ADS_TWR_rev2

Ioan Domuta [1]

[1]Technical University of Cluj Napoca

October 30, 2023

Abstract

The asymmetrical double-sided two-way ranging

means uneven session duration, as a result clocks errors

propagate differently in the two TWR measurement sessions. The article proposes a method of the errors compensation by weighted mean of two TWR measurements and minimization of error expectation. By this method, the offset error is cancelled, the full-scale error is not diminished, the cumulated noise variance is attenuated, and the correlated noises variance is strongly diminished. The error analysis shows that the minimum

lower band is reached by SDS-TWR.

Probabilistic Approach of Asymmetrical Double-Sided Two-Way Ranging

Ioan Domuta and Tudor Petru Palade

Abstract— The asymmetrical double-sided two-way ranging means uneven session duration, as a result clocks errors propagate differently in the two TWR measurement sessions. The article proposes a method of the errors compensation by weighted mean of two TWR measurements and minimization of error expectation. By this method, the offset error is cancelled, the full-scale error is not diminished, the cumulated noise variance is attenuated, and the correlated noises variance is strongly diminished. The error analysis shows that the minimum lower band is reached by SDS-TWR.

Index Terms— two-way ranging (TWR), ultra-wideband (UWB), probabilistic

I. Introduction

Nowadays need of high accuracy and high time resolution for the targets geolocation (e.g. autonomous navigation) faces the wireless networks limited capabilities and the rules imposed for the networks coexistence. The Asymmetrical Double-Sided Two-Way Ranging (ADS-TWR) is a trade-off of error compensation provided by the Symmetrical Double-Sided Two-Way Ranging (SDS-TWR), and the need of reducing transmission time in order to increase battery life and keep compliance with the rules imposed by the standards related to networks coexistence [1].

The research in the field is oriented both to accuracy improving and emission power reduction. In order to reduce the number of exchanged packets, [2] proposes multiple acknowledgments of the first ranging message and averaging the estimated time. This estimator is biased, and the error variance is attenuated by the number of acknowledgments. The other biased estimator [3] exploits the inequality of delay times of ADS-TWR, assuming that the ranging initiator responds instantly to ranging partner reply. The preamble and start frame delimiter (SFD) contained in the low-rate UWB frame limits the reply time, so the method is impracticable in such networks.

An alternative to SDS-TWR is the Double Two-Way Ranging algorithm [4]. This algorithm gives an unbiased estimation and has the advantage that both ranging sessions are executed from initiator side, but, like SDS_TWR, it is inefficient for ranging to several anchors.

A strong engineering approach of error propagation is found in [5]. The paper proves the error reduction in Cooperative Position Protocol [6] by applying the Alternative Double-Sided Two-Way Ranging method [7] for range estimation.

The current letter is focused on variances propagation in the ADS-TWR estimation method and proposes the error compensation by minimizing error expectation, without accomplishing a deep analysis of channel and receiver errors. The channel errors are unpredictable, but by the fusion of the information stipulated by the 802.15.4 standard [8], MCPS-DATA.confirm and MCPS-DATA.indication ranging parameters, the errors can be drastically reduced. The radio transfer of all ranging parameters conflicts with the need for transmission time reduction. By the proposed method, the propagation time is computed at the ranging initiator side, the error correction can be performed locally and only the useful information is to be submitted to the location processor.

II. Weighted Double-Sided Two-Way Ranging Method

The method proposes the estimation of propagation time *Te* by the weighted mean of the two TWR session measured time Tm_0, Tm_1 (1),

$$Te = w_0\, Tm_0 + w_1\, Tm_1; \quad w_0 + w_1 = 1\,; \tag{1}$$

where w_0, w_1 are the weighting coefficients.

This paper analyzes the error sources and the error propagation, and finally the weighting coefficients are computed in order to cancel the error expectation *E(err) = 0* (2),

$$w_0 = \frac{Td_0}{Td_0 + Td_1}\,; \quad w_1 = \frac{Td_1}{Td_0 + Td_1}\,; \tag{2}$$

where Td_0, Td_1 are the delay time.

The method is inferred in a low-rate ultra-wideband (UWB) network with Time Domain Multiple Access (TDMA)

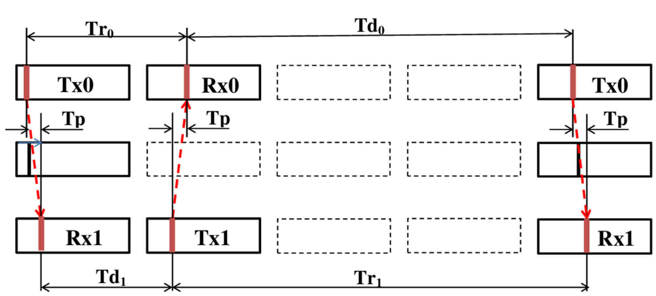

Fig. 1. Asymmetrical Double Sided TWR

TxX, RxX – Communication Slots

Tp - propagation time, Td_X – delay time, Tr_X – round time

Submitted on dd/mm/yyyy

Ioan Domuta is with Communication Department, Technical University of Cluj-Napoca (e-mail ioan.domuta@2dd.ro)

Tudor Petru Palade is with Communication Department, Technical University of Cluj-Napoca (e-mail: palade@com.utcluj.ro)

scenario, as depicted in Fig. 1, where the Tx0, Tx1 constitute the transmission slots and Rx0, Rx1 are the reception slots. One session of Two-Way Ranging (TWR) consists of two message exchanges, one from the ranging initiator to the ranging partner, Tx0 to Rx1, and one delayed reply message from the ranging partner to the initiator, Tx1 to Rx0. The SDS-TWR entails two sequences of TWR. In the second sequence the roles are interchanged, the responder becomes the initiator, and the same value of reply delay is maintained. In the ADS-TWR the reply message from the ranging partner, Tx1 to Rx0, constitutes the originating message of the second TWR session, and the reply delay of the second session, Td_0, differs from the first one, Td_1.

The basic equation for one TWR session is given by (3).

$$Tp = \frac{Tr - Td}{2} \tag{3}$$

The round time *Tr* is measured at session initiator side, the delay time *Td* is measured at ranging partner side and *Tp* is the useful information. Because $Td >> Tp$, the measuring method is highly sensitive to clock offset and clock noise.

A. Noise propagation

Consider the clock frequency *f*, having nominal frequency f_0, frequency offset off_{ck} and frequency noise n_{ck} (4):

$$f = f_0\,(1 + off_{ck} + n_{ck}) \tag{4}$$

Using the Taylor expansion, the clock period $T = 1/f$ is (5):

$$T = T_0\,(1 - off_{ck} - n_{ck} + 0(off_{ck}, n_{ck})) \tag{5}$$

The clock noise n_{ck} integrates the temperature drift, crystal noise, oscillator jitter, PLL noise etc. Because the temperature drift is much slower than the DS-TWS session, it is rejected as noise and has to be incorporated into the total offset.

The time *Tec*, estimated by the counter, is the *N* sum of the clock period T_i (6):

$$Tec = \sum_{i=0}^{N-1} T_i = N\,T_0 - N\,off_{ck} - \sum_{i=0}^{N-1} T_0\,n_{ck_i} \tag{6}$$

where *N* is the counter value.

The noise propagated through the counter n_{clk} is modeled considering the clock noise as a string of independent random variables, therefore the resulting noise variance $\sigma_{ckn}{}^2$ is the *N* sum of clock variances $\sigma^2_{ck_i}$ (7):

$$\sigma^2_{ckn} = \sum_{i=0}^{N-1} T_0^2\,\sigma^2_{ck_i} = N\,T_0^2\,\sigma^2_{ck} = Tc\,T_0\,\sigma^2_{ck} \tag{7}$$

where $Tc = NT_0$ is the true counted time and $\sigma_{ck_i} = \sigma_{ck}$ is the clock standard deviation expressed as relative to unit.

Beside the clock noise, which acts as a multiplicative one, the measurements are corrupted by additive receiver noise n_r and channel noise n_{ch}. There are many published data, not referenced in the letter, regarding the improvement of UWB reception quality, and the studies show that for the coherent receiver the bit error rate is strongly dependent on the clocks alignment and on the length of synchronization frame.

The delay time is counted by the receiving station starting from ranging marker reception, and therefore the delay time is corrupted by receiver and channel noises too.

Considering that the return propagation path is the same as the direct path one can conclude that the noises are correlated, to a certain extent.

B. Ranging algorithm

The estimation of propagation time *Te*, as shown in (1), is the weighted mean of the two TWR measurements Tm_0, and Tm_1.

The measurements, corrupted by noise, of the first TWR session are (8)

$$Tr^m{}_0 = 2\,Tp\,\left(1 - off_{ck_0} - n_{ck_0}\right) + Td_1\,\left(1 - off_{ck_0} - n_{ck_0}\right) + n_{r_0} + n_{ch_0}$$

$$Td^m{}_1 = Td_1\,\left(1 - off_{ck_1} - n_{ck_1}\right) + n_{r_1} + n_{ch_1} \tag{8}$$

where Tr^m is the measured round time, Td^m is the measured delay time, *Td* is the imposed delay time, and the subscript number represents the station which performs the measurement.

The second TWR session is similar to the first one, but it has the inverted station number in its formulas.

The error terms $off_{ck}+n_{ck}$ induce a full-scale error and being much smaller than one it is neglected. Combining (3) and (8), the noisy measured time (9) is inferred:

$$Tm_0 = Tp + Td_1\,\delta o + n_{00} - n_{01}$$

$$\delta o = \frac{1}{2}\,(off_{ck_1} - off_{ck_0})\,;\; n_0 = n_{00} - n_{01}\,;$$

$$n_{00} = \frac{-Td_1\,n_{ck_0} + n_{r_0} + n_{ch_0}}{2}\;;\quad n_{01} = \frac{-Td_1\,n_{ck_1} + n_{r_1} + n_{ch_1}}{2}\;; \tag{9}$$

where δo is the clock offset difference between the two ranging stations, n_0 is the the total noise of the first TWR session, and n_{ij} is the noise of station *j* at TWR session *i*.

Assuming the process is wide-sense stationary and that the random variables are uncorrelated, zero-mean, white Gaussian noise, the sum of the noises is also zero-mean, with the variances σ^2_{00}, σ^2_{01} (10),

$$n_{00} \sim N(0, \sigma^2_{00})\;;\quad \sigma^2_{00} = \frac{Td_1\,T_0\,\sigma^2_{ck_0} + \sigma^2_{r_0} + \sigma^2_{ch_0}}{4}\;;$$

$$n_{01} \sim N(0, \sigma^2_{01})\;;\quad \sigma^2_{01} = \frac{Td_1\,T_0\,\sigma^2_{ck_1} + \sigma^2_{r_1} + \sigma^2_{ch_1}}{4}\;; \tag{10}$$

where the clock variance is given by (7).

The total noise n_0 of the first TWR session (11) is

$$n_0 \sim \mathrm{N}(0, \sigma^2_0)\;;\quad \sigma^2_0 = \sigma^2_{00} + \sigma^2_{01} - 2\,r_0\,\sigma_{00}\,\sigma_{01} \tag{11}$$

where r_0 is the correlation coefficient between direct and replied exchanges.

Using (8) to (11) the final error $err = Te\text{-}Tp$ (12),

$$err = w_0\, Td_1\, \delta o - w_1\, Td_0\, \delta o + w_0 n_0 + w_1 n_1 \tag{12}$$

the expectation value (13),

$$E[err] = w_0\, Td_1\, \delta o - w_0\, Td_0\, \delta o \tag{13}$$

and the weighting coefficients (2), are inferred.

The final measurement noise nf (14) is

$$nf \sim N(0,\ {w_0}^2\, \sigma_0^2 + {w_1}^2\, \sigma_1^2) \tag{14}$$

The minimum variance σ^2_{min} (15) is

$$\sigma_{min}^2 = \frac{\sigma_0^2\, \sigma_1^2}{\sigma_0^2 + \sigma_1^2} \tag{15}$$

and it is reached if (16)

$$w_0 = \frac{\sigma_1^2}{\sigma_0^2 + \sigma_1^2} \tag{16}$$

From (15) and (16) we conclude that the error lower band is reached if $\sigma^2_0 = \sigma^2_1 = \sigma^2$, $w_0 = w_1 = 1/2$, so by SDS-TWR method, and the low limit is (17):

$$\sigma_{lb}^2 = \sigma^2/2 \tag{17}$$

Fig. 2 shows the probability density functions (pdfs) of the errors occurring during estimation. The pdfs of the first TWR session (TWR-0) and second session (TWR-1) have a large variance and are biased. By using the proposed method, the error expectation is cancelled, the uncorrelated noise variance is diminished (ADS-TWR uncorrelated) and the variance of correlated noises (ADS-TWR correlated) is strongly reduced.

III. Comparative Study of the Proposed Method Efficiency

This section analyzes the estimation error of the inferred method depending of the estimated range.

The weighting method performances are compared with the

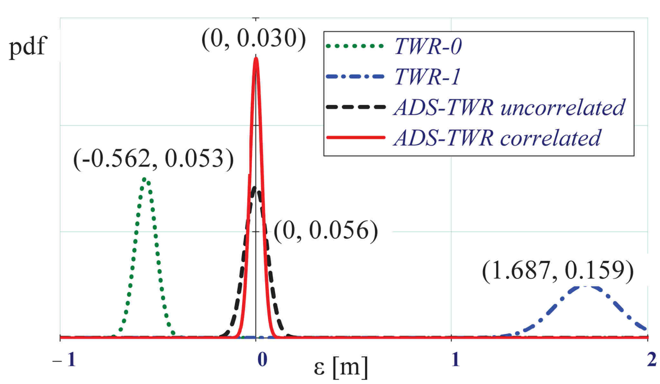

Fig. 2. Errors probability density functions

The brackets (μ, σ) embody the mean μ and standard deviation σ

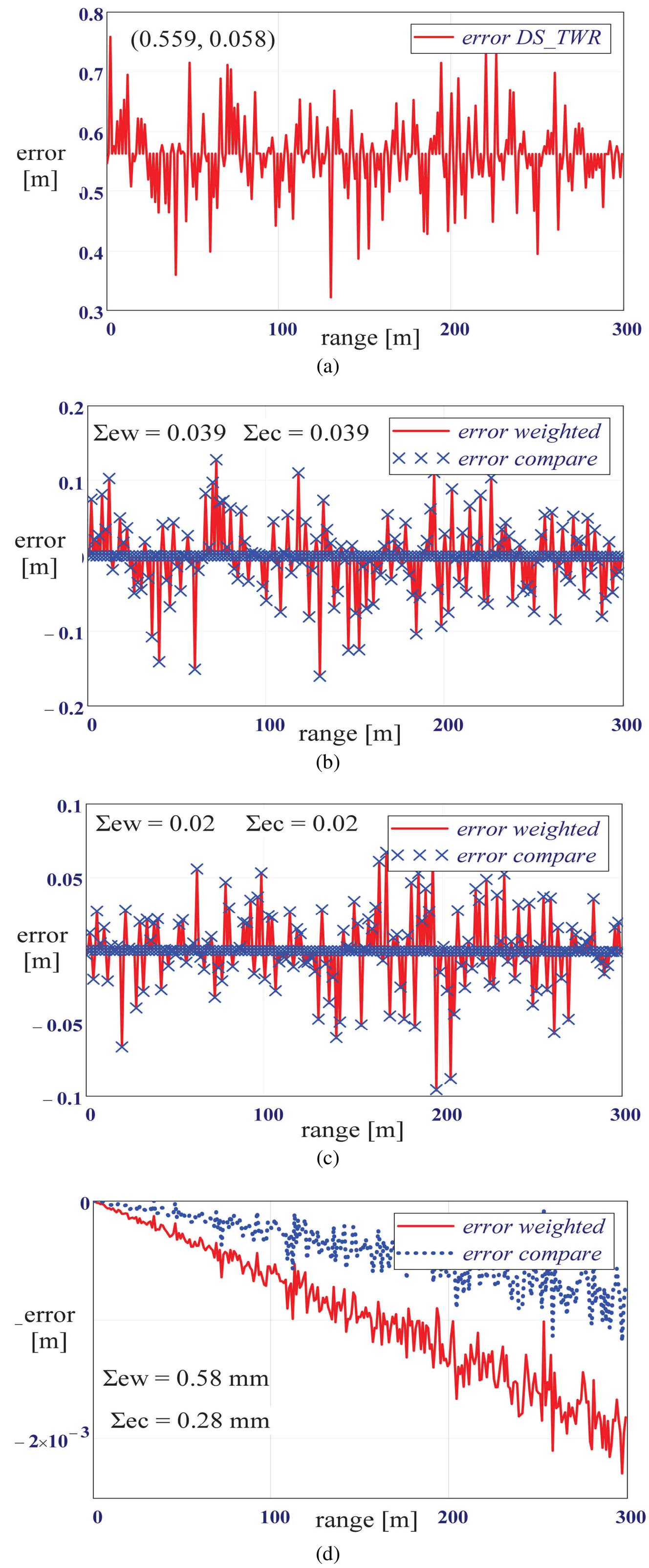

Fig. 3 Estimation error depending of the range

The brackets (μ, σ) embody the mean μ and standard deviation σ

Σew is the error standard deviation in the weighting method

Σec is the error standard deviation in the referenced method

method proposed in [7], as, to the best of our knowledge, this method offers the lowest errors in asymmetrical TWR. Ref. [7] gives the following estimation equation (18):

$$Tec = \frac{Tr_0\, Tr_1 - Td_0\, Td_1}{Tr_0 + Tr_1 + Td_0 + Td_1} \tag{18}$$

The correlated random variables, having the imposed variance σ^2, is generated as the weighted sums of two independent variables ni_0, ni_1 of variance σ^2 (19)

$$n_0 = \omega_0\, ni_0 + \omega_1\, ni_1 \;;\; n_1 = \omega_1\, ni_0 + \omega_0\, ni_1$$

$$\omega_0{}^2 + \omega_1{}^2 = 1 \;;\; r01 = 2\,\omega_0\,\omega_1 \;;\; Var(n) = \sigma^2 \tag{19}$$

where ω_0, ω_1 are the weighting coefficients and *r01* is the correlation coefficient.

It has to be stressed that the product between the counted time *Td* and the noise clock n_{ck} has to be computed as the sum of the independent random variables *ni* (20).

$$Td\, n_{ck} = \sum_0^{Td/T_0} ni_i \tag{20}$$

The average of the two TWR session errors, Fig. 3a, shows the error level without applying the compensation methods. The Fig. 3b and Fig. 3c show that the weighting method (error weighted track), and the reference method (error compare track) give offset zero and the same variance. The variances are attenuated for the uncorrelated noises, Fig. 3b, and are strongly diminished for correlated noises, Fig. 3c.

Fig.3d, achieved by total noise correlation, shows the full-scale error. It should be noted that both methods give low errors, but the reference method behaves better for long distances.

IV. Conclusion

The proposed ranging algorithm leads to offset error cancelation, but it does not reduce the full scale error.

The method joins the information of the two TWR sessions, and by analyzing the sources of error and their propagation it is shown that the estimation variance is attenuated and is strongly diminished for correlated noises.

References

[1] ETSI, *Electromagnetic compatibility and Radio spectrum Matters (ERM); Short Range Devices (SRD) using Ultra Wide Band (UWB); Transmission characteristics; Part 2: UWB mitigation techniques*, 2014

[2] H. Kim, “Double-sided two-way ranging algorithm to reduce ranging time,” *IEEE Commun. Lett.*, vol. 13, no. 7, pp. 486–488, 2009.

[3] Y. Jiang and V. C. M. Leung, “An asymmetric double sided two-way ranging for crystal offset,” in *Conference Proceedings of the International Symposium on Signals, Systems and Electronics*, 2007, pp. 525–528.

[4] M. Kwak and J. Chong, “A new double two-way ranging algorithm for ranging system,” in *Proceedings - 2010 2nd IEEE International Conference on Network Infrastructure and Digital Content, IC-NIDC 2010*, 2010, pp. 470–473.

[5] K. A. Horváth, G. Ill and Á. Milánkovich, “Passive Extended Double-Sided Two-Way Ranging with Alternative Calculation”, in *Proceedings of 2017 IEEE 17th International Conference on Ubiquitous Wireless Broadband (ICUWB)*, 2017

[6] Z. Sahinoglu and S. Gezici, “Enhanced position estimation via node cooperation,” in *IEEE International Conference on Communications*, 2010.

[7] D. Neirynck, E. Luk, and M. McLaughlin, “An alternative double-sided two-way ranging method,” in *Proceedings of the 2016 13th Workshop on Positioning, Navigation and Communication, WPNC 2016*, 2017.

[8] IEEE, *IEEE Standard for Local and metropolitan area networks, Part 15.4: Low-Rate Wireless Personal Area Networks*. 2011.