The MIMO data transfer line with three-frequency quaternion carrier

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Abstract

With the development of communication systems and the Internet, as well as with the growth of services created on the basis of communication networks, the need to increase the capacity of communication channels is increasing. Theoretically, it is shown that it is possible to increase the capacity of communication channels and exceed the “Shannon limit” by moving from the real space of signals on a plane to a multidimensional one with dimension M. In a multidimensional space, each signal is a multidimensional vector, and when such a signal passes through a channel, a MIMO (Many-Input – Many-Output) scheme is formed. As an alternative to existing methods for implementing a MIMO scheme in a physical space with multiple antennas at the input and output of a communication channel, a method for transmitting information using a MIMO scheme in a hypercomplex vector space with one antenna for transmission and one for reception is proposed for wireless communication systems and communication cables.

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Abstract

With the development of communication systems and the Internet, as well as with the growth of services created on the basis of communication networks, the need to increase the capacity of communication channels is increasing. Theoretically, it is shown that it is possible to increase the capacity of communication channels and exceed the "Shannon limit" by moving from the real space of signals on a plane to a multidimensional one with dimension M . In a multidimensional space, each signal is a multidimensional vector, and when such a signal passes through a channel, a MIMO (Many-Input – Many-Output) scheme is formed.

As an alternative to existing methods for implementing a MIMO scheme in a physical space with multiple antennas at the input and output of a communication channel, a method for transmitting information using a MIMO scheme in a hypercomplex vector space with one antenna for transmission and one for reception is proposed for wireless communication systems and communication cables.

It is known that hypercomplex numbers are an extension of complex numbers through the doubling procedure and form a hypercomplex space on imaginary orthogonal axes and one scalar axis orthogonal to them. For example, a quaternion in algebraic form is written as $q= s+xi+yj+zk$, где s, x, y, z – real numbers, i, j, k – imaginary units. A quaternion forms a four-dimensional (4D) space. Hypercomplex numbers are also widely known, such as the octonion in 8D and sedenion in 16D spaces. Accordingly, based on these numbers, MIMO schemes with dimensions of 4D, 8D, 16D are implemented.

Let us represent the mathematical model of a MIMO channel in a hypercomplex space using a square channel matrix of dimension MxM. From an energy point of view, this MIMO channel model is equivalent to the antenna diversity MIMO model. With an orthogonal channel matrix, maximum capacity is ensured. To synthesize the channel matrix, an exponential function of the quaternion and a polar form of representation of exponentials of imaginary units were used. To get rid of imaginary units in the algebraic form of writing a quaternion and for the purpose of forming a channel matrix, it is represented as a real matrix of dimension 4x4, i.e. three-frequency fundamental matrix $\mathbf{\Phi}(\omega_i,\omega_j.\omega_k,t)$.

Using trigonometry formulas, the channel matrix is decomposed into 4 single-frequency matrices of combination frequencies: $\Omega_1 =\omega_i+\omega_j+\omega_k$, $\Omega_2 =\omega_i+\omega_j-\omega_k$,, $\Omega_3 =\omega_i-\omega_j+\omega_k$,, $\Omega_4 =\omega_i-\omega_j-\omega_k$. A three-frequency channel matrix will, accordingly, be equal to the sum of single-frequency matrices. Modulation of subcarrier frequencies is carried out by multiplying the channel matrix by information vectors: $\mathbf{y}(\omega_i,\omega_j.\omega_k,t)=\mathbf{\Phi}(\omega_i,\omega_j.\omega_k,t)\mathbf{x}(0)$. As a result of multiplication, we obtain QPSK modulation for each combination frequency. When adding frequencies, we obtain a multi-frequency oscillation in each element of the modulated vector. Moreover, each element of the output vector $\mathbf{y}(\omega_i,\omega_j.\omega_k,t)$ contains information about all elements of the information vector $\mathbf{x}(0)$, transmitted at all 4 combination frequencies.

Elements of the modulated vector are transmitted sequentially as information elements arrive, and the speed of information transmission at the output is equal to the speed of information arrival at the input of the transmitter. Moreover, each multi-frequency element consumes the entire transmitter power, which is distributed between 4 frequencies and 4 spatial orthogonal coordinate axes. In addition, in the proposed solution only the elements of the multi-frequency vector are transmitted, and not the elements of the channel matrix, as in existing methods. Thus, using the space-time channel matrix synthesized on the basis of a hypercomplex number, we implement the

MIMO scheme in $M=2^n$ - dimensional vector space, where n=2,3,4,… for the number of frequencies $F=2^{M-2}$.

According to the transmission model, interference is added to each pulse of the modulated vector when passing through the communication channel. It is clear that the interferences are not correlated, and with a constant dispersion of the interferences, the interference vector has circular symmetry. Consequently, the interference power is distributed along orthogonal axes and frequencies. Since the interference is white noise, the optimal receiver for the received vector will be a correlator using transposed basis matrices for each combinational frequency.

By multiplying incoming modulated multi-frequency elements in the sum with interference by single-frequency basis matrices with subsequent integration, we obtain an estimate of each information element with uncorrelated interference and at different frequencies. Since when summing signals add up by energy, and noise - by power, the gain in the signal-to-noise ratio (SNR) based on different basic matrices for the quaternion will be equal to 4. After adding up the obtained estimates for different frequencies, we also obtain a gain in SNR by another 4 times. The total gain in SNR will be 16. It is possible to increase the information transmission speed by an appropriate number of times at a given transmission power. By expanding the frequency band, it is also possible to increase the noise immunity and secrecy of transmitter operation. Since the same information is transmitted in each symbol of a multi-frequency vector, the noise immunity to signal fading in time and frequency increases.

I. Introduction

The need to increase the capacity of communication lines is increasing. For example, in the new next-generation 5G mobile communications standard, it is necessary to provide downlink speeds of up to 20 Gbit/s and subscriber speeds of up to 100 Mbit/s. The use of traditional modulation schemes requires a transition to a higher wave range (more than 20 GHz), which is associated with an increase in the power of transmitting devices or a significant decrease in the communication distance.

Theoretically, increased capacity is possible by using a Many-Input – Many-Output (MIMO) scheme [1-4]. This requires that the signal and noise be multidimensional Gaussian processes. Maximum capacity is achieved with a square orthogonal channel matrix. If the dimension of the channel matrix is MxM, then the throughput of the MIMO channel will be M times greater than the throughput of a channel using real 1D or 2D signals.

To increase the speed of information transmission, the multi-frequency information transmission is mainly used, for example, orthogonal frequency division multiplexing - (OFDM). However, such a signal has a high crest factor and, accordingly, increases transmission power costs.

The transmission speed in 6G is expected to increase from 1 Tbit/s. To increase capacity, it is planned to use massive and ultra-massive MIMO, in which hundreds and thousands of active antennas in the form of multi-element digital antenna arrays are connected to base stations. At the same time, multi-element digital antenna arrays must also be implemented on user terminals.

To increase capacity in 6G, it is also planned to use the terahertz range from 300 GHz to 3 THz. However, the communication distance at terahertz waves decreases significantly due to increased propagation losses in free space. There is also an increase in propagation losses through obstacles, such as urban areas, and losses due to rain or fog. In fact, communications on terahertz waves are limited by line-of-sight range.

Currently MIMO scheme is implementing in physical space using many antennas for transmission and reception, which significantly complicates its use. As the number of antennas increases, the difficulties in implementing this method increase. In addition, this implementation makes it impossible to use MIMO in wireline communications.

In works [5, 6], methods were proposed for transmitting and receiving information using the MIMO scheme not in physical space with many antennas, but in complex and hypercomplex spaces using one antenna for transmission and one for reception. It is shown that when using complex numbers in a matrix representation, we obtain a 2-fold gain in noise immunity, and when using quaternions, a 4-fold gain compared to BPSK. Accordingly, it is possible to increase the speed of information transfer by the same amount. Moreover, this method of using the MIMO scheme can also be applied in wired communication channels.

However, to increase the capacity of communication channels, it is also necessary to expand the frequency band. In this case, problems arise regarding the frequency efficiency of the proposed methods. As already mentioned, the use of the OFDM method leads to a large crest factor and, as a consequence, to a loss of power in the amplifiers.

The purpose of this work is to present a method for increasing the capacity using a MIMO scheme in the hypercomplex space of a three-frequency quaternion.

II. MATERIALS AND METHODS FOR SOLVING THE PROBLEM

The mathematical model of a MIMO channel with the same number of inputs and outputs is written in the form [1-4]:

$$\mathbf{s}(t) = \mathbf{H}(t)\mathbf{x}(0) + \mathbf{n}(t), \tag{1}$$

where $\mathbf{H}(t)$ – square channel matrix of dimension *M*x*M*, $\mathbf{x}(0)$ - *M* - dimensional vector information symbols, $\mathbf{n}(t)$ - zero-mean noise vector with circular symmetry.

To implement the MIMO channel model (1) in a hypercomplex space, we formulate the basic requirements for the channel matrix $\mathbf{H}(t)$:

1) Based on the requirement for maximum throughput, the channel matrix $\mathbf{H}(t)$ must be orthogonal. Physically, this means that the signal space must have a maximum volume, which is achieved when the coordinate axes of such a space are orthogonal. It is clear that a space of a given dimension and maximum volume makes it possible to increase the diversity of signals to the maximum extent;

2) Since the channel matrix $\mathbf{H}(t)$ is used as an *M*-dimensional function, which is modulated by *M*-dimensional information vectors $\mathbf{x}(0)$, and then radiated into space, the *M*-dimensional function must be continuous and harmonic;

3) It is known that the main way to increase capacity is to increase the frequency band. Therefore, the channel matrix must be multi-frequency and decomposed into single-frequency matrices;

4) Since optimal signal reception under the influence of white noise is carried out using a dual basis and correlation processing using basis functions, the channel matrix must be decomposed into basis matrices.

Let's consider the problem of synthesizing a multi-frequency channel matrix using a quaternion as a hypercomplex number. In algebraic form we write the quaternion as [6]:

$$q = s + xi + yj + zk, \tag{2}$$

where *s*, *x*, *y*, *z* – real numbers.

Let's present operations with imaginary units in the form of a table.

Table 1. *Quaternion imaginary unit multiplication operations*.

$\times$	1	i	j	k
1	1	i	j	k
i	i	-1	k	$-j$
j	j	$-k$	-1	i
k	k	j	$-i$	-1

For the quaternion (2) we write the exponential function:

$$\mathrm{e}^{q} = \mathrm{e}^{s+xi+yj+zk} = \mathrm{e}^{s}\left(\cos x + i\sin x\right)\left(\cos y + j\sin y\right)\left(\cos z + k\sin z\right). \tag{3}$$

For a single-frequency quaternion $x = y = z$ therefore expression (3) will take the form:

$$e^q = e^{s+x(i+j+k)} = e^s e^{x(i+j+k)} = e^s e^{x\hat{i}} = e^s\left(\cos x + \hat{i}\sin x\right), \tag{4}$$

where $\hat{i} = (i+j+k)/\sqrt{3}$ - imaginary unit. Expression (4) was also used in [7 - 10] to obtain spectra from a single-frequency quaternion and as a channel matrix for a 4×4 MIMO scheme [6].

Just as for a complex number in exponential representation, the coefficients for the imaginary units of the quaternion in the polar form of notation have the physical meaning of the rotation angle. In addition, frequency conversions in radio engineering problems are considered mainly for time-varying signals. Therefore, we write the angles *x*, *y*, *z* in (3) as functions of time: $x(t) = \omega_i t$, $y(t) = \omega_j t$, $z(t) = \omega_k t$, where ω_i, ω_j, ω_k are the angular frequencies on the axes *i*, *j*, *k*. The radius of rotation in 4D space is calculated as $r = \sqrt{s^2 + x^2 + y^2 + z^2}$. After multiplying the expressions in parentheses in formula (3) and grouping by the real and imaginary parts, we obtain an exponential function in the form of a three-frequency quaternion, which we denote as

$$f\left(q(\omega_i,\omega_j,\omega_k,t)\right) = \tag{5}$$
$$= p(\omega_i,\omega_j,\omega_k,t) + iu(\omega_i,\omega_j,\omega_k,t) + jv(\omega_i,\omega_j,\omega_k,t) + kw(\omega_i,\omega_j,\omega_k,t).$$

After grouping similar terms, the components in expression (5) will take the form:

$$p(\omega_i,\omega_j,\omega_k,t) = \cos\omega_i t\cos\omega_j t\cos\omega_k t - \sin\omega_i t\sin\omega_j t\sin\omega_k t, \tag{6}$$
$$u(\omega_i,\omega_j,\omega_k,t) = \sin\omega_i t\cos\omega_j t\cos\omega_k t + \cos\omega_i t\sin\omega_j t\sin\omega_k t,$$
$$v(\omega_i,\omega_j,\omega_k,t) = \cos\omega_i t\sin\omega_j t\cos\omega_k t - \sin\omega_i t\cos\omega_j t\sin\omega_k t,$$
$$w(\omega_i,\omega_j,\omega_k,t) = \sin\omega_i t\sin\omega_j t\cos\omega_k t + \cos\omega_i t\cos\omega_j t\sin\omega_k t.$$

As can be seen from (6), we received 8 combinations of products of cosines and sines of different frequencies. Let us transform expressions (6) using well-known trigonometry formulas for products of three combinations with sines and cosines. According to these formulas, the products of three sines and cosines in various combinations are converted into the sum of sines and cosines from the sum of frequencies in various combinations. Let us denote the combinations of angular frequencies obtained as a result of the expansion as

$$\Omega_1 = \omega_i + \omega_j + \omega_k = 2\pi(f_i + f_j + f_k),\ \Omega_2 = \omega_i + \omega_j - \omega_k = 2\pi(f_i + f_j - f_k), \tag{7}$$
$$\Omega_3 = \omega_i - \omega_j + \omega_k = 2\pi(f_i - f_j + f_k),.\ \Omega_4 = \omega_i - \omega_j - \omega_k = 2\pi(f_i - f_j - f_k).$$

So, for three generating (reference) frequencies ω_i, ω_j, ω_k we get $2^{4-2} = 2^2 = 4$ combinations of positive frequencies Ω_n, $n = 1,2,3,4$. Negative frequencies will appear when the signal spectrum is transferred to the carrier frequency ω_c. After decomposing the products of sines and cosines (6) into sums of sines and cosines, bringing similar terms and grouping by combination frequencies, we obtain the following expressions of functions (6) for combination frequencies (7):

$$4p(\Omega_1,\Omega_2,\Omega_3,\Omega_4,t) = \tag{8}$$
$$= \cos(\Omega_1 t) + \sin(\Omega_1 t) + \cos(\Omega_2 t) - \sin(\Omega_2 t) + \cos(\Omega_3 t) - \sin(\Omega_3 t) + \cos(\Omega_4 t) + \sin(\Omega_4 t),$$
$$4u(\Omega_1,\Omega_2,\Omega_3,\Omega_4,t) =$$
$$= -\cos(\Omega_1 t) + \sin(\Omega_1 t) + \cos(\Omega_2 t) + \sin(\Omega_2 t) + \cos(\Omega_3 t) + \sin(\Omega_3 t) - \cos(\Omega_4 t) + \sin(\Omega_4 t),$$
$$4v(\Omega_1,\Omega_2,\Omega_3,\Omega_4,t) =$$
$$= \cos(\Omega_1 t) + \sin(\Omega_1 t) - \cos(\Omega_2 t) + \sin(\Omega_2 t) + \cos(\Omega_3 t) - \sin(\Omega_3 t) - \cos(\Omega_4 t) - \sin(\Omega_4 t),$$
$$4w(\Omega_1,\Omega_2,\Omega_3,\Omega_4,t) =$$
$$= -\cos(\Omega_1 t) + \sin(\Omega_1 t) - \sin(\Omega_2 t) - \cos(\Omega_2 t) + \sin(\Omega_3 t) + \cos(\Omega_3 t) + \cos(\Omega_4 t) - \sin(\Omega_4 t).$$

Thus, we have obtained expressions for the exponential function (3) of a three-frequency quaternion in the form of a sum of functions (5), which are represented by 8 combinations of products of cosines and sines (6) of different reference frequencies (7). Also, using trigonometry formulas for the products of three combinations with sines and cosines, we obtained the exponential function (3) in the form of sums of cosines and sines (8) from various combination frequencies (7).

To get rid of imaginary units in the algebraic form of writing the quaternion (2), we present it in the form of a real matrix of dimension 4×4 [6-10]:

$$\mathbf{Q}=\begin{bmatrix} s & x & y & z \\ -x & s & -z & y \\ -y & z & s & -x \\ -z & -y & x & s \end{bmatrix}. \tag{9}$$

Matrix (9) is decomposed into *basis matrices* **E**, **I**, **J**, **K** and quaternion (9) is written as a sum of matrices:

$$\mathbf{Q}=\mathbf{E}s+\mathbf{I}x+\mathbf{J}y+\mathbf{K}z,$$

where

$$\mathbf{E}=\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},\ \mathbf{I}=\begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{bmatrix},\ \mathbf{J}=\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{bmatrix},\ \mathbf{K}=\begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{bmatrix}.$$

The corresponding basis matrices of the quaternion, as well as the elements *i*, *j* and *k*, are related by the multiplication rules presented in Table 2:

Table 2. *Multiplication operations of quaternion basis matrices*.

×	**E**	**I**	**J**	**K**
E	**E**	**I**	**J**	**K**
I	**I**	–**E**	**K**	–**J**
J	**J**	–**K**	–**E**	**I**
K	**K**	**J**	–**I**	–**E**

(10)

As can be seen from (10) the structure of the basis matrices **E**, **I**, **J**, **K**, they are *spatial basis matrices*, i.e. bases of 4D space. These matrices are also permutation matrices with a change of sign of 2 elements. The basis matrices will be orthogonal: $\mathbf{II}^{\mathrm{T}}=\mathbf{E}$, $\mathbf{JJ}^{\mathrm{T}}=\mathbf{E}$, $\mathbf{KK}^{\mathrm{T}}=\mathbf{E}$. Note that the basis matrices of a quaternion are also quaternions in matrix representation.

It is convenient to represent the information transfer model as a model in state space using the dynamics equation in the form [11]:

$$\dot{\mathbf{x}}(t)=\mathbf{A}\mathbf{x}(t). \tag{11}$$

We write the *state transition matrix* **A** using the imaginary part of matrix (9) in the form:

$$\mathbf{A}=\omega_i\mathbf{I}+\omega_j\mathbf{J}+\omega_k\mathbf{K}. \tag{12}$$

Let's call matrix **A** as *a matrix of quaternion reference frequencies*, where $\omega_i=2\pi/T_i=2\pi f_i$ is the angular frequency of the imaginary axis *i*, radians/s; $T_i=1/f_i$ – period of frequency f_i, s.; $\omega_j=2\pi/T_j=2\pi f_j$ - angular frequency of the imaginary axis *j*, radian/s; $T_j=1/f_j$ – period of frequency f_j, s.; $\omega_k=2\pi/T_k=2\pi f_k$ - angular frequency of the imaginary axis *k*, radian/s; $T_k=1/f_k$ – period of frequency f_k, s.

The matrix of quaternion reference frequencies **A** is a differential operator for the state space model (11). The solution to equation (11) will be the exponent of matrix (12) for functions (6):

$$e^{\mathbf{A}t}=e^{(\omega_i\mathbf{I}+\omega_j\mathbf{J}+\omega_k\mathbf{K})t}=\mathbf{\Phi}(\omega_i,\omega_j,\omega_k,t)= \tag{13}$$
$$=\mathbf{E}p(\omega_i,\omega_j,\omega_k,t)+\mathbf{I}u(\omega_i,\omega_j,\omega_k,t)+\mathbf{J}v(\omega_i,\omega_j,\omega_k,t)+\mathbf{K}w(\omega_i,\omega_j,\omega_k,t).$$

Since matrix (13) is a solution to the differential equation (11), the matrix $\mathbf{\Phi}(\omega_i,\omega_j,\omega_k,t)$ is called the *fundamental matrix*.

Substituting expressions (6) into the matrix $\mathbf{\Phi}(\omega_i,\omega_j,\omega_k,t)$, we obtain a fundamental matrix in the form of sums of products of cosines and sines in various combinations:

$$\mathbf{\Phi}(\omega_i,\omega_j,\omega_k,t)= \tag{14}$$

$$=\left[\begin{matrix} \cos\omega_i t\cos\omega_j t\cos\omega_k t-\sin\omega_i t\sin\omega_j t\sin\omega_k t & \sin\omega_i t\cos\omega_j t\cos\omega_k t+\cos\omega_i t\sin\omega_j t\sin\omega_k t \\ -(\sin\omega_i t\cos\omega_j t\cos\omega_k t+\cos\omega_i t\sin\omega_j t\sin\omega_k t) & \cos\omega_i t\cos\omega_j t\cos\omega_k t-\sin\omega_i t\sin\omega_j t\sin\omega_k t \\ -(\cos\omega_i t\sin\omega_j t\cos\omega_k t-\sin\omega_i t\cos\omega_j t\sin\omega_k t) & \cos\omega_i t\cos\omega_j t\sin\omega_k t+\sin\omega_i t\sin\omega_j t\cos\omega_k t \\ -(\cos\omega_i t\cos\omega_j t\sin\omega_k t+\sin\omega_i t\sin\omega_j t\cos\omega_k t) & -(\cos\omega_i t\sin\omega_j t\cos\omega_k t-\sin\omega_i t\cos\omega_j t\sin\omega_k t) \end{matrix}\right.$$

$$\left.\begin{matrix} \cos\omega_i t\sin\omega_j t\cos\omega_k t-\sin\omega_i t\cos\omega_j t\sin\omega_k t & \cos\omega_i t\cos\omega_j t\sin\omega_k t+\sin\omega_i t\sin\omega_j t\cos\omega_k t \\ -(\cos\omega_i t\cos\omega_j t\sin\omega_k t+\sin\omega_i t\sin\omega_j t\cos\omega_k t) & \cos\omega_i t\sin\omega_j t\cos\omega_k t-\sin\omega_i t\cos\omega_j t\sin\omega_k t \\ \cos\omega_i t\cos\omega_j t\cos\omega_k t-\sin\omega_i t\sin\omega_j t\sin\omega_k t & -(\sin\omega_i t\cos\omega_j t\cos\omega_k t+\cos\omega_i t\sin\omega_j t\sin\omega_k t) \\ \sin\omega_i t\cos\omega_j t\cos\omega_k t+\cos\omega_i t\sin\omega_j t\sin\omega_k t & \cos\omega_i t\cos\omega_j t\cos\omega_k t-\sin\omega_i t\sin\omega_j t\sin\omega_k t \end{matrix}\right]$$

As in the case of a communication line with a single-frequency quaternion [6], the three-frequency matrix (14) will serve as the channel matrix of the MIMO scheme. Therefore, by analogy with a single-frequency matrix, let's call it a *three-frequency quaternion carrier*.

Let us denote the basis matrices for the fundamental matrix (14) of the reference angular frequencies $\omega_i,\omega_j,\omega_k$, as

$$\begin{aligned} &\mathbf{\Phi_E}(\omega_i,\omega_j,\omega_k,t)=\Big(\cos(\omega_i t)\cos(\omega_j t)\cos(\omega_k t)-\sin(\omega_i t)\sin(\omega_j t)\sin(\omega_k t)\Big)\mathbf{E}, \\ &\mathbf{\Phi_I}(\omega_i,\omega_j,\omega_k,t)=\Big(\sin(\omega_i t)\cos(\omega_j t)\cos(\omega_k t)+\cos(\omega_i t)\sin(\omega_j t)\sin(\omega_k t)\Big)\mathbf{I}, \\ &\mathbf{\Phi_J}(\omega_i,\omega_j,\omega_k,t)=\Big(\cos(\omega_i t)\sin(\omega_j t)\cos(\omega_k t)-\sin(\omega_i t)\cos(\omega_j t)\sin(\omega_k t)\Big)\mathbf{J}, \\ &\mathbf{\Phi_K}(\omega_i,\omega_j,\omega_k,t)=\Big(\cos(\omega_i t)\cos(\omega_j t)\sin(\omega_k t)+\sin(\omega_i t)\sin(\omega_j t)\cos(\omega_k t)\Big)\mathbf{K}. \end{aligned} \tag{15}$$

Using notation (15), we write the three-frequency fundamental matrix (14) as a sum of basis matrices:

$$\begin{aligned} &\mathbf{\Phi}(\omega_i,\omega_j,\omega_k,t)= \\ &=\mathbf{\Phi_E}(\omega_i,\omega_j,\omega_k,t)+\mathbf{\Phi_I}(\omega_i,\omega_j,\omega_k,t)+\mathbf{\Phi_J}(\omega_i,\omega_j,\omega_k,t)+\mathbf{\Phi_K}(\omega_i,\omega_j,\omega_k,t). \end{aligned} \tag{16}$$

The fundamental matrix (16) is orthogonal, since

$$\mathbf{\Phi}^{\mathrm{T}}(\omega_i,\omega_j,\omega_k,t)\mathbf{\Phi}(\omega_i,\omega_j,\omega_k,t)=\mathbf{\Phi}(\omega_i,\omega_j,\omega_k,t)\mathbf{\Phi}^{\mathrm{T}}(\omega_i,\omega_j,\omega_k,t)=\mathbf{E}.$$

Using expressions (8), we decompose the fundamental matrix (14) of the reference angular frequencies ω_i, ω_j, ω_k into single-frequency matrices of frequency combinations (7).

Let's group by frequencies and write single-frequency matrices through the basis matrices **E**, **I**, **J**, **K** in the form:

$$\begin{aligned} &\mathbf{\Phi}_1(\Omega_1,t)=\frac{1}{4}\Big[\Big(\cos(\Omega_1 t)+\sin(\Omega_1 t)\Big)\mathbf{E}+\Big(-\cos(\Omega_1 t)+\sin(\Omega_1 t)\Big)\mathbf{I}+ \\ &\qquad+\Big(\cos(\Omega_1 t)+\sin(\Omega_1 t)\Big)\mathbf{J}+\Big(-\cos(\Omega_1 t)+\sin(\Omega_1 t)\Big)\mathbf{K}\Big], \\ &\mathbf{\Phi}_2(\Omega_2,t)=\frac{1}{4}\Big[\Big(\cos(\Omega_2 t)-\sin(\Omega_2 t)\Big)\mathbf{E}+\Big(\cos(\Omega_2 t)+\sin(\Omega_2 t)\Big)\mathbf{I}+ \\ &\qquad+\Big(-\cos(\Omega_2 t)+\sin(\Omega_2 t)\Big)\mathbf{J}+\Big(-\cos(\Omega_2 t)-\sin(\Omega_2 t)\Big)\mathbf{K}\Big], \\ &\mathbf{\Phi}_3(\Omega_3,t)=\frac{1}{4}\Big[\Big(\cos(\Omega_3 t)-\sin(\Omega_3 t)\Big)\mathbf{E}+\Big(\cos(\Omega_3 t)+\sin(\Omega_3 t)\Big)\mathbf{I}+ \\ &\qquad+\Big(\cos(\Omega_3 t)-\sin(\Omega_3 t)\Big)\mathbf{J}+\Big(\cos(\Omega_3 t)+\sin(\Omega_3 t)\Big)\mathbf{K}\Big], \end{aligned} \tag{17}$$

$$\mathbf{\Phi}_4(\Omega_4,t)=\frac{1}{4}\Big[\big(\cos(\Omega_4 t)+\sin(\Omega_4 t)\big)\mathbf{E}+\big(-\cos(\Omega_4 t)+\sin(\Omega_4 t)\big)\mathbf{I}+$$
$$+\big(-\cos(\Omega_4 t)-\sin(\Omega_4 t)\big)\mathbf{J}+\big(\cos(\Omega_4 t)-\sin(\Omega_4 t)\big)\mathbf{K}\Big].$$

From this we can conclude that the fundamental three-frequency quaternion matrix (14) is equal to the sum of single-frequency quaternion matrices:

$$\mathbf{\Phi}(\omega_i,\omega_j,\omega_k,t)=\mathbf{\Phi}_1(\Omega_1,t)+\mathbf{\Phi}_2(\Omega_2,t)+\mathbf{\Phi}_3(\Omega_3,t)+\mathbf{\Phi}_4(\Omega_4,t)\,. \qquad (18)$$

In accordance with (7), we present the matrix for converting reference frequencies ω_i, ω_j, ω_k into combinational ones Ω_n, $n=1,2,3,4$, as:

$$\mathbf{\Omega}=\begin{bmatrix}1 & 1 & 1\\ 1 & 1 & -1\\ 1 & -1 & 1\\ 1 & -1 & -1\end{bmatrix}.$$

We write the reference frequencies as a vector $\mathbf{f}=\begin{bmatrix}f_1 & f_2 & f_3\end{bmatrix}^{\mathrm{T}}$ and the vector of combination frequencies, as $\mathbf{F}=\begin{bmatrix}F_1 & F_2 & F_3 & F_4\end{bmatrix}^{\mathrm{T}}$. From here $\mathbf{F}=\mathbf{\Omega f}$.

The matrix for converting combination frequencies into reference ones will look like:

$$\mathbf{\Omega}^{\mathrm{T}}=\frac{1}{4}\begin{bmatrix}1 & 1 & 1 & 1\\ 1 & 1 & -1 & -1\\ 1 & -1 & 1 & -1\end{bmatrix}.$$

For example, at $\mathbf{f}=\begin{bmatrix}6 & -2 & -1\end{bmatrix}^{\mathrm{T}}$ we get

$$\mathbf{F}=\mathbf{\Omega f}=\begin{bmatrix}1 & 1 & 1\\ 1 & 1 & -1\\ 1 & -1 & 1\\ 1 & -1 & -1\end{bmatrix}\begin{bmatrix}6\\ -2\\ -1\end{bmatrix}=\begin{bmatrix}3\\ 5\\ 7\\ 9\end{bmatrix},$$

and

$$\mathbf{f}=\frac{1}{4}\mathbf{\Omega}^{\mathrm{T}}\mathbf{F}=\frac{1}{4}\begin{bmatrix}1 & 1 & 1 & 1\\ 1 & 1 & -1 & -1\\ 1 & -1 & 1 & -1\end{bmatrix}\begin{bmatrix}3\\ 5\\ 7\\ 9\end{bmatrix}=\begin{bmatrix}6\\ -2\\ -1\end{bmatrix}.$$

As can be seen, the matrix $\mathbf{\Omega}^{\mathrm{T}}$ is a pseudo-inverse. Thus, we can obtain the necessary values of the reference and combination frequencies.

III. INFORMATION TRANSMISSION LINE DIAGRAM

Modulation of three-frequency quaternion carrier

We will modulate the three-frequency carrier in the same way as in [6], by multiplying the information vector $\mathbf{x}(0)=\begin{bmatrix}x_0 & x_1 & x_2 & x_3\end{bmatrix}^{\mathrm{T}}$ by the fundamental matrix (14), which acts as a channel matrix in the MIMO scheme (1). As a result, we obtain the output modulated vector

$$\mathbf{y}(\omega_i,\omega_j,\omega_k,t)=\mathbf{\Phi}(\omega_i,\omega_j,\omega_k,t)\mathbf{x}(0)\,. \qquad (19)$$

Since the channel matrix is the fundamental matrix for the dynamics equation in state space (11), then, by definition, the information vector $\mathbf{x}(0)$ is also the vector of the initial state of the dynamic system.

Let's consider the case of a binary information vector $\mathbf{x}(0)=[\pm 1 \quad \pm 1 \quad \pm 1 \quad \pm 1]^{\mathrm{T}}$. We write all possible combinations of information vectors in the form of a matrix:

$$\mathbf{x}(0)=\begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 \\ 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 \\ 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 \\ 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \end{bmatrix}. \quad (20)$$

More generally, the initial vector can take on any value in 4D space. A binary vector $\mathbf{x}(0)$ can take on 16 different values. We will depict possible combinations of information vectors as states in two 3D spaces, as shown in Figures 1 and 2. Positive values of the most significant bit of the information vector are shown in red in the form of a point mass (charge), and negative values in blue.

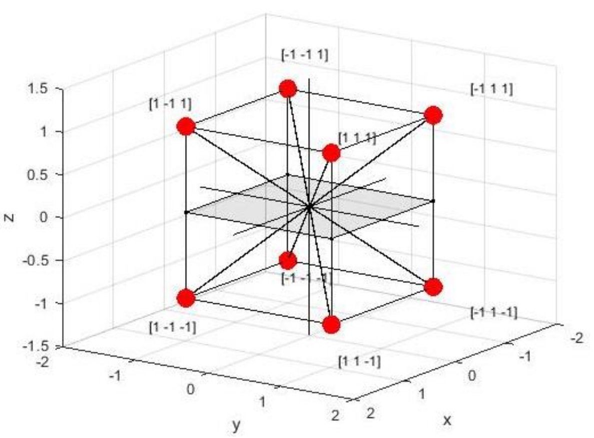


Figure 1. Vectors of information pulses in the form of a quaternion for positive values of the first element of the information vector

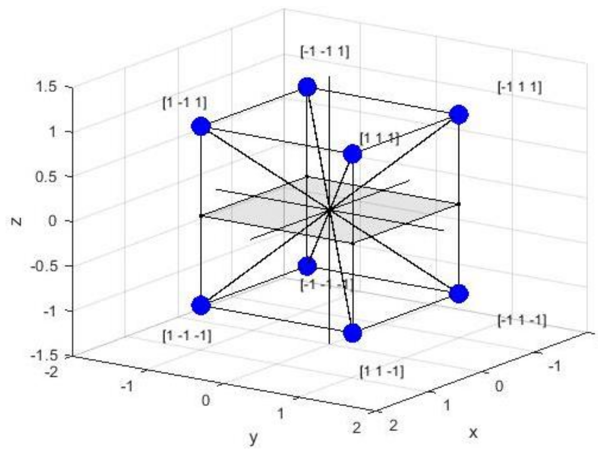


Figure 2. Vectors of information pulses in the form of a quaternion for negative values of the first element of the information vector

Since the three-frequency matrix is equal to the sum of single-frequency matrices (18), then modulation by single-frequency matrices will, accordingly, have the form:

$$\mathbf{y}(\Omega_1,\Omega_2,\Omega_3,\Omega_4,t)=\mathbf{\Phi}(\omega_i,\omega_j,\omega_k,t)\mathbf{x}(0)=\mathbf{y}(\omega_i,\omega_j,\omega_k,t)= \quad (21)$$
$$=\mathbf{\Phi}_1(\Omega_1,t)\mathbf{x}(0)+\mathbf{\Phi}_2(\Omega_2,t)\mathbf{x}(0)+\mathbf{\Phi}_3(\Omega_3,t)\mathbf{x}(0)+\mathbf{\Phi}_4(\Omega_4,t)\mathbf{x}(0)=$$
$$=\mathbf{y}_1(\Omega_1,t)+\mathbf{y}_2(\Omega_2,t)+\mathbf{y}_3(\Omega_3,t)+\mathbf{y}_4(\Omega_4,t).$$

Since channel matrix $\mathbf{\Phi}(\omega_i,\omega_j,\omega_k,t)$ is orthogonal, the power of the output vector will be the same as the input one.

Figure 3 shows a visualization of the formation of a modulated signal in a transmitting device with an information vector of $\mathbf{x}(0)=[1 \quad 1 \quad -1 \quad 1]^{\mathrm{T}}$. The input of the transmitting device receives binary data pulses 0, 1. This data sequence is converted into bipolar pulses by replacing $0\rightarrow 1$ and $1\rightarrow -1$. The first 4 pulses of the bipolar sequence, as they arrive, are converted into a 4D vector by delaying the 1st received pulse by a pulse duration of 3*T*, the second by 2*T*, the third by *T*, and the last pulse arrives in real time. During the arrival of the 4th pulse, the three-frequency quaternion carrier is modulated in the form of a 4x4 matrix (14) by multiplying it by the vector of incoming pulses. Multiplication occurs by four rows of the matrix during the arrival and formation of the next 4D vector of pulses.

Since a three-frequency matrix is equal to the sum of single-frequency ones (18), modulation is easier to implement by multiplying the vector by single-frequency channel matrices followed by summation (21).

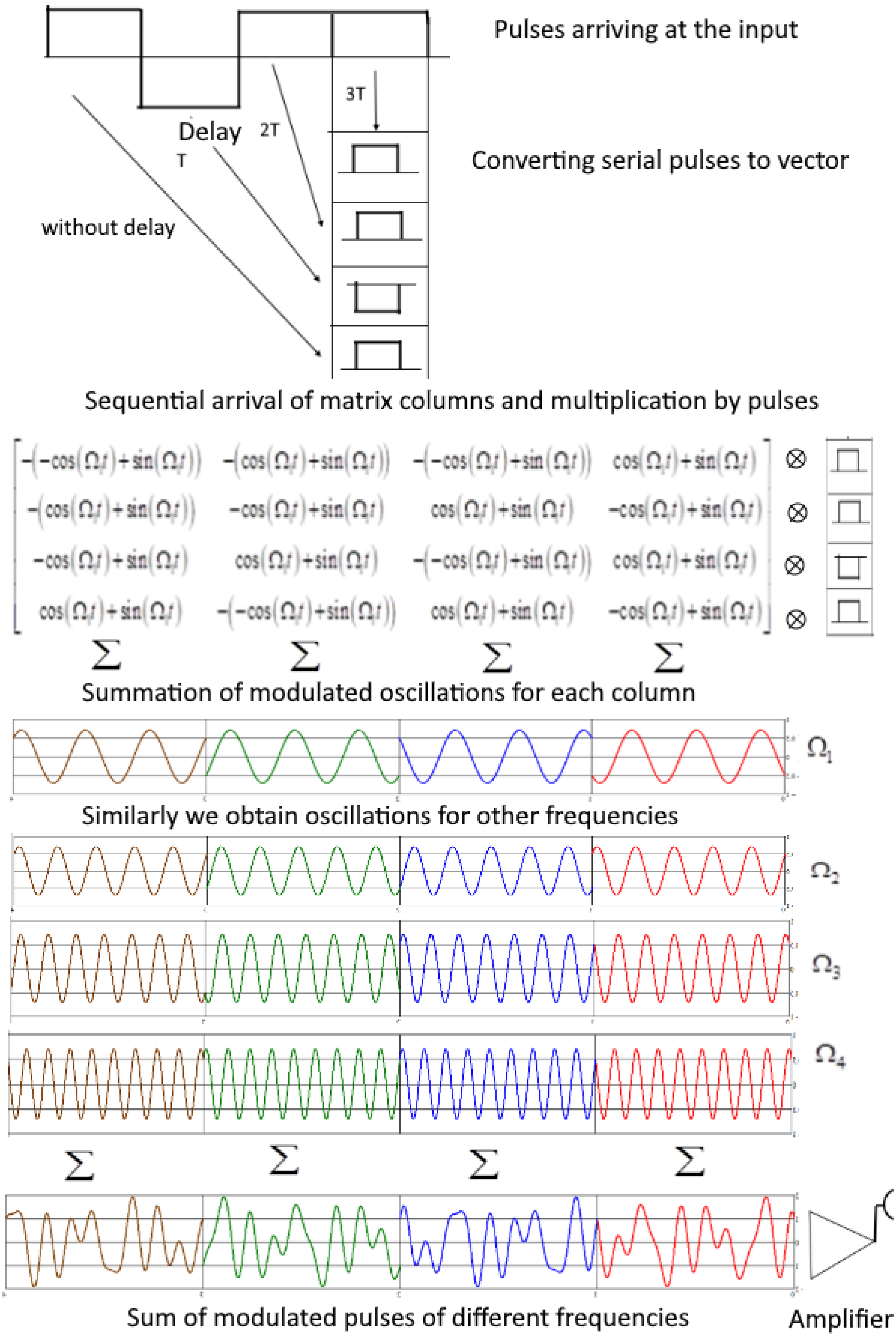


Figure 3. Modulator diagram

Let us first consider multiplying the information vector by a channel matrix with frequency Ω_1:

$$\mathbf{\Phi}_1(\Omega_1,t)=$$

$$=\frac{1}{4}\begin{bmatrix} \cos(\Omega_1 t)+\sin(\Omega_1 t) & -\cos(\Omega_1 t)+\sin(\Omega_1 t) & \cos(\Omega_1 t)+\sin(\Omega_1 t) & -\cos(\Omega_1 t)+\sin(\Omega_1 t) \\ -\left(-\cos(\Omega_1 t)+\sin(\Omega_1 t)\right) & \cos(\Omega_1 t)+\sin(\Omega_1 t) & -\left(-\cos(\Omega_1 t)+\sin(\Omega_1 t)\right) & \cos(\Omega_1 t)+\sin(\Omega_1 t) \\ -\left(\cos(\Omega_1 t)+\sin(\Omega_1 t)\right) & -\cos(\Omega_1 t)+\sin(\Omega_1 t) & \cos(\Omega_1 t)+\sin(\Omega_1 t) & -\left(-\cos(\Omega_1 t)+\sin(\Omega_1 t)\right) \\ -\left(-\cos(\Omega_1 t)+\sin(\Omega_1 t)\right) & -\left(\cos(\Omega_1 t)+\sin(\Omega_1 t)\right) & -\cos(\Omega_1 t)+\sin(\Omega_1 t) & \cos(\Omega_1 t)+\sin(\Omega_1 t) \end{bmatrix}.$$

To obtain modulated elements in real time, i.e., sequentially for each clock period, it is necessary to represent all the rows in the form of columns, i.e., transpose the matrix, and write these columns in reverse order:

$$\overset{\leftrightarrow}{\mathbf{\Phi}}{}_1^{\mathrm{T}}(\Omega_1,t)=$$

$$=\frac{1}{4}\begin{bmatrix} -\left(-\cos(\Omega_1 t)+\sin(\Omega_1 t)\right) & -\left(\cos(\Omega_1 t)+\sin(\Omega_1 t)\right) & -\left(-\cos(\Omega_1 t)+\sin(\Omega_1 t)\right) & \cos(\Omega_1 t)+\sin(\Omega_1 t) \\ -\left(\cos(\Omega_1 t)+\sin(\Omega_1 t)\right) & -\cos(\Omega_1 t)+\sin(\Omega_1 t) & \cos(\Omega_1 t)+\sin(\Omega_1 t) & -\cos(\Omega_1 t)+\sin(\Omega_1 t) \\ -\cos(\Omega_1 t)+\sin(\Omega_1 t) & \cos(\Omega_1 t)+\sin(\Omega_1 t) & -\left(-\cos(\Omega_1 t)+\sin(\Omega_1 t)\right) & \cos(\Omega_1 t)+\sin(\Omega_1 t) \\ \cos(\Omega_1 t)+\sin(\Omega_1 t) & -\left(-\cos(\Omega_1 t)+\sin(\Omega_1 t)\right) & \cos(\Omega_1 t)+\sin(\Omega_1 t) & -\cos(\Omega_1 t)+\sin(\Omega_1 t) \end{bmatrix}.$$

As you can see, matrix elements $\pm\left(\cos(\Omega_1 t)+\sin(\Omega_1 t)\right)$ and $\pm\left(\cos(\Omega_1 t)-\sin(\Omega_1 t)\right)$ are conjugate oscillations with the initial phase $\pi/4$, $3\pi/4$, $5\pi/4$ and $7\pi/4$. Thus, when an information vector is multiplied by a matrix, QPSK modulation occurs.

As shown in Figure 3, simultaneously with the time-sequential multiplication of the information vector by the elements of the columns at the duration of the cycle, summation occurs, as a result of which we obtain oscillations with the corresponding initial state. In 4 clock cycles we obtain a sequence of modulated oscillations with a frequency with the corresponding initial states that determine the initial phases of the oscillations.

In the same way, at the same time, we obtain modulated oscillations with frequencies Ω_2, Ω_3, and Ω_4. As shown in the lower part of Figure 3, modulated oscillations with different frequencies are added in real time, resulting in a vector of elements from combination frequencies: $\mathbf{y}(\Omega_1,\Omega_2,\Omega_3,\Omega_4,t)=\mathbf{y}(\omega_i,\omega_j,\omega_k,t)$. Since the three-frequency fundamental matrix is orthogonal, the power of each resulting three-frequency element will be equal to the power of the information element, i.e. in our case 1. According to (17), the power of each single-frequency element will be equal to 1/4. Note that for some combinations of information elements we can obtain single-frequency signals not only with phase modulation, but also with amplitude and space modulation.

In the process of obtaining combined oscillations, they enter, according to Figure 3, a power amplifier and are sequentially emitted into space or transmitted through wires. First, from right to left, the 1st element of the vector is emitted $y_1(\Omega_1,\Omega_2,\Omega_3,\Omega_4,t)$, then the 2nd $y_2(\Omega_1,\Omega_2,\Omega_3,\Omega_4,t)$, 3rd $y_3(\Omega_1,\Omega_2,\Omega_3,\Omega_4,t)$ and 4th $y_4(\Omega_1,\Omega_2,\Omega_3,\Omega_4,t)$.

It is possible to use a separate power amplifier for each single-frequency oscillation, and summation should be done after amplification. In the process of emitting oscillations, a new information vector is formed from newly received impulses and the process is repeated. Thus, the modulation scheme requires a time delay of 3T.

Demodulation of a three-frequency quaternion carrier

When modulated oscillations from combination frequencies pass through the communication channel, interference is added to them. As a noise, consider 4D white noise with circular symmetry. Let's imagine the interference as a 4D vector and write the received signal (19) or (21) as:

$$\mathbf{s}(t)=\begin{bmatrix} y_1(\Omega_1,\Omega_2,\Omega_3,\Omega_4,t) \\ y_2(\Omega_1,\Omega_2,\Omega_3,\Omega_4,t) \\ y_3(\Omega_1,\Omega_2,\Omega_3,\Omega_4,t) \\ y_4(\Omega_1,\Omega_2,\Omega_3,\Omega_4,t) \end{bmatrix}+\begin{bmatrix} n_1(t) \\ n_2(t) \\ n_3(t) \\ n_5(t) \end{bmatrix}. \tag{22}$$

Just as in [6], demodulation will be carried out using transposed basic single-frequency matrices. When separating the signal spectra and at a constant spectral density of the interference power, we obtain that in the signal band for each element of the output vector the interference will be 4 times less than the total power. That is, the interference dispersion vector when distributing the interference power over the frequencies of each element can be written as $\boldsymbol{\sigma}^2=\frac{1}{4}\begin{bmatrix}\sigma^2 & \sigma^2 & \sigma^2 & \sigma^2\end{bmatrix}^{\mathrm{T}}$. As stated, 4 information elements are distributed along orthogonal axes within each single-frequency element. The circular symmetry of the interference in the form of white noise means that the noise on the orthogonal coordinate axes is not correlated. Consequently, the interference acting on them is also distributed in power over 4 orthogonal axes and, accordingly, has 4 times less power. Hence, the vector of interference dispersion acting on a separate information element, taking into account the distribution over frequencies and orthogonal axes, has the form $\boldsymbol{\sigma}^2=\frac{1}{16}\begin{bmatrix}\sigma^2 & \sigma^2 & \sigma^2 & \sigma^2\end{bmatrix}^{\mathrm{T}}$.

Thus, when using the MIMO scheme in a 3-frequency hypercomplex quaternion space, the power of both the signal and the interference is distributed over 4 frequencies and 4 orthogonal axes so that each information element and the interference acting on it have a power of 16 times less and maintain the signal-to-noise ratio (SNR).

In accordance with the theory of optimal reception in white noise, demodulation will be carried out using transposed basic single-frequency matrices. Let's first consider the demodulation circuit for combination frequency $\Omega_1=\omega_i+\omega_j+\omega_k$. Let us decompose the single-frequency channel matrix (17) for frequency Ω_1 into basis matrices, which we denote as

$$\mathbf{E}_1^{\mathrm{T}}(\Omega_1,t)=\left(\cos(\Omega_1 t)+\sin(\Omega_1 t)\right)\mathbf{E}^{\mathrm{T}},\ \mathbf{I}_1^{\mathrm{T}}(\Omega_1,t)=\left(-\cos(\Omega_1 t)+\sin(\Omega_1 t)\right)\mathbf{I}^{\mathrm{T}}, \tag{23}$$
$$\mathbf{J}_1^{\mathrm{T}}(\Omega_1,t)=\left(\cos(\Omega_1 t)+\sin(\Omega_1 t)\right)\mathbf{J}^{\mathrm{T}},\ \mathbf{K}_1^{\mathrm{T}}(\Omega_1,t)=\left(-\cos(\Omega_1 t)+\sin(\Omega_1 t)\right)\mathbf{K}^{\mathrm{T}}.$$

Since white noise is considered as interference, the optimal receiver for each element is a correlator. Demodulation will be carried out by multiplying the received vector (22) by transposed basis matrices (23) with subsequent integration over the pulse duration *T*. Since during integration high-frequency components are compensated, at the end of integration the basis matrices (23) will be orthogonal:

$$\int_0^T \mathbf{E}_1{}^{\mathrm{T}}(\Omega_1,t)\mathbf{E}_1(\Omega_1,t)dt=\int_0^T\left(\cos(\Omega_1 t)+\sin(\Omega_1 t)\right)^2\mathbf{E}dt=\mathbf{E}, \tag{24}$$
$$\int_0^T \mathbf{I}_1{}^{\mathrm{T}}(\Omega_1,t)\mathbf{I}_1(\Omega_1,t)dt=\int_0^T\left(\cos(\Omega_1 t)-\sin(\Omega_1 t)\right)^2\mathbf{E}dt=\mathbf{E},$$
$$\int_0^T \mathbf{J}_1{}^{\mathrm{T}}(\Omega_1,t)\mathbf{J}_1(\Omega_1,t)dt=\int_0^T\left(\cos(\Omega_1 t)+\sin(\Omega_1 t)\right)^2\mathbf{E}dt=\mathbf{E},$$
$$\int_0^T \mathbf{K}_1{}^{\mathrm{T}}(\Omega_1,t)\mathbf{K}_1(\Omega_1,t)dt=\int_0^T\left(\cos(\Omega_1 t)-\sin(\Omega_1 t)\right)^2\mathbf{E}dt=\mathbf{E}.$$

The basis matrices for matrices (17) with frequencies Ω_2 ,Ω_3 ,Ω_4 will also be orthogonal.

The demodulation circuit is shown in Figure 4. As shown in the diagram, multiplication of the received sequence of elements of vector $\mathbf{s}(t)$ (22) by elements of basic single-frequency matrices (23) and their integration (24) with subsequent summation of the results is carried out in real time. Multipliers and integrators are grouped into 4 groups in accordance with the number of

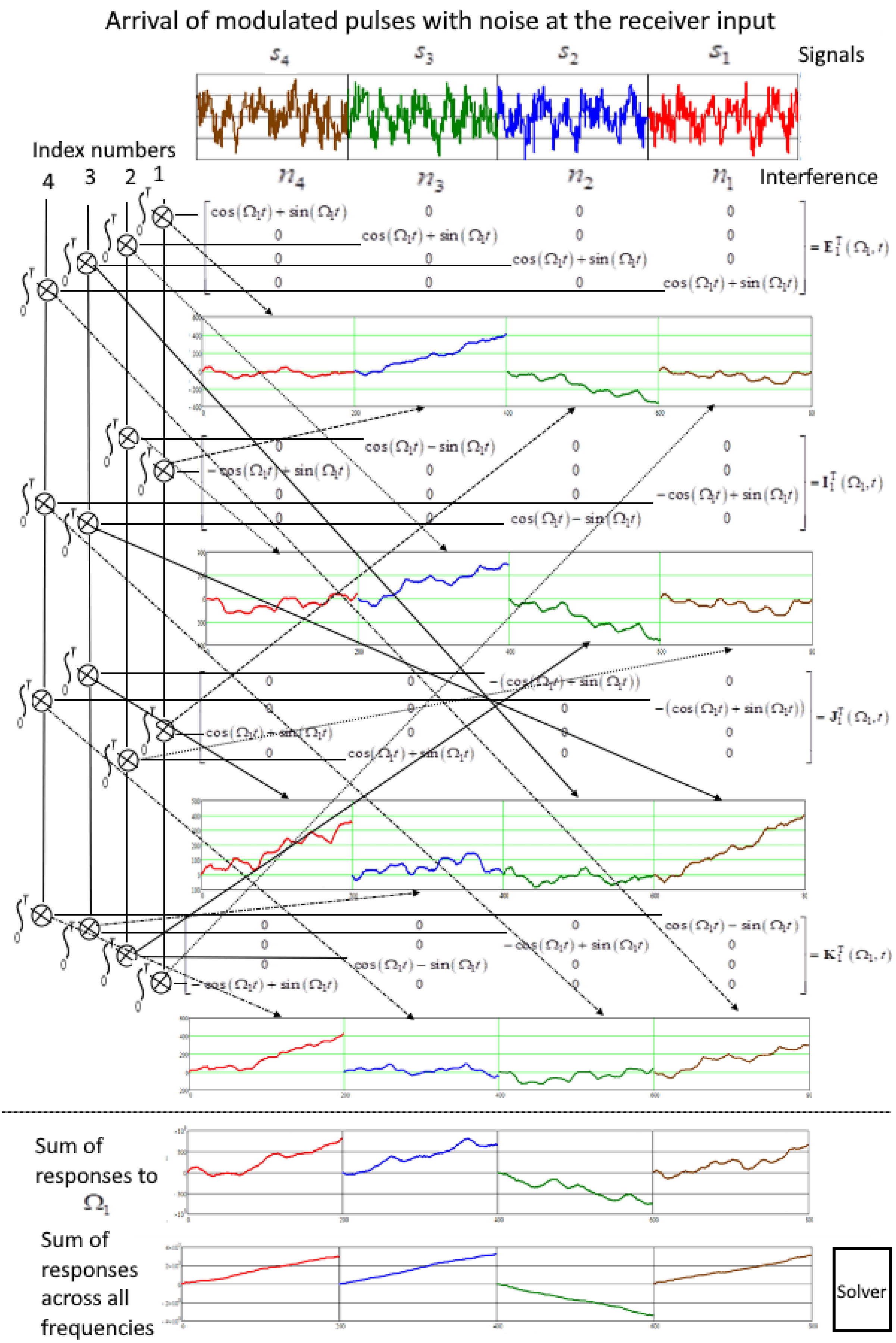


Figure 4. Demodulator diagram

elements in one bus. The first received element $s_1 = y_1(\Omega_1,\Omega_2,\Omega_3,\Omega_4,t) + n_1(t)$ is fed to bus 1 and multiplied by the elements of the transposed basis matrices located in the first columns of the basis matrices. At the same time, we obtain from the first element s_1 estimates of all 4 elements of information vector $\mathbf{x}(0)$.

In Figure 4, the arrows show that from the first bus, the accumulation results are distributed to the corresponding locations of the pulses on the graph located at the bottom of the basis matrix $\mathbf{E}_1^{\mathrm{T}}\left(\Omega_1,t\right)$. Since the final result of accumulation is important to us, at the end of accumulation all 4 samples are counted and stored in RAM.

Next, the second element $s_2 = y_2(\Omega_1,\Omega_2,\Omega_3,\Omega_4,t) + n_2(t)$ arrives and is fed to bus 2. Similarly, during the arrival process, it is multiplied by the elements of the basis matrices in the second columns. At the same time, we also obtain from the second element s_2 estimates of all 4 elements of the information vector. In the figure, the arrows show that from the 2nd bus, the accumulation results are distributed on the graph located at the bottom of the basis matrix $\mathbf{I}_1^{\mathrm{T}}\left(\Omega_1,t\right)$. At the end of accumulation, all 4 counts are counted and stored.

The following elements $s_3 = y_3(\Omega_1,\Omega_2,\Omega_3,\Omega_4,t) + n_3(t)$ and $s_4 = y_4(\Omega_1,\Omega_2,\Omega_3,\Omega_4,t) + n_4(t)$ are supplied to buses 3 and 4, and are multiplied by the elements of the basis matrices in columns 3 and 4, respectively. The results of integration for each matrix are shown in the figure directly below matrices $\mathbf{J}_1^{\mathrm{T}}\left(\Omega_1,t\right)$ and $\mathbf{K}_1^{\mathrm{T}}\left(\Omega_1,t\right)$. Below is the sum of the results for the single-frequency fundamental matrix $\mathbf{\Phi}_1(\Omega_1,t)$ as a whole.

As can be seen from the diagram, each input element produces 4 outputs and thus the MIMO scheme is implemented. At the same time, the estimation of each information element occurs under different noise conditions. Since interference affects signal elements at different times, they are not correlated. When adding the integration results, the information components are added by energy, and the interference components are added by power. The signal amplitude increases 4 times.

Demodulation occurs in a similar way for single-frequency matrices $\mathbf{\Phi}_2(\Omega_2,t)$, $\mathbf{\Phi}_3(\Omega_3,t)$, $\mathbf{\Phi}_4(\Omega_4,t)$. If the frequencies are sufficiently separated, the interference will also be uncorrelated. As shown at the bottom of Figure 4, when summing up the evaluation results, we obtain a 16-fold increase in the signal amplitude. Accordingly, the ratio of signal power to noise power will increase by 16 times. The total results of the readings are sent to the decision device (solver), in which the received information elements are evaluated.

Diversity of the symbols

The task of the solver is to determine the vector that was transferred from all possible 16 vectors in (20). We will make the decision using the maximum likelihood criterion. In this case, we must compare the received vector with all possible information vectors (20) using some norm. To do this, it is necessary to determine the distance between the received vector and all vectors. We will also present the resulting estimate as a vector $\hat{\mathbf{x}}(0) = \mathbf{x}(0) + \tilde{\mathbf{x}}(0)$, where $\tilde{\mathbf{x}}(0)$ is the estimation error caused by the noise. We calculate the distance between any pair of vectors as $d_{nm} = \sqrt{2}\sqrt{\left\|\mathbf{s}_n\right\|^2 - \left(\mathbf{s}_m,\mathbf{s}_n\right)}$, $n,m = 1,2,...,16$, where $\left(\mathbf{s}_m,\mathbf{s}_n\right)$ scalar product of the vector.

In our case, we receive the transmitted vector using the basis matrices with subsequent addition, leading to an increase in the SNR. Consequently, as a result of adding the estimates of information vectors, we obtain the amplitude of each information element 4 times greater than the amplitude of the original element. Consider the scalar product of vectors that differ in one symbol:

$\mathbf{x}_1 = 4[1 \ \ -1 \ \ 1 \ \ 1]$ and $\mathbf{x}_2 = 4[-1 \ \ -1 \ \ 1 \ \ 1]$. Minimum distance: $d_{nm} = \sqrt{2}\sqrt{\|\mathbf{s}_n\|^2 - (\mathbf{s}_m, \mathbf{s}_n)} = \sqrt{2}\sqrt{64-32} = 8$. As you can see, the minimum distance between vectors has increased 4 times.

As described above, we transmit information vectors in 4D space using matrices (17). From the point of view of separability of information symbols, it is important to note that the fundamental matrix (14) is decomposed into basic matrices (17), i.e. is equal to the sum of the basis matrices. Therefore, when multiplying one of the possible information vectors (20) by the basis matrices **E**, **I**, **J**, **K**, we obtain 4 orthogonal information vectors.

Figure 5 shows the corresponding 4 vectors in 3D space obtained after multiplying the information vector $\mathbf{x}(0) = [1 \ \ -1 \ \ 1 \ \ 1]$ by the basis matrices when depicting the scalar as a point mass. Figure 6 shows 4 vectors for the information vector $\mathbf{x}(0) = [-1 \ \ -1 \ \ 1 \ \ 1]$, which differs from the original vector by the value of the 1st element.

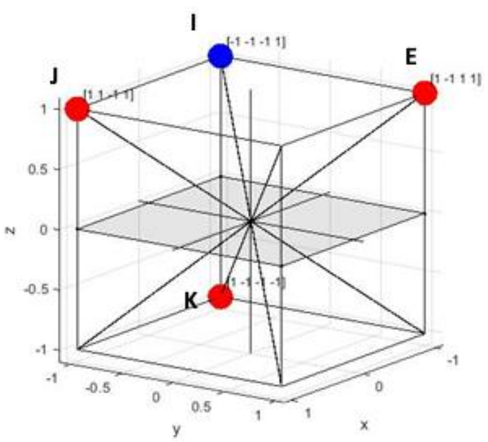


Figure 5. Information vectors obtained from the initial state vector [1-111] by multiplication by spatial basis matrices

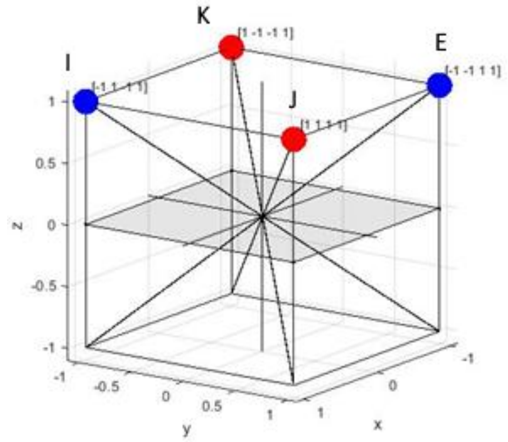


Figure 6 Information vectors obtained from the initial state vector [-1-111] by multiplication by spatial basis matrices

As can be seen from Figures 5, 6, the difference between four spatial vectors in 4D is more significant than the difference between two vectors that differ only in one symbol, determined by the scalar product. Let's write these vectors in the form of matrices:

$$\mathbf{F}_5 = \begin{bmatrix} 1 & -1 & 1 & 1 \\ -1 & -1 & -1 & 1 \\ 1 & 1 & -1 & 1 \\ 1 & -1 & -1 & -1 \end{bmatrix}, \quad \mathbf{F}_{13} = \begin{bmatrix} -1 & -1 & 1 & 1 \\ -1 & 1 & -1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & -1 & -1 & 1 \end{bmatrix},$$

where the matrix indices show the column numbers in (20) for the initial state vector, which is written in the first row.

The matrix data is orthogonal because $\mathbf{F}_5^{\mathrm{T}}\mathbf{F}_5 = \mathbf{F}_5\mathbf{F}_5^{\mathrm{T}} = 4\mathbf{E}$ и $\mathbf{F}_{13}^{\mathrm{T}}\mathbf{F}_{13} = \mathbf{F}_{13}\mathbf{F}_{13}^{\mathrm{T}} = 4\mathbf{E}$.

Thus, we are faced with the task of distinguishing not two vectors, but two matrices for different hypotheses. This problem is solved by calculating the square of the Euclidean norm of the matrix or the Frobenius norm.

It is shown that all 16 resulting matrices will be orthogonal. The squared Frobenius norm of these matrices will be $\|\mathbf{F}_n\|_F^2 = 16$, or $\mathrm{tr}\{\mathbf{F}_5^{\mathrm{T}}\mathbf{F}_5\} = \mathrm{tr}\{\mathbf{F}_5\mathbf{F}_5^{\mathrm{T}}\} = 16$ and $\mathrm{tr}\{\mathbf{F}_{13}^{\mathrm{T}}\mathbf{F}_{13}\} = \mathrm{tr}\{\mathbf{F}_{13}\mathbf{F}_{13}^{\mathrm{T}}\} = 16$. The squared Frobenius norms for two matrices formed from opposite vectors will be equal -16, for example, for a matrix $\mathbf{F}_5$ and $\mathbf{F}_{10}$ we get $\mathrm{tr}\{\mathbf{F}_5^{\mathrm{T}}\mathbf{F}_{10}\} = -16$. Having calculated all possible

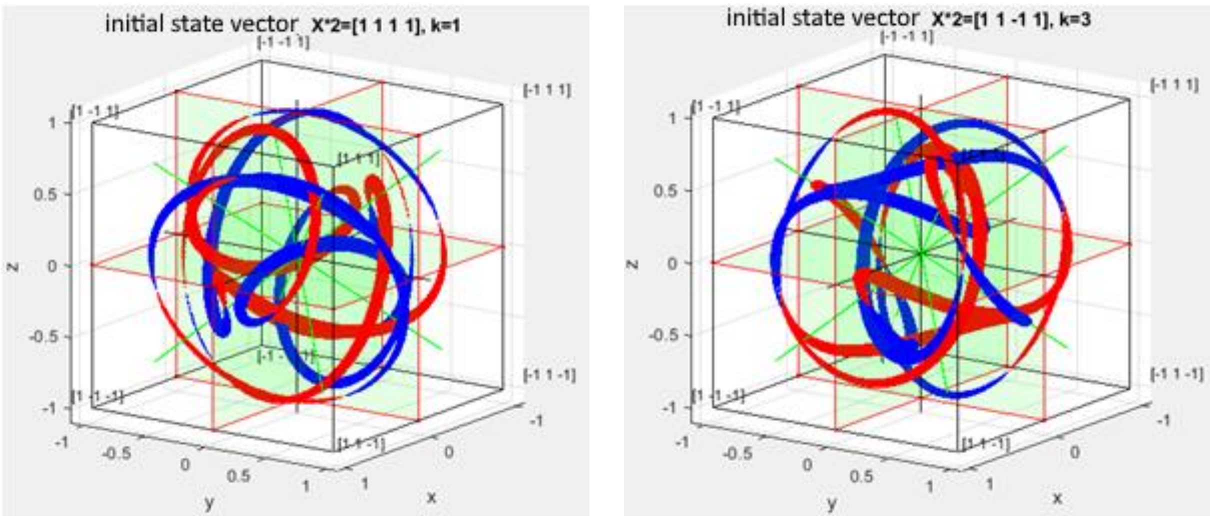
Frobenius norms for matrix products, we obtain 4 norms 8, 4 norms -8, 6 norms 0 and 1 norm -16. Therefore, the minimum distance between norms will be equal to 16 – 8=8.

Thus, we have obtained that the result of calculating the maximum likelihood does not depend on whether we calculate the distance between two vectors with amplitude 4 times larger using the scalar product or between two matrices using the Frobenius norm.

Orbits of rotation of the vector of initial states in the three-frequency space of the quaternion

Of the 16 possible combinations of information bipolar vectors, we have 8 vectors with a positive scalar part and 8 with a negative one. Figure 7 shows various orbits of rotation of the scalar part of the quaternion for initial states equal to the numbers of information vectors 1, 3, 10, 12 for reference frequencies $f_1 = 6$, $f_2 = -2$, $f_3 = -1$ Hz. The corresponding combination frequencies are $F_1 = 3$, $F_2 = 5$, $F_3 = 7$, $F_4 = 9$ Hz. From 16 orbits, identical orbits are selected and grouped into four with positive and negative scalar parts, such as 1, 6 and 11,16; 2, 3 and 1, 15; 4, 7 and 10, 13; 5, 8 and 9,12. Thus, we got 4 different orbits for 4 different initial states of each.

For a single-frequency quaternion, we also obtained 4 orbits for 4 different initial states of each [6]. However, a comparison of these orbits with the orbits of a three-frequency quaternion shows that the orbits of a three-frequency quaternion occupy a significantly larger volume of quaternion space and, as a consequence, the signals have a significantly greater variety. As is known, the greater the diversity of the signal, the less susceptible it is to interference and the higher the information transmission speed it is possible to obtain. In the limiting case, when the signal is similar to white noise and occupies the entire volume, we obtain the throughput [3].



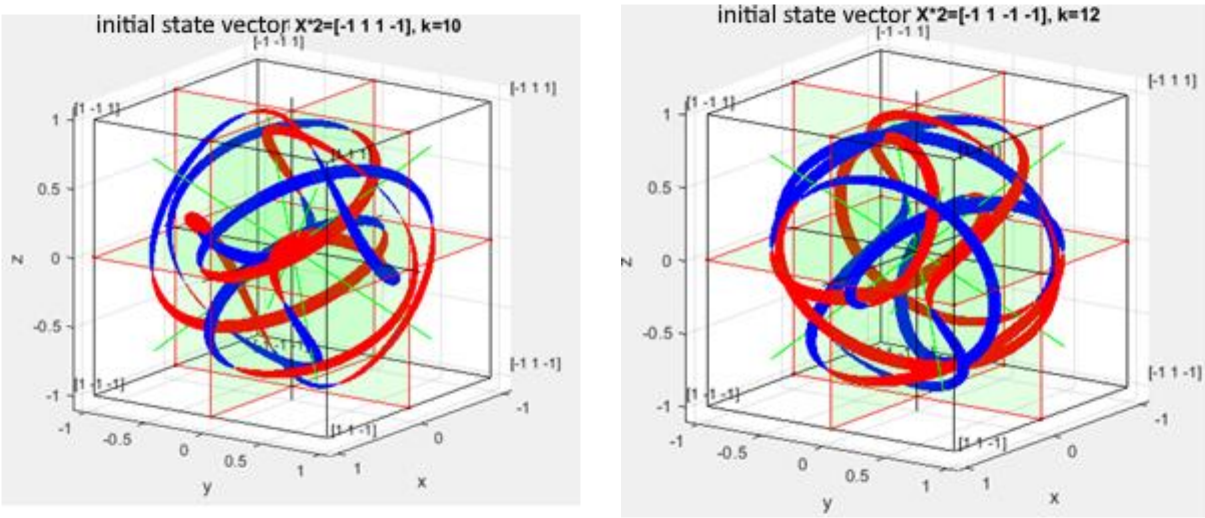


Figure 7. Orbits of rotation of the scalar part of the quaternion

From Figure 7 you can also visually draw the following conclusion. Since in the MIMO scheme each information pulse will be simultaneously transmitted at different frequencies and at different times, the influence of frequency selective fading and time selective fading will also be reduced.

V. CONCLUSION

Thus, in contrast to existing technologies for increasing the capacity of communication channels using a MIMO scheme with many antennas at the input and output, the proposed technology using a MIMO scheme in a hypercomplex vector space, which has a patent [12], has significant advantages:

Instead of transferring the entire *M*x*M* matrix to the space-time code, i.e. channel matrix, a multi-frequency modulated vector is transmitted, obtained by multiplying the information vector by the channel matrix. In this case, all phase relationships between elements obtained during modulation are preserved, and the entire transmitter power is spent on each multi-frequency element of the vector;

Instead of *M* antennas for transmission and reception, one antenna is used for transmission and reception, and the MIMO signal is generated in a multidimensional hypercomplex space, which significantly reduces the complexity of implementing MIMO schemes and instead of *M* antennas, *M* multipliers and adders are used for transmission and reception;

As the dimension of the MIMO scheme increases, the degree of increase in throughput does not decrease, as is the case in physical space due to the limitation of its dimension;

There is no need to increase the size of mobile radios and mobile phones when increasing the MIMO dimension, since there is no need to place multiple antennas in them;

Since the phase relationships are preserved during the formation of the modulated vector, there is no need to know the phase shift of the signal from *M* transmitting antennas in each of the *M* receiving antennas. Instead of a complex synchronization system with a return channel, only knowledge of the carrier frequency and phase synchronization with it is required, as in the case of coherent reception of a one-dimensional signal;

Obtaining a gain in noise immunity of M times allows us to provide the necessary speed of information transmission in 5G and 6G without the need to switch to the terahertz range and, accordingly, without reducing the communication range to line-of-sight range;

Without a transition to the terahertz range there will be no significant impact on the human body, and the technological difficulties of creating electronic elements in this range will not increase;

The absence of a large crest factor in hypercomplex signals, unlike multi-frequency signals, does not require an increase in amplifier power;

The use of MIMO in the hypercomplex space allows for increased throughput in wireline communication systems;

When using the MIMO scheme in hypercomplex space, each multi-frequency pulse contains information about all *M* information elements, which significantly reduces the impact of frequency and time selective fading.

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