

Understanding Photovoltaic Cell Dynamic Resistance Behavior with Changing Incident Light Intensity - draftv1

FRANCISCO J. GARCIA-SANCHEZ ¹

¹Universidad Simón Bolívar

October 30, 2023

Abstract

A theoretical examination of the general behavior that should be expected to be displayed by the magnitude of the dynamic resistance of a conventional illuminated photovoltaic device within the power-generating quadrant of its I - V characteristics, when measured in quasi-static conditions from the short-circuit point to the open-circuit point, at various incident illumination intensities. The analysis is based on assuming that the photovoltaic device in question may be adequately described by a simple conventional d-c lumped-element single-diode equivalent circuit solar cell model, which includes significant constant series and shunt resistive losses, but lacks any other secondary effects. Using explicit analytic expressions for the dynamic resistance, we elucidate how its magnitude changes as a function of the terminal variables, the incident illumination intensity and the model's equivalent circuit elements' parameters.

Understanding Photovoltaic Cell Dynamic Resistance Behavior with Changing Incident Light Intensity

Francisco J. García-Sánchez, *Senior Member, IEEE*

Abstract—We present a theoretical examination of the general behavior that should be expected to be displayed by the magnitude of the dynamic resistance of a conventional illuminated photovoltaic device within the power-generating quadrant of its I - V characteristics, when measured in quasi-static conditions from the short-circuit point to the open-circuit point, at various incident illumination intensities. The analysis is based on assuming that the photovoltaic device in question may be adequately described by a simple conventional d-c lumped-element single-diode equivalent circuit solar cell model, which includes significant constant series and shunt resistive losses, but lacks any other secondary effects. Using explicit analytic expressions for the dynamic resistance, we elucidate how its magnitude changes as a function of the terminal variables, the incident illumination intensity and the model's equivalent circuit elements' parameters.

Index Terms—Photovoltaic device, solar cell model, dynamic resistance, Lambert W function.

I. INTRODUCTION

THE negative value of the reciprocal slope of the I - V characteristics of an illuminated photovoltaic cell is usually called its dynamic resistance, r . This is a very useful parameter to measure when analyzing and optimizing the cell's performance capabilities [1]. It is also essential when defining a solar cell's maximum power point (MPP). Here we will look, from a basic theoretical point of view, into some fundamental issues regarding the dynamic resistance of conventional photovoltaic cells, with the intention of better understanding its expected general behavior as incident light intensity changes. The questions to be addressed are:

- 1) Does the magnitude of the dynamic resistance r of conventional photovoltaic cells depend on the intensity of the incident illumination? Or, equivalently, is r dependent on the magnitude of the photo-current, I_{ph} , produced by the separation and collection of photo-generated charge carriers?
- 2) Does the magnitude of r measured under short-circuit or open-circuit conditions depend on incident light intensity?
- 3) If so, how are these dependencies?

To obtain theoretical answers to the above fundamental queries, we will start from the mathematical equations of the

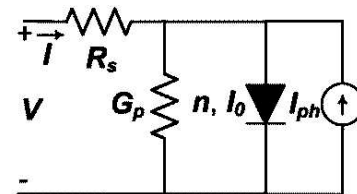


Fig. 1. Equivalent lumped-element dc circuit model used to describe the current-voltage characteristics at the terminals of a conventional illuminated cell.

simplest model that can be used to describe the most significant physical phenomena responsible for the typical shape of conventional photovoltaic cells' static I - V characteristics under illumination. That simplest realistic model is an equivalent condensed-element (a.k.a. lumped-parameter) dc electric circuit [1], whose configuration is that shown in Fig. 1.

The electric elements of this circuit consist of a parallel-connected combination of: a single diode (characterized by the linked pair of lumped parameter values n and I_0 , which merge the most significant charge carrier transport mechanisms at the cell's junction); a photo-generated current source (with magnitude defined by its lumped parameter I_{ph}) oriented in the direction of the diode's reverse current, which accounts for the photo-current produced by all collected photo-generated carriers; and a shunt resistor (specified by the value of its lumped parameter $R_p=1/G_p$) that includes all resistive losses across the cell's junction. As shown in Fig. 1, this parallel combination is further connected in series to a second resistor (specified by the value of its lumped parameter R_s), which combines all series resistive losses caused by the cell's contacts and terminals. Both resistive elements are assumed to stay approximately constant with respect to the circuit's variables and illumination intensity.

We will assume that the magnitude of the dynamic resistance, r , embodied by the value of the illuminated photovoltaic cell I - V characteristics' reciprocal slope, can be experimentally determined at quasi-static conditions, either by performing a small-signal ac measurement at very low frequency, or by numerical calculation from two or more consecutive data points taken around the location of interest on the measured static I - V characteristic [2].

II. METHODS

The first step to theoretically study the behavior of the dynamic resistance is to write the equation that describes the simplest equivalent circuit model capable of adequately representing the conventional non-ideal photovoltaic cell with series and parallel resistive losses ($R_s \neq 0$, $G_p \neq 0$). We have chosen to use the single-diode model whose lumped-element equivalent circuit [1] is shown in Fig. 1. The relationship between its terminals' current and voltage is mathematically expressed using Kirchoff's Laws and Shockley's diode equation [3], which results in the following implicit transcendental linear-exponential equation:

$$I = I_{ph} - I_0 \left[\exp\left(\frac{IR_s + V}{n v_{th}}\right) - 1 \right] - (IR_s + V)G_p . \quad (1)$$

Notice that in writing (1) we have defined the current as coming out of the positive terminal, that is, in the opposite direction (sign) as that shown in Fig. 1. We do so for convenience and without loss of generality, in order to adhere to the sign convention normally used by photovoltaic power engineers, where the power generating quadrant is presented as the first (instead of as the fourth) quadrant of the illuminated cell's I - V characteristics (see Fig. 2).

A. Explicit solutions

Closed-form solutions of the cell's terminal current and voltage equations as explicit functions of each other are preferred over (1) from simulation and analysis points of views. Such explicit solutions are possible to be obtained from (1) thanks to the use of the Lambert W function [4]. The solution of (1) for the terminal current as an explicit function of the terminal voltage is [5]:

$$I(V) = -\frac{n v_{th}}{R_s} W_0 \left\{ \frac{I_0 R_s \exp\left[\frac{(I_0 + I_{ph})R_s + V}{n v_{th}(G_p R_s + 1)}\right]}{n v_{th}(G_p R_s + 1)} \right\} - \frac{-(I_0 + I_{ph})R_s + V}{R_s} . \quad (2)$$

where W_0 represents the principal branch of the Lambert W function [4]. To find the corresponding expression for the terminal current, I_i , of an ideal cell, i.e., a cell with negligible series and parallel resistive losses, substituting $R_s=0$ and $G_p=0$ directly into (2) will not work. Strictly, we would have to find the limit of (2) as $R_s \rightarrow 0$ and $G_p \rightarrow 0$. However, a direct route is to simply substitute $R_s=0$ and $G_p=0$ into (1):

$$I_i(V_i) = I_{ph} - I_0 \left[\exp\left(\frac{V_i}{n v_{th}}\right) - 1 \right] . \quad (2i)$$

Likewise, the solution of (1) for the terminal voltage as an explicit function of the terminal current can be written also in terms of the Lambert W function [5]:

$$V(I) = -n v_{th} W_0 \left[\frac{I_0 \exp\left(\frac{I_0 + I_{ph} - I}{G_p n v_{th}}\right)}{G_p n v_{th}} \right] + \frac{I_0 + I_{ph} - I}{G_p} - IR_s , \quad (3)$$

The corresponding expression for the terminal voltage, V_i , of the ideal cell is quickly found by inverting (2i):

$$V_i(I_i) = n v_{th} \ln \left(\frac{I_{ph} + I_0 - I_i}{I_0} \right) . \quad (3i)$$

B. Short-circuit and open-circuit expressions

An expression for the short circuit current I_{sc} can be written by evaluating (1) at $V=0$:

$$I_{sc} = -\frac{n v_{th}}{R_s} W_0 \left[\frac{I_0 R_s \exp\left(\frac{(I_0 + I_{ph})R_s}{n v_{th}(G_p R_s + 1)}\right)}{n v_{th}(G_p R_s + 1)} \right] + \frac{(I_0 + I_{ph})}{(G_p R_s + 1)} , \quad (4)$$

To find the corresponding expression for the short circuit current, I_{sc} , of an ideal cell, substituting $R_s=0$ and $G_p=0$ directly into (4) will not work. Strictly, we would have to find the limit of (4) as $R_s \rightarrow 0$ and $G_p \rightarrow 0$. A much more direct route is to simply evaluate (2i) and at $V=0$:

$$I_{isc} = I_{ph} , \quad (4i)$$

The expression for the open circuit voltage V_{oc} is found by evaluating (2) at $I=0$:

$$V_{oc} = -n v_{th} W_0 \left[\frac{I_0 \exp\left(\frac{I_0 + I_{ph}}{G_p n v_{th}}\right)}{G_p n v_{th}} \right] + \frac{I_0 + I_{ph}}{G_p} . \quad (5)$$

Again, to find the corresponding expressions for the open-circuit voltage, V_{ioc} , of an ideal cell, substituting $R_s=0$ and $G_p=0$ directly into (5) will not work. We would need to find the limit of (5) as $R_s \rightarrow 0$ and $G_p \rightarrow 0$. However, a simpler direct route is to evaluate (3i) and at $I=0$:

$$V_{ioc} = n v_{th} \ln \left(\frac{I_{ph} + I_0}{I_0} \right) . \quad (5i)$$

The above equations clearly indicate, that the short-circuit current and the open-circuit voltage of an ideal or real photovoltaic cell are both functions of incident light intensity (represented by the value of the photo-current I_{ph}). They also depend on the equivalent circuit's lumped elements parameter values, except that V_{oc} does not depend on the value of R_s , since the voltage drop across R_s is zero when $I=0$.

C. A hypothetical photovoltaic cell example

We will use a hypothetical photovoltaic cell, with known equivalent circuit lumped-elements' parameter values, as an example to generate synthetic I - V characteristics that will help to analyze and graphically illustrate the behavior of the dynamic resistance. The following parameters values of the elements of the circuit model shown in Fig. 1 will be used as example: diode's junction quality factor $n=1.5$, diode's reverse saturation

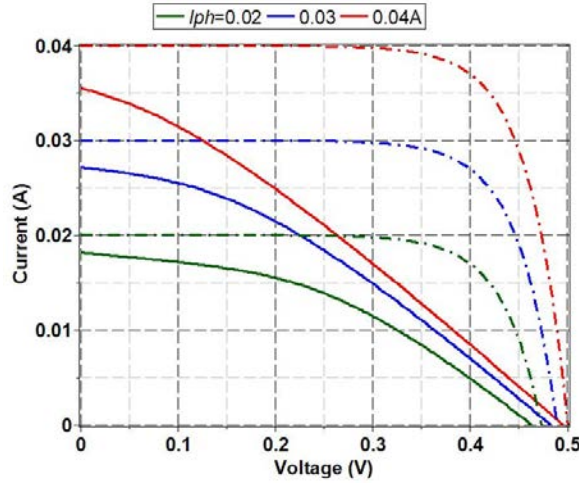


Fig. 2. Power-generating quadrant of synthetic current-voltage characteristics of a hypothetical illuminated real and ideal photovoltaic cell, with (solid lines) and without (dashed lines) resistive losses, respectively. The cell is modeled at room temperature by the equivalent circuit of Fig. 1, using the diode's parameters given in the text. Solid lines: $1/G_p = R_p = 100\Omega$ and $R_s = 10\Omega$; Dashed lines: $G_p = 0$ and $R_s = 0$ (ideal cell). Curves calculated for three indicated values of photo-current. The negative of the curves' reciprocal slope at any point is called the dynamic resistance.

current $I_0 = 10^{-7}A$, shunt resistive loss given by the stated value of $R_p = 1/G_p$, and series resistive loss given by the stated value of R_s . We assume room temperature operation, i.e. a thermal voltage $v_{th} = k_B T/q = 0,02586V$.

Three synthetic I - V characteristics of this hypothetical cell, calculated using the explicit solution for the current given by (2), are presented in Fig. 2, for three intensities of incident light which generate within the cell corresponding photo-current magnitudes of $I_{ph} = 0.02, 0.03$, and $0.04A$. Matching I - V curves of an ideal cell, alike but without shunt or series resistive losses ($R_p = G_p = 0$, $R_s = 0$), are also shown for visual comparison.

The numeric values of short circuit current and open circuit voltage, calculated with (4), (4i), (5) and (5i) for this particular example, at three given incident light intensities, are presented in Table I. It is worthwhile to point out that the sometimes hastily stated assumption that $I_{sc} \approx I_{ph}$ is not always a valid approximation, as (4) clearly implies. Its validity depends on the relative magnitudes of the cell's parallel and series resistive losses.

III. RESULTS

A. Behavior of the dynamic resistance along the I - V curves

The magnitude of the dynamic resistance, r , was already defined in Section I as the negative of the reciprocal slope of the synthetic I - V characteristics. Explicit analytic expressions for r as a function of the terminal current, and of its reciprocal the dynamic conductance, g , are obtained by taking the corresponding derivatives of the explicit solutions (2) and (1) with respect to current and voltage, respectively. This can be accomplished without difficulty because the Lambert W function is readily differentiable [4]. The resulting explicit expressions for r and g are:

TABLE I
CALCULATED SHORT-CIRCUIT CURRENT AND OPEN-CIRCUIT VOLTAGE VALUES OF HYPOTHETICAL REAL AND IDEAL SOLAR CELLS AT THREE VALUES OF INCIDENT LIGHT INTENSITY

I_{ph} (mA)	I_{sc} (mA)		V_{oc} (V)	
	real	ideal	real	ideal
20	18.2	20	0.463	0.473
30	27.2	30	0.482	0.489
40	35.5	40	0.495	0.500

Equivalent circuit elements' parameters: $v_{th} = 0,02586V$, $n = 1.5$, $I_0 = 10^{-7}A$, Real: $R_p = 100\Omega$, and $R_s = 10\Omega$; Ideal: $G_p = 0$, and $R_s = 0$;

$$r(I) = \frac{dV(I)}{dI} = -\frac{1}{G_p} \frac{1}{1 + W_0 \left[\frac{I_0 \exp\left(\frac{I_0 + I_{ph} - I}{G_p n v_{th}}\right)}{G_p n v_{th}} \right]} - R_s. \quad (6)$$

and

$$g(V) = \frac{dI(V)}{dV} = -\frac{W_0 \left[\frac{I_0 R_s \exp\left(\frac{(I_0 + I_{ph})R_s + V}{n v_{th}(G_p R_s + 1)}\right)}{n v_{th}(G_p R_s + 1)} \right] + \frac{G_p R_s}{(G_p R_s + 1)}}{R_s \left\{ 1 + W_0 \left[\frac{I_0 R_s \exp\left(\frac{(I_0 + I_{ph})R_s + V}{n v_{th}(G_p R_s + 1)}\right)}{n v_{th}(G_p R_s + 1)} \right] \right\}}. \quad (7)$$

Fig. 3 presents the resulting evolution of the dynamic resistance r as a function of voltage, calculated along the I - V curve of the illustrative example cell, from the short-circuit point ($V=0$) to the open circuit point ($V=V_{oc}$), at three levels of photo-current, I_{ph} , intensity. Fig. 3 indicates that the magnitude of the dynamic resistance r , for any illumination level, decreases as the terminal voltage increases from the short-circuit point to the open-circuit point. Alternative expressions of r as a function of both the terminal voltage and current can be obtained by direct differentiation of the implicit (1):

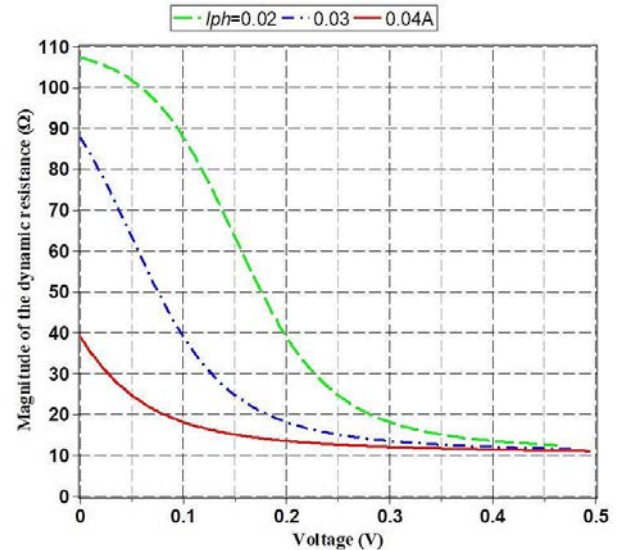


Fig. 3. Magnitude of the dynamic resistance r of the hypothetical illuminated cell with $R_p = 100\Omega$, and $R_s = 10\Omega$, calculated with either (6) or (7) at three values of photo-current intensity I_{ph} , from the short-circuit point ($V=0$) to the open-circuit point ($V=V_{oc}$). These values of r correspond to the negative reciprocals of the I - V curves' slopes shown in Fig. 2.

$$1/r(V, I) = \frac{dI}{dV} = -\left(1 + \frac{dI}{dV} R_s\right) \left[G_p + \frac{I_0}{n v_{th}} \exp\left(\frac{IR_s + V}{n v_{th}}\right)\right]. \quad (8)$$

Solving (8) for dI/dV and rearranging yields $r(V, I)$:

$$r(V, I) = \frac{1}{dI/dV} = \frac{dV}{dI} = -R_s - \frac{n v_{th}}{G_p n v_{th} + I_0 \exp\left(\frac{IR_s + V}{n v_{th}}\right)}. \quad (9)$$

Substitution of $R_s=0$ and $G_p=0$ into (9) yields the dynamic resistance r_i of an ideal photovoltaic cell:

$$r_i(V) = -\frac{n v_{th}}{I_0 \exp\left(\frac{V}{n v_{th}}\right)}. \quad (9i)$$

Another expression for the dynamic resistance $r(V, I)$ as a function of both terminal variables can be obtained by substitution of (1) into (9):

$$r(V, I) = \frac{dV}{dI} = -R_s - \frac{n v_{th}}{G_p(n v_{th} - V) + I_{ph} + I_0 - I(G_p R_s + 1)}. \quad (10)$$

Substitution of $R_s=0$ and $G_p=0$ into (10) produces another form of dynamic resistance, r_i , of an ideal cell equivalent to (9i):

$$r_i(I) = -\frac{n v_{th}}{I_0 + I_{ph} - I}. \quad (10i)$$

B. Dynamic resistance dependence on incident light intensity

The most important general conclusion that can be drawn from Fig. 3 is that the magnitude of the dynamic resistance r , at any voltage in the power-generating quadrant, always decreases as incident light intensity increases. Another conclusion from Fig. 3 worth of analysis is that for any incident light intensity, r decreases as voltage increases from the short-circuit point ($V=0$) to the open-circuit point ($V=V_{oc}$). Furthermore, although it might not be fully obvious from Fig. 3, because of the insufficient variation of light intensities used (not wide enough range of values of I_{ph}), it seems to suggest that the high and low limiting values of r at any voltage are approximately given by $R_p + R_s$ and R_s . This behavior of the dynamic resistance, r , with incident light intensity can be more conveniently visualized by examining the magnitude of r at the open-circuit point and at the short-circuit points of the I - V curves as a function of photo-generated current I_{ph} .

C. Dynamic resistance at the open-circuit point

To calculate the magnitude of the dynamic resistance at the open-circuit point, we evaluate (9) at $V=V_{oc}$ and $I=0$:

$$r_{oc}(V_{oc}) = -R_s - \frac{n v_{th}}{G_p n v_{th} + I_0 \exp\left(\frac{V_{oc}}{n v_{th}}\right)}. \quad (11)$$

For the case of an ideal cell, evaluation of (9i) at $V=V_{oc}$, or substituting $R_s=0$ and $G_p=0$ into (11) yields:

$$r_{ioc}(V_{oc}) = -\frac{n v_{th}}{I_0 \exp\left(\frac{V_{oc}}{n v_{th}}\right)}. \quad (11i)$$

Alternatively, evaluating (10) at $V=V_{oc}$ and $I=0$ yields an expression equivalent to (11):

$$r_{oc}(V_{oc}, I_{ph}) = -R_s - \frac{n v_{th}}{G_p(n v_{th} - V_{oc}) + I_{ph} + I_0}. \quad (12)$$

For the case of an ideal cell, evaluation of (10i) at $I=0$, or substitution of $R_s=0$ and $G_p=0$ into (12) yields an expression equivalent to (11i):

$$r_{ioc}(I_{ph}) = -\frac{n v_{th}}{I_{ph} + I_0}. \quad (12i)$$

Table II presents three values of $|r_{ioc}|$ and $|r_{oc}|$ as calculated with (12i) and (12), respectively, for three levels of incident light intensity, represented by the given three values of I_{ph} .

The variation of the dynamic resistance's magnitude, r_{oc} , calculated with (11) or (12) at the open-circuit point ($V=V_{oc}$, $I=0$), is presented in Fig. 4 as a function of photo-generated current intensity, I_{ph} , (which is a direct consequence of the incident light intensity), for $R_p=100\Omega$ and three values of series resistance: $R_s=1, 4$, and 10Ω .

Fig. 4 indicates that as I_{ph} becomes high (e.g. $I_{ph} > 0.04A$ in this example), the curves start to flatten, tending the magnitude of r_{oc} to values near those of R_s ($\approx 1, 4$, and 10Ω in this example). This observation is consistent with Fig. 3, which shows the case for $R_s=10\Omega$, where we see that as $V \rightarrow V_{oc} \approx 0.5V$ (open-circuit point) all three curves of r (calculated at three values of $I_{ph}=0.02, 0.03$, and $0.04A$) approach the value of $R_s=10\Omega$.

Fig. 4 also reveals that the magnitudes of r_{oc} increase as I_{ph} decreases until they saturate to constant values around $R_p + R_s$ ($\approx 101, 104$, and 110Ω in the present example), as I_{ph} reaches very low values (about an order of magnitude lower in the present example). Notice that the saturation condition ($r_{oc} \approx R_p + R_s \approx 110\Omega$ in this example) cannot be observed at the open-circuit point of curves shown in Fig. 3, since none of those three r_{oc} curves correspond to a low enough value of I_{ph} . The lowest value of I_{ph} in Fig. 3 is $0.02A$, and according to the curve for $R_s=10\Omega$ in Fig. 4, the value of r_{oc} at $I_{ph}=0.02A$ is only slightly larger than R_s , still far from being able to reach saturation with $r_{oc} \approx R_p + R_s$.

TABLE II
MAGNITUDE OF THE DYNAMIC RESISTANCE AT OPEN-CIRCUIT CONDITIONS ($V=V_{oc}$, $I=0$) OF THE HYPOTHETICAL IDEAL AND REAL SOLAR CELLS, FOR THREE VALUES OF INCIDENT LIGHT INTENSITY

Incident light intensity, expressed as photo-current I_{ph} (mA)	Ideal $ r_{ioc} $ (Ω) $G_p=0, R_s=0$	Real $ r_{oc} $ (Ω) $R_p=100\Omega, R_s=10\Omega$
20	1.939	12.462
30	1.293	11.517
40	0.970	11.095

Equivalent circuit model diode element's parameters:
 $v_{th}=0.02586V$, $n=1.5$, $I_0=10^{-7}A$.

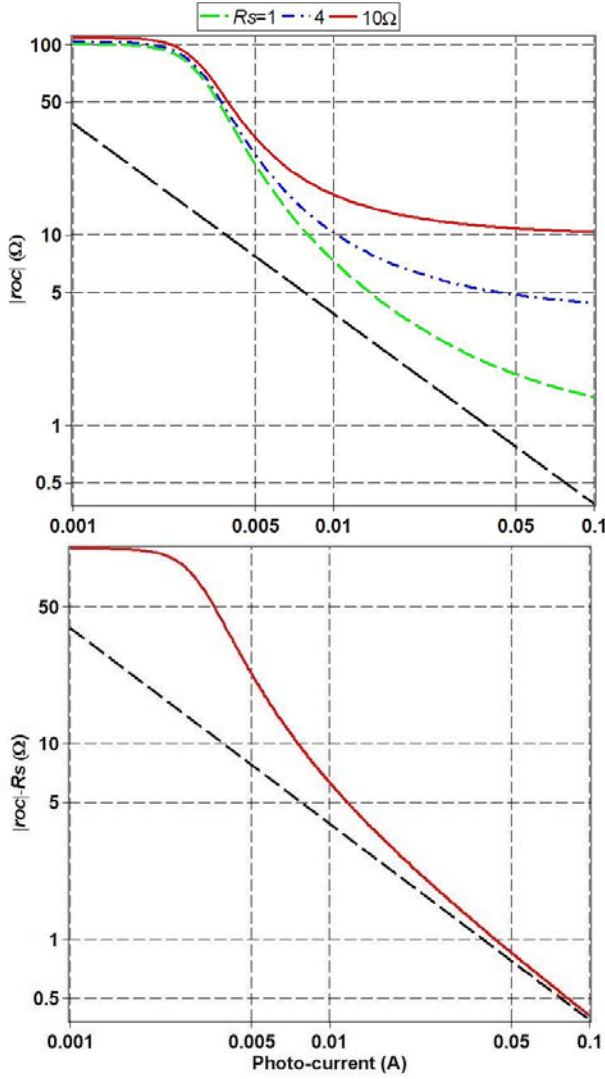


Fig. 4. Log-log plot of the magnitude of the dynamic resistance $|roc|$, calculated with (11) or (12) at the open-circuit point as a function of photo-current I_{ph} (upper plot), and the value of $|roc| - R_s$ (lower plot), for three values of R_s and $R_p = 100 \Omega$. Also shown for comparison as a black dash straight line is the magnitude of the dynamic resistance $|rioc|$ for the corresponding ideal cell ($G_p = 0$ and $R_s = 0$) as calculated with (12i).

The three curves of $|roc|$ shown in the upper pane of Fig. 4 for different values of R_s , may be made to collapse into a single curve by subtracting from $|roc|$ the corresponding value of R_s , as (11) and (12) suggest, and the lower pane of Fig. 4 illustrates.

We do not present plots of $|roc|$ vs I_{sc} because the difference with the plots of $|roc|$ vs I_{ph} shown in Fig. 4 would be almost imperceptible, since only the horizontal axis would need to be shifted by the small difference between I_{sc} and I_{ph} given by (4). In the case of the ideal cell that difference does not exist since (4i) compels $I_{isc} = I_{ph}$. Instead, we present in Fig. 5 the magnitude of the dynamic resistance $|roc|$ at the open-circuit point, as a function of the reciprocal short-circuit current $1/I_{sc}$, calculated with (11) or (12) for the same three values of R_s and R_p . The value of I_{sc} is calculated using (4). Also shown in Fig. 5 for comparison is the $|rioc|$ vs $1/I_{isc}$ of the ideal cell ($G_p = 0$ and $R_s = 0$), calculated with (11i) or (12i), with the value of $I_{isc} = I_{ph}$ according to (4i).

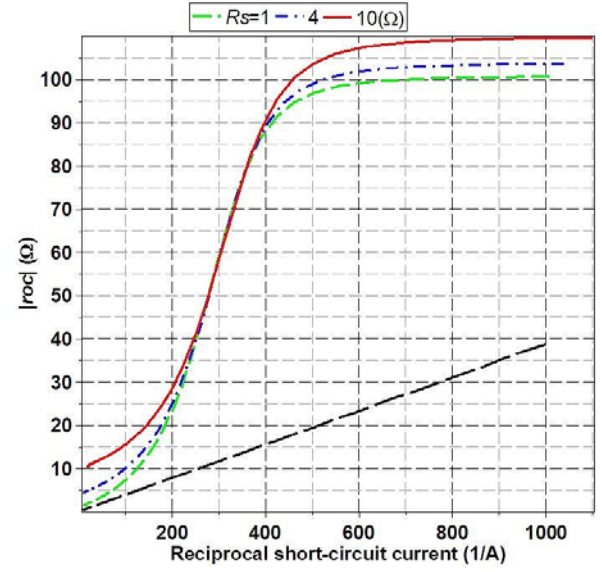


Fig. 5. Linear plots of the magnitude of the dynamic resistance $|roc|$ as a function of reciprocal short-circuit current $1/I_{sc}$, calculated at the open-circuit point with (11) or (12) for three values of R_s and $R_p = 100 \Omega$. The value of $1/I_{sc}$ is calculated from I_{ph} using (4). Also shown as a black dash straight line, is the $|rioc|$ of the ideal cell ($G_p = 0$ and $R_s = 0$) vs $1/I_{isc}$, calculated with (11i) or (12i), using $I_{isc} = I_{ph}$ according to (4i).

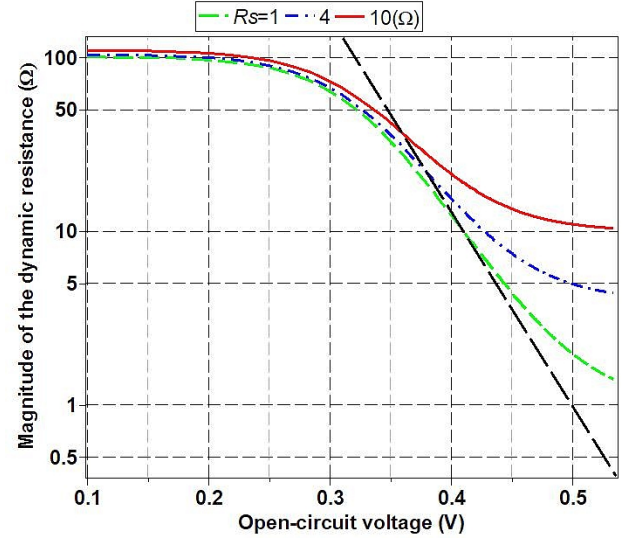


Fig. 6. Semi-logarithmic plot of the magnitude of the dynamic resistance roc at the open-circuit point, as a function of open-circuit voltage, V_{oc} , corresponding to increasing values of I_{ph} , calculated using (11) or (12) for three values of R_s and $R_p = 100 \Omega$. The value of V_{oc} is calculated using (5). Also shown for comparison as a black dash straight line is the dynamic resistance $|rioc|$ of the ideal cell ($G_p = 0$ and $R_s = 0$), calculated with (11i).

The variation of the dynamic resistance's magnitude, $|roc|$, at the open-circuit point is shown in Fig. 6 also as a function of open-circuit voltage, V_{oc} , whose value according to (5) corresponds to increasing levels of incident light intensity. It is calculated using (11) with $R_p = 100 \Omega$ at the three values of $R_s = 1, 4$, and 10Ω . All three curves shown in Fig. 6 are seen to saturate to a value of $\approx R_p + R_s$ at low V_{oc} (very low levels of incident light intensity), and to a value of $\approx R_s$ at high V_{oc} (very high levels of incident light intensity). Also shown in Fig. 6 for comparison is the magnitude of the dynamic resistance $|rioc|$ of the corresponding ideal cell ($G_p = 0$ and $R_s = 0$), calculated with

(11i). It is easy to visualize in Fig. 6 how all three curves of $|roc|$ with $Gp \neq 0$ and $Rs \neq 0$ would transform into the single straight line representing $|rioc|$ if their resistive losses $Gp \rightarrow 0$ and $Rs \rightarrow 0$.

The semi-logarithmic nature of Fig. 6 indicates that since the plot of the magnitude of the ideal cell's $|rioc|$ as a function of V_{ioc} shows up as a straight line with negative slope, the value of $|rioc|$ must vary (decrease) as an exponential function of negative open-circuit voltage, V_{ioc} , which in fact does according to (11i).

On the other hand, Fig. 6 also indicates that, in the case of a real cell ($Gp \neq 0$ and $Rs \neq 0$), the dependence of $|roc|$ on V_{oc} happens to be in general considerably different from such purely exponential behavior. How much it differs depends on how significant the magnitudes of Rs and Gp are, as clearly implied by (11), and illustrated in Fig. 6 by way of the observed saturations to $|roc| \rightarrow R_p + R_s$ at low V_{oc} and to $|roc| \rightarrow R_s$ at high V_{oc} . Obviously, the smaller the values of Rs and Gp are, the nearer (11) becomes to (11i), and thus, the closer the behavior of $|roc|$ would be to that of $|rioc|$, which behaves as a decreasing function of the open-circuit voltage.

D. Dynamic resistance at the sort-circuit point

Let us now look at the other end of the power-generating quadrant, i.e., the short-circuit point, defined by the coordinates ($V=0$, $I=I_{sc}$). The magnitude of the dynamic resistance $|r_{sc}|$ at the sort-circuit point is also a useful parameter to look at. To calculate it we need to evaluate (9) at $V=0$ and $I=I_{sc}$:

$$r_{sc}(I_{sc}) = -R_s - \frac{n v_{th}}{G_p n v_{th} + I_0 \exp\left(\frac{I_{sc} R_s}{n v_{th}}\right)} . \quad (13)$$

In the case of an ideal cell, substitution of $Rs=0$ and $Gp=0$ into (13) yields a constant value of $|r_{isc}|$:

$$r_{isc} = -\frac{n v_{th}}{I_0} , \quad (13i)$$

Alternatively, evaluation of (10) at $V=0$ and $I=I_{sc}$ yields an expression equivalent to (13):

$$r_{sc} = r(I_{ph}, I_{sc}) = -R_s - \frac{n v_{th}}{G_p n v_{th} - I_{sc}(G_p R_s + 1) + I_{ph} + I_0} , \quad (14)$$

For the case of an ideal cell, substitution of $Rs=0$ and $Gp=0$ into (14) yields the same constant as in (13i):

$$r_{isc} = -\frac{n v_{th}}{I_0} . \quad (14i)$$

The light intensity I_{ph} dependence of the magnitude of dynamic resistance $|r_{sc}|$ calculated at the sort-circuit point is presented in Fig. 7, for the same values of $R_p=100\Omega$ and $Rs=1, 4$, and 10Ω . A quick look reveals that $|r_{sc}|$ is clearly a function of light intensity I_{ph} , in a way that in general qualitatively resembles that of $|roc|$ shown in Fig.4. But this similarity refers only the fact that when I_{ph} reaches very large values, $|r_{sc}|$ exhibits saturated values near those of Rs ($\approx 1, 4$, and 10Ω in

this example); and that as the value of I_{ph} starts to decrease, $|r_{sc}|$ starts to increase until at low values of I_{ph} , $|r_{sc}|$ saturates to constant values around those of $R_p + R_s$ ($\approx 101, 104$, and 110Ω in the present example).

Table III presents values of $|r_{sc}|$ and $|r_{isc}|$ for three values of I_{ph} . Notice that in the case of the ideal cell, the value of $|r_{isc}|$ is very large and constant at any incident light intensity (represented by I_{ph}).

Fig. 7 is perfectly compatible with the behavior observed in Fig. 3. Notice, e.g., that when $I_{ph}=0.02A$, the r_{sc} curve corresponding to $Rs=10\Omega$ (solid red line in Fig. 7) reaches a constant value of $\approx R_p + R_s$ ($\approx 107\Omega$ in this example). This is the same value that the $|r|$ curve that corresponds to $I_{ph}=0.02A$ (dashed green line in Fig.3) reaches when $V \rightarrow 0$ (short-circuit point) in Fig. 3.

IV. DISCUSION

We have just established that the magnitudes of the dynamic resistance at the short-circuit and the open-circuit points, $|r_{sc}|$ and $|r_{sc}|$, both experience a transition when I_{ph} changes as a consequence of a change of the incident light intensity. They both decrease from $\sim R_p + R_s$ at very low incident light intensities (very low I_{ph}) to $\sim R_s$ at very high incident light

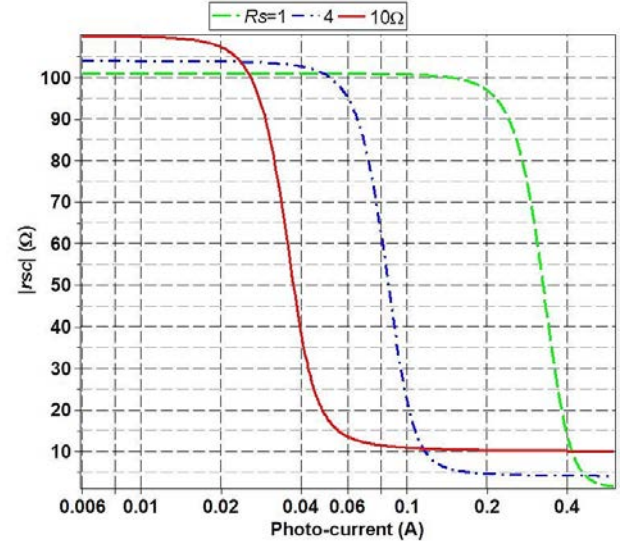


Fig. 7. Semi-logarithmic plot of the magnitude of the dynamic resistance r_{sc} calculated at the short-circuit point ($V=0$, $I=I_{sc}$) on the I - V characteristics of the illuminated hypothetical cell, as a function of photo-current I_{ph} , for $R_p=100\Omega$ and three values of R_s . The plot of the dynamic resistance $|r_{isc}|$ of the ideal cell ($Gp=0$ and $Rs=0$), calculated with (13i), may not be included here for comparison, since it is a very large constant (see Table III) that falls outside the bounds of this figure.

TABLE III
MAGNITUDE OF THE DYNAMIC RESISTANCE AT SHORT-CIRCUIT CONDITIONS ($V=0$, $I=I_{sc}$) OF THE HYPOTHETICAL IDEAL AND REAL SOLAR CELL, FOR THREE VALUES OF INCIDENT LIGHT INTENSITY

Incident light intensity, expressed as photo-current I_{ph} (mA)	Ideal $ r_{isc} $ (k Ω) $Gp=0$, $Rs=0$	Real $ r_{sc} $ (Ω) $Rp=100\Omega$, $Rs=10\Omega$
20	387.9	107.28
30	387.9	87.87
40	387.9	39.11

Equivalent circuit model diode element's parameters:
 $v_{th}=0.02586V$, $n=1.5$, $I_0=10^{-7}A$.

intensities (very high I_{ph}). Although the observed behavior of these two magnitudes appear to follow similar sigmoid-function type of behavior, a more careful comparison of Figs. 4 and 7 clearly reveals that their transitions from high to low values as the light intensity increases differ from each other. Several differences can be visualized in the present example. The first observation is that the transition of $|r_{oc}|$ (Fig. 4) occurs at lower values of I_{ph} than the transition of $|r_{sc}|$ (Fig. 7), and this is so for all values of R_s .

The second observation is that a similar change of the value of R_s causes a much larger shift on the I_{ph} axis (wider spread) of the transition of $|r_{sc}|$ (Fig. 7) than that of $|r_{oc}|$ (Fig. 4). The shift produced by increasing the value of R_s by the same amount is smaller in the case of $|r_{oc}|$ (Fig. 4) than in the case of $|r_{sc}|$ (Fig. 7).

The third and most obvious observation refers to how the magnitudes of $|r_{oc}|$ and $|r_{sc}|$ vary in different ways. In the case of $|r_{sc}|$ that difference is a consequence of the crossovers of the curves, which can mean a total reversal of the dependence direction, as can be seen in the central region of Fig. 7. For example, assuming a value of I_{ph} of 0.08A, the curves presented in Fig. 7 for $|r_{sc}|$ indicate that if the value R_s increases from 1, to 4, and then to 10 Ω , the corresponding magnitude of $|r_{sc}|$ experiences a direct reduction from ~101, to ~62, and then to ~12 Ω . This is the opposite dependence that occurs at very high and low values of I_{ph} , shown at the right and left sides, respectively, of Fig. 7. Even stranger behavior of $|r_{sc}|$ can be expected when R_s changes at certain values of I_{ph} , e.g. at $I_{ph}=0.2A$ in Fig. 7. This type of effect does not exist in the case of $|r_{oc}|$, whose value always increases as R_s increases at any value of I_{ph} , as shown Fig. 4.

A common mistake is falling to the temptation of *a priori* assuming that $I_{ph} \approx I_{sc}$ as a good approximation regardless of any considerations about the presence of significant parasitic resistive losses; an assumption that is in general disavowed by (4). Another approximation mistake that unfortunately is often made about the dynamic resistance of illuminated solar cells is to naively assume that is always approximately equal to R_s when measured at the open-circuit point, or on the end of the power-producing quadrant, assuming that the dynamic resistance measured at the short-circuit point is always approximately equal to R_p [6]. While these two approximations could be perfectly assumable in many instances, the hazard is, as we have shown here, to assume that these approximations are always justified, regardless of the magnitude of parasitic resistive losses and the intensity of the incident light, as Figs. 4 and 7 clearly indicate. The results of this analysis, therefore, call attention to the need to handle these assumptions about the dynamic resistance with care, always checking their validity before proceeding to draw conclusions that depend on them.

V. CONCLUSIONS

The following conclusions may be drawn about the expected theoretical behavior of the dynamic resistance of illuminated solar cells:

1) In general, the magnitude of the dynamic resistance of an

illuminated photovoltaic cell measured (or calculated) at any point of its static I - V characteristic, depends on the illumination intensity incident upon the cell's surface. This includes, of course, the dynamic resistance measured at the points of the I - V curve corresponding to short circuit, open circuit, and MPP conditions. The incident light dependence of the dynamic resistance, therefore, can affect the ability to effectively track the MPP of a solar cell or panel as the intensity of the solar illumination changes hourly, or because of cloud shadowing, during the course of the day.

- 2) If the intensity of incoming illumination is allowed to increase, e.g. through light concentration, so that the photocurrent can reach high enough levels, the value of the dynamic resistance can eventually become approximately equal to the value of the lumped series resistive loss (R_s) at any operating point along the I - V characteristic, including the short-circuit, the open-circuit, and the MPP points.
- 3) Likewise, if the level of illumination decreases drastically, e.g. at night or under cloud cover in the case of terrestrial solar cells, so that the intensity of the photocurrent becomes low enough, the value of the dynamic resistance will approach and eventually will saturate to a value approximately equal to the sum of lumped series and parallel resistive losses ($R_s + R_p$) at any operating point along the I - V characteristic, including the short-circuit, the open-circuit, and the MPP points.
- 4) Although a trivial fact, it is important to keep in mind that changing the illumination level might significantly change the values of both I_{sc} and V_{oc} , which behave as linear and logarithmic functions of the photo-generated current, respectively.

ACKNOWLEDGMENT

Helpful discussions with countless unnamed students and colleagues have contributed over time to better understand the complex implications of this issue.

REFERENCES

- [1] A. Ortiz-Conde, F. J. García-Sánchez, J. Muci, A. Sucre-González, "A review of diode and solar cell equivalent circuit model lumped parameter extraction procedures," *Facta Universitatis, Series: Electronics and Energetics*, vol. 27, no. 1, pp. 57-102, March 2014. <http://dx.doi.org/10.2298/FUEE1401057O>
- [2] J. Thongpran, K. Kirtikara, C. Jivacate, "A method for the determination of dynamic resistance of photovoltaic modules under illumination," *Solar Energy Materials and Solar Cells*, vol. 90, no. 18-19, pp. 3078-3084, Nov. 2006. <https://doi.org/10.1016/j.solmat.2006.06.029>
- [3] W. Shockley, "The theory of p-n junction in semiconductors and p-n junction transistors," *Bell Syst Tech J.*, vol. 28, pp. 435-489, 1949. <https://doi.org/10.1002/j.1538-7305.1949.tb03645.x>
- [4] R. M. Corless, G. H. Gonnet, D. E. G. Hare, D. J. Jeffrey, D. E. Knuth, "On the Lambert W Function," *Adv Comput Math*, vol. 5, no. 1, pp. 329 - 359, Dec. 1996. <https://doi.org/10.1007/BF02124750>
- [5] A. Jain, A. Kapoor, "Exact analytical solutions of the parameters of real solar cells using Lambert W-function," *Solar Energy Materials and Solar Cells*, vol. 81, no. 2, pp. 269-277, 6 Feb 2004. <https://doi.org/10.1016/j.solmat.2003.11.018>
- [6] K. Tada, "Calculation of error in series/shunt resistance estimated from current-voltage slope using exact analytical expressions with Roberts g-function," *IEEJ Trans. Electrical & Electronic Engineering*, vol. 14, no. 2, pp. 333-334, Feb. 2019. <https://doi.org/10.1002/tee.22813>