# Enforcing Local Power Conservation for Metasurface Design Using Electromagnetic Inversion

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# Enforcing Local Power Conservation for Metasurface Design Using Electromagnetic Inversion

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***Abstract*—A method based on electromagnetic inversion is extended to facilitate the design of passive, lossless, and reciprocal metasurfaces. More specifically, the inversion step is modified to ensure that the field transformation satisfies local power conservation, using available knowledge of the incident field. This paper formulates a novel cost functional to apply this additional constraint, and describes the optimization procedure used to find a solution that satisfies both the user-defined field specifications and local power conservation. Lastly, the method is demonstrated with a two-dimensional (2D) example.**

***Index Terms*—Electromagnetic metasurfaces, inverse problems, inverse source problems, optimization, antenna design**

## I. Introduction

Over the past decade, metasurfaces have emerged as useful devices for controlling electromagnetic radiation [1]–[6]. These subwavelength thin metamaterials can support arbitrary field transformations in a systematic fashion by imposing appropriate surface boundary conditions, providing a level of control over some *desired* field produced by a known *incident* field. This fundamental ability has led to a variety of applications, including generalized refraction and reflection [7], polarization manipulation [8], [9], spatial processing [10], and others.

In order to design a metasurface to support a field transformation, the tangential electric and magnetic fields must be known on either side of the boundary imposed by the metasurface. Most existing design procedures are limited to problems in which the output field is known analytically on the output side of the metasurface, which is satisfactory for well-defined problems such as plane wave refraction [11]. We recently developed a design method which facilitates output field specifications in a less restrictive manner [12]. Using this method, the field specifications can be at arbitrary locations external to the metasurface, either with or without phase information. Furthermore, the desired field can also be specified as a set of *performance* criteria, such as main beam direction(s), null location(s), beamwidth, or polarization. While this method allows for more general field specifications, it does not take advantage of prior knowledge of the incident field and consequently requires loss and/or gain to support the resulting field transformation.

In this work, we extend the method presented in [12] to allow for the design of lossless, passive, and reciprocal metasurfaces. This method uses electromagnetic inversion to solve for a set of tangential output (transmitted) fields that produce some user-specified field. This work modifies the inversion process by incorporating an additional step that penalizes solutions that do not satisfy local power conservation (LPC) using the known information about the incident field. Once an appropriate solution is found that satisfies both the field specifications and LPC, surface susceptibilities can be computed to support the transformation.

In this paper, we begin by presenting a brief review of the design procedure without enforcing LPC in Section II. In Section III we discuss and derive the constraint used to enforce LPC, and Section IV describes how the inversion process is modified to account for this new constraint. A preliminary example is presented in Section V, followed by some conclusions and a discussion of possible extensions to this work.

## II. Inverse Source Design Framework

Herein, we present a brief review of the design method presented in [12], in which the main goal is to find tangential fields on the *output* side of the metasurface that satisfy some set of user-defined field specifications $\mathsf{S}$ in some external region of interest (ROI). An overview of the problem is depicted in Figure 1. We denote the input and output surface boundaries of the metasurface as $\Sigma^-$ and $\Sigma^+$, respectively. The tangential fields (denoted as such by the subscript $\mathsf{t}$) that we require to design the metasurface consist of the total fields on $\Sigma^-$, $\overline{\mathsf{E}}_\mathsf{t}^-$ and $\overline{\mathsf{H}}_\mathsf{t}^-$ (consisting of the incident and reflected fields), and the transmitted fields on $\Sigma^+$, $\overline{\mathsf{E}}_\mathsf{t}^+$ and $\overline{\mathsf{H}}_\mathsf{t}^+$. The user-defined specifications $\mathsf{S}$ fall into three general categories, ordered from most to least specific (i.e., most to least information):

1) Complex (amplitude and phase) field distributions (either in the near-field or far-field regions)
2) Phaseless field distributions (i.e., amplitude-only, power pattern)
3) Far-field performance criteria (i.e., main beam directions(s), null locations, beamwidth, etc.)

First, an electromagnetic inverse source problem is solved to find a set of equivalent electric ($\overline{\mathsf{J}}$) and magnetic ($\overline{\mathsf{M}}$) currents that produce the field specifications in the ROI. The domain upon which the equivalent currents are reconstructed,

Fig. 1. Visual overview of the metasurface design problem. The internal and external surface boundaries of the metasurface are denoted by $^{-}$ and $^{+}$, respectively. Some source generates an incident field $\sim^{inc}$ which interacts with the metasurface, producing both a reflected field $\sim^{ref}$ and a transmitted field $\sim^{tr}$. The tangential components of the electric and magnetic fields on $^{-}$ are denoted as $E_t^{-}$ and $H_t^{-}$, while the tangential fields on $^{+}$ are denoted as $E_t^{+}$ and $H_t^{+}$. The user-defined field specifications $S$ are defined on some of interest (ROI) external to the metasurface. Since the metasurface may be of arbitrary shape, we define the local coordinate system $(\hat{u}, \hat{v}, \hat{n})$ on $^{+}$, where $\hat{n}$ is the unit outward normal to $^{+}$. c 2019 IEEE. Reprinted, with permission, from [12] with minor modifications.

commonly referred to in inverse source problems as the 'reconstruction surface', is chosen to coincide with the physical boundary imposed by the metasurface. These currents are found by minimizing a cost functional, which we denote herein as $C_1(J, M)$, using the conjugate gradient method. This functional quantifies the difference between the fields generated by the equivalent currents and the field specifications, although the exact form depends on the category of field specifications listed above (for more details see (12), (13), and (20) in [12]).

If Love's equivalence condition is enforced (i.e., enforcing that the equivalent currents produce null fields on the input side of the metasurface), then the resulting equivalent currents will be related to the desired transmitted fields as

$$H_t^{+} = \hat{n} \; J \quad \text{and} \quad E_t^{+} = \hat{n} \; M; \qquad (1)$$

where is a real-valued scaling parameter. Introducing does not affect the characteristics of the *normalized* radiated field, but allows for some flexibility that will be utilized in Section III.

Once the desired tangential transmitted fields are known, the generalized sheet transition conditions (GSTCs) [13] can be utilized to determine a set of surface susceptibilities to support the discontinuity from the (known) incident field and (desired) reflected field [6]. Assuming a time-dependency of $e^{j!t}$ and free space on either side of the metasurface, the relationship can be written as

$$\begin{pmatrix} H_v \\ H_u \end{pmatrix} = j!\,_0 \begin{pmatrix} {}^{uu}_{ee} & {}^{uv}_{ee} \\ {}^{vu}_{ee} & {}^{vv}_{ee} \end{pmatrix} \begin{pmatrix} E_{u;av} \\ E_{v;av} \end{pmatrix} + j!\,p\overline{\ _0\ _0\ } \begin{pmatrix} {}^{uu}_{em} & {}^{uv}_{em} \\ {}^{vu}_{em} & {}^{vv}_{em} \end{pmatrix} \begin{pmatrix} H_{u;av} \\ H_{v;av} \end{pmatrix} \qquad (2a)$$

$$\begin{pmatrix} E_u \\ E_v \end{pmatrix} = j!\,_0 \begin{pmatrix} {}^{vv}_{mm} & {}^{vu}_{mm} \\ {}^{uv}_{mm} & {}^{uu}_{mm} \end{pmatrix} \begin{pmatrix} H_{v;av} \\ H_{u;av} \end{pmatrix} + j!\,p\overline{\ _0\ _0\ } \begin{pmatrix} {}^{vv}_{me} & {}^{vu}_{me} \\ {}^{uv}_{me} & {}^{uu}_{me} \end{pmatrix} \begin{pmatrix} E_{v;av} \\ E_{u;av} \end{pmatrix}; \qquad (2b)$$

where ! is the angular frequency of the time harmonic fields, and $_0$ and $_0$ are the permittivity and permeability of free space.[1] The subscripts and superscripts $u$ and $v$ denote the tangential components of the local coordinate system of each unit cell defined by $\hat{u} \quad \hat{v} = \hat{n}$ and $\hat{u} ? \hat{v}$. The terms represent the electric/magnetic (first subscript) surface susceptibility components in the presence of an electric/magnetic (second subscript) field excitation [14]. The difference and average fields are defined as

$$\sim \; , \; \sim^{tr} \qquad \sim^{inc} + \sim^{ref} \qquad (3)$$

$$\sim_{av} \; , \; \frac{\sim^{tr}j_{+} + \quad \sim^{inc}j \quad + \sim^{ref}j}{2}: \qquad (4)$$

The final step in the design procedure is solving (2) for the non-zero susceptibility terms (depending on the problem, some terms may be assumed to be zero).

## III. ENFORCING LOCAL POWER CONSERVATION

The main limitation of the procedure presented in Section II and [12] is that the synthesized susceptibilities may require (undesirable) loss and/or gain. To overcome this limitation, we first note that a necessary condition for a passive and lossless metasurface is that the input and output fields must satisfy LPC [15], [16]. That is, the real power incident on each unit cell must be equal to the real power transmitted from each unit cell, as indicated by the following equation that must hold along the metasurface:

$$\frac{1}{2}\mathrm{Re}(E_t^{-} \quad H_t^{-}) = \frac{1}{2}\mathrm{Re}(E_t^{+} \quad H_t^{+}): \qquad (5)$$

From this point onwards, we will assume 2D TE$_z$ polarized fields and a 1D metasurface along the line $x = 0$ (i.e., $\hat{u} = \hat{z}$, $\hat{v} = \hat{y}$, and $\hat{n} = \hat{x}$) for notational simplicity, although the formulation would still hold for arbitrarily-shaped metasurfaces and 3D fields. We denote the left hand side of (5) evaluated at the $i^{th}$ unit cell as

$$p_i = \frac{1}{2}\mathrm{Re}(E_y \quad H_z)\Big|_{\text{unit cell } i}: \qquad (6)$$

[1] The formulation shown here assumes that the normal components of the polarization densities are zero, both for mathematical convenience and since the tangential components are enough to uniquely define the fields.