbf{v}_t \mathbf{v}_t^T] = \mathbf{I}, \quad (12)$$

$$\text{Cov}[\mathbf{v}_t, \mathbf{v}_{t+j}] = E[\mathbf{v}_t \mathbf{v}_{t+j}^T] = \mathbf{0}, \quad (13)$$

for all $j \neq 0$, and $\mathbf{I}$ the identity matrix. For $j > 0$ the vector $\mathbf{v}_{t+j}$ is uncorrelated with $\mathbf{z}_t$,

$$\text{Cov}[\mathbf{z}_t, \mathbf{v}_{t+j}] = E[\mathbf{z}_t \mathbf{v}_{t+j}^T] = \mathbf{0} \text{ for } j > 0.$$

The second order correlation structure of the process $\mathbf{z}_t$ depends on the matrices $\mathbf{M}_0 = \text{Var}[\mathbf{z}_t] = E[\mathbf{z}_t \mathbf{z}_t^T]$ and $\mathbf{M}_1 = \text{Cov}[\mathbf{z}_{t+1}, \mathbf{z}_t] = E[\mathbf{z}_{t+1} \mathbf{z}_t^T]$, with components

$$\begin{aligned} \mathbf{M}_0 &= \begin{pmatrix} \text{Var}[x_t] & \text{Cov}[x_t, y_t] \\ \text{Cov}[y_t, x_t] & \text{Var}[y_t] \end{pmatrix} \\ \mathbf{M}_1 &= \begin{pmatrix} 1 & \rho_{xy} \\ \rho_{xy} & 1 \end{pmatrix}, \end{aligned} \quad (14)$$

that is symmetric and t -independent. Whereas

$$\begin{aligned} \mathbf{M}_1 &= \begin{pmatrix} \text{Cov}[x_{t+1}, x_t] & \text{Cov}[x_{t+1}, y_t] \\ \text{Cov}[y_{t+1}, x_t] & \text{Cov}[y_{t+1}, y_t] \end{pmatrix} \\ \mathbf{M}_1 &= \begin{pmatrix} \rho_x & \rho_{x_{t+1}, y_t} \\ \rho_{y_{t+1}, x_t} & \rho_y \end{pmatrix} \end{aligned} \quad (15)$$

is not symmetric.

To find expressions for the parametric matrices post-multiply eq. (10) by $\mathbf{z}_t$ and take the expected value

$$\mathbf{M}_1 = \mathbf{A} \mathbf{M}_0,$$

therefore

$$\mathbf{A} = \mathbf{M}_1 \mathbf{M}_0^{-1}. \quad (16)$$

Similarly, post-multiply eq. (10) by $\mathbf{z}_{t+1}$ and take expected values to get

$$\mathbf{M}_0 = \mathbf{M}_1 \mathbf{M}_0^{-1} \mathbf{M}_1^T + \mathbf{B} \mathbf{B}^T. \quad (17)$$

One can then obtain $\mathbf{B}$ from Eq. 17 assuming it is triangular. A simple way is to assume that $\rho_{x_{t+1}, y_t} = \rho_{xy} \rho_x$ and $\rho_{y_{t+1}, x_t} = \rho_{xy} \rho_y$ (Matalas, 1967). In that case, $\mathbf{M}_1 = \mathbf{D} \mathbf{M}_0$, with $\mathbf{D}$ a diagonal

Figure 1. Expected value of the total variation of an univariate auto-regressive process as a function of the lag-one auto-correlation coefficient. The points are the average of 100 simulations. The continuous line is eq. (21).

matrix

$$\mathbf{D} = \begin{pmatrix} \rho_x & 0 \\ 0 & \rho_y \end{pmatrix}.$$

Therefore eq. (16) is now

$$\mathbf{A} = \mathbf{M}_1 \mathbf{M}_0^{-1} = \mathbf{D}. \quad (18)$$

Similarly, eq. (17) simplifies to

$$\begin{aligned} \mathbf{B} \mathbf{B}^T &= \mathbf{M}_0 - \mathbf{D} \mathbf{M}_0 \mathbf{D} \\ \mathbf{B} \mathbf{B}^T &= \begin{pmatrix} 1 - \rho_x^2 & \rho_{xy}(1 - \rho_x \rho_y) \\ \rho_{xy}(1 - \rho_x \rho_y) & 1 - \rho_y^2 \end{pmatrix} \end{aligned} \quad (19)$$

that one can solve for $\mathbf{B}$

$$\mathbf{B} = \begin{pmatrix} \sqrt{1 - \rho_x^2} & 0 \\ \frac{\rho_{xy}(1 - \rho_x \rho_y)}{\sqrt{1 - \rho_x^2}} & \sqrt{1 - \rho_y^2 - \frac{\rho_{xy}^2(1 - \rho_x \rho_y)^2}{1 - \rho_x^2}} \end{pmatrix}. \quad (20)$$

The expected value of the total variation of an univariate auto-regressive process (the first component of eq. (10)) is

$$E \left[\bigvee_1^n x_t \right] = (n - 1) \times \frac{2}{\sqrt{\pi}} \sqrt{1 - \rho_x}. \quad (21)$$

Fig. 1 shows the expected value as a function of the lag-one autocorrelation coefficient ρ_x .

The result in eq. (21) comes realizing that $|x_{t+1} - x_t| = |\sqrt{1 - \rho^2} v_{t+1} - (1 - \rho)x_t|$, where both x_t and v_t are independent random variables, with

Figure 2. Expected value of the total variation of the sum of two AR(1) series as a function of ρ_{xy} for different values of $\rho_y = -\rho_x$ (left panel), and as a function of $\rho_y = -\rho_x$ for different values of ρ_{xy} (right panel). Points are average of 100 simulations and the continuous line is eq. (24).

zero mean and unit variance. In addition, one usually assumes that their distribution is Gaussian. Therefore, the expected value of $\bigvee_1^n(x_t)$ is

$$\begin{aligned} E \left[\bigvee_1^n x_t \right] &= E \left[\sum_{k=0}^{n-2} |x_{t+1} - x_t| \right] \\ E \left[\bigvee_1^n x_t \right] &= \sum_{k=0}^{n-2} E \left[|\sqrt{1-\rho^2}v_{t+1} - (1-\rho)x_t| \right]. \end{aligned} \quad (22)$$

285 y_t :

$$E \left[\bigvee_1^n (x_t + y_t) \right] = \sum_{k=0}^{n-2} E \left[|(b_{1,1} + b_{2,1})v_{t+1} + b_{2,2}w_{t+1} - (1 - a_{1,1})x_t - (1 - a_{2,2})y_t| \right] \quad (23)$$

where the $a_{i,j}$ and $b_{i,j}$ are respectively the components of the matrices **A** and **B** in eqs. (18) and (20). All the random variables in eq. (23) are Gaussian, with zero mean and unit variance. v_{t+1} and w_{t+1} are mutually independent and independent of x_t, y_t . The covariance between x_t and y_t is ρ_{xy} . Therefore, using properties of the Gaussian distribution, the absolute value in eq. (23) corresponds to a Gaussian distribution with zero mean and variance equal to $(b_{1,1} + b_{2,1})^2 + b_{2,2}^2 + (1 - a_{1,1})^2 + (1 - a_{2,2})^2 + 2(1 - a_{1,1})(1 - a_{2,2})\rho_{xy} = 2(1 + \rho_{xy})(2 - \rho_x - \rho_y)$.

Finally, the expected value of the total variation of the sum is

$$E \left[\bigvee_1^n (x_t + y_t) \right] = (n-1) \frac{2}{\sqrt{\pi}} \sqrt{(1 + \rho_{xy})(2 - \rho_x - \rho_y)}. \quad (24)$$

A simple calculation of the expected value in eq. (22) gives eq. (21).

Similar arguments apply to the calculation of the expected value of the total variation of the sum $x_t +$

Figure 2 illustrates this result.

Using the above results, eqs. (21) and (24), the total variation complementarity index between two auto-regressive time series with lag-one correlation coefficients ρ_x and ρ_y , and cross correlation ρ_{xy} , is

$$\begin{aligned} \phi(x_t, y_t) &= 1 - \frac{\bigvee_1^n (x_t + y_t)}{\bigvee_1^n (x_t) + \bigvee_1^n (y_t)} \\ &= 1 - \frac{\sqrt{(1 + \rho_{xy})(2 - \rho_x - \rho_y)}}{\sqrt{1 - \rho_x} + \sqrt{1 - \rho_y}}. \end{aligned} \quad (25)$$

Figure 3 illustrate some particular cases of this result. We are interested in the comparison of ϕ with ϕ' . Recall that the unit variance of x_t and y_t implies that $\phi'(x_t, y_t) = (1 - \rho_{xy})/2$.

Figure 3. Expected value of total variation complementarity index as a function of ρ_x , ρ_y and ϕ' for the sum of two AR(1) series: left panel, $\rho_y = \rho_x$; center panel, $\rho_{xy} = -0.7$, $\phi' = 0.85$; and right panel, $\rho_y = -\rho_x$. Notice a region of negative values that correspond to non-admissible parameter values for the AR(1) process.

A first observation is that for $\rho_{xy} = -1$ both ϕ and ϕ' are one, that is perfect complementarity. If $\rho_x = \rho_y$ the total variation complementarity is independent of ρ_x , with $\phi = 1 - \sqrt{2(1 + \rho_{xy})}/2 \leq \phi' = (1 - \rho_{xy})/2$ (left panel of fig. 3). Still within the equality of the lag-one correlation coefficients, the case $\rho_{xy} = 0$ and therefore $\phi = 1 - \sqrt{2}/2 \approx 0.29$ is interesting in comparison with $\phi' = 0.5$. That high a variance complementarity index in the absence of cross-correlation may come from how the range of the index was put between zero and one in eq. (9). In fact, positive cross correlations give $0 < \phi' < 1/2$, something that seems high. An alternative to eq. (9) could be $1 - \rho_{xy}$ for negative correlations and zero for positive ones. We think that both alternatives are inferior than the total variation complementarity index. For null cross correlation, of the order of 30% of the total variation of the sum cancels out just by chance, whereas either 50% or 0% seem off.

For constant $\rho_{xy} = -0.7$ in the center panel of fig. 3, the total variation complementarity does depend on both ρ_x and ρ_y although $\phi' = 0.85$.

For the case $\rho_x = -\rho_y$ (right panel of fig. 3) ϕ depends on ρ_x except for ϕ' close to one. In all cases ϕ is less than ϕ' .

We conclude from this comparison that the total variation complementarity index has more information than the variance complementarity.

3.2 Case of study 2: wind vs. hydro in the Colombian electricity market

The total installed capacity for electricity generation in Colombia is approximately 17.5 GW, of which 68.3% is hydro while the only one wind plant accounts for near 0.1%. In fact, this big scale difference impacts the complementarity assessment.

To quantify the complementarity between wind and hydro-resources in the Colombian electricity system, we use daily time series provided by XM, the electric system administrator and operator in Colombia. For hydro-resources, we analyze natural inflows (in units of energy) to the largest 15 hydropower reservoirs, with a total effective installed capacity of 7,668 MW ($\approx 70\%$ of total hydro-capacity) and records from 2000 to 2020. These rivers are San Lorenzo, Nare, Guatapé, San Carlos, Tenche, Guadalupe, Grande, Porce II, Miel I, Urrá, Betania, Batá, Guavio, Alto Anchicayá, and Salvajina. For details on the hydropower system of Colombia, see XM the internet site. For wind resources, we analyze daily wind power generation from the Jepirachi plant, with 19.5 MW of effective installed capacity and records for the period 2004–2020. Notice that physical dimensions of both series are the same (energy).

Figure 4 shows the estimation of ϕ and ϕ' indices to the daily series and the annual cycles of hydro-inflows and wind generation with moving averages of 31 days (top panel). Considering the large scale

Figure 4. Comparison of ϕ and ϕ' indices between wind and hydraulic resources in Colombia electricity market, with smoothed moving averages of 31 days. The scale factor in the case of scaled annual cycle is 1/100 for hydro-inflow series and 1/10 for wind-power generation series.

365 difference of the accumulated hydro-inflows and the
 366 Jepirachi plant, the scaled series's complementarity
 367 was also studied (bottom panel), being the scale
 368 factor 1/100 for hydro-inflow series and 1/10 for
 369 wind generation series. Table 1 presents the results
 370 for all analyzed cases.

As expected, because of the scale differences, ϕ
 does not show complementarity, while ϕ' index
 indicates a complementarity of 50% for the daily
 and annual cycles. When one uses the scaled
 375 series, the ϕ index increases to 39%. We interpret
 395 this scaled analysis to evaluate the resources'
 potential complementarity if the resources' scale
 were comparable.

The difference in the results may come because
 380 the ϕ index depends on each series's serial
 correlation, while ϕ' does not. Besides, there is
 a tendency of ϕ' to exceed ϕ , overestimating the
 complementarity quantification.

3.3 Case of study 3: two hydropower sources in the Colombian electricity market

390 Nare and Guavio are two important rivers of the
 hydropower reservoir system of Colombia. The
 Andes mountain range separates the two basins,
 causing important differences in their hydroclimatic
 patterns. For instance, seasonality of rainfall is
 unimodal in the Guavio basin and bimodal in the
 Nare basin (Urrea et al., 2019), and streamflows
 show these regimes too (Fig. 5). According to XM,
 these rivers belong to different hydrological regions
 (see details in the XM website).

We computed the complementarity indices
 between the daily natural inflows (in energy units)
 of Nare and Guavio rivers for the period 2000–2020.
 Figure 5 shows the results, where the left panel
 shows the 31-day moving average daily inflows and
 the right panel the annual cycle at daily time-step for
 both rivers. Table 2 lists the results for all analyzed
 cases.

Table 1. Complementarity between wind and hydro resources in Colombia. The daily time series were smoothed, obtaining moving averages of 31 days. Variance has units of GWh^2 in the case of smoothed time series and annual cycle. In all other cases, the variables are dimensionless. ρ_1 represents the exponential value of the slope of the linear regression for the auto-correlograms of both resources.

Group	σ_{Hydro}^2	$\rho_{1\text{Hydro}}$	σ_{Wind}^2	$\rho_{1\text{Wind}}$	$\rho_{\text{Hydro,Wind}}$	ϕ	ϕ'
Smoothed time series	1,508	0.98	0.01	0.98	-0.12	0.00	0.50
Annual cycle	878	-	0.00	-	-0.26	0.00	0.50
Scaled series	0.09	-	0.13	-	-0.26	0.39	0.63

Table 2. Complementarity between 31-day moving average daily streamflows of Nare and Guavio rivers. Variance has units of GWh^2 in the case of smoothed time series and annual cycle. In all other cases, the variables are dimensionless. ρ_1 represents the linear regression slope for the auto-correlograms of both resources.

Group	σ_{Nare}^2	$\rho_{1\text{Nare}}$	σ_{Guavio}^2	$\rho_{1\text{Guavio}}$	$\rho_{\text{Nare,Guavio}}$	ϕ	ϕ'
Smoothed time series	62	0.98	129	0.98	0.25	0.20	0.38
Annual cycle	17	-	101	-	0.49	0.18	0.33
Scaled series	0.46	-	1.45	-	0.49	0.19	0.29

The scale of both resources is not as different as in the previous case. Consequently, the ϕ index is similar for the daily and scaled series with values close to 20%. Using the equivalent to eq. (25) that considers the values of variance (not shown), with the parameters contained in table 2, one gets $\phi = 0.20$. The same value that one obtains using the time series and eqs. (4) and (5). This correspondence suggests that the auto-regressive model is good to estimate complementarity. Again, ϕ' shows a higher result than ϕ for all cases, quantifying the complementarity around 38%. The pattern is that the variance complementarity index tends to overestimate complementarity.

3.4 Case of study 4: Hydropower integrated sources in the Colombian electricity market

One of the main advantages of the ϕ index is that one can evaluate the complementarity not just between two but among any number of series. To illustrate this capability, we quantify the complementarity among the 15 largest rivers energy inflows in the Colombian hydroelectric system (described in section 3.2). These rivers are in different Colombia regions with different

Figure 5. Assessment of complementarity by the ϕ and ϕ' indices between the Nare and Guavio rivers. Left panel: 31-day moving average daily inflows. Right panel: annual cycle at daily time-step.

Figure 6. Quantification of the complementarity with ϕ Index among the energy inflows of the main 15 rivers of the Colombian electrical system, with smoothed moving averages of 31 days.

geographic and climatic characteristics, making the complementarity analysis relevant.

Figure 6 present the daily series and annual cycle of the integrated hydro system with moving averages of 31 days. The ϕ index shows a complementarity of 34% for the smoothed series and 30% for the annual cycles. For the unsmoothed series (not shown), the complementarity is somewhat higher, reaching 38% and 46%, respectively.

4 Conclusions

Correlation coefficients between time series are the common practice to evaluate and quantify complementarity. Pearson's correlation coefficient, which is the most frequently used metric of complementarity, has several fundamental issues that undermine its usefulness for that purpose. As it is scaled independent, one could use it for non-dimensionally homogeneous variables, which does

not make any sense. Also, if the series's scale is very different, the correlation coefficient may mislead the real complementarity. We showed that negative correlation is a necessary but not sufficient condition to uphold complementarity, demonstrated that scale has a large effect on complementarity magnitude. Also, that dimensional homogeneity between the variables is a requisite to provide physical sense to the concept of complementarity.

We propose a novel method based on the concept and main mathematical properties of Total Variation. The total variation complementarity index (ϕ) is a consistent and satisfactory metric for complementarity between any number of series. Index ϕ measures the sum of the variables' regularity and is unaffected by the issues pointed out regarding correlation coefficients. Some essential properties of ϕ are: 1) ϕ is sensitive to the scale of the variables; 2) ϕ works only with dimensionally homogeneous variables; 3) ϕ can be used with any number of variables.

Given the unreliability of renewable energy sources, consistent evaluation of complementarity is a critical issue. Our proposed index fulfills this need, making a substantial contribution to energy planning.

Inspired by the total variation complementarity index, we proposed the variance complementarity index to correct the issues identified about Pearson's correlation coefficient in complementarity quantification.

We obtained analytical results for the expected value of two cross-correlated auto-regressive series, demonstrating that the variance complementarity index does not capture all the complementarity's dependencies. In particular, the example shows that serial auto-correlation does affect complementarity. Besides, the applications to the Colombian power system show that the variance complementarity overestimates complementarity.

Finally, the proposed complementarity indices quantify the Colombian electricity system's complementarity due to its hydrological and climatic differences. The total variation complementarity index among the daily series of the hydro resources is 30%. This figure is the first estimation of the

- complementarity among the 15 main rivers feeding the Colombian power system. The total variation complementarity index, ϕ , between hydro and wind resources is almost null due to the wind component's negligible contribution. When one uses scaled variables, it is 39%, representing the potential complementarity between these resources and could stimulate a higher level of investment in wind capacity development. The corresponding figures for the variance complementarity index between hydro and wind resources are 50% and 63%. The ϕ index between Nare and Guavio is 20%, whereas $\phi' = 38\%$. These two rivers are often considered complementary because of their annual cycles. The indices put a solid basis and a concrete number on those intuitive ideas.
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