Wavelet Transform for IoT – A Signal Processing Perspective

I. Dey, S. Siddiqui and N. Marchetti

Abstract—Denoising of signals in an Internet-of-Things (IoT) network is critically challenging owing to the diverse nature of the nodes generating them, environments through which they travel, characteristics of noise plaguing the signals and the applications they cater to. In order to address the above-mentioned challenges, we conceptualize a generalized framework combining wavelet packet transform (WPT) and energy correlation analysis. WPT decomposes both the low-frequency and high-frequency components of the received signals in different time scales and wavelet spaces. Noise components are identified, removed through filtering and the signal components are predicted back after filtering using inverse wavelet packet transform (IWPT). Next energy of the reconstructed signal components are compared with that of the original transmitted signal to modify the characteristics of the decomposed signal components. Using the modified details, the signal components are reconstructed back again and the noise components are filtered out. This process is repeated until noise is completely removed. Initial results suggest that, our proposed framework offers improvement in error probability performance of a medium-scale IoT network over traditional discrete wavelet transform (DWT) and WPT based techniques by around 3 dB and 7 dB respectively.

Index Terms—signal denoising; IoT networks; wavelets; wavelet transform; wavelet packet transform; generalized framework.

I. INTRODUCTION

Internet-of-Things (IoT) has emerged as a rapidly growing network formed by a diverse range of smart devices, which support a varied range of applications in the domains of healthcare, agriculture, transportation, manufacturing industry and many more. IoT networks promise a wonderland of unparalleled coverage, quality-of-service (QoS) and flexibility. Such an ecosystem can only be achieved through the coexistence of a multitude of low size weight and power (SWaP) devices (like relays, sensors, actuators etc.) in varied environments (indoor, outdoor, underground, underwater etc.). Within the network, the devices will transmit large volumes of data of varying length, rate and kind using different

signalling formats simultaneously. Transmitted signals are obstructed and scattered, corrupted by noise and distorted by intrinsic interference on their way to the IoT gateway or the receiving devices. Noise can be contributed by the environment, internal electronics of the devices or frequency-dependent anomalies [1], and interference is caused by the superposition of the transmitted signals when sent simultaneously over the multiple access channel [2].

Noise of any kind has a profound negative effect on the real-time information processing received from multiple devices and, in turn, on the information fusion and data aggregation [3]. In order to regain accurate and reliable information on the receiver side, it is therefore absolutely crucial to filter out the noise and reconstruct back the original transmitted signal before further processing like information fusion, aggregation etc.

Denoising refers to the method of eliminating noise from the signal while retaining the transmitted one without distorting the originality and resolution of information [4]. Filtering (low or band-pass) is the fundamental technique for denoising but is only effective in removing out-of-band noise. Removing noise that falls within the signal frequency range is challenging and cannot be accomplished just by filtering. Denoising is also quite a formidable task owing to the varying origin, nature and properties of noise.

Researchers have proposed many denoising techniques over the years. One of the most traditional methods is based on *Fourier Transform (FT)*, where noise is perceived as high frequency components, followed by applying FT and eventually filtering them out [5]. However, FT-based denoising can only restore the transmit signal partially and is vulnerable to the Gibbs phenomenon [6]. Another drawback with FT is that it offers high resolution in the frequency domain but zero resolution in the time and/or spatial domain. As a result, it can efficiently distinguish frequencies of the noise components but cannot identify the location of those components.

Another popular method is the *Modulus Maxima* technique [7] where maxima of the corrupted signal are projected onto a space. The projection space is divided into multiple scales and it has been demonstrated that the maxima of the signal and noise components exhibit different characteristics. Based on these characteristics, it is easy to differentiate the noise components which are eventually filtered out. Modulus maxima, though efficient performance-wise, suffers from high computational complexity. Moreover, as signal and noise maxima are projected on space, it is easy to locate them but very difficult to gain knowledge about the properties of the

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Figure 1: An example set of wavelets where x-axis plots time and y-axis plots amplitude. Here Morlet, Mexican-Hat and Gaussian wavelets are continuous wavelet functions; while Daubechies, Symlet and Coiflet wavelets are discrete ones.

signal and noise components themselves.

In order to circumvent the shortcomings of the above-mentioned denoising techniques, *Wavelet Transform (WT)* has recently emerged as a powerful tool for signal denoising [8]. WT uses transformation both in frequency (like FT) and in spatial domains (like modulus maxima method) in order to extract useful information about the signal and noise and eliminate noise accordingly [9]. WT can not only identify the frequencies of the noise components, but also can point out their exact location in space. WT analyzes the signal by looking at it through different window sizes. It starts with a large window to extract bulk characteristics and gradually decreases the window size to extract more and more detailed features.

The success of WT in denoising signals, that vary frequently with time and frequency, motivates us to embrace the benefits of WT and develop a conceptual architecture for employing it for denoising signals within an IoT network. Therefore, the primary contribution of this article is to conceptualize a general framework for denoising signals in an IoT network. The framework combines *Wavelet Packet Transform (WPT)* [10], a special kind of WT, with energy correlation analysis (comparing the energy content within reconstructed and transmitted signal components) to extract weak IoT signals out of the noise blanket. WPT implements tree-structured filter-banks to iteratively analyze signal attributes, while energy correlation analysis strengthens the similarities between the denoised signal and the original one iteratively. In a nutshell, the proposed framework i) decomposes the signal both in different time scales and wavelet space, ii) identifies noise components based on the energy content within the decomposed received signal components and then iii) eliminates the noise components through suitable filtering. This combination improves the denoising performance owing to its invariance to the heterogeneity of signals in an IoT network.

First, we introduce the readers to the basic concept of WT and the fundamental types of WT. Next we explain what the different ways of employing WT for denoising signals are. Then we present our proposed framework

for denoising signals within IoT networks and, finally we open up some research problems and future directions associated with our proposed framework.

II. WAVELET TRANSFORM

A. Wavelets

Practical data or signals vary slowly with time, punctuated and interrupted by abrupt changes. These sudden changes in signal characteristics often convey important information and interesting insights. Though extensively used for data analysis, FT is unable to characterize such abrupt changes as it uses sum of sinusoids to represent any kind of signal. Sinusoids oscillate for infinite duration of time and therefore, cannot analyze fast and abrupt changes accurately. Unlike sinusoids, wavelets oscillate for a finite duration of time and decay rapidly after a certain interval of time. Intuitively, wavelets are localized in time while sine waves are not; a characteristics that allows WT to acquire both spatial and frequency information.

A wide range of wavelets are available that are of different shapes and extend over different durations of time; an important factor responsible for the applicability of wavelet-based analysis over a large array of signal processing applications. It is also easy to generate novel wavelet types by satisfying the 'zero-mean constraint' and 'finite energy content'. The zero-mean constraint refers to zero mean in time domain and zero magnitude at zeroth frequency. Finite energy content refers to the localization property of wavelets in both time and frequency making the wavelet function integrable such that its inner product with the signal always exists. Moreover, it is possible to generate new wavelet functions from existing ones. As a result, at any point-of-time, we are equipped with an array of different kinds of wavelets, which are orthogonal, bi-orthogonal, symmetric, complex, unit-energy or none of the above. We have presented some widely used wavelets in Fig. 1 (including Morlet, Daubechies, Coiflets, Mexican Hat, Symlets and Gaussian).

B. Wavelet Transform

The process of decomposing a signal (or a signal function) into a set of wavelets is termed as WT. This is done by convolving the signal with an appropriate wavelet function at different locations in time. In essence, the original wavelet function is slowly moved across the signal from its beginning to its end. Once the process is completed using the principal wavelet function (also referred to as 'mother wavelet'), the wavelet can be scaled in magnitude and the process can be repeated. A comprehensive schematic overview of time and frequency resolution of FT and WT of a continuous signal is presented in Fig. 2. As evident from Fig. 2, the time-series representation of the signal offers high resolution in time domain and none in frequency domain. FT offers high resolution in frequency domain, while none in time domain. WT offers high resolution in time-domain for time-dependent features and it allows high resolution for frequency-dependent features in frequency domain.

Figure 2: A graphical representation of a continuous-time signal, its time-series representation, its Fourier Transform and its Wavelet Transform. Size and orientation of the block are congruent to the size of resolution. Movement from lighter to darker shades of colour represents increase in time and/or frequency.

The two fundamental properties of WT are scaling and shifting. Scaling deals with stretching or shrinking of signal with time and is controlled by a scaling factor. Scaling factor is inversely proportional to frequency of operation. That means low frequency corresponds to large scaling factor and a stretched wavelet, while high frequency matches to a small scaling factor and a shrunk wavelet. For example, if the scaling factor of a sine wave is 2, the original frequency is reduced by half or an octave. Therefore, it is possible to construct different scales that are inversely proportional to the frequency components of a signal of interest. Shifting deals with delaying or advancing of a wavelet along the duration of the signal of interest. Shifting characterizes the location of the wavelets in both time and/or space. Shifting of a wavelet can be used to align with either the time-dependent or the frequency-dependent features of any signal. Depending on how wavelets are scaled and shifted to analyze a signal, WT can be broadly classified into Continuous WT (CWT) [11] and Discrete WT (DWT) [12].

The type of WT to be used depends on the application scenario. For example, CWT is useful for time-frequency analysis and frequency component extraction from any signal, while DWT is suitable for signal, data or image compression and denoising. The main difference between CWT and DWT is that CWT uses an infinite set of wavelets defined over a continuous set of scales, shifts and locations while DWT corresponds to a finite set of wavelets at discrete sets of scales, locations and shifts.

C. Continuous WT

CWT of a signal yields coefficients that are function of scale ($1/\text{frequency}$) and time. Since a scaling factor of 2 for a wavelet results in frequency reduction by an octave, it is possible to add intermediate scales between a pair of octaves through the application of CWT. The number of intermediate scales is referred to as 'scales per octave'; the higher the number, the more detailed is the analysis. The scales are multiplied by the samples of the signal of interest for each wavelet scale at each time step, or in other words, a signal is convolved with a set of wavelets at a variety of scales. If each scaled wavelet is shifted in time along the signal duration and compared with the original signal, coefficients ensue that are functions of wavelets' scale and shift parameters.

D. Discrete WT

The DWT process, on the other hand, is equivalent to comparing a signal with discrete multi-rate filter banks. Any signal is initially subdivided into a low and a high frequency sub-band by passing it through a lowpass and a highpass filters respectively. In the next stage, the low frequency sub-band is filtered again using low and highpass filters to result in narrower sub-bands. This process continues iteratively until we are left with a few large magnitude DWT coefficients for the actual signal and a very few small magnitude coefficients for the noise (or anomalies in the signal). Therefore, it is possible to analyze signals at increasingly narrower sub-bands of different resolutions (refer to Fig. 3(a)).

E. Wavelet Packet Transform

The major disadvantage with DWT is that it iteratively decomposes the low frequency sub-bands to analyse the signal as accurately as possible. Noise is assumed to be high frequency in nature and the DWT therefore does not operate on the high frequency components. However, environmental noise exists also in low frequency range (about 10 Hz to 200 Hz), especially in industrial environment (industrial IoT networks), where there are numerous machines, turbines and instruments operating simultaneously (producing strong 'whoosing' sounds, for example). If noise appears both in low and high frequency sub-bands, then WPT is a more suitable option for signal analysis. WPT iteratively applies WT on both the low frequency and the high frequency components. WPT uses wavelet packets (WPs) and WPs are formed from a linear combination of wavelet functions where the WPs acquire properties of their corresponding wavelet functions. Therefore, the process of level-by-level decomposition of a signal (or a signal function) into a set of WPs is termed as WPT (refer to Fig. 3(b)).

III. WAVELET TRANSFORM FOR DENOISING

A. Denoising with DWT

DWT is always preferred over CWT for signal denoising as CWT suffers from two major drawbacks of redundancy and impracticality. 'Redundancy' occurs in CWT as the scaling and shifting factors are continuous and therefore, there can exist infinite number of wavelets.

Figure 3: Basic Schematic Representation for (a) DWT and IDWT Technique and (b) WPT and IWPT Technique. (HPAF: high-pass analysis filter, LPAF: low-pass analysis filter, HPSF: high-pass synthesis filter, LPSF: low-pass synthesis filter, DS: down-sampling, US: up-sampling)

The availability of such large number of wavelets is not efficient at all leading to the ‘impracticality’ in using CWT for analysing stationary and harmonic signals. Therefore, in this section, we will summarize the process of applying DWT for signal denoising. The first step is to split the signal into its low frequency components (also referred to as ‘approximation’ levels or coefficients) and its high frequency components (also referred to as ‘detail’ levels or coefficients). The high frequency components consist of the noise corrupting the signal as well as sudden changes and abrupt anomalies within the signal. These changes and aberrations in the signal often carry important information that need preserving.

B. Threshold Computation

For any denoising technique, noise is invariably assumed to be Gaussian distributed with real values that are independent and identically distributed. The main focus of the denoising process is to retain the useful information while removing the corrupting Gaussian distributed noise entangled with the high frequency signal components. The best way to accomplish this is to scale the detail coefficients by a threshold. The four most common methods for threshold computation are Universal, Minimax, Stein’s Unbiased Estimation of Risk (SURE) and Minimum Description Length (MDL) [13].

The Universal threshold is calculated using the length of the signal vector after sampling and the standard deviation of noise. The Minimax threshold is formulated corresponding to each set of data using the estimates of the minimum and the maximum risk bounds for the transformed wavelets. For the SURE algorithm, the threshold value that minimizes the SURE value is chosen for scaling, where the SURE value is the unbiased estimate of the variance between filtered and unfiltered signal after sampling. The MDL method computes the threshold using the number of largest coefficients retained after filtering, filter type, wavelet coefficients from WT and the largest coefficients in amplitude retained after filtering.

C. Thresholding

Once the threshold is computed, the next step is to apply thresholding operation to the ‘detail coefficients’. The common methods of thresholding are: soft and hard thresholding. For both the techniques, coefficients with magnitudes less than the threshold are set to null. In case of soft thresholding, the threshold value is subtracted from the coefficients with magnitude larger than the threshold thereby reducing those coefficients. This particular process ensures that there are no discontinuities in the resulting signal after threshold application. In case of hard thresholding, coefficients with value larger than the threshold are left unchanged. After the thresholding operation, new detail level coefficients are obtained which can be used for reconstructing back the signal.

D. Inverse DWT (IDWT)

The final step in the denoising process is reconstructing the signal back from the detail level coefficients after the thresholding application. This is achieved by a technique called ‘Inverse DWT (IDWT)’ [15]. IDWT restores back the signal from the modified detail coefficients by combining the process of folding and lifting. Folding combines approximation and detail coefficients obtained from consecutive intermediate decomposition levels to generate new set of coefficients. Lifting factorizes the coefficients to predict the signal components back from them. The factorization process in lifting can be replaced by poly-phase decomposition filtering to regain back the signal components. The newly obtained signal components, in turn, can be merged to get the original discrete or time-series representation of the signal using shift-and-add scheme. A schematic diagram for IDWT and Inverse WPT (IWPT) is presented in Fig. 3(a) and Fig 3(b) respectively. It is worth-mentioning here that the steps followed in case of DWT and IDWT for signal denoising can also be applied to WPT and IWPT when used for denoising. A brief demonstration of what happens when DWT and WPT are applied for denoising is presented in Fig. 4.

Figure 4: (a) Denoising with DWT. (b) Denoising with WPT and WPT-ECorr. Our proposed framework combining WPT and energy correlation i.e. WPT-ECorr is capable of removing noise more effectively than DWT and WPT techniques; where WPT offers better denoising than DWT.

IV. SIGNAL DENOISING IN IOT NETWORKS

Unlimited sequences of time-varying data generated by massive number of communicating nodes (or end-points) in an IoT network are used to take real-time decisions on a certain observation/phenomenon. If the detected signal energy received from the nodes is higher than a predefined level, the target is present or a certain phenomenon is happening and vice-versa. However, node signals can face innumerable obstacles resulting in data getting corrupted by noise, outdated, inaccurate and contradictory, on their way to the receiver or data center where they are processed further. Moreover, node signals suffer from interference due to partial overlap of multiple signals and superposition of signals when sent over a multiple access channel (MAC). Such interference is also considered as noise and needs to be eliminated for accurate decision making.

In such a scenario, the WPT method will be the best option for denoising, given that noise can be entangled with both the high and the low frequency components. The WPT method decomposes both the approximation and the detail coefficients thereby revealing more details about both the signal and the noise. If WPT is combined with energy correlation analysis (WPT-ECorr), it will be possible to eliminate noise by investigating the signal characteristics at each different decomposition level.

A. WPT-ECorr Framework

Here, we present a universal framework for denoising node signals in IoT networks which involves a two-step removal of noise components and reconstruction of the transmitted signals.

- 1) In the first step, approximation and detailed coefficients are obtained through WPT and threshold computation and application to reconstruct the signal received from the IoT nodes using IWPT.
- 2) In the second step, the reconstructed signal is compared to the one actually received to generate

a modified set of approximation and detail coefficients. The modified set of coefficients is then used to compute the threshold, which in turn, is used to scale the detail coefficients using soft-thresholding technique. The scaled detailed coefficients are then used to reconstruct back the transmit signal using IWPT again.

The entire process continues iteratively until there are only few low magnitude noise components left that can be either nullified or filtered out. The number of iterations is initially set to be equal to the number of decomposition levels used for the WPT. At the end of the last iteration, if the signal is not maximally denoised, the framework will increase the number of decomposition levels, and thereby the number of iterations. A flow diagram for the above framework is depicted in Fig. 5 and Fig. 4(b) demonstrates what happens when it is applied for denoising.

The main advantage of this framework is its adaptability to the condition of the received signals from the IoT nodes. If the signal is noisy, the framework adopts a higher number of decomposition levels as compared to the case where the received signal is less corrupted. This provides an edge to the proposed framework over the very few ones existing in literature [14] in terms of both energy efficiency and computational complexity.

B. An Example Scenario

Next we simulate the performance of a synthetic IoT network, where M number of IoT nodes are randomly deployed and uniformly distributed in a circular annulus around an IoT Gateway in order to demonstrate how the error probability in activity/target/phenomenon detection gets improved manifold when our proposed framework is used for signal denoising. The local observations of the IoT nodes are transmitted simultaneously over a Gamma-shadowed and Rayleigh block-faded MAC. It is worth-mentioning here that the IoT Gateway is equipped with N antennas to exploit spatial diversity

Figure 5: Flow-diagram of the general framework of WPT-ECorr for signal denoising.

Figure 6: (a) An example scenario for the considered IoT networks. (b) Comparative detection error probability performance as a function of signal-to-noise ratio (SNR) for the example IoT network scenario with DWT, WPT and WPT-ECorr aided signal denoising and without the signal denoising procedure.

and opportunistic throughput. Maximal ratio combining (MRC) is used for information fusion at the IoT Gateway. Information processing and signal denoising take place at the Gateway before forwarding it to the suitable application endpoint.

The example scenario depicted in Fig. 6(a) and Fig. 6(b) shows the error probability performance for an under-loaded ($M = 10$ and $N = 60$, $M < N$) configuration. The IoT nodes transmit their observations over a MAC experiencing a moderate pathloss and shadowing. Combined WPT and energy correlation (WPT-ECorr) aided denoising offers significant improvement in performance as compared to no denoising. Our proposed framework offers considerable performance gain over DWT-based and WPT-only based denoising procedures. It is worth-mentioning here that in our considered scenario, IoT nodes are transmitting sinusoidal signals which are corrupted with Gaussian distributed noise on their way to the gateway. At the gateway Symlet4 [7] wavelet function is used for producing the WPs for employing the WPT. It is worth-mentioning here that Symlet is a family of symmetric discrete wavelet

functions, and we have used order 4 Symlet wavelets for our signal analysis as they have an approximately linear phase response. This means that the wavelet used for signal analysis preserves and retains the shape and phase-relationships of the original input signal.

C. Note on Computational Complexity

Energy efficiency is one of the most crucial factors in IoT networks as most IoT nodes are low size, weight and power devices, battery operated and untethered to any energy source. Specific applications, like embedding IoT nodes within concrete structure or nodes spread over an agricultural field, call for a careful analysis of lifetime-extending strategies, identifying the key components that can lead to energy efficiency in IoT networks. Apart from developing more energy efficient nodes, the data processing in the network needs to operate with low computation complexity in order to enhance energy conservation within the network.

The main advantage of our proposed framework, towards this end, is that the architectural design of the framework can be optimized to offer low critical

path latency, fast convergence and low complexity, as it is an iterative process. If 100% utilization of processing resources is ensured, the framework can be implemented with minimum number of registers and operators, thereby reducing hardware cost and control complexity (both computational and logical complexities). It is worth-mentioning that actual utilization of the resources varies depending on the amount and type of processing required.

V. OPEN RESEARCH ISSUES AND FUTURE DIRECTIONS

In this paper, we have introduced a framework for denoising signals received from multiple communicating nodes transmitting simultaneously their information to a gateway within a standard IoT network. The framework combines the techniques of WPT and energy correlation analysis for noise removal and accurate signal restoration at the IoT gateway. Initial simulation results reveal a profound improvement in error probability performance over no denoising, DWT and WPT-only based denoising techniques. The results are encouraging and therefore, in future, we plan to extend our simulation set-up to include different kinds of noise, signals, wavelets and WT techniques and validate the benefits offered by our proposed framework in different communication scenarios. Below, we outline some future directions that we need to work upon to establish and mature the initial concept presented in this paper.

A. Different kinds of Noise

We simulate performance considering that the noise is additive in nature and the real-valued components are Gaussian distributed. However, in practical communication networks, noise can be additive or multiplicative. The real-valued noise components can be Gaussian, Laplace or Poisson distributed depending on the application scenario and the kind of signal handled within a particular network. In certain power-line and acoustic networks, different kinds of noise like impulse, speckle or impact noise are encountered, the components of which cannot be modelled with a Gaussian distribution. Therefore, performance of the proposed framework needs to be validated against different kinds of noise and noise distributions.

B. Different kinds of Signal

The signal generated within the IoT network has been considered to be sinusoidal in nature. However, with varying IoT applications, different system requirements and propagation environments, different kinds of signal are generated. Signal can be digital pulse or square wave in nature, exponentially increasing or decaying, impulse shoot, unitary/Heavy-side step function, Poisson or Laplace distributed etc. What kind of signal will be encountered in a particular application and propagation scenario can only be accurately characterized through a measurement campaign. Therefore, detailed measurement campaigns need to be conducted in different

scenarios to model the signal, which in turn, can be incorporated in our proposed framework for validation of performance.

C. Different kinds of Wavelets

The results generated in this paper use Symlet function for WP creation. A large variety of wavelet functions is available including Haar, Daubechies, Coiflets, biorthogonal, Meyer, Gaussian etc. The kind of wavelet function that is best suitable for denoising depends on the scenario, signal and noise characteristics within a specific IoT network and application. Therefore, it is necessary to validate the versatility of the framework against different WP functions at different scales/ levels of decomposition.

D. Different kinds of WT

Finally, it is worth-investigating what kinds of WT can be combined with energy correlation analysis to provide improved denoising performance. Advanced WT techniques like complex WT (CWT), dual-tree CWT (DTCWT), second-generation WT (SGWT) or harmonic WT (HWT) need to be employed to check the denoising capability of the framework. Performance of these techniques have to be evaluated against metrics like computational complexity, energy efficiency and network throughput to arrive at the best possible choice of WT technique.

VI. CONCLUSIONS

Over the last couple of years, IoT has become an integral part of our lives and its applications are spread across many important segments of our life, like health, finance, business, transportation, homes, offices, etc. As a result, signals generated in an IoT network are of varying characteristics and length, and are crippled with different kinds of noise, resulting in incorrect, outdated and useless information. In this paper, we develop an initial framework for eliminating this noise at the IoT gateway before further processing, decision making and forwarding information to the destination endpoint. The framework uses WPT for signal construction out of WP decomposition levels and coefficients, followed by correlating reconstructed signal energy with the received signal energy, in order to generate a modified set of WP decomposition coefficients and reconstructing the signal from the modified coefficients. We simulate performance of the proposed framework against sinusoidal signal corrupted by Gaussian distributed noise using Symlet4 function for WP generation. Results show marked enhancement in detection probability performance over scenarios where no denoising, DWT or WPT based denoising technique are used at the gateway. In future, we will test our framework against a variety of signals, different kinds of noise, and different wavelet functions. The framework also can be generalized to include other advanced WT techniques like DTCWT, HWT or SGWT.

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