Exploiting Soft Constraints within Decomposition and Coordination Methods for Sub-hourly Unit Commitment

Niranjan Raghunathan [1], Mikhail Bragin [2], Bing Yan [1], Peter Luh [1], Khosrow Moslehi [1], Xiaoming Feng [1], Chien-Ning Yu [1], and Chia-Chun Tsai [1]

[1]Affiliation not available
[2]University of Connecticut

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Abstract

Unit commitment (UC) is an important problem solved on a daily basis within a strict time limit. While hourly UC problems are currently considered, they may not be flexible enough with the fast-changing demand and the increased penetration of intermittent renewables. Sub-hourly UC is therefore recommended. This, however, will significantly increase problem complexity even under the deterministic setting, and current methods may not be able to obtain good solutions within the time limit. In this paper, deterministic sub-hourly UC is considered, with the innovative exploitation of soft constraints – constraints that do not need to be strictly satisfied, but with predetermined penalty coefficients for their violations. The key idea is the "surrogate optimization" concept that ensures multiplier convergence within "surrogate" Lagrangian relaxation as long as the "surrogate optimality condition" is satisfied without the need to optimally solve the "relaxed problem." Consequently, subproblems can still be formed and optimized when soft constraints are not relaxed, leading to a drastically reduced number of multipliers and improved performance. To further enhance the method, a parallel version is developed. Testing results on the Polish system demonstrate the effectiveness and robustness of both the sequential and parallel versions at finding high-quality solutions within the time limit.

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***Abstract*--Unit commitment (UC) is an important problem solved on a daily basis within a strict time limit. While hourly UC problems are currently considered, they may not be flexible enough with the fast-changing demand and the increased penetration of intermittent renewables. Sub-hourly UC is therefore recommended. This, however, will significantly increase problem complexity even under the deterministic setting, and current methods may not be able to obtain good solutions within the time limit. In this paper, deterministic sub-hourly UC is considered, with the innovative exploitation of soft constraints – constraints that do not need to be strictly satisfied, but with predetermined penalty coefficients for their violations. The key idea is the "surrogate optimization" concept that ensures multiplier convergence within "surrogate" Lagrangian relaxation as long as the "surrogate optimality condition" is satisfied without the need to optimally solve the "relaxed problem." Consequently, subproblems can still be formed and optimized when soft constraints are not relaxed, leading to a drastically reduced number of multipliers and improved performance. To further enhance the method, a parallel version is developed. Testing results on the Polish system demonstrate the effectiveness and robustness of both the sequential and parallel versions at finding high-quality solutions within the time limit.**

***Index Terms*-- Sub-hourly Unit Commitment, Soft Constraints, Surrogate Lagrangian Relaxation (SLR), Surrogate Absolute-Value Lagrangian Relaxation (SAVLR), Parallel Algorithms**

I. Introduction

Unit commitment (UC) is to decide when to turn on or off generators and their generation levels to minimize the total cost subject to system demand, reserve, transmission, and unit-level constraints. The problem is generally formulated as a mixed-integer linear programming (MILP) problem, with linear cost function and constraints, which include both binary and continuous decision variables. There are variations in how constraints are formulated. When constraints are required to be strictly satisfied (e.g., system demand), they are "hard" constraints. Otherwise, they are "soft" constraints (e.g., reserve and transmission capacity constraints), where violations incur penalties with predetermined coefficients. Such problems are solved daily with a strict time limit (e.g., 15 or 20 minutes).

In the current practice, hourly intervals are used, with a planning horizon typically of 24 hours. With the fast-changing demand and the increasing penetration of intermittent renewables, high sub-hourly variations cannot be adequately accounted for. Hourly UC may obtain expensive solutions and may result in renewable curtailment and load shedding [2]. Sub-hourly UC with, e.g., 15 minutes as the interval, is therefore recommended for higher flexibility and efficiency.

The major difficulty of sub-hourly UC is that it has a larger number of binary variables than the hourly UC (quadrupled for the 15-minute case), significantly increasing problem complexity. Also, the ramp rate per interval of a unit is much smaller than that of the hourly UC, rendering more ramping constraints active with increased computational requirements. As a result, sub-hourly UC is difficult to solve even under the deterministic setting, and current methods may not be able to obtain good solutions within the required time limit.

In this paper, deterministic sub-hourly UC is considered, with 15 minutes as the time interval. After reviewing the literature [2-6] in Section II, the problem formulation is presented in Section III, with system demand modeled as "hard" and reserve requirements and transmission capacities modeled as "soft."

In Section IV, soft constraints are innovatively exploited within a novel decomposition and coordination framework of Surrogate Lagrangian Relaxation (SLR) [6] and Surrogate Absolute-Value Lagrangian Relaxation (SAVLR) [7], which overcame all the major difficulties of traditional Lagrangian relaxation (LR) of long subproblem solving times, zigzagging of multipliers, and the need to guesstimate the optimal dual values. SAVLR, however, generally relaxes all system-wide constraints, resulting in a very large number of multipliers for the sub-hourly UC. To overcome this difficulty, only hard system demand constraints are relaxed, but not soft reserve and transmission capacity constraints. This is possible since, under the recent "surrogate optimization" concept, the multipliers will

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Niranjan Raghunathan, Mikhail A. Bragin, and Peter B. Luh are with the Department of Electrical and Computer Engineering, University of Connecticut, Storrs, CT 06269-4157, USA (e-mail: niranjan.raghunathan@uconn.edu, mikhail.bragin@uconn.edu and peter.luh@uconn.edu).

Bing Yan is with the Department of Electrical and Microelectronic Engineering at Rochester Institute of Technology, Rochester, NY 14623, USA (e-mail: bxyeee@rit.edu).

Khosrow Moslehi, Xiaoming Feng, Chien-Ning Yu, and Chia-Chun Tsai are with ABB (e-mail: khosrow.moslehi@hitachi-powergrids.com, xiaoming.feng@hitachi-powergrids.com, chien-ning.yu@hitachi-powergrids.com, chia-chun.tsai@hitachi-powergrids.com).

Yaowen Yu was with ABB, and is now with the Key Laboratory of Image Processing and Intelligent Control, School of Artificial Intelligence and Automation, Huazhong University of Science and Technology, Wuhan 430074, China (e-mail: yaowen_yu@hust.edu.cn).

still converge as long as the "surrogate optimality condition" is satisfied without the need to optimally solve the "relaxed problem." Consequently, subproblems can still be formed and optimized when soft constraints are not relaxed. This is a unique feature of SAVLR and is not possible under the traditional LR framework. To further improve performance, a parallel version is also implemented, which utilizes multiple cores of processors to construct and solve subproblems in parallel.

In Section V, three numerical examples are presented. The first is an hourly UC problem for a 6-bus system, and is used to demonstrate the impacts of the innovative exploitation of soft constraints within the SAVLR framework. The second example is a 15-minute UC problem based on the 2383-bus Polish system, and is used to demonstrate the ability of our methods to solve large-scale practical problems. In the third example, Monte Carlo testing based on six load profiles is performed for the same Polish system. Testing results demonstrate the effectiveness and robustness of both the sequential and parallel versions at finding high-quality solutions within the time limit. These methods are generic and may be used to solve other complex MILP problems in power systems and beyond.

II. LITERATURE REVIEW

In subsection II-A, the use of soft constraints in UC is presented. In subsection II-B, research on sub-hourly UC from the literature is reviewed. In subsection II-C, Surrogate Lagrangian Relaxation (SLR) and Surrogate Absolute-Value Lagrangian Relaxation (SAVLR), which are dual methods for solving MILP problems, are reviewed.

A. Use of soft constraints in unit commitment

In [1], the use of soft constraints by Independent System Operators (ISOs) in the United States is reviewed. For UC problems, constraints are modeled by soft constraints when they do not need to be strictly satisfied. For example, ISO-New England models reserve requirements with soft constraints, with different predetermined penalty coefficients for different types of reserves. These coefficients are typically high enough so that requirements are only significantly violated when satisfying them is difficult or impossible. Practical procedures for selecting the values of penalty coefficients for reserve and transmission capacity constraints are different for each ISO, and detailed information may not be available.

Certain ISOs also model the demand constraints as soft. The predetermined penalty coefficient for not satisfying the demand constraints serves as a cap on the energy market price. Reference [1] provides an analysis of the market implications of using soft constraints. While soft constraints are often used in power systems and other fields, their use within the LR methods for MILP problems has not been explored.

B. Sub-hourly unit commitment

With the fast-changing demand and the increasing penetration of intermittent renewables, high sub-hourly variations cannot be adequately accounted for in hourly UC. Sub-hourly UC has therefore been presented to address the difficulties in [2 - 5].

In [2], deterministic and stochastic UC based on the IEEE 118-bus system with sub-hourly intervals (i.e., 15 or 20 minutes) are compared with the hourly UC. Results obtained by using a modified Benders decomposition method show that costs can be significantly reduced if sub-hourly UC is considered for both deterministic and stochastic problems.

In [3], the deterministic UC problems based on the Irish system (72 units and 1 pumped storage plant) with sub-hourly time intervals (i.e., 5, 15, 30 min) are compared with the hourly model. System demand and reserve constraints are considered, but not transmission capacity constraints. The study by using FICO Xpress-MP shows that increased temporal resolution captures more variability in system load and renewable generation, leading to more realistic estimations in total generation costs. It also captures the significant cycling and ramping of units. Thus, if the intermittent nature of wind and solar is explicitly modeled, the sub-hourly UC should obtain more economic solutions than the hourly model.

In addition to the above, the impact of sub-hourly UC on power system dynamics was discussed in [4] based on the IEEE 39-bus system using GUROBI; and the impact of sub-hourly UC on spinning reserve in the presence of intermittent renewables was investigated for an isolated island (ten diesel generators, three wind turbines, and one Photovoltaic power plant) and the IEEE 118-bus system using dynamic programming plus priority listing in [5].

C. Surrogate Lagrangian relaxation and Surrogate Absolute-Value Lagrangian Relaxation

For a system consisting of multiple interconnected subsystems, an optimization problem is referred to as "separable" if its objective function and the constraints coupling the subsystems are additive in terms of subsystem variables. LR has traditionally been a price-based decomposition and coordination approach to exploit such separable structures, especially for problems with discrete variables. Within the approach, constraints that couple subsystems are relaxed by using Lagrangian multipliers to form the "relaxed problem," with its objective function called "Lagrangian." The relaxed problem is decomposed into subproblems, one for each subsystem. Each subproblem is of much-reduced complexity as compared to that of the original problem because of its reduced size. For a given set of multipliers, all subproblems are optimized, and the resulting levels of constraint violation form a subgradient, which is used to update the multipliers, and the process iterates. At the convergence of the multiplier updating process, heuristics are used to obtain a feasible solution satisfying all constraints.

Traditional LR methods, however, suffer from major difficulties of long subproblem solving times, zigzagging of multipliers, and the need to guesstimate the optimal dual values. One of the fundamental reasons for these difficulties is that the relaxed problem is required to be fully optimized. This renders long subproblem solving times, and drastic changing of subgradient directions from one iteration to the next because of the geometry of the dual functions of MILP problems, causing multipliers to zigzag. To what extent should the relaxed problem be optimized? It was proved in the recent "Surrogate Lagrangian Relaxation" (SLR) method [6] that the relaxed problem does not need to be fully optimized, but only partially optimized subject to a simple inequality constraint – the "surrogate optimality condition" (see equation (10) in subsection IV-A). Consequently, only a subset of decision variables needs to be optimized at each time, and the resulting levels of constraint violation form a "surrogate" subgradient (as opposed to a true subgradient), which can be used to update multipliers. The subset of decision variables to be optimized is then rotated while keeping other decision variables at their latest available values. Such an iterative process requires much-reduced computation effort, much less changing of surrogate subgradient directions from one iteration to the next, and much-reduced multiplier zigzagging as compared to those of the traditional LR. Also, the convergence proof of SLR is based on contraction mapping and does not require the optimal dual value. There is no need to guesstimate the optimal dual values in its step-sizing rule either. Consequently, all major difficulties of the traditional LR have been overcome by SLR.

From a different perspective, subproblems can still be formed within the SLR framework. The objective function of a subproblem is obtained by collecting from the Lagrangian all the terms related to the subsystem under consideration, and is minimized with respect to the decision variables belonging to the subsystem while keeping other decision variables at their latest available values, subject to all non-relaxed constraints. In this way, problem separability is no longer required, the concept of subproblems is extended, and the LR approach is established for non-separable problems. Also, not all subproblems need to be solved to obtain a subgradient to update multipliers. Rather, only one subproblem needs to be solved, and even that subproblem does not need to be optimally solved – only subject to the surrogate optimality condition. The resulting surrogate subgradient is used to update multipliers without the knowledge of the optimal dual value, leading to much reduced computational effort and zigzagging of multipliers.

Very recently, the convergence of SLR has been accelerated through the adding of absolute-value penalties on constraint violations, resulting in the Surrogate Absolute-Value Lagrangian Relaxation (SAVLR) [7].

For the sub-hourly UC problems under consideration, SAVLR may still have difficulties to obtain near-optimal solutions within the time limit for practical cases because of the inherent difficulties as introduced in Section I. When all the coupling constraints are relaxed, a large number of multipliers will be introduced, leading to convergence difficulties. Our idea is to relax hard demand constraints only while managing reserve and transmission capacity constraints through innovative exploitation of their softness based on the surrogate optimization concept of SAVLR. This will be presented after a brief introduction to the problem formulation.

III. Problem Formulation

Consider a power system which is divided into areas, and a large area can be further divided into subareas. The 15-minute UC is formulated as a mixed-binary linear programming problem. The formulation is similar to that of the hourly UC, with ramp rates and other parameters appropriately scaled. The objective is to minimize the sum of operating costs (generation, startup, no-load, and reserve costs) and soft constraint penalties (on reserve and transmission capacity), subject to unit-level, area-level, and system-level constraints. The objective function is formulated as:

$$\min_{\{p,u,x,r,\upsilon\}} \left\{ \sum_t \left(\sum_j \left(\sum_{b\in B_j} C_{b,j}\, p_{b,j}(t) + C_j^{\mathrm{SU}} u_j(t) + C_j^{\mathrm{NL}} x_j(t) + \sum_m C_{j,m}^{\mathrm{R}} r_{j,m}(t) \right) + \sum_a \sum_n C_{a,n}^{\mathrm{P,R}} \upsilon_{a,n}^{\mathrm{R}}(t) + \sum_l C_l^{\mathrm{P,TC}} \left(\upsilon_l^{\mathrm{TC},+}(t) + \upsilon_l^{\mathrm{TC},-}(t) \right) \right) \right\}. \tag{1}$$

In the above, unit j has the following decision variables: $p_{b,j}(t)$ the continuous generation-level of block $b \in B_j$ at time t, $x_j(t)$ the binary unit commitment variable, $u_j(t)$ the binary start-up variable, $r_{j,m}(t)$ the continuous variable for reserve level of type m from a unit's perspective, including regulation, 10-minute spinning, ten-minute non-spinning, thirty-minute spinning, and thirty-minute non-spinning of decreasing quality. Area a has the following decision variable: $\upsilon_{a,n}^{\mathrm{R}}$ a continuous variable for soft reserve violation of type n from an area's perspective, including regulation, 10-minute spinning, 10-minute total, and 30-minute total, each can be satisfied by unit contributions of equal or higher quality. Line l has the following decision variables: $\upsilon_l^{TC,+}(t)$ and $\upsilon_l^{TC,-}(t)$ the continuous variables for soft transmission capacity violations on the positive and negative directions, respectively. Cost and penalties within (1) include generation cost $C_j p_j$, start-up cost $C_j^{SU} u_j$, no-load cost $C_j^{NL} x_j$, reserve cost $C_{j,m}^{R} r_{j,m}$, soft reserve penalty $C_{a,n}^{\mathrm{P,R}} \upsilon_{a,n}^{\mathrm{R}}$, and soft transmission capacity penalty $C_l^{\mathrm{P,TC}}(\upsilon_l^{TC,+}(t) + \upsilon_l^{TC,-}(t))$.

The problem is subject to the following unit-level constraints: generation capacity, ramp rates, start-up, min up/down-time, and reserve capacity, which are given in (1), (4-5), (11-13), (14), and (15-18) of [8], respectively.

Area-wise soft regulation constraints (type n = 1) for individual areas are formulated as:

$$\sum_j \left(r_{1,j}(t) \times \mathrm{pa}_{j,a} \right) + \upsilon_{a,1}(t) \ge \mathrm{RR}_{a,1}(t), \ \forall a, t, \tag{2}$$

where $\mathrm{pa}_{j,a}$ is the binary area participation indicator of unit j in area a, which is 1 if the unit is in area a and 0 otherwise, and $\mathrm{RR}_{a,1}$ is the regulation requirement for area a . Reserve constraints for other types are formulated similarly.

The system-level demand constraint is given as

$$\sum_j p_j(t) = \sum_i P_i^{\mathrm{D}}(t) \quad \forall t, \tag{3}$$

where $p_j(t)$ the total generation level which is the sum of outputs of all the blocks, and P_i^{D} is the demand at node i. Soft transmission capacity constraints are formulated as:

$$-f_l^{\max} - \upsilon_l^{\mathrm{TC},-}(t) \le \sum_i \alpha_{i,l} \left(\sum_{j\in J_i} p_j(t) - P_i^{\mathrm{D}}(t) \right) \le f_l^{\max} + \upsilon_l^{\mathrm{TC},+}(t) \ \ \forall t, l, \tag{4}$$

where $f_l^{\max}$ is the capacity of transmission line l; J_i is the set of generators at node i; and $\{\alpha_{i,l}\}$ are generation shift factors, specifying the relationship between the power injection at node i and the power flow through line l.

IV. Solution Methodology

In subsection IV-A, soft constraints are exploited within the SAVLR framework to solve the problem with step-by-step derivations. In IV-B, initialization, parameter tuning, and solving problems with hard reserve and transmission capacity constraints are presented. To speed up computation, the parallel version of SAVLR is developed in IV-C.

A. Exploiting soft constraints within the SAVLR framework

When solving a large sub-hourly UC, relaxing all coupling constraints will lead to an excessive number of multipliers and slow convergence. To significantly reduce the number of multipliers, only system demand constraints are relaxed, but not soft reserve and transmission capacity constraints. This is not possible under the traditional LR framework but is possible under SAVLR (and SLR) as presented below.

Relaxing system demand constraints

With system demand constraints (3) relaxed by using multipliers $\{\lambda(t)\}_t$ and constraint violations penalized by using the penalty coefficient c, the relaxed problem is:

$$\min_{\{p,u,x,r,\upsilon\}} L_c, \text{ where}$$

$$\begin{aligned} L_c = \sum_t \Bigg(\sum_j \Bigg(\sum_{b\in B_j} C_{b,j} p_{b,j}(t) + C_j^{\mathrm{SU}} u_j(t) + C_j^{\mathrm{NL}} x_j(t) \\ + \sum_m C_{j,m}^{\mathrm{R}} r_{j,m}(t) \Bigg) + \sum_a \sum_n C_{a,n}^{\mathrm{P,R}} \upsilon_{a,n}^{R}(t) \\ + \sum_{l\in L} C_l^{\mathrm{P,TC}} \left(\upsilon_l^{\mathrm{TC},+}(t) + \upsilon_l^{\mathrm{TC},-}(t) \right) \\ + \lambda(t) \left(\sum_i P_i^{\mathrm{D}}(t) - \sum_j p_j(t) \right) \\ + c \left| \left(\sum_i P_i^{\mathrm{D}}(t) - \sum_j p_j(t) \right) \right| \Bigg), \end{aligned} \tag{5}$$

subject to individual unit-level constraints and soft reserve and transmission capacity constraints. In the above, L_c is the Lagrangian.

Linearizing absolute value penalties

The relaxed problem (5) is non-linear due to the absolute-value penalty terms. To make it linear, the absolute-value terms are linearized in a standard way [9]. The relaxed problem after linearization is:

$$\min_{\{p,u,x,r,\upsilon,z\}} L_c', \text{ where}$$

$$\begin{aligned} L_c' = \sum_t \Bigg(\sum_j \Bigg(\sum_{b\in B_j} C_{b,j} p_{b,j}(t) + C_j^{\mathrm{SU}} u_j(t) + C_j^{\mathrm{NL}} x_j(t) \\ + \sum_m C_{j,m}^{\mathrm{R}} r_{j,m}(t) \Bigg) + \sum_a \sum_n C_{a,n}^{\mathrm{P,R}} \upsilon_{a,n}^{R}(t) \\ + \sum_{l\in L} C_l^{\mathrm{P,TC}} \left(\upsilon_l^{\mathrm{TC},+}(t) + \upsilon_l^{\mathrm{TC},-}(t) \right) \\ + \lambda(t) \left(z^+(t) - z^-(t) \right) + c \left(z^+(t) + z^-(t) \right) \Bigg), \end{aligned} \tag{6}$$

subject to individual unit-level constraints and soft reserve and transmission capacity constraints, which are not relaxed. In the above, $z^+(t)$ and $z^-(t)$ are the continuous linearization variables satisfying the following newly introduced constraints:

$$z^+(t) - z^-(t) = \sum_i P_i^{\mathrm{D}}(t) - \sum_j p_j(t), \ \forall t. \tag{7}$$

Formulating and solving area or subarea subproblems

The overall system is divided into areas, and a large area can be further divided into subareas. To simplify the terminology, let a "subsystem" be an area or a subarea if it exists. let S be the set of all subsystems with $S = \{1, 2, .. s, .., |S|\}$. A subproblem for a subsystem is formed by collecting all the terms from the Lagrangian L_c' in (6) related to that subsystem as the objective function. This objective function is minimized with respect to the decision variables belonging to that subsystem while keeping all other decision variables at their latest available values following the procedures of SAVLR as reviewed in subsection II-C. The subproblem s solved at iteration k is:

$$\min_{\{p,u,x,r,\upsilon,z\}} L_s^k, \text{ where}$$

$$\begin{aligned} L_s^k = \sum_t \Bigg(\sum_{j:j\in G_s} \Bigg(\sum_{b\in B_j} C_{b,j} p_{b,j}(t) + C_j^{SU} u_j(t) + C_j^{NL} x_j(t) \\ + \sum_m C_{j,m}^{R} r_{j,m}(t) \Bigg) + \sum_a \sum_n C_{a,n}^{\mathrm{P,R}} \upsilon_{a,n}^{R}(t) \\ + \sum_{l\in L} C_l^{\mathrm{P,TC}} \left(\upsilon_l^{\mathrm{TC},+}(t) + \upsilon_l^{\mathrm{TC},-}(t) \right) \\ + \lambda^k(t) \left(z^{k,+}(t) - z^{k,-}(t) \right) + c^k \left(z^{k,+}(t) + z^{k,-}(t) \right) \Bigg), \end{aligned} \tag{8}$$

where G_s is the set of units belonging to subsystem s. For brevity, constant terms are omitted from (8). This subproblem is subject to unit-level constraints for units belonging to the subsystem, soft reserve and transmission capacity constraints (which are not relaxed), and the following linearization constraints which are not relaxed:

$$\begin{aligned} z^{k,+}(t) - z^{k,-}(t) = \sum_i P_i^D(t) \\ - \left(\sum_{j:j\notin G_s} p_j^{k-1}(t) + \sum_{j:j\in G_s} p_j^k(t) \right), \end{aligned} \tag{9}$$

where $p_j^{k-1}(t)$ denote the most recent decision variable obtained at iteration k - 1.

Subproblems are solved by using B&C subject to the satisfaction of the surrogate optimality condition [7]:

$$\tilde{L}_{c^k,\lambda^k}^{k} < \tilde{L}_{c^k,\lambda^k}^{k-1}, \tag{10}$$

where $\tilde{L}_{c^k,\lambda^k}^{k}$ is the "surrogate dual value," which is the Lagrangian (6) evaluated at the solutions of generators in this subsystem at iteration k, the solutions of generations in other subsystems at iteration k - 1, multipliers λ^k, and the penalty coefficient c^k; and $\tilde{L}_{c^k,\lambda^k}^{k-1}$ is (6) evaluated at subproblem solutions at iteration k - 1, multipliers λ^k, and the penalty coefficient c^k.

Updating multipliers and the penalty coefficient

If the surrogate optimality condition (10) is satisfied, then the "surrogate subgradient" is obtained as:

$$\tilde{g}^k(t) = \sum_i P_i^{\mathrm{D}}(t) - \sum_{j \in G_s} p_j^k(t) - \sum_{j \notin G_s} p_j^{k-1}(t), \quad \forall t, \tag{11}$$

and the multiplier $\lambda(t)$ is updated as:

$$\lambda^{k+1}(t) = \lambda^k(t) + s^k \cdot \tilde{g}^k(t), \quad \forall t. \tag{12}$$

In (12), the step size s^k is obtained as:

$$s^k = \alpha_k \frac{s^{k-1} \left\| \tilde{g}^{k-1} \right\|_2}{\left\| \tilde{g}^k \right\|_2}, \quad 0 < \alpha_k < 1, \tag{13}$$

where

$$\alpha_k = 1 - \frac{1}{M k^{\rho}}, \; \rho = 1 - \frac{1}{k^r}, \; M \geq 1, \; 0 \leq r \leq 1. \tag{14}$$

The penalty coefficient c is updated as:

$$c^{k+1} = \min\left(c^{\mathrm{ub}}, \beta \cdot c^k\right), \; \beta > 1, \tag{15}$$

where c^{ub} is an upper bound on c, preventing c from getting too large.

If the surrogate optimality condition is not satisfied, the multiplier $\lambda(t)$ and the penalty coefficient c are not updated, and the next subproblem is solved. If the surrogate optimality condition cannot be satisfied after $|S|$ consecutive subproblems, c is deemed too large, and is reduced follows [7]:

$$c^{k+1} = \frac{c^k}{\beta}, \; \beta > 1. \tag{16}$$

The penalty coefficient c is also decreased following (16) if subproblem solutions, when put together, are feasible to the original problem. This is because in this case there is no longer a need to increase it, and decreasing it may facilitate coordination and prevent premature termination of the algorithm. After updating the multipliers, stepsize, and penalty coefficient, the next subproblem is solved, and the process is repeated until a time limit is reached or the multiplier converges based on appropriate criteria.

Comments

With soft reserve and transmission capacity constraints not relaxed, subproblems can still be formed and optimized subject to the surrogate optimality condition. Furthermore, the ability of subproblem solutions to satisfy the surrogate optimality condition is high since constraint requirements are not hard but soft, and are easy to be satisfied (at the expense of incurring penalties). Therefore, this unique "surrogate" feature inherent to SAVLR allows to establish convergence and to reduce the number of multipliers at the same time. This is not possible under the traditional LR framework since the Lagrangian L_c' in (6) cannot be decomposed into independent terms belonging to subsystems. To perform decomposition, soft reserve and transmission capacity constraints must be relaxed.

Finding feasible solutions and evaluating solution quality

Once the time limit is reached or appropriate stopping criteria are satisfied for the iterative subproblem solving and multiplier updating process, the binary decision variables are fixed at their subproblem solution values, and the resulting LP problem is solved to obtain a feasible solution satisfying all the constraints. If a feasible solution cannot be obtained, then more complicated heuristics will be needed, e.g., by fixing a subset of binary decisions and solving the entire problem. This, however, was not encountered in our testing to be reported in Section V.

To measure the quality of a feasible solution, the following optimality gap is used:

$$\text{Gap (\%)} = 100 \times \frac{\left(f^{\text{best}} - lb^{\text{best}}\right)}{f^{\text{best}}}, \tag{17}$$

where f^{best} is the best feasible cost and lb^{best} is the best known lower bound, which is obtained by using pure B&C in our numerical testing to be presented in Section V.

B. Initialization, Parameter Tuning and Solving Problems with Hard Constraints

In this subsection, initialization of subproblem solutions, tuning stepsizing parameters, and solving problems with hard reserve and transmission capacity constraints are presented.

Initializing subproblem solutions

Initial subproblem solutions are needed to start the iterative process (see the flowchart in Fig. 1). To do this, startup and generation level variables are chosen for each time interval such that generation capacity (1), ramp rate (4-5), startup (11-13), and minimum up/down-time (14) constraints are satisfied, while other constraints are ignored.

Tuning stepsizing parameters

Stepsizing parameters $M, r,$ and s^0 require tuning. The choice of these parameters depends on system topography and time limit. Once a set of parameters is tuned for a particular instance of a system, e.g., the Polish system, then the same set of parameters should be used for different instances of the system with varying system parameters and load. For tuning, the method is run on a grid of parameters for one instance of the problem, and the set of parameters that gives the lowest cost is selected.

Solving problems with hard constraints

UC problems formulated with hard reserve and transmission capacity constraints can also be solved by using our approach developed above. This is done by converting hard constraints to soft constraints at the beginning of the solution process, with the requirement of satisfying hard constraints at the final stage of finding a feasible solution.

The key consideration is the selection of predetermined penalty coefficients. If the coefficients are too high, then the converted soft constraints behave like hard constraints, impeding the coordination of subproblem solutions. If they are too low (e.g., the order of their magnitude is similar to that of multipliers or smaller), then soft constraints may be excessively violated, impacting the value of the optimal multipliers, and making it difficult to find feasible solutions to the original problem at the end. To prevent these difficulties, predetermined penalty coefficients are set to be an order of magnitude higher than the expected upper bound of multipliers. To find the expected upper bound of demand multipliers, initial multiplier estimates are obtained by priority list commitment and dispatch of [10] in our numerical testing.

A flowchart for SAVLR is given in Fig. 1.

C. Synchronous parallel Surrogate Absolute-Value Lagrangian Relaxation

The method presented above can be further improved by taking advantages of multiple cores available in modern

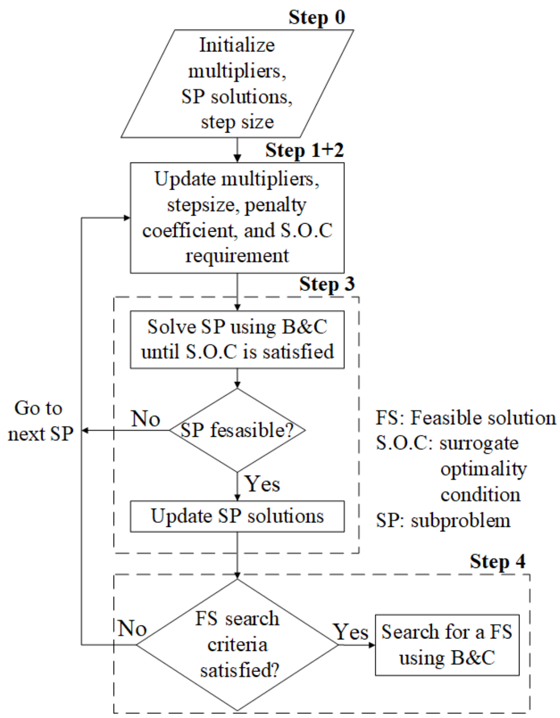

Figure 1: SAVLR flowchart

processors. The formulation of the relaxed problem and the decomposition process are identical to those for the sequential SAVLR. Subproblems are grouped in batches, and subproblems within a batch are assigned to individual cores of a multi-core processor. Construction and solving of multiple subproblems is performed in parallel.

After a batch of subproblems is solved, their solutions are combined. Although individual solutions may satisfy the surrogate optimality condition, the combined solution may not. If the combined solution satisfies the surrogate optimality condition, then the multipliers $\{\lambda(t)\}_t$ and the penalty coefficient c are updated as for sequential SAVLR, and the next batch of subproblems is solved. If the combined solution does not satisfy the surrogate optimality condition, then combinations of subsets of solutions are checked, with priority given to the largest combinations. Once a combination satisfying the surrogate optimality condition is found, multipliers and the penalty coefficient are updated, and the next batch is solved. Since updating is done after all the subproblems in the batch are solved, the method is synchronous.

The number of subproblems solved in parallel should not be too large. To see this, consider the extreme case where all the subproblems are solved in parallel. Then this is essentially the traditional LR framework: solving all subproblems, obtaining a subgradient (rather than a surrogate subgradient), and updating multipliers. All the shortcomings of the traditional LR will show up. Consequently, only a fraction of subproblems is solved in parallel to speed up the process while avoiding the difficulties of traditional LR methods.

V. Numerical Testing and Analysis

Three UC problems are solved to demonstrate the performance of our method. The first is an hourly UC problem over a time horizon of 24 hours for a 6-bus system to demonstrate the impacts of the exploitation of soft constraints within the SAVLR framework. The second problem is a 15-minute UC based on the 2383-bus Polish system, with hard demand, reserve, and transmission capacity constraints. After hard reserve and transmission capacity constraints are converted to soft constraints, the ability of the sequential SAVLR to solve such large problems is presented. Finally, to demonstrate the robustness and performance of both sequential and parallel algorithms, Monte Carlo testing based on six load profiles is performed for the 15-minute UC problem of the Polish system with hard demand constraints and soft reserve and transmission capacity constraints. All testing is performed on an Intel Xeon CPU 3.1 GHz, 4 Cores, 32 GB laptop, with MATLAB R2018a and CPLEX 12.8.

Example 1: 6-bus system

An hourly UC problem over a 24-hour time horizon for a 6-bus system with 9 units and 11 transmission lines is considered as shown in Fig. 2. The problem is subject to the standard unit-level constraints, hard system demand constraints, and soft transmission capacity constraints. For simplicity, reserves are not considered. Predetermined penalties of $1000/MW are set for not meeting the transmission capacity requirements.

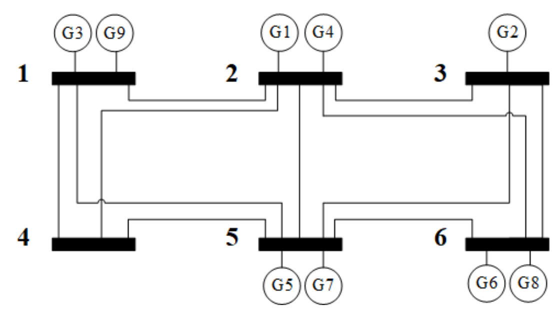

Figure 2: 6-bus system

The problem is solved by using B&C, and sequential and parallel SAVLR. For SAVLR, the problem is decomposed into 5 subproblems, which are solved by using B&C. For parallel SAVLR, the subproblems are grouped into 3 batches, with 2 batches containing 2 subproblems, and 1 batch consisting of a single subproblem. For comparison purposes, the problem is also solved by using sequential SAVLR with the soft constraints relaxed. The feasible costs obtained by each method, and performance metrics (i.e., algorithm run time, optimality gap, and the number of iterations performed) are given in Table I.

Because of the low complexity of the problem, B&C finds a solution with .01% gap in a fraction of a second. Both sequential and parallel SAVLR, when only hard demand constraints are relaxed, find the same solution in 25 seconds. They take longer because, within it, multipliers and subproblem solutions are set to update for 25 seconds before searching for a feasible solution. The sequential SAVLR solved 286 subproblems, of which 219 satisfied the surrogate optimality condition. The parallel SAVLR solved 219 batches, of which 11 didn't satisfy the surrogate optimality condition after combining their subproblem solutions. As a result, 390 subproblems are solved. When soft transmission capacity constraints are relaxed, convergence is slow because of 528 additional multipliers, and the feasible cost is also higher.

TABLE 1. RESULTS FOR EXAMPLE I

Method	Feasible Cost	Gap	Time	Iterations
B&C	$623,956	0.01%	< .1s	-
Sequential SAVLR – not relaxing soft constraints	$623,956	0.01%	25s	40
Parallel SAVLR – not relaxing soft constraints	$623,956	0.01%	25s	66
Sequential SAVLR – relaxing soft constraints	$666,280	6.05%	43s	68

Example 2: The 2386-bus Polish system with hard demand, reserve and transmission capacity constraints

In this example, the effectiveness of our sequential method is demonstrated. A 15-minute UC problem over a 24-hour planning horizon is considered based on an available 2383-bus Polish system power flow test case, modeled with 327 units and 2895 transmission lines with hard system demand, reserve and transmission capacity constraints [11]. Based on the results of Example 1, hard reserve and transmission capacity constraints are first converted to soft constraints. Predetermined penalty coefficients are set at $500/MW, which is an order of magnitude higher than the expected upper bound on system demand multipliers of $50/MW. To increase problem complexity, the ramp rates of units are reduced to 60% of their nominal values. The problem is decomposed into 28 subproblems, each consisting of 9 to 12 units.

Both B&C and sequential SAVLR are run for 2 hours. For sequential SAVLR, feasible solution searches are performed every 10 minutes. The feasible costs and gaps obtained over time by each method is plotted in Fig. 3. SAVLR obtains a feasible solution with cost $32.78 million and gap of 1.52% in 20 minutes. B&C does not find a feasible solution with cost under $36 billion until after 115 minutes, at which point, it finds a solution with cost $32.3 million with a gap of .04%.

To demonstrate the effects of the predetermined soft constraint penalty coefficient, the problem is also solved with their values set at $200/MW and $50,000/MW. Performance metrics including norm squared of the relaxed constraints, incurred soft penalty, feasible cost, and optimality gap for the 3 cases are presented in Table II.

TABLE II
PERFORMANCE WITH DIFFERENT PENALTIES FOR EXAMPLE 2

Sequential SAVLR with different soft constraint penalties				
Soft constraint penalty	norm^2 (MW^2)	Incurred penalty	Feasible cost	Gap
$500/MW	170	$866	$\$3.278 \times 10^7$	1.52 %
$200/MW	4,118	$3616	No feasible sol.	-
$50,000/MW	7.9×10^6	~ 0	No feasible sol.	-

The soft penalty incurred in column 3 is informative about how much subproblem solutions violate the original hard reserve and transmission capacity constraints. By dividing the penalty incurred by the predetermined penalty coefficient, the total amount of violation of the original reserve and transmission capacity constraints can be calculated. With the value of $200/MW, the amount by which subproblem solutions violate reserve and transmission capacity constraints is 18.1

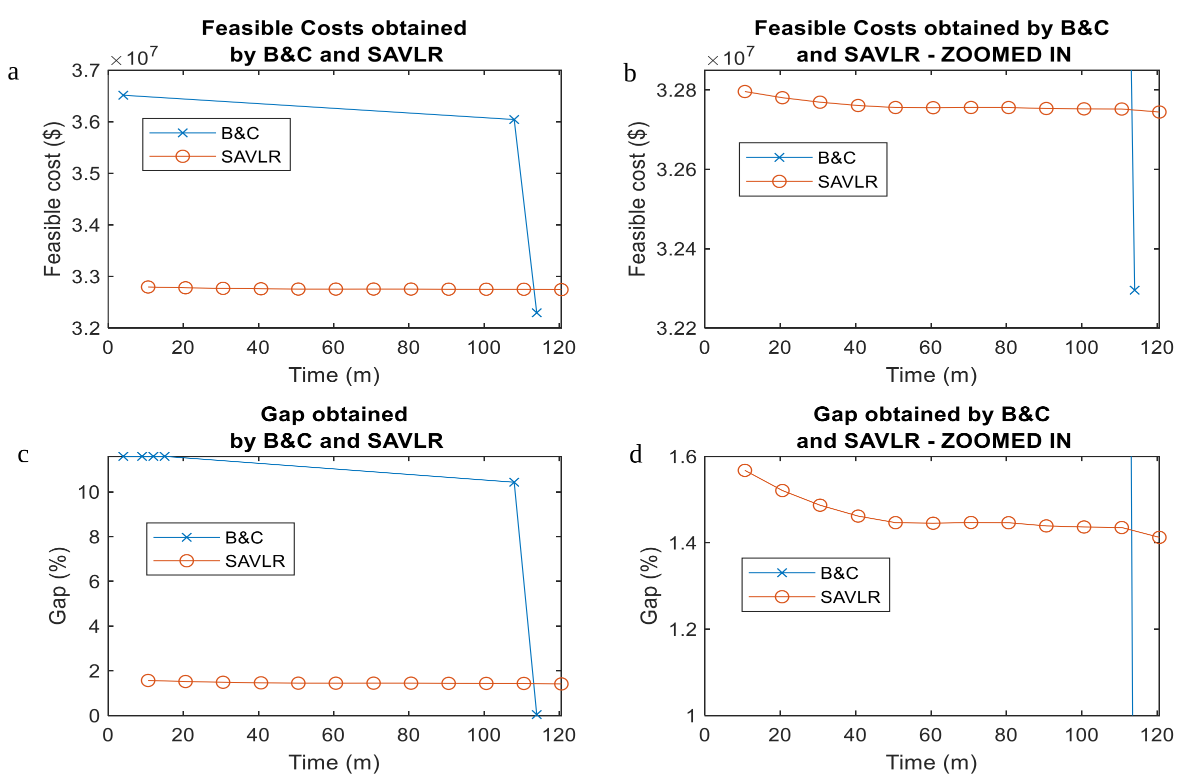

Figure 3: a) Feasible costs. b) Feasible costs obtained by SAVLR decrease over time. c) Optimality gaps. d) Optimality gaps zoomed in.

MW. With such a high level of violation, a feasible solution to the original problem is not found. With a soft constraint penalty of \$50,000/MW, the amount by which subproblem solutions violate the original reserve and transmission capacity constraints is negligible. However, with such a high coefficient, the soft constraints behave like hard constraints, impeding the effective coordination of subproblem solutions. The norm squared of the violation of system demand constraint remains large (7.9×10^6 MW^2) even after 20 minutes, and a feasible solution to the original problem is not found. With a coefficient of \$500/MW, the amount by which reserve and transmission capacity constraints are violated is 1.73 MW, and the norm squared value is $170\ \mathrm{MW}^2$, which are lower than the other cases, and a feasible solution with a gap of 1.52% is obtained.

Example 3: Monte Carlo simulations for the 2383-bus system, with soft reserve and transmission capacity constraints

In this example, the robustness and performance of both the sequential and the parallel methods are examined by performing Monte Carlo testing of the 15-minute UC problem over a 24-hour planning horizon for the Polish system. For the parallel SAVLR, 8 batches with 3 subproblems each and 1 batch with 4 subproblems are formed. Cases are based on scenarios with seasonal and weekday/weekend variations following Table 4 of [12]. For each case, three scenarios are created by varying the load at each node by up to 3%. All methods are run for 20 minutes. The average results of three scenarios are shown in Table III.

TABLE III
RESULTS FOR EXAMPLE 3

Season – weekday/weekend	Average feasible cost (average gap)		
	Sequential SAVLR ($\times 10^7$)	Parallel SAVLR ($\times 10^7$)	B&C ($\times 10^7$)
Winter - weekday	3.2435 (2.68%)	3.2476 (2.80%)	3.5859 (11.97%)
Winter - weekend	3.2535 (3.06%)	3.2531 (3.05%)	3.5504 (11.17%)
Summer - weekday	3.2518 (2.95%)	3.2809 (3.77%)	3.1690 (.39%)*
Summer - weekend	3.2363 (2.64%)	3.2385 (2.71%)	3.2881 (3.88%)
Spring/Fall - weekday	3.2493 (2.61%)	3.2625 (3.01%)	3.5631 (11.19%)
Spring/Fall - weekend	3.2691 (2.6%)	3.2783 (2.88%)	3.5648 (10.68%)
*Average values are based on 2 scenarios. No solution found for the 3rd scenario			

For 5 out of the 6 cases, sequential and parallel SAVLR find lower average costs than those of B&C. B&C finds the best average cost for the "Summer - weekday" case. However, only 2 out of 3 scenarios were solved, and no solution was found for the 3rd instance (the average cost and gap were calculated using the 2 scenarios for which solutions were found). Both versions of SAVLR give feasible solutions for all scenarios of all cases. SAVLR consistently finds solutions with less than a 4% gap in 20 minutes, while B&C struggles for most scenarios due to the high problem complexity.

Sequential SAVLR finds solutions that cost less than those found by the parallel version for 5 out of 6 cases. This is because the parallel version still suffers from the convergence issues mentioned in subsection IV-C, although it solves more subproblems than those solved by the sequential version.

VI. CONCLUSION

The soft reserve and transmission capacity constraints are innovatively exploited within the SAVLR framework. With a much-reduced number of multipliers, both the sequential and the parallel versions consistently provide near-optimal solutions in a computationally efficient manner for the 15-minute UC problem. These methods are generic and may be used to solve other complex MILP problems in power systems and beyond.

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