A photonic microwave and RF integrator based on a transversal filter

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Abstract

We demonstrate a photonic RF integrator based on an integrated soliton crystal micro-comb source. By multicasting and progressively delaying the input RF signal using a transversal structure, the input RF signal is integrated discretely. Up to 81 wavelengths are provided by the microcomb source, which enable a large integration time window of ˜6.8 ns, together with a time resolution as fast as ˜84 ps. We perform signal integration of a diverse range of input RF signals including Gaussian pulses with varying time widths, dual pulses with varying time intervals and a square waveform. The experimental results show good agreement with theory. These results verify our microcomb-based integrator as a competitive approach for RF signal integration with high performance and potentially lower cost and footprint.

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Abstract—We demonstrate a photonic RF integrator based on an integrated soliton crystal micro-comb source. By multicasting and progressively delaying the input RF signal using a transversal structure, the input RF signal is integrated discretely. Up to 81 wavelengths are provided by the microcomb source, which enable a large integration time window of ~6.8 ns, together with a time resolution as fast as ~84 ps. We perform signal integration of a diverse range of input RF signals including Gaussian pulses with varying time widths, dual pulses with varying time intervals and a square waveform. The experimental results show good agreement with theory. These results verify our microcomb-based integrator as a competitive approach for RF signal integration with high performance and potentially lower cost and footprint.

Index Terms—Integrator, Kerr micro-comb, RF signal processing.

I. INTRODUCTION

Temporal integration is a basic function that has a wide range of applications in signal processing systems. In contrast to electrical integrators that are subject to the electronic bandwidth bottleneck, photonic techniques offer distinctive advantages such as the broad bandwidth, strong immunity to electromagnetic interference, and low loss [1-3], thus holding great promise to address the limitations of their electrical counterparts.

Extensive effort has been made to achieve photonic integrators

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(see Table 1 for comparison of existing photonic integrators), such as those based on gratings [4-6], and micro-ring resonators (MRRs) [7-9]. These approaches achieve optical signal integration with a time resolution as fast as 8 ps [7] and a large time-bandwidth product with high-Q resonant structures.

However, these approaches still face limitations. Many are not reconfigurable in terms of the temporal resolution and the length of the integration window, preventing processing of RF signals with different bandwidths and varying integration time windows. In addition, many approaches process optical signals, instead of the RF signals directly, in which case they require electro-optical interfaces that limit their performance.

Approaches to photonic integrators based on transversal structures offer high reconfigurability and accuracy owing to the parallel scheme where each path can be controlled independently [10-12]. By tailoring the progressive delay step, RF signal integration with a reconfigurable operation bandwidth can be achieved [10-12]. Yet these integrators are still limited by the number of channels. To increase the number of wavelength channels, discrete laser arrays or electro-optical comb sources can be employed, although these approaches have a trade-off between the number of wavelengths and system complexity, ultimately leading to a limited number of channels and time-bandwidth product.

Recently, [13-19] a novel multi-wavelength source—integrated microcombs— has attracted great interest in RF photonic systems [20, 21]. Microcombs arise from optical parametric oscillation in ultra-high-Q monolithic MRRs and offer many advantages over traditional multi-wavelength sources, including a much higher number of wavelengths and a greatly reduced footprint and complexity for the system. A wide range of RF applications have been demonstrated based on microcombs, such as RF true time delays [22, 23], transversal signal processors [24-29], frequency conversion [30], phase-encoded signal generators [31], and channelizers [32, 33] and Hilbert transforms [34].

In this paper, we demonstrate a highly reconfigurable photonic RF integrator using an integrated soliton crystal micro-comb source [35, 36] with a low comb spacing of 49GHz. The input RF signal is multicast onto the flattened microcomb lines and progressively delayed via dispersion, and then summed upon detection to achieve temporal integration. The large number of wavelengths - up to 81 - offered by the microcomb enable a large integration time window of ~6.8 ns with a time resolution as fast as ~84 ps. A comb shaping system is developed to compensate for the non-flat spectral output of the soliton crystal microcomb. We successfully test the system on a range of different input signals. The experimental results match well with theory, verifying the performance and feasibility of our approach to achieving photonic RF integration with a large time window and potentially lower cost and footprint.

Table 1. Parameters of existing photonic RF integrators

Method	Reconfigurability	Time window	Time resolution	Bandwidth	Time-bandwidth product
Fiber Bragg grating [4]	×	N.A.	6ps	N.A.	N.A.
Active fiber Bragg grating [5]	×	~5 ns	50 ps	20 GHz	100
apodized uniform period fiber Bragg grating [6]	×	60 ps	2.5 ps	400 GHz	24
Passive high-Q MRR [7]	×	800 ps	8 ps	200 GHz	100
InP-InGaAsP MRR [8]	×	6331 ps	< 54 ps	>15 GHz	>117
Passive MRR [9]	×	12.5 ps	1.9 ps	400 GHz	6.6
Incoherent light source [10]	√	200 ps	7 ps	>18 GHz	28.5
Multi-wavelength fiber laser [11]	√	1.24 ns	~27 ps	36.8 GHz	~45.9
Incoherent light source and fiber Bragg grating [12]	√	7 ns	344 ps	2.9 GHz	20.3
Microcomb (this work)	√	6.8 ns	84 ps	11.9 GHz	81

II. EXPERIMENTAL RESULTS

Figure 1 illustrates the operation principle of photonic RF integrator. The integration process can be achieved via a discrete time-spectrum convolution operation between the RF input signal $f(t)$ and the flattened microcomb. With a delay step of Δt, the operation can be described as:

$$y(t) = \sum_{k=1}^{N} f(t + k \cdot \Delta t)$$

(1)

where N is the total number of wavelength channels. After the replicas of $f(t)$ are delayed progressively and summed together, the integration of $f(t)$ can be achieved, with a time feature [10] of Δt and a total integration time window of $T = N \times \Delta t$.
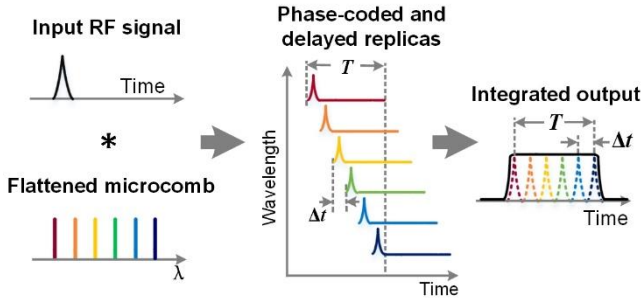

Fig. 1. Schematic diagram of the photonic RF integration.

Figure 2 shows the experimental up of the photonic RF integrator using a microcomb source. The microcomb was generated by pumping a nonlinear high-Q MRR (Q factor > 1.5 million, free spectral range = ~0.4nm or ~48.9 GHz) with a continuous-wave (CW) laser. As the pump power and wavelength detuning were adjusted to provide sufficient parametric gain, soliton crystal microcombs were generated. The soliton crystal microcombs [21] with tightly packaged solitons circulating in the MRR were generated in our experiments due to a mode crossing at ~1552 nm of the MRR. The distinctive palm-like comb spectra (Fig. 3) is a result of the spectral interference between the circulating solitons. We then flattened the microcomb spectral lines in the C band with two stages of WaveShapers (Finisar 4000S). The input RF signal was imprinted onto the comb lines, generating replicas across all wavelength channels. The replicas were progressively delayed by a spool of

standard single-mode fibre (~13km) and summed upon photodetection using a high-speed photodetector (Finisar, 40 GHz bandwidth).
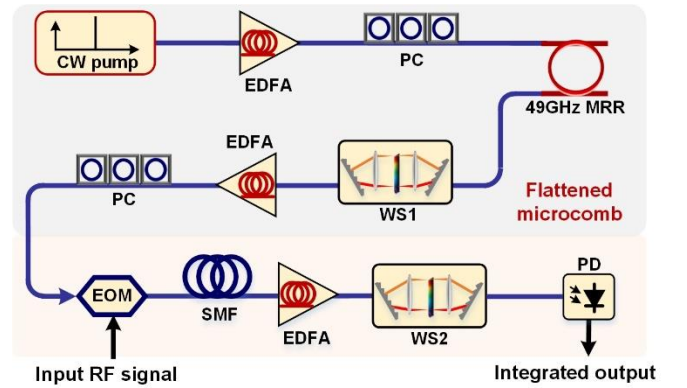

Fig. 2. Experimental setup of the photonic RF integrator based on a microcomb source. EDFA: erbium-doped fibre amplifier. PC: polarization controller. MRR: micro-ring resonator. WS: WaveShaper. EOM: Mach-Zehnder modulator. SMF: single mode fibre. PD: photodetector.
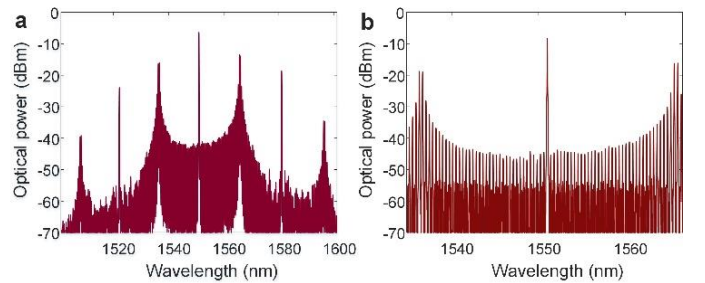

Fig. 3. Optical spectra of the generated soliton crystal microcomb with spans of (a) 100 nm and (b) 30 nm.

The delay step between the adjacent wavelength channels Δt, or referred as the time feature of the integrator [10], was measured to be ~ 84 ps, which was determined by the dispersion and length of the fibre and the spectral spacing between the comb lines. We note that the fastest time feature is determined by the delay step of the wavelength channels, and thus in theory can become arbitrarily small by reducing the amount of dispersion, although with a tradeoff that the integration window also decreases proportionally.

The first WaveShaper (WS1) pre-flattened the optical spectrum of the microcomb to acquire a high link gain and a high signal-to-noise ratio. The second WaveShaper was employed for accurate comb shaping assisted by feedback control. The error signal was generated by reading and comparing the comb lines' power with the desired channel weights, which are all equal for the integrator. The measured optical spectrum after comb shaping is shown in Fig. 4.
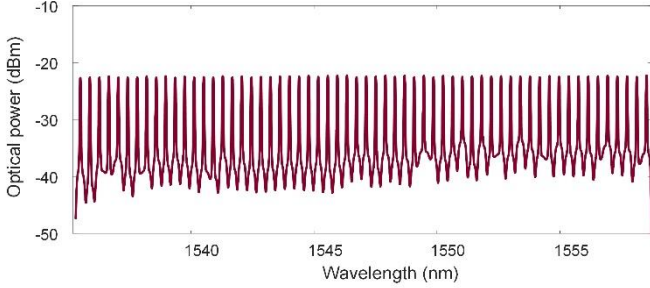

Fig. 4. Optical spectrum of the shaped microcomb.

We selected 60 comb lines in the C-band for the integration, yielding a total integration time window ($T=N \times \Delta t$) reached up to 60 × 84 ps = 5.04 ns, as denoted by the yellow shaded region in Fig. 5. To verify the performance of our approach, we performed signal integration for different RF input signals. The red curves in Fig. 5(a-c) show the integration results of Gaussian pulses (blue curves) with a full width at half maximum (FWHM) varying from 0.20 ns to 0.94 ns, where the demonstrated integration window T (~5 ns) matched well with theoretical calculations (5.04 ns). Fig. 5(d-e) shows the integration results of dual Gaussian pulses with different time intervals of 1.52 ns and 3.06 ns, respectively. The measured results (red curves) clearly illustrate the performance of our integrator by exhibiting three distinct intensity steps in the integration waveforms. The left step corresponds to the integration

of the first pulse while the middle step indicates the integration of both of the two initial pulses, and the right step shows the integration of only the second pulse since it is beyond the integration window of the first pulse. Moreover, the performance of the integrator is further demonstrated by an input signal with a rectangular input waveform with its width equal to the integration window (5 ns). The measured integrated waveform exhibits a triangular shape that matches well with ideal integration results (gray curve).
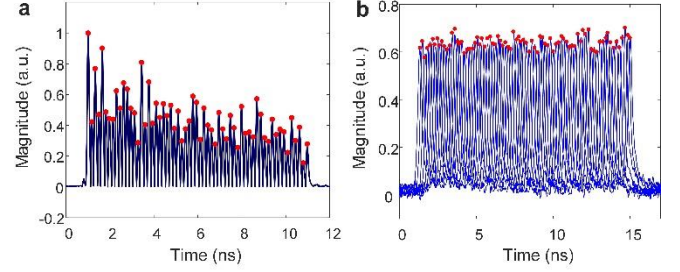

Fig. 6. Measured impulse response of the integrator (a) after comb optical power shaping and (b) after impulse response shaping using a Gaussian RF input pulse.

III. RESULT ANALYSIS AND OPTIMIZATION

As shown in Fig. 5, we note that discrepancies between the measured (red curves) and the ideal (grey curves) results can be observed. Considering the optical power of the comb lines has been well flattened, we deduce that the errors were introduced by the non-ideal impulse response of the system, caused by the non-flat transmission response of the optical amplifier, the modulation and the photodetector across the wavelength channels. To verify our deduction, we measured the impulse response of the system with a Gaussian pulse input. Considering that the time resolution of the
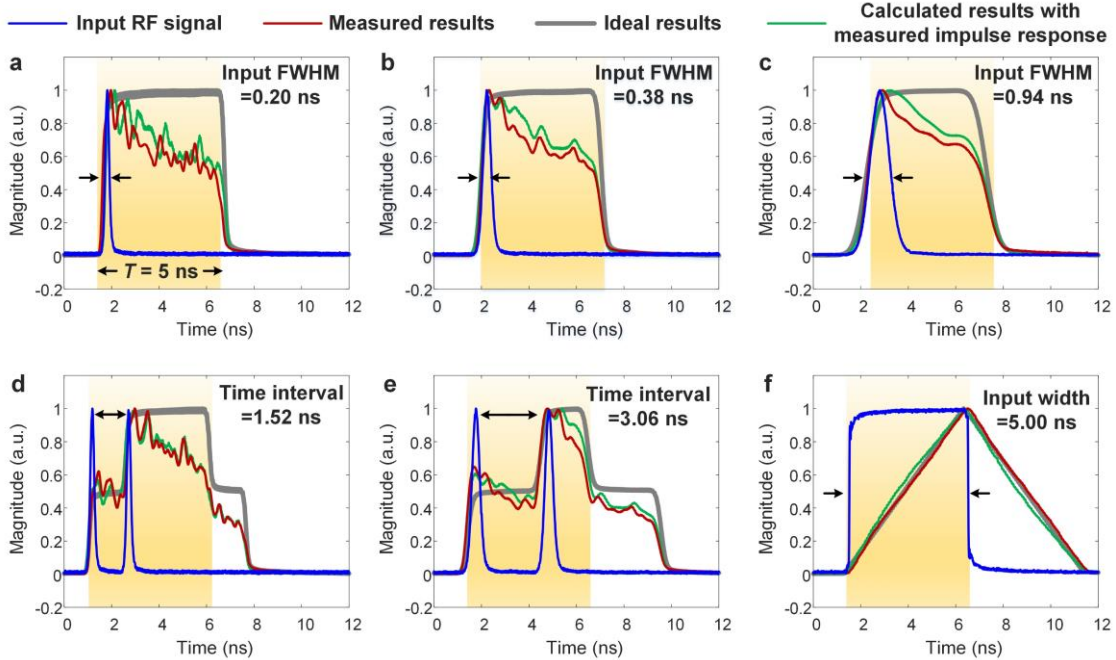

Fig. 5. Experimental results of the microcomb-based RF integrator after comb optical power shaping for input (a-c) Gaussian pulses with FWHM of 0.20, 0.38 and 0.94 ns, (d-e) dual Gaussian pulses with time intervals of 1.52 and 30.6 ns, and (f) a triangular waveform with a width of 5.00 ns. The blue curves denote the input signal, the red curves denote the measured integration results, the gray curves denote the ideal integration results, and the green curves denote the integration results calculated with measured impulse response of the system.
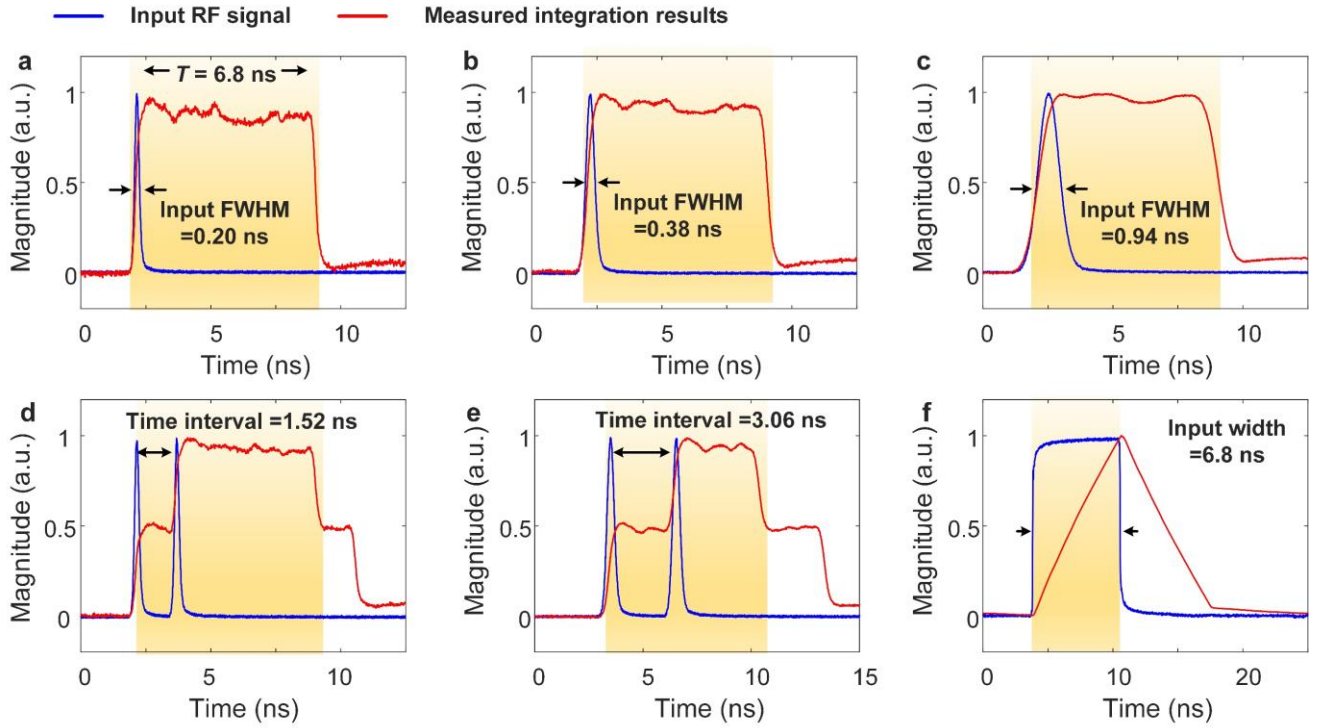
Fig. 7. Experimental results of the microcomb-based RF integrator after impulse response shaping for input (a-c) Gaussian pulses with FWHM of 0.20, 0.38 and 0.94 ns, (d-e) dual Gaussian pulses with time intervals of 1.52 and 3.06 ns, and (f) a triangular waveform with a width of 5.00 ns. The blue curves denote the input signal, the red curves denote the measured integration results.

system (~ 84 ps) was much smaller than the duration of the input pulse, we separated the wavelength channels into multiple subsets (each with a much larger spacing between the adjacent comb lines and thus obtaining a temporal resolution larger than the input pulse duration), and measured their impulse responses sequentially. Figure 6(a) shows the measured impulse response of the system, which was not flat in magnitude even when the comb lines were perfectly flattened. We used the measured impulse response and the input RF signal in Fig. 5 to calculate corresponding integral output, with the results matching the experimentally measured integration well — verifying our deductions that the experimental errors were induced by the non-ideal impulse responses of the system.

In order to reduce the errors mentioned above, we developed a much more accurate comb shaping approach, where the error signal of the feedback loop was generated directly by the measured impulse response, instead of the optical power of the comb lines. As a result, the flattened impulse response is shown as Fig. 6(b), which is much closer to the ideal impulse response than Fig. 6(a).

We then performed integration with the same RF inputs as previous measurements, the results are shown in Fig. 7. Note that during this measurement, 81 wavelength channels were enabled by the impulse response shaping process, as such the integration time window ($T = N \times \Delta t$) increased to $81 \times 84 ps = 6.804$ ns, resulting in an operation bandwidth of $1/84 ps = 11.9$ GHz and a time-bandwidth product of $6.804 ns \times 11.9 GHz = \sim 81$ (approximately equal to the number of channels N). The measured integrated results (red curves, Fig. 6) show significantly fewer discrepancies and agree well with the theoretical predictions, indicating the success of the impulse response shaping method, and the feasibility of our approach to photonic RF integration based on microcombs.

IV. CONCLUSIONS

In conclusion, we demonstrate a photonic RF integrator using an integrated soliton crystal micro-comb source. Through broadcast and delay processes employing 81 wavelength channels generated by the microcomb source, discrete temporal integration of RF signals is achieved. A large integration time window of 6.8 ns is demonstrated, together with a time feature as fast as 84 ps. An impulse response shaping approach was developed to compensate for the non-flat optical transmission response of the system to guarantee uniform channels weights for the integrator. Different input signals were successfully integrated. The experimental results verify that our RF integrator is a competitive approach towards integrating high-speed photonic RF signals with high performance and potentially reduced cost and footprint.

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